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Relativistic Quantum Mechanics.

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§ 1. *Introduction.*

The steady development of the quantum theory that has taken place during the present century was made possible only by continual reference to the Correspondence Principle of Bohr, according to which, classical theory can give valuable information about quantum phenomena in spite of the essential differences in the fundamental ideas of the two theories. A masterful advance was made by Heisenberg in 1925, who showed how equations of classical physics could be taken over in a formal way and made to apply to quantities of importance in quantum theory, thereby establishing the Correspondence Principle on a quantitative basis and laying the foundations of the new Quantum Mechanics. Heisenberg's scheme was found to fit wonderfully well with the Hamiltonian theory of classical mechanics and enabled one to apply to quantum theory all the information that classical theory supplies, in so far as this information is consistent with the Hamiltonian form. Thus one was able to build up a satisfactory quantum mechanics for dealing with any dynamical system composed of interacting particles, provided the interaction could be expressed by means of an energy term to be added to the Hamiltonian function.

This does not exhaust the sphere of usefulness of the classical theory. Classical electrodynamics, in its accurate (restricted) relativistic form, teaches us that the idea of an interaction energy between particles is only an approximation and should be replaced by the idea of each particle emitting waves,

which travel outward with a finite velocity and influence the other particles in passing over them. We must find a way of taking over this new information into the quantum theory and must set up a relativistic quantum mechanics, before we can dispense with the Correspondence Principle.

A preliminary attack on the question of relativistic quantum mechanics has been made through the solution of the problem of a single charged particle moving in a specified classical field. For the treatment of this problem it is essential to use Schrödinger's form of quantum mechanics, according to which the motion of the particle is described by a wave function involving the space and time co-ordinates in a symmetrical manner. The solution is satisfactory from the point of view of the Correspondence Principle, although it involves a difficulty owing to the appearance of possible negative energy values for the particle. The difficulty is not due to a misuse of classical information and will not concern us here.

The extension of this wave-function method to two or more particles can easily be made so long as we keep to the idea of a given classical field in which the particles are moving. The resulting theory is logically satisfactory, but is, of course, incomplete, as it gives no interaction between the particles. It becomes necessary then to abandon the idea of a given classical field and to have instead a field which is of dynamical significance and acts in accordance with quantum laws.

An attempt at a comprehensive theory on these lines has been made by Heisenberg and Pauli.* These authors regard the field itself as a dynamical system amenable to Hamiltonian treatment and its interaction with the particles as describable by an interaction energy, so that the usual methods of Hamiltonian quantum mechanics may be applied. There are serious objections to these views, apart from the purely mathematical difficulties to which they lead. If we wish to make an observation on a system of interacting particles, the only effective method of procedure is to subject them to a field of electromagnetic radiation and see how they react. Thus the rôle of the field is to provide a means for making observations. *The very nature of an observation requires an interplay between the field and the particles.* We cannot therefore suppose the field to be a dynamical system on the same footing as the particles and thus something to be observed in the same way as the particles. The field should appear in the theory as something more elementary and fundamental.

Again, the field equations are always linear and thus of the form typical of

* 'Z. Physik,' vol. 56, p. 1, and vol. 59, p. 168 (1929).

the wave equation of quantum theory. This suggests deep-lying connections and possibilities for simplification and unification which are entirely lacking in the Heisenberg-Pauli theory.

In the present paper a scheme is proposed which gives the interplay between particles and field apparently correctly and in a surprisingly simple manner. Full use is made of all the information supplied by the classical theory. The general ideas are applicable with any kind of simple harmonic wave transmitting the interaction between particles and providing the means of observation of particles (*e.g.*, with longitudinal waves like sound waves) and not merely for the electromagnetic case, though presumably only the latter is of interest in atomic theory.

§ 2. *Relativistic Observations.*

A definite advance in the relativistic theory of the interaction of two electrons is contained in a recent paper by Möller,* where it is shown that in the calculation of the mutual scattering of two colliding electrons by Born's method of approximation, one may describe the interaction with retarded potentials and use relativistic ideas throughout, without getting any ambiguity in the scattering coefficient to the first order of approximation. This lack of ambiguity is ground of presumption of the correctness of the result. When, however, one tries to apply similar methods to the higher approximations or to more general problems, one meets very definitely with ambiguities.

The method by which Möller obtained his result may be compared with the methods of the Correspondence Principle in use before the introduction of Heisenberg's matrix theory, for calculating Einstein's A and B coefficients from classical models. In certain cases the result obtained was unambiguous (usually those cases for which the result was zero) and was then presumed to be correct. In general, however, there was ambiguity, so that one could get no reliable accurate result.

This analogy suggests that it would be useless to try to extend Möller's method by setting up rules to provide a definite interpretation for ambiguous quantities. Any attempts in this direction would be just as futile as the attempts made in the pre-Heisenberg epoch to calculate Einstein's A's and B's from some sort of mean of classical quantities referring to the initial and final states. One ought to proceed on quite different lines, namely by following the methods introduced by Heisenberg in 1925, which have already met with such great success for non-relativistic quantum mechanics.

* 'Z. Physik,' vol. 70, p. 786 (1931).

Heisenberg put forward the principle that one should confine one's attention to observable quantities, and set up an algebraic scheme in which only these observable quantities appear. Strictly speaking, it is not the observable quantities themselves (the Einstein A's and B's) that formed the building stones of Heisenberg's algebraic scheme, but rather certain more elementary quantities, the matrix elements, having the observable quantities as the squares of their moduli. The extra phase quantities introduced in this way are essential.

Let us see what are the corresponding quantities in relativistic theory. To make a relativistic observation on a system of particles we must, as mentioned in the introduction, send in some incident electromagnetic radiation and examine the scattered radiation. The numerical quantity that we observe is thus the probability of occurrence of a certain radiative transition process. This process may be specified by the intensities of the various monochromatic components of the ingoing and of the outgoing fields of radiation. (We shall ignore the purely mathematical difficulty that the total number of these components is an infinity of a high order.) The phases must not be specified together with the intensities, as this would violate well-established quantum principles.

In non-relativistic quantum mechanics the probability of occurrence of any transition process is always given as the square of the modulus of a certain quantity, of the nature of a matrix element or simply a transformation function, referring to the initial and final states. It appears reasonable to assume that this will still be the case in relativistic quantum mechanics. Thus the relativistic observable quantities, which are always transition probabilities, will all appear as the squares of the moduli of certain quantities. These quantities, which we shall refer to as probability amplitudes, will then be the building stones analogous to Heisenberg's matrix elements. We should expect to be able to *set up an algebraic scheme involving only the probability amplitudes and to translate the equations of motion of relativistic classical theory directly into exact equations expressible entirely in terms of these quantities.*

The information that classical theory supplies is thus to be used to give relations between the probability amplitudes of *different* physical processes, rather than to enable one to calculate a particular one of them. Only in very special cases, of which Möller's paper provides an example, is it possible to evaluate a relativistic transition probability without at the same time evaluating a whole series of them, referring to all the possible ways in which the particles under consideration can react with the radiation field.

A point of special importance about the building stones of the new theory

is that each of them refers to one field of ingoing waves and one field of outgoing waves, or to one initial field of a transition process and one final field. Quantities referring to two initial fields, or to two final fields, are not allowed. This shows a departure from the theory of Heisenberg and Pauli, according to which, if one is given any quantity referring to one initial field and one final field, one can obtain from it a quantity referring to two initial fields, or to two final fields, by a straightforward application of the transformation theory of quantum mechanics. The Heisenberg-Pauli theory thus involves many quantities which are unconnected with results of observations and which must be removed from consideration if one is to obtain a clear insight into the underlying physical relations.

§ 3. *Equations of Motion.*

We shall now consider in detail the question of how the information contained in classical electrodynamics can be taken over into the quantum theory. We meet at once with the difficulty that the classical theory itself is not free from ambiguity.

To make the discussion precise, let us suppose we have a single electron interacting with a field of radiation and consider the radiation resolved into ingoing and outgoing waves. The classical problem is, given the ingoing radiation and suitable initial conditions for the electron, determine the motion of the electron and the outgoing radiation. The classical equations which deal with this problem are of two kinds, (i) those that determine the field produced by the electron (which field is just the difference of the ingoing and outgoing fields) in terms of the variables describing the motion of the electron, and (ii) those that determine the motion of the electron. Equations (i) are quite definite and unambiguous, but not so equations (ii). The latter express the acceleration of the electron in terms of field quantities at the point where the electron is situated and these field quantities in the complete classical picture are infinite and undefined.

In the usual approximate treatment of the problem one takes for these field quantities just the contributions of the ingoing waves. This treatment is necessarily only approximate, since it does not take into account the reaction on the electron of the waves it emits. We should expect in an accurate treatment, that the field determining the acceleration of the electron would be in some way associated with both the ingoing and outgoing waves. Classical attempts have been made to improve the theory by assuming a definite structure for the electron and calculating the effect on one part of it of the field produced by the rest, but such methods are not permissible in modern physics.

We must recognise at this point that we have reached the limit of classical electromagnetic theory. We have quite definite equations for determining the motion of the electron in terms of field quantities, but we cannot interpret these field quantities in the usual classical picture and the most we can say about them is that they are related in some non-classical way to two fields, namely, those of the ingoing and of the outgoing waves. Further advance can be made only by introducing quantum ideas.

Let us make the assumption that *the passage from the field of ingoing waves to the field of outgoing waves is just a quantum jump performed by one field.* This assumption is permissible on account of the fact, discussed in the preceding section, that all the quantities in relativistic quantum mechanics are of the nature of probability amplitudes referring to one ingoing field and one outgoing field, so that we may associate, say, the right-hand sides of the probability amplitudes with ingoing fields and the left-hand sides with outgoing fields. In this way we automatically exclude quantities referring to two ingoing fields or to two outgoing fields and make a great simplification in the foundations of the theory.

The significance of the new assumption lies in the fact *that the classical picture from which we derive our equations of motion must contain no reference to quantum jumps.* This classical picture must therefore involve just one field, a field composed of waves passing undisturbed through the electron and satisfying everywhere Maxwell's equations for empty space. With this picture the equations of motion for the electron are perfectly definite and unambiguous. There are no equations of motion for the field, as the field throughout space-time is pictured as given. Thus the interaction between electron and field is introduced into the equations in only one place.

The quantisation of the equations of motion derived from this picture may conveniently be carried out in two stages. Let us first quantise only the variables describing the electron. We then get just the usual quantum theory of the motion of an electron in a given classical field, with the difference that in the present case the field must necessarily be resolvable into plane waves and must therefore contain nothing of the nature of a Coulomb force. We have a Schrödinger equation of the form

$$\mathrm{F}\psi = 0,$$

where the operator F is, neglecting spin

$$\mathrm{F} = \left(ih\frac{\partial}{\partial t} + e\mathrm{A}_0\right)^2 - \left(ihc\frac{\partial}{\partial x} - e\mathrm{A}_x\right)^2 - \ldots - m^2c^4. \qquad (1)$$

It should be remembered that the wave-function ψ involves not only the variables x, y, z, t describing the electron, but also a large number of parameters describing the field, which parameters may conveniently be taken to be the intensities J and phases w of the various Fourier components of the field. The potentials A occurring in F are likewise functions, not only of the variables x, y, z, t describing the momentary position of the electron, but also of the parameters J and w.

In the second stage of the quantisation we assume that the J's and w's occurring in ψ and the A's are not numerical, but are operators satisfying the usual quantum conditions governing the intensities and phases of the Fourier components of the electromagnetic field in empty space. The new wave equation obtained in this way is to be treated on the same lines as the previous one. In particular, it may be used to determine matrix elements associated with electron jumps. Each such matrix element will now be a function of the non-commuting J's and w's, so that, when we take a representation of the J's and w's, it becomes a set of quantities, each referring to two states of the field as well as the two electronic states, and thus being of the nature of the probability amplitudes of § 2.

For the problem of the interaction of two electrons, we require a wave-function ψ which is a function of the variables x_1, y_1, z_1, t_1 and x_2, y_2, z_2, t_2 describing the two electrons and of one set of J's and w's describing one field. This ψ must satisfy the two wave equations

$$\mathrm{F}_1\psi = 0, \qquad \mathrm{F}_2\psi = 0, \tag{2}$$

where F_1 is the operator obtained from F by substituting $\partial/\partial t_1$, etc., for $\partial/\partial t$, etc., and taking for the A's their values at the point x_1, y_1, z_1, t_1, and similarly for F_2. These two wave equations describe completely the relations between the two electrons and the field. No terms of the type of a Coulomb interaction energy are required in the operators of the wave equations. The interaction of the two electrons is due to the motions of both being connected with the same field. This interaction manifests itself mathematically through the fact that, if we take a wave function ψ_1, a function only of x_1, y_1, z_1, t_1 and the J's and w's, satisfying

$$\mathrm{F}_1\psi_1 = 0, \tag{3A}$$

and a second wave-function ψ_2, a function only of x_2, y_2, z_2, t_2 and the J's and w's, satisfying

$$\mathrm{F}_2\psi_2 = 0, \tag{3B}$$

then neither of the products $\psi_1\psi_2$ and $\psi_2\psi_1$ will satisfy both the wave equations (2). The solution of equations (2) is an essentially different and more complicated problem than the solution of (3A) and (3B).

§ 4. *Interaction between Two Particles in One Dimension.*

It may seem rather surprising that a theory in which all the fields are resolvable into plane waves can give anything of the nature of the usual electrostatic forces between electrons. We shall therefore illustrate by a simple example the fact that these forces really are contained in our wave equations. We shall take the case of two particles moving in a field in one-dimensional space and shall proceed to solve equations (2), making various approximations that are permissible when we are not interested in relativistic effects.

Suppose the field to be describable by a potential function V satisfying the wave equation

$$\frac{\partial^2 V}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = 0,$$

and the classical expression for the energy to be

$$H = \frac{1}{8\pi}\int\left\{\left(\frac{\partial V}{\partial x}\right)^2 + \frac{1}{c^2}\left(\frac{\partial V}{\partial t}\right)^2\right\} dx.$$

If we resolve V into its Fourier components, thus

$$V = \int_{-\infty}^{\infty} \{a_\nu e^{i\nu(t+x/c)} + b_\nu e^{i\nu(t-x/c)}\}\, d\nu, \tag{4}$$

the expression for the energy will go over into

$$H = \frac{1}{c}\int_0^\infty \nu^2 \{a_\nu a_{-\nu} + b_\nu b_{-\nu}\}\, d\nu. \tag{5}$$

Let us now see what are the Poisson bracket relations between the Fourier coefficients a and b. These relations must be chosen such that the quantities $a_\nu e^{i\nu t}$, $b_\nu e^{i\nu t}$, considered as dynamical variables, satisfy equations of motion of the Hamiltonian form with the Hamiltonian function (5), thus

$$\frac{d}{dt}(a_\nu e^{i\nu t}) = [a_\nu e^{i\nu t}, H]$$

or

$$i\nu a_\nu = [a_\nu, H],$$

and similarly for b_ν. It is easily verified that we must have

$$[a_\nu, a_{\nu'}] = [b_\nu, b_{\nu'}] = ic/\nu \,.\, \delta(\nu + \nu')$$

$$[a_\nu, b_{\nu'}] = 0.$$

In the quantum theory these relations become

$$\left.\begin{aligned} a_\nu a_{\nu'} - a_{\nu'} a_\nu = b_\nu b_{\nu'} - b_{\nu'} b_\nu = -hc/\nu \,.\, \delta(\nu + \nu') \\ a_\nu b_{\nu'} - b_{\nu'} a_\nu = 0 \end{aligned}\right\}. \tag{6}$$

We now introduce two particles, of masses m_1 amd m_2 and "charges" ε_1 and ε_2, and suppose the interaction of each with the field can be described by an interaction energy equal to its charge multiplied by the value of V at the point where it is situated. Thus, if we neglect the relativistic variation of mass with velocity, we have the two wave equations

$$\left\{ih\frac{\partial}{\partial t_1} + \frac{h^2}{2m_1}\frac{\partial^2}{\partial x_1^2} - \varepsilon_1 \mathrm{V}(x_1 t_1)\right\}\psi = 0,$$

$$\left\{ih\frac{\partial}{\partial t_2} + \frac{h^2}{2m_2}\frac{\partial^2}{\partial x_2^2} - \varepsilon_2 \mathrm{V}(x_2 t_2)\right\}\psi = 0.$$

By putting $t_1 = t_2 = t$, we can reduce these to the one wave equation

$$\left\{ih\frac{\partial}{\partial t} + \frac{h^2}{2m_1}\frac{\partial^2}{\partial x_1^2} + \frac{h^2}{2m_2}\frac{\partial^2}{\partial x_2^2} - \varepsilon_1 \mathrm{V}(x_1 t) - \varepsilon_2 \mathrm{V}(x_2 t)\right\}\psi = 0. \tag{7}$$

We shall proceed to obtain a solution of this equation in the form of a power series in the ε's. Thus we put

$$\psi = \psi_0 + \psi_1 + \psi_2 + \ldots$$

where

$$\left\{ih\frac{\partial}{\partial t} + \frac{h^2}{2m_1}\frac{\partial^2}{\partial x_1^2} + \frac{h^2}{2m_2}\frac{\partial^2}{\partial x_2^2}\right\}\psi_0 = 0 \tag{8}$$

$$\left\{ih\frac{\partial}{\partial t} + \frac{h^2}{2m_1}\frac{\partial^2}{\partial x_1^2} + \frac{h^2}{2m_2}\frac{\partial^2}{\partial x_2^2}\right\}\psi_1 = \{\varepsilon_1 \mathrm{V}(x_1 t) + \varepsilon_2 \mathrm{V}(x_2 t)\}\psi_0 \tag{9}$$

$$\left\{ih\frac{\partial}{\partial t} + \frac{h^2}{2m_1}\frac{\partial^2}{\partial x_1^2} + \frac{h^2}{2m_2}\frac{\partial^2}{\partial x_2^2}\right\}\psi_2 = \{\varepsilon_1 \mathrm{V}(x_1 t) + \varepsilon_2 \mathrm{V}(x_2 t)\}\psi_1. \tag{10}$$

We take as the solution of (8)

$$\psi_0 = e^{ip_1 x_1/h} e^{ip_2 x_2/h} e^{-iWt/h} \delta_{J0},$$

where

$$\mathrm{W} = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}, \tag{11}$$

representing a state for which the particles have the momenta p_1 and p_2, and all the J's, *i.e.*, the intensities of the Fourier components of the field, vanish. Now the operator $\varepsilon_1 V(x_1 t) + \varepsilon_2 V(x_2 t)$ occurring on the right-hand sides of (9) and (10), if expressed as a matrix in a representation in which the J's are diagonal, would contain only matrix elements referring to transitions in which just one of the J's changes by one quantum. It follows that ψ_1 must consist of a sum of terms each referring to a state of the field in which just one oscillation is excited by one quantum. Similarly ψ_2 must consist of a sum of terms each referring either to a two-quantum or to a zero-quantum state of the field. The latter are the ones that interest us here, as they may be compared with the terms that would arise from the insertion of an interaction energy between the two particles into the operator of the wave equation (7).

We can obtain the solution of equation (9) by expanding the right-hand side in terms of its Fourier components by means of (4) and dividing each component by the number to which the operator on the left-hand side of (9) is equivalent when it operates on that component. This gives

$$\psi_1 = \varepsilon_1 \int_{-\infty}^{\infty} \left\{ \frac{a_\nu e^{i\nu(t+x_1/c)}}{W - h\nu - (p_1 + h\nu/c)^2/2m_1 - p_2^2/2m_2} + \frac{b_\nu e^{i\nu(t-x_1/c)}}{W - h\nu - (p_1 - h\nu/c)^2/2m_1 - p_2^2/2m_2} \right\} d\nu \cdot \psi_0$$

$$+ \varepsilon_2 \int_{-\infty}^{\infty} \left\{ \frac{a_\nu e^{i\nu(t+x_2/c)}}{W - h\nu - p_1^2/2m_1 - (p_2 + h\nu/c)^2/2m_2} + \frac{b_\nu e^{i\nu(t-x_2/c)}}{W - h\nu - p_1^2/2m_1 - (p_2 - h\nu/c)^2/2m_2} \right\} d\nu \cdot \psi_0.$$

If we use (11) and also neglect terms like $p_1/m_1 c$, $h\nu/m_1 c^2$ compared with unity, as is permissible when we are not interested in relativistic effects, this reduces to

$$\left.\begin{aligned} \psi_1 = &- \frac{\varepsilon_1}{h} \int_{-\infty}^{\infty} \{a_\nu e^{i\nu(t+x_1/c)} + b_\nu e^{i\nu(t-x_1/c)}\} \frac{d\nu}{\nu} \cdot \psi_0 \\ &- \frac{\varepsilon_2}{h} \int_{-\infty}^{\infty} \{a_\nu e^{i\nu(t+x_2/c)} + b_\nu e^{i\nu(t-x_2/c)}\} \frac{d\nu}{\nu} \cdot \psi_0 \end{aligned}\right\} \quad (12)$$

When we substitute this value for ψ_1 in the right-hand side of (10) and also substitute for V its expansion given by (4), we obtain an expression consisting of an operator, which is a homogeneous quadratic function of the a's and b's, operating on ψ_0. We must evaluate that particular part of the expression that refers to the unexcited state of the field. Only those terms of the operator

involving products like $a_\nu a_{-\nu}$ or $b_\nu b_{-\nu}$ will contribute anything to that part. To obtain the contribution of a term involving $a_\nu a_{-\nu}$, we observe that, for $\nu > 0$, a_ν and $a_{-\nu}$ are like the quantities $p + iq$ and $p - iq$ respectively in the problem of the simple harmonic oscillator. Thus $a_\nu a_{-\nu}$, with $\nu > 0$, is proportional to twice the energy of the corresponding oscillation (without zero-point energy) so that it gives no contribution when multiplied into ψ_0. The first of the quantum conditions (6) now shows that, to get the contribution from a term involving $a_{-\nu} a_\nu$ with $\nu > 0$, we must count $a_{-\nu} a_\nu$ as equal to $hc/\nu \,.\, \delta(\nu - \nu')$. In the same way we find that we must count $b_\nu b_{-\nu} = 0$ and $b_{-\nu} b_\nu = hc/\nu \,.\, \delta(\nu - \nu')$ with $\nu > 0$.

The term on the right-hand side of (10) arising from the product of $\varepsilon_2 \mathrm{V}(x_2 t)$ with the first of the terms for ψ_1 in (12) may be written

$$-\frac{\varepsilon_1 \varepsilon_2}{h} \int_{-\infty}^{\infty} d\nu' \{a_{-\nu'} e^{-i\nu'(t+x_2/c)} + b_{-\nu'} e^{-i\nu'(t-x_2/c)}\} \times \int_{-\infty}^{\infty} \frac{d\nu}{\nu} \{a_\nu e^{i\nu(t+x_1/c)} + b_\nu e^{i\nu(t-x_1/c)}\} \,.\, \psi_0.$$

That part of it referring to the unexcited state of the field is, by the foregoing rules

$$-\frac{\varepsilon_1 \varepsilon_2}{h} \int_0^{\infty} d\nu' \int_0^{\infty} \frac{d\nu}{\nu} \frac{hc}{\nu} \delta(\nu - \nu') \{e^{-i\nu'(t+x_2/c)} e^{i\nu(t+x_1/c)} + e^{-i\nu'(t-x_2/c)} e^{i\nu(t-x_1/c)}\} \,.\, \psi_0$$

$$= -2\varepsilon_1 \varepsilon_2 c \int_0^{\infty} \frac{d\nu}{\nu^2} \cos \nu (x_1 - x_2)/c \,.\, \psi_0.$$

The coefficient of ψ_0 here differs only by an infinitely great constant (independent of x_1 and x_2) from

$$2\varepsilon_1 \varepsilon_2 c \int_0^{\infty} \frac{d\nu}{\nu^2} \{1 - \cos \nu (x_1 - x_2)/c\} = \pi \varepsilon_1 \varepsilon_2 |x_1 - x_2|.$$

The other terms on the right-hand side of (10) may be dealt with in the same way and give for the complete part referring to the unexcited state of the field

$$\{2\pi \varepsilon_1 \varepsilon_2 |x_1 - x_2| + \mathrm{K}\} \psi_0, \tag{13}$$

where K is an infinite constant.

Equation (10) with expression (13) on its right-hand side is just what we should get if we were solving the wave equation

$$\left\{ih \frac{\partial}{\partial t} + \frac{h^2}{2m_1} \frac{\partial^2}{\partial x_1^2} + \frac{h^2}{2m_2} \frac{\partial^2}{\partial x_2^2} - 2\pi \varepsilon_1 \varepsilon_2 |x_1 - x_2| - \mathrm{K}\right\} \psi = 0,$$

by a method of approximation through expansion in powers of $\varepsilon_1 \varepsilon_2$. Thus

our wave-equation (7) contains implicitly an interaction between the particles, expressible approximately by the interaction energy $2\pi\epsilon_1\epsilon_2|x_1 - x_2|$. This interaction energy agrees numerically with what we should expect from a one-dimensional electrostatic theory. There is, however, a mistake in sign, as it gives an attractive force between like charges.

Summary.

A quantum theory is proposed in which the interaction between particles takes place by means of vibrations of an intervening medium transmitted with a finite velocity. The fundamental relations involve only quantities having observational significance, account being taken of the fact that an act of observation necessarily involves an interplay between particles and field. A detailed solution of a one-dimensional problem is given in order to show that forces of electrostatic nature are implicitly contained in the theory.

[*Note added, April 20th.*—It has been pointed out to me by Professor Heisenberg that the sign of the interaction energy given by the above calculation is really quite correct, since with the one-dimensional, longitudinal waves there used the classical theory also requires an attractive force between like charges.]