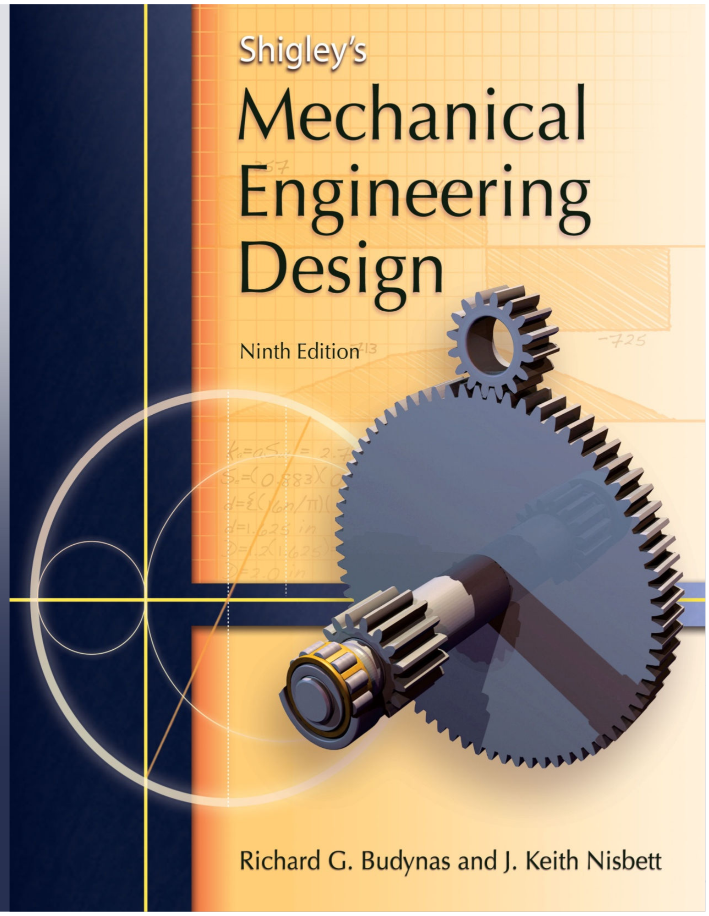
Lecture Slides

Chapter 14

Spur and Helical Gears



AGMA Method

- The American Gear Manufacturers Association (AGMA) provides a recommended method for gear design.
- It includes *bending stress* and *contact stress* as two failure modes.
- It incorporates modifying factors to account for various situations.
- It imbeds much of the detail in tables and figures.

- **Procedure:**
  - First, the bending and contact stresses are computed.
  - Then, the bending and contact allowables are evaluated.
  - Finally, the safety factors are calculated.

STRESS CALCULATIONS

AGMA Bending Stress

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases} \tag{14–15}$$

where for U.S. customary units (SI units),

W^t is the tangential transmitted load, lbf (N)
K_o is the overload factor
K_v is the dynamic factor
K_s is the size factor
P_d is the transverse diametral pitch
F (b) is the face width of the narrower member, in (mm)
K_m (K_H) is the load-distribution factor
K_B is the rim-thickness factor
J (Y_J) is the geometry factor for bending strength (which includes root fillet stress-concentration factor K_f)
(m_t) is the transverse metric module

AGMA Contact Stress

$$\sigma_c = \begin{cases} C_p\sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F}\dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E\sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b}\dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases} \tag{14–16}$$

where W^t, K_o, K_v, K_s, K_m, F, and b are the same terms as defined for Eq. (14–15). For U.S. customary units (SI units), the additional terms are

C_p (Z_E) is an elastic coefficient, $\sqrt{\text{lbf/in}^2}$ ($\sqrt{\text{N/mm}^2}$)
C_f (Z_R) is the surface condition factor
d_P (d_{w1}) is the pitch diameter of the *pinion,* in (mm)
I (Z_I) is the geometry factor for pitting resistance

Overload Factor K_o

- To account for likelihood of increase in nominal tangential load due to particular application.
- Examples include variations in torque from the mean value due to firing of cylinders in an internal combustion engine or reaction to torque variations in a piston pump drive.
- **Recommended values:**

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Dynamic Factor K_v

- Accounts for increased forces with increased speed.
- Dynamic Factor equation

$$K_v = \begin{cases} \left(\dfrac{A+\sqrt{V}}{A}\right)^B & V \text{ in ft/min} \\ \left(\dfrac{A+\sqrt{200V}}{A}\right)^B & V \text{ in m/s} \end{cases} \tag{14–27}$$

$$A = 50 + 56(1 - B)$$
$$B = 0.25(12 - Q_v)^{2/3} \tag{14–28}$$

- Maximum recommended velocity for a given quality number,

$$(V_t)_{\max} = \begin{cases} [A + (Q_v - 3)]^2 & \text{ft/min} \\ \dfrac{[A + (Q_v - 3)]^2}{200} & \text{m/s} \end{cases} \tag{14–29}$$

Dynamic Factor K_v

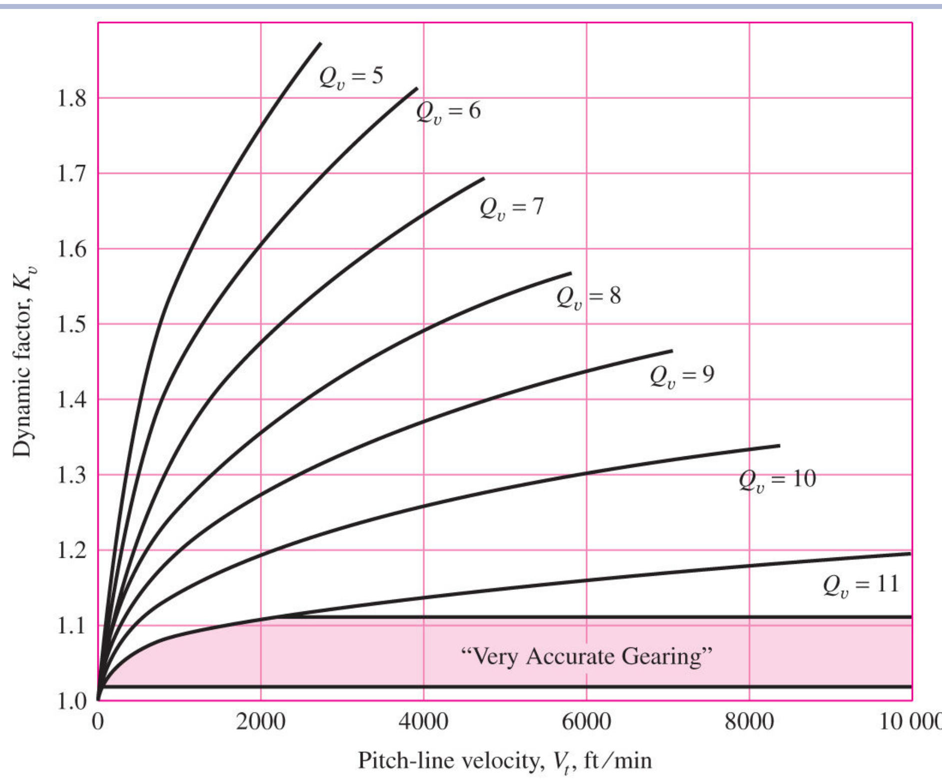


Fig. 14–9

Size Factor K_s

- Accounts for fatigue size effect, and non-uniformity of material properties for large sizes
- AGMA has not established size factors
- Use 1 for normal gear sizes
- Could apply fatigue size factor method from Ch. 6, where this size factor is the reciprocal of the Marin size factor k_b. Applying known geometry information for the gear tooth,

$$K_s = \frac{1}{k_b} = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535}$$

Values of Lewis Form Factor Y

Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

Table 14–2

Load-Distribution Factor K_m (K_H)

- Accounts for non-uniform distribution of load across the line of contact
- Depends on mounting and face width
- Load-distribution factor is currently only defined for
  - Face width to pinion pitch diameter ratio $F/d \leq 2$
  - Gears mounted between bearings
  - Face widths up to 40 in
  - Contact across the full width of the narrowest member

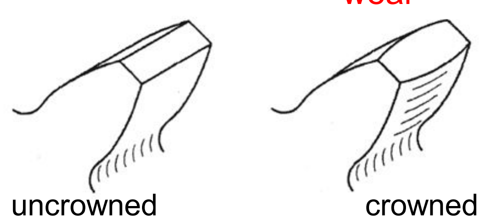
Load-Distribution Factor K_m (K_H)

- Face load-distribution factor

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

To reduce noise and wear

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases}$$



$$C_{pf} = \begin{cases} \dfrac{F}{10d} - 0.025 & F \leq 1 \text{ in} \\ \dfrac{F}{10d} - 0.0375 + 0.0125F & 1 < F \leq 17 \text{ in} \\ \dfrac{F}{10d} - 0.1109 + 0.0207F - 0.000\,228F^2 & 17 < F \leq 40 \text{ in} \end{cases} \qquad (14\text{–}32)$$

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases} \qquad (14\text{–}35)$$

Load-Distribution Factor K_m (K_H)

$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \geq 0.175 \end{cases} \qquad (14\text{–}33)$$

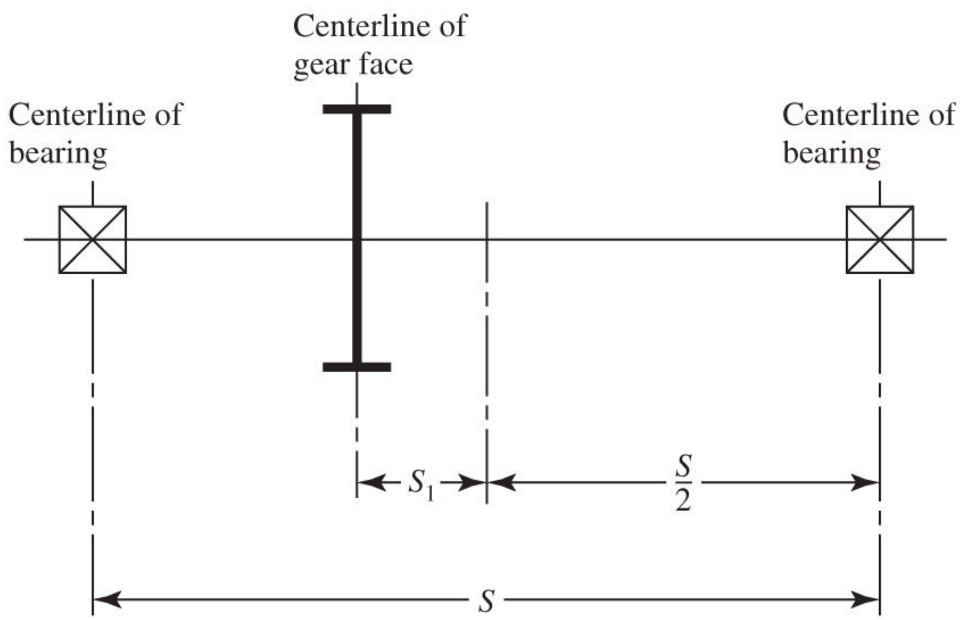


Fig. 14–10

Load-Distribution Factor K_m (K_H)

- C_{ma} can be obtained from Eq. (14–34) with Table 14–9

$$C_{ma} = A + BF + CF^2 \qquad \text{(see Table 14–9 for values of } A, B, \text{ and } C) \qquad (14\text{–}34)$$

Table 14–9

Empirical Constants A, B, and C for Eq. (14–34), Face Width F in Inches*

Source: ANSI/AGMA 2001-D04.

Condition	A	B	C
Open gearing	0.247	0.0167	$-0.765(10^{-4})$
Commercial, enclosed units	0.127	0.0158	$-0.930(10^{-4})$
Precision, enclosed units	0.0675	0.0128	$-0.926(10^{-4})$
Extraprecision enclosed gear units	0.00360	0.0102	$-0.822(10^{-4})$

*See ANSI/AGMA 2101-D04, pp. 20–22, for SI formulation.

- Or can read C_{ma} directly from Fig. 14–11

Load-Distribution Factor K_m (K_H)

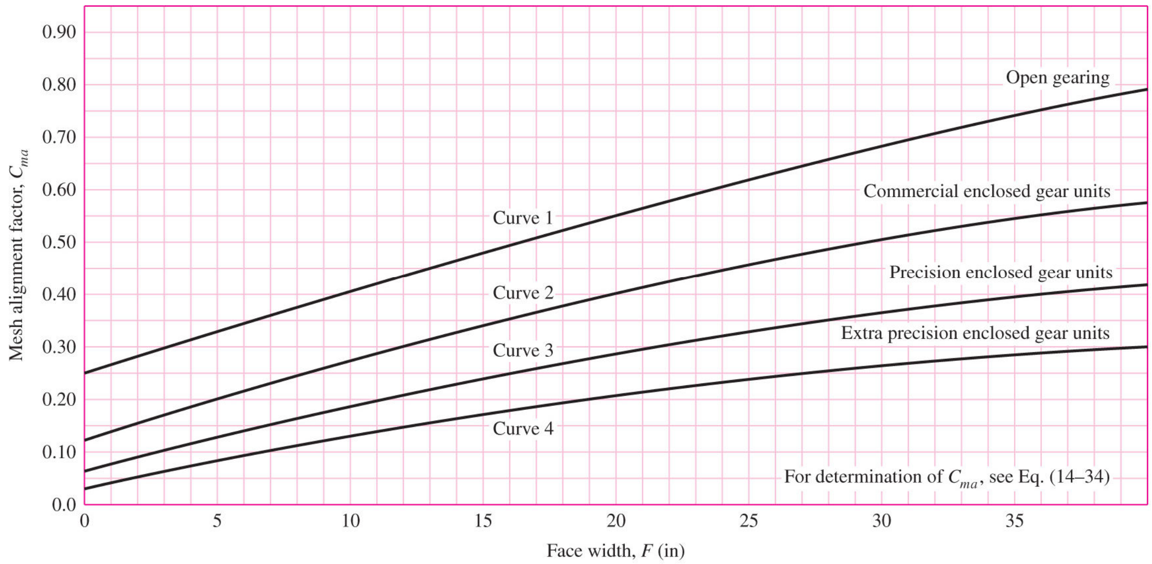


Fig. 14–11

Rim-Thickness Factor K_B

- When the rim thickness is not sufficient to provide full support for the tooth root, the location of the bending fatigue failure may be through the rim rather than at the tooth fillet.

$$K_B = \begin{cases} 1.6 \ln \dfrac{2.242}{m_B} & m_B < 1.2 \\ 1 & m_B \geq 1.2 \end{cases} \qquad (14\text{–}40)$$

$$m_B = \frac{t_R}{h_t} \qquad (14\text{–}39)$$

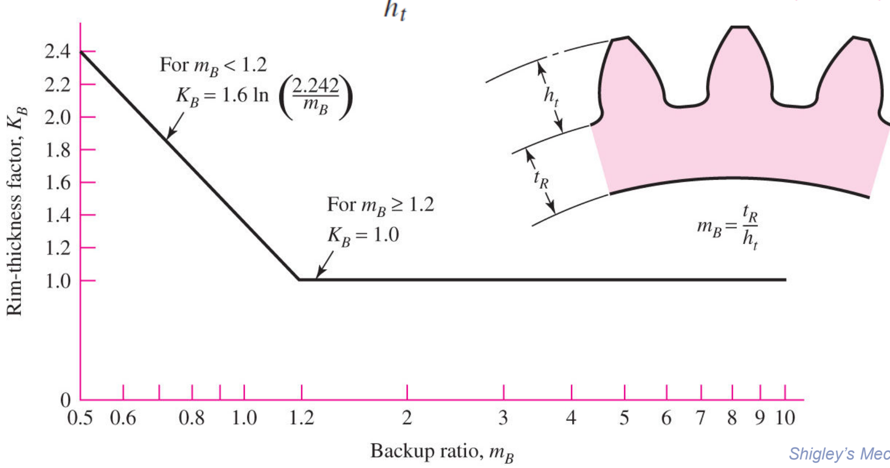


Fig. 14–16

Geometry Factor J (Y_J in metric)

- Accounts for shape of tooth in bending stress equation
- Includes
  - A modification of the Lewis form factor Y
  - Fatigue stress-concentration factor K_f
  - Tooth *load-sharing ratio* m_N
- AGMA equation for geometry factor is

$$J = \frac{Y}{K_f m_N} \qquad (14\text{–}20)$$

$$m_N = \frac{p_N}{0.95Z} \qquad (14\text{–}21)$$

- Values for Y and Z are found in the AGMA standards.
- For most common case of spur gear with 20º pressure angle, J can be read directly from Fig. 14–6.
- For helical gears with 20º normal pressure angle, use Figs. 14–7 and 14–8.

Spur-Gear Geometry Factor J

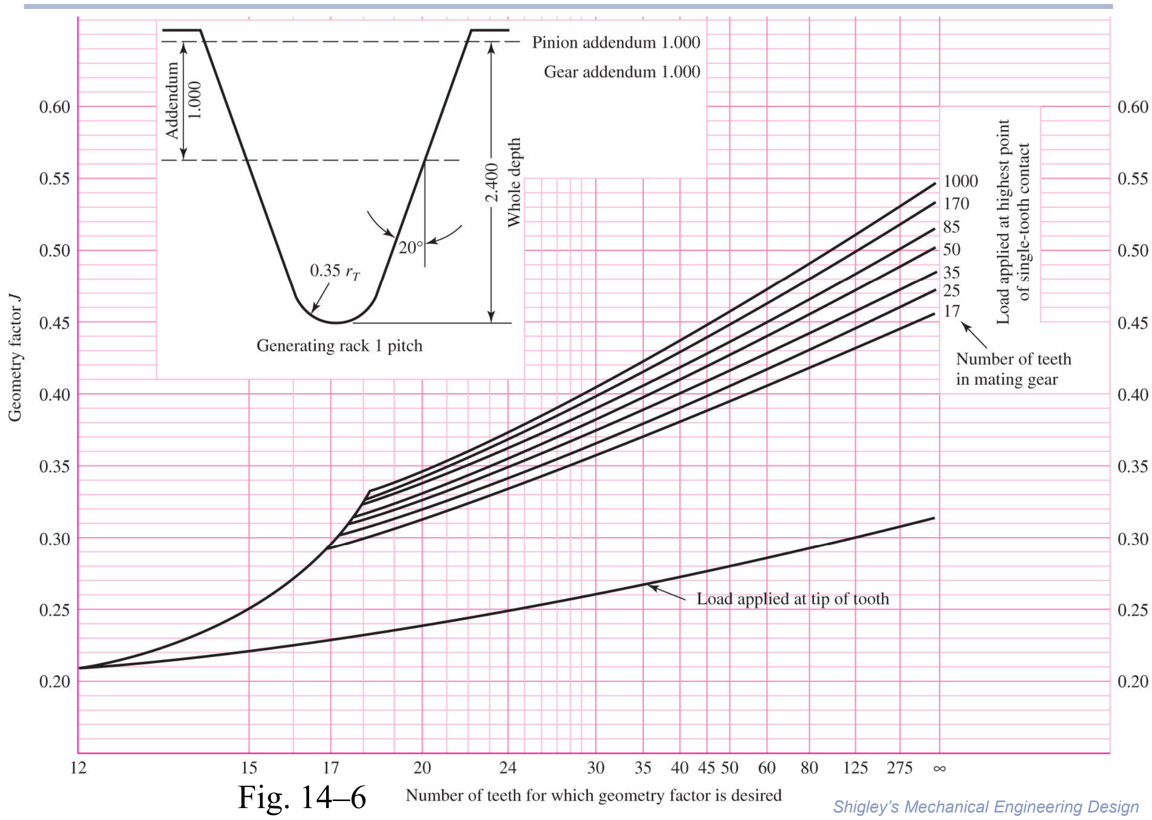


Fig. 14–6

Helical-Gear Geometry Factor *J*

- Get J' from Fig. 14–7, which assumes the mating gear has 75 teeth
- Get multiplier from Fig. 14–8 for mating gear with other than 75 teeth
- Obtain J by applying multiplier to J'

Ex: ψ=30°, N_P=17, N_G=52
→ J_P'=0.45, J_G'=0.53

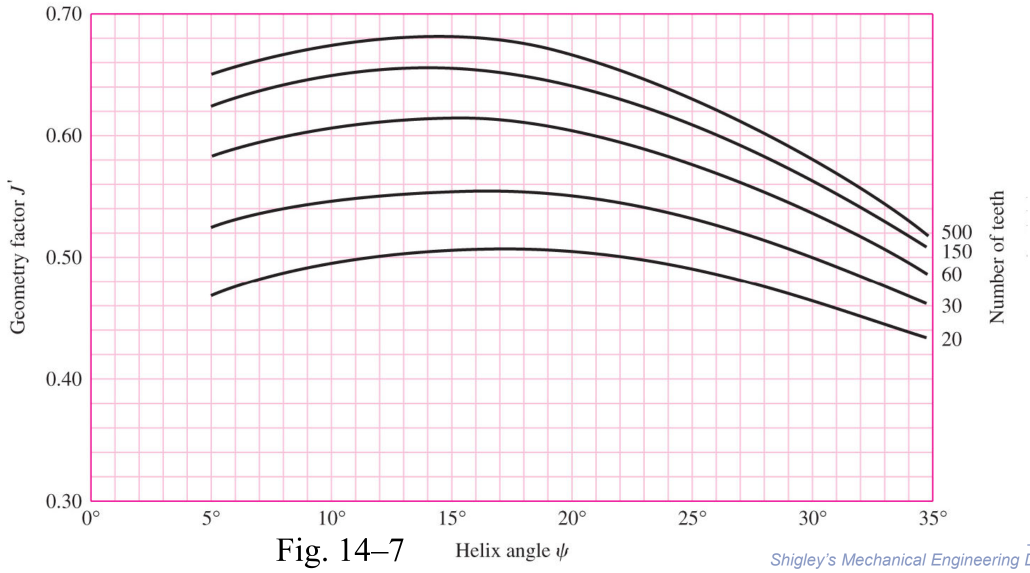


Fig. 14–7

Modifying Factor for *J*

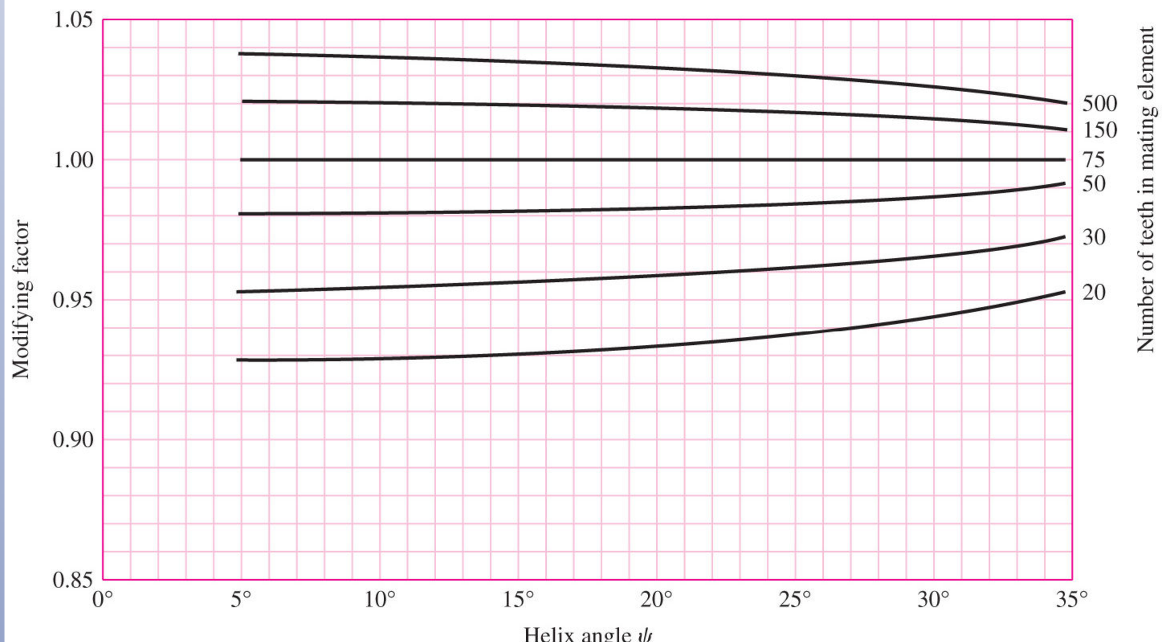


Fig. 14–8

Ex: ψ=30°, N_P=17, N_G=52
→ MF_P=0.94, MF_G=0.99

Elastic Coefficient C_P (Z_E)

- Elastic coefficient can be obtained from

$$C_p = \left[\frac{1}{\pi \left(\frac{1 - \nu_P^2}{E_P} + \frac{1 - \nu_G^2}{E_G} \right)} \right]^{1/2} \qquad (14\text{–}13)$$

Surface Strength Geometry Factor I (Z_I in metric)

- Called *pitting resistance geometry factor* by AGMA

$$I = \begin{cases} \dfrac{\cos\phi_t \sin\phi_t}{2m_N} \dfrac{m_G}{m_G + 1} & \text{external gears} \\[2ex] \dfrac{\cos\phi_t \sin\phi_t}{2m_N} \dfrac{m_G}{m_G - 1} & \text{internal gears} \end{cases} \qquad (14\text{–}23)$$

$$m_G = \frac{N_G}{N_P} = \frac{d_G}{d_P} \qquad (14\text{–}22)$$

$$m_N = \frac{p_N}{0.95Z} \quad (m_N\text{=1 for spur gears}) \qquad (14\text{–}21)$$

$$p_N = p_n \cos\phi_n \qquad (14\text{–}24)$$

$$Z = \left[(r_P + a)^2 - r_{bP}^2\right]^{1/2} + \left[(r_G + a)^2 - r_{bG}^2\right]^{1/2} - (r_P + r_G)\sin\phi_t \qquad (14\text{–}25)$$

$$r_b = r\cos\phi_t \qquad (14\text{–}26)$$

Surface Condition Factor C_f (Z_R)

- To account for detrimental surface finish
- No values currently given by AGMA
- **Use value of 1 for normal commercial gears**

STRENGTH CALCULATIONS

AGMA Strengths

- The gear strength values are only for use with the AGMA stress values, and should not be compared with other true material strengths.

- Representative values of typically available bending strengths are given in Table 14–3 for steel gears and Table 14–4 for iron and bronze gears.
- Figs. 14–2, 14–3, and 14–4 are used as indicated in the tables.
- Tables assume repeatedly applied loads at 10^7 cycles and 0.99 reliability.

Allowable Bending Stress

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F}\dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\[2ex] \dfrac{S_t}{S_F}\dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \qquad (14\text{–}17)$$

where for U.S. customary units (SI units),

S_t is the allowable bending stress, lbf/in^2 (N/mm^2)
Y_N is the stress cycle factor for bending stress
K_T (Y_θ) are the temperature factors
K_R (Y_Z) are the reliability factors
S_F is the AGMA factor of safety, a stress ratio

Allowable Contact Stress

$$\sigma_{c,\mathrm{all}} = \begin{cases} \dfrac{S_c}{S_H} \dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\[2ex] \dfrac{S_c}{S_H} \dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \qquad (14\text{–}18)$$

S_c is the allowable contact stress, lbf/in^2 (N/mm^2)
Z_N is the stress cycle life factor
C_H (Z_W) are the hardness ratio factors for pitting resistance
K_T (Y_θ) are the temperature factors
K_R (Y_Z) are the reliability factors
S_H is the AGMA factor of safety, a stress ratio

Bending Strengths for Through-hardened Steel Gears

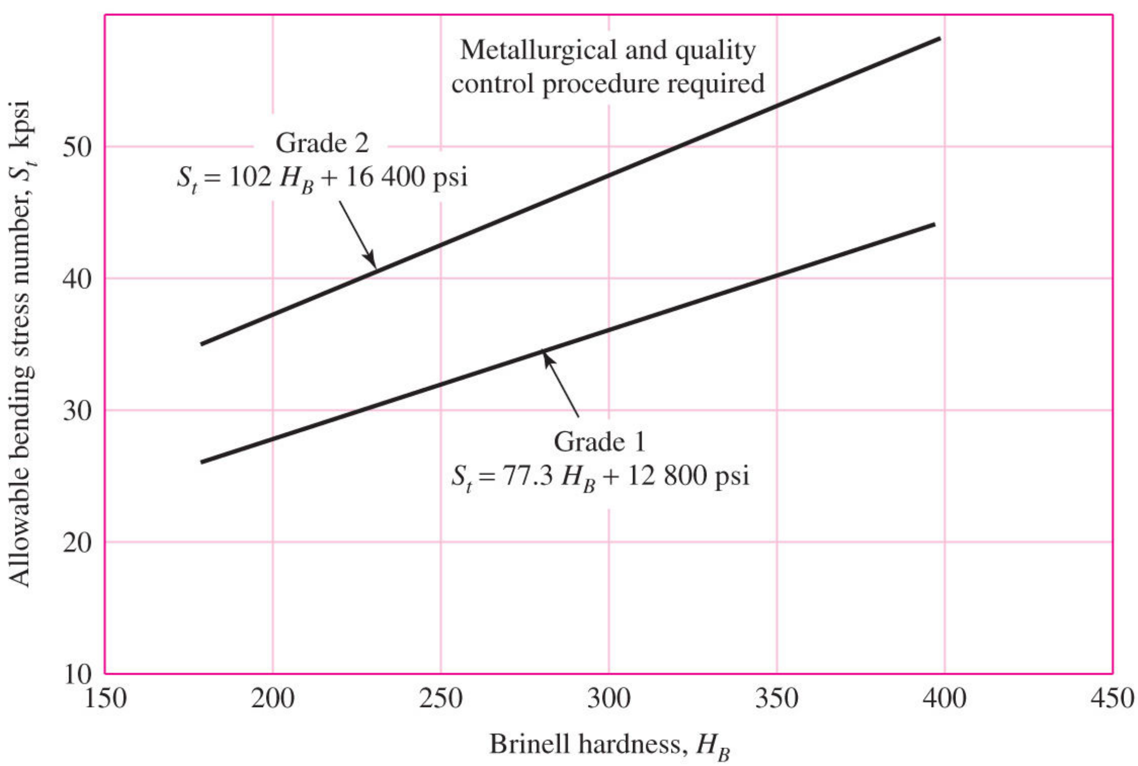


Fig. 14–2

Contact Strength for Through-hardened Steel Gears

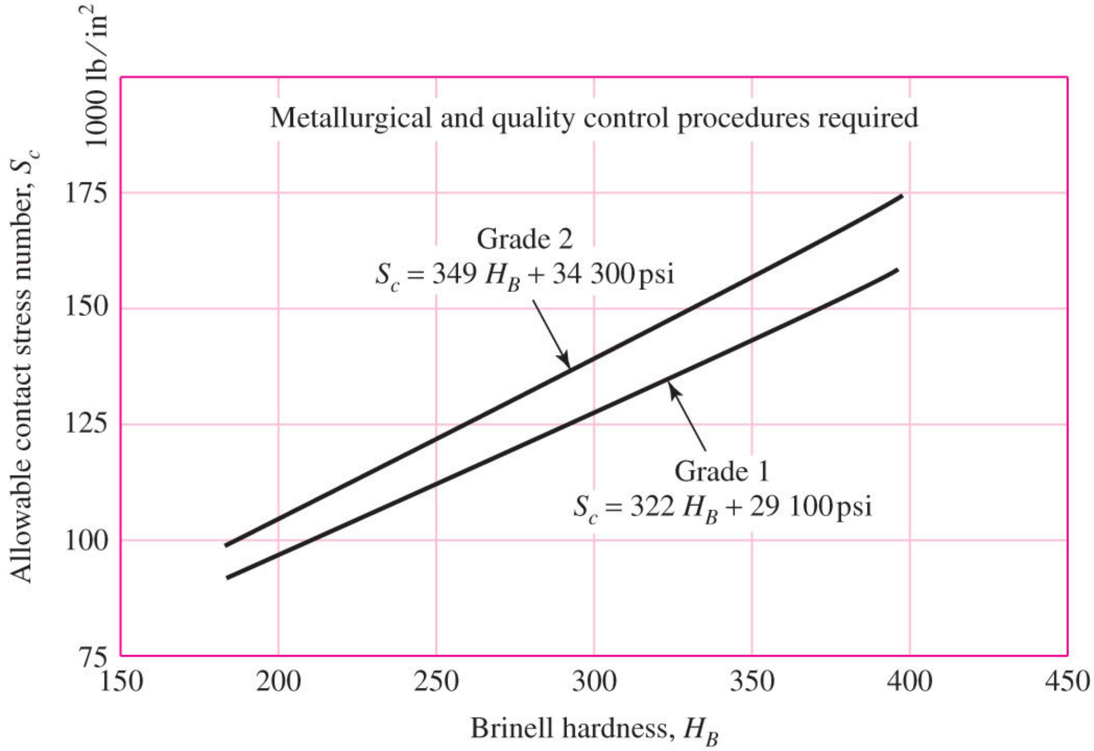


Fig. 14–5

Stress-Cycle Factor Y_N

- AGMA strengths are for 10^7 cycles
- Stress-cycle factors account for other design cycles

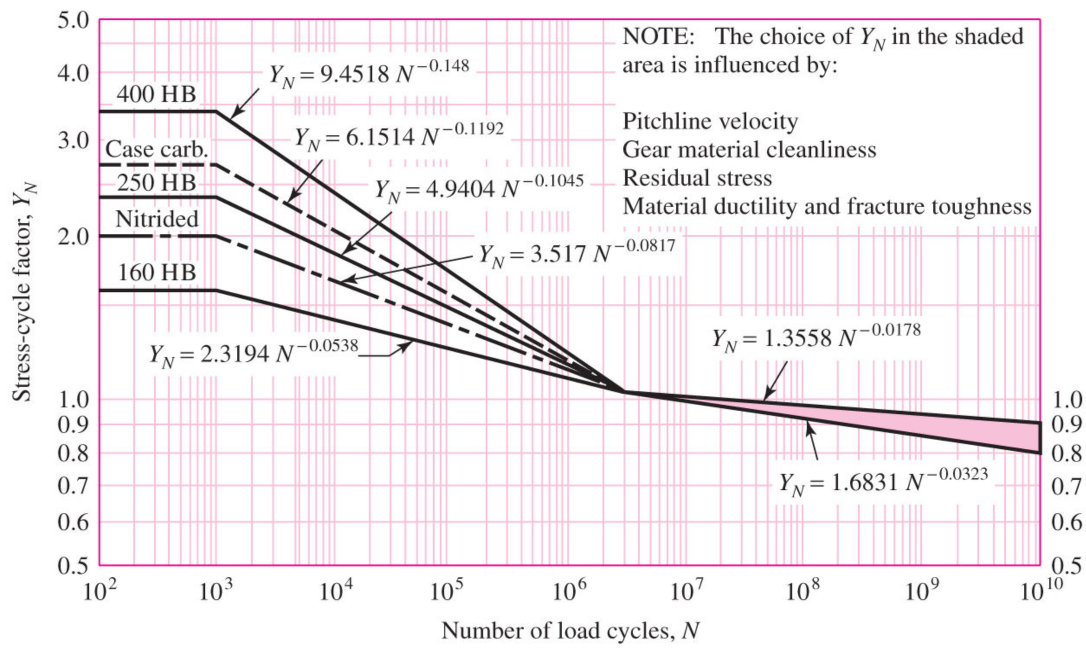


Fig. 14–14

Stress-Cycle Factor Z_N

- AGMA strengths are for 10^7 cycles
- Stress-cycle factors account for other design cycles

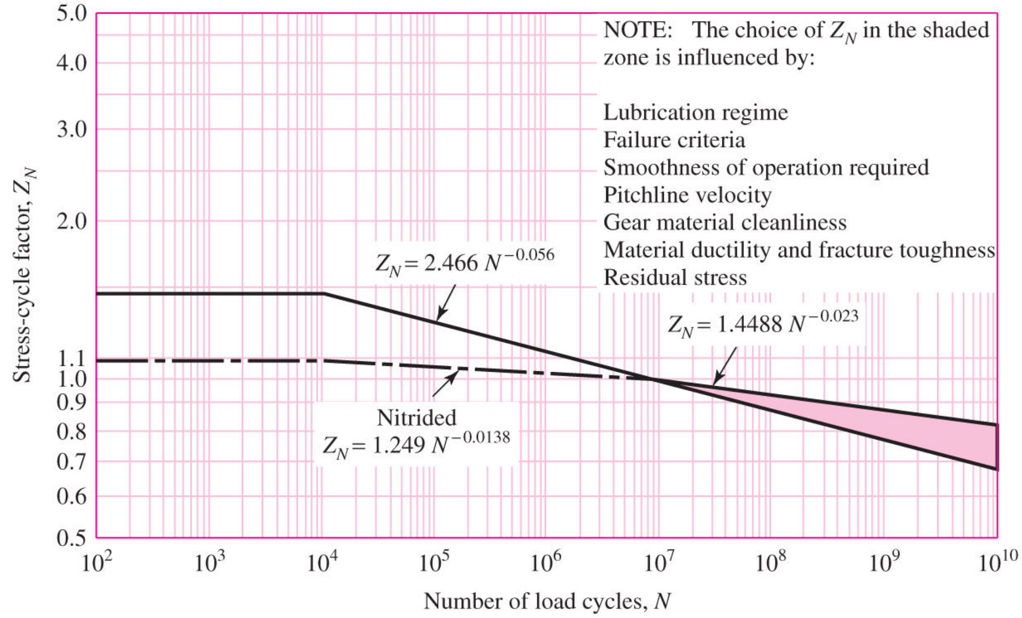


Fig. 14–15

Temperature Factor K_T (Y_θ)

- AGMA has not established values for this factor.
- For temperatures up to 250ºF (120ºC), $K_T = 1$ is acceptable.

Reliability Factor K_R (Y_Z)

- Accounts for statistical distributions of material fatigue failures
- Does not account for load variation
- Use Table 14–10
- Since reliability is highly nonlinear, if interpolation between table values is needed, use the least-squares regression fit,

$$K_R = \begin{cases} 0.658 - 0.0759 \ln(1 - R) & 0.5 < R < 0.99 \\ 0.50 - 0.109 \ln(1 - R) & 0.99 \le R \le 0.9999 \end{cases} \quad (14\text{–}38)$$

Reliability	K_R (Y_Z)
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

Table 14–10

Hardness-Ratio Factor C_H (Z_W)

- Since the pinion is subjected to more cycles than the gear, it is often hardened more than the gear.
- The hardness-ratio factor accounts for the difference in hardness of the pinion and gear.
- C_H is only applied to the gear. That is, $C_H = 1$ for the pinion.
- For the gear,

$$C_H = 1.0 + A'(m_G - 1.0) \quad (14\text{–}36)$$

$$A' = 8.98(10^{-3})\left(\frac{H_{BP}}{H_{BG}}\right) - 8.29(10^{-3}) \quad \text{for } 1.2 \le \frac{H_{BP}}{H_{BG}} \le 1.7$$

$A' = 0$ for $H_{BP}/H_{BG} < 1.2$

$A' = 0.00698$ for $H_{BP}/H_{BG} > 1.7$

SAFETY FACTOR CALCULATIONS

Safety Factors S_F and S_H

- Included as design factors in the strength equations
- Can be solved for and used as factor of safety

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \tag{14-41}$$

$$S_H = \frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c} = \frac{\text{fully corrected contact strength}}{\text{contact stress}} \tag{14-42}$$

- Or, can set equal to unity, and solve for traditional factor of safety as $n = \sigma_{\text{all}}/\sigma$

Summary for Bending of Gear Teeth

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33\,000\ \mathrm{H}}{V}$$

1 [or Eq. (a), Sec. 14–10]; p. 759

Eq. (14–30); p. 759

Eq. (14–40); p. 764

Gear bending stress equation Eq. (14–15)

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

Fig. 14–6; p. 753

Eq. (14–27); p. 756

Table below

${}_{0.99}(S_t)_{10^7}$ Tables 14–3, 14–4; pp. 748, 749

Gear bending endurance strength equation Eq. (14–17)

$$\sigma_{\mathrm{all}} = \frac{S_t}{S_F} \frac{Y_N}{K_T K_R}$$

Fig. 14–14; p. 763

Table 14–10, Eq. (14–38); pp. 763, 764

1 if $T < 250°\mathrm{F}$

Bending factor of safety Eq. (14–41)

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma}$$

Fig. 14–17

Summary for Surface Wear of Gear Teeth

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33\,000\ \mathrm{H}}{V}$$

1 [or Eq. (a), Sec. 14–10]; p. 759

Eq. (14–30); p. 759

1

Gear contact stress equation Eq. (14–16)

$$\sigma_c = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I} \right)^{1/2}$$

Eq. (14–23); p. 755

Eq. (14–27); p. 756

Eq. (14–13), Table 14–8; pp. 744, 757

Table below

${}_{0.99}(S_c)_{10^7}$ Tables 14–6, 14–7; pp. 751, 752

Fig. 14–15; p. 763

Section 14–12, gear only; pp. 761, 762

Gear contact endurance strength Eq. (14–18)

$$\sigma_{c,\mathrm{all}} = \frac{S_c Z_N C_H}{S_H K_T K_R}$$

Table 14–10, Eq. (14–38); pp. 763, 764

1 if $T < 250°\mathrm{F}$

Gear only

Wear factor of safety Eq. (14–42)

$$S_H = \frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c}$$

Fig. 14–18

Example 14–4 (SPUR GEAR)

A 17-tooth 20° pressure angle spur pinion rotates at 1800 rev/min and transmits 4 hp to a 52-tooth disk gear. The diametral pitch is 10 teeth/in, the face width 1.5 in, and the quality standard is No. 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion is a grade 1 steel with a hardness of 240 Brinell tooth surface and through-hardened core. The gear is steel, through-hardened also, grade 1 material, with a Brinell hardness of 200, tooth surface and core. Poisson's ratio is 0.30, $J_P = 0.30$, $J_G = 0.40$, and Young's modulus is $30(10^6)$ psi. The loading is smooth because of motor and load. Assume a pinion life of 10^8 cycles and a reliability of 0.90, and use $Y_N = 1.3558N^{-0.0178}$, $Z_N = 1.4488N^{-0.023}$. The tooth profile is uncrowned. This is a commercial enclosed gear unit.

(*a*) Find the factor of safety of the gears in bending.
(*b*) Find the factor of safety of the gears in wear.
(*c*) By examining the factors of safety, identify the threat to each gear and to the mesh.

Example 14–5 (HELICAL GEAR)

A 17-tooth 20° normal pitch-angle helical pinion with a right-hand helix angle of 30° rotates at 1800 rev/min when transmitting 4 hp to a 52-tooth helical gear. The normal diametral pitch is 10 teeth/in, the face width is 1.5 in, and the set has a quality number of 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion and gear are made from a through-hardened steel with surface and core hardnesses of 240 Brinell on the pinion and surface and core hardnesses of 200 Brinell on the gear. The transmission is smooth, connecting an electric motor and a centrifugal pump. Assume a pinion life of 10^8 cycles and a reliability of 0.9 and use the upper curves in Figs. 14–14 and 14–15.

(*a*) Find the factors of safety of the gears in bending.
(*b*) Find the factors of safety of the gears in wear.
(*c*) By examining the factors of safety identify the threat to each gear and to the mesh.