Transport Phenomena of Non-Newtonian Fluids

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Lecture - 34
Mass Transfer Combined with Chemical Reactions

Welcome to the MOOCs course Transport Phenomena of Non-Newtonian Fluids. The title of today's lecture is Mass Transfer Combined with Chemical Reactions. So, in the 1st lecture of mass transfer part what we have seen? If there is a chemical reaction, so, then according to reaction type the contribution from the reaction in the flux has to be brought in differently because of the different nature of the reactions because if it is homogeneous reaction, it is occurring in the entire volume.

So, then that contribution of the reaction in the mass transfer whatever is there or the flux whatever is there that should be brought as a kind of source term in the you know species conservation equation or the balance equation cell balance equation whatever we do. If it is a heterogeneous reaction it must be taking place at specified locations like you know catalyst surface etcetera.

So, under such conditions what happens? The contribution from the reaction in the flux whatever is there that should be brought through the boundary condition because it is occurring at a specified location. Since the nature of the reactions are different they are occurring in different locations different way. So, then their contribution in the mass flux would also be coming differently so, then corresponding mathematical representation would also be different, that is what we have seen.

So, now, what we do? We are taking two different problem one problem is the mass transfer combined with heterogeneous reaction because that would be simple one, simple in the sense the contribution from the reaction would be coming into the picture as a kind of boundary condition only. Then second problem we take mass transfer combined with homogeneous reaction, where the reaction part is would be appearing in the in a balanced equation species conservation equation as a source term.

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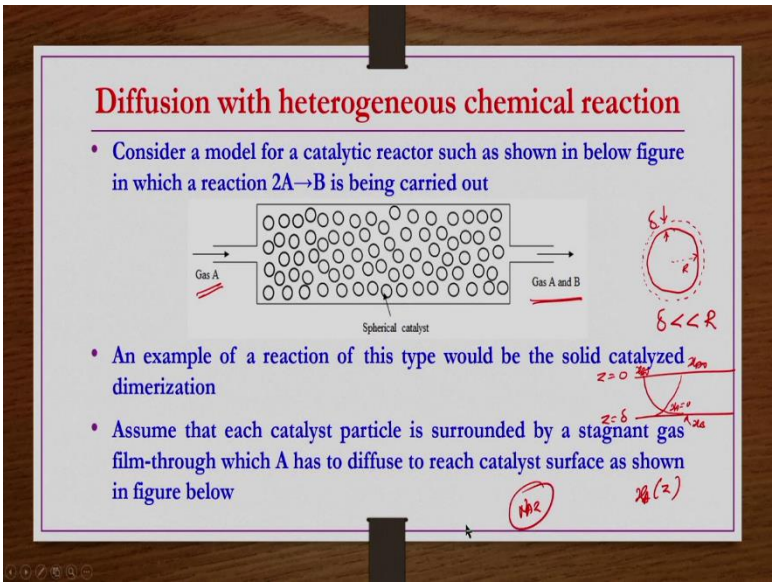

So, diffusion with heterogeneous chemical reaction we take, we are taking only the diffusive mass transfer because when you include the forced to convective mass transfer combined with chemical reactions, the equations complexity would increase and then you have to go for a numerical solution.

So, that is the reason in order to keep the complexity up to tractable level we are taking only diffusive mass transfer. Consider a model for a catalytic reactor such as shown in below figure in which reaction 2A tends to B is being carried out. We have a catalytic bit in which gas A is coming in and then that is reacting on the surface of catalyst particles and then product B is forming and then outlet unreacted gas A and then product B are coming out, right.

An example of reaction of this type would be the solid catalyst dimerization reactions which are very common in polymers polymer industries in general. Assume that each catalyst particle is surrounded by a stagnant gas film through which A has to diffuse to reach catalyst surface as shown in figure below that we have in the next slide.

So, that is if you enlarge this catalytic particle. So, if it is a particle. So, each particle is surrounded by a gas film thin gas film, ok. So, let us say thickness is δ film thickness is δ, but compared to the radius of catalyst particle, this δ is very very small that is δ is very very smaller than R.

So, under such conditions what we can do? We do not need to consider the curvature effect and then we can have a Cartesian coordinate representation of these thing because now this entire all number of catalysts the complexity of the packing bed etcetera the all that we are not taking place all that we are not considering here, we are taking only one catalyst particle on which the reaction is taking place.

So, that simple case we are taking. So, now, for that we are taking like this. So, if it is a catalyst surface let us say we will locate $z = \delta$ and then hypothetical film boundary whatever is there that we take $z = 0$, right. At $z = 0$ the concentration of A would be maximum let us say x_{A0} and then as it moves through the reaction takes place at $z = \delta$ and then the depending on the fastness order of the reaction rate constant of the reaction.

So, then this disappearance what is the concentration at of x at boundary on this catalytic surface that you know accordingly that concentration will be there. If it is an instantaneous reaction; instantaneous reaction in the sense instantaneous and irreversible that is whatever A is there that is immediately reacting and then forming B within very short time.

So, then at the catalyst surface virtually x_A would be 0. If it is an instantaneous reaction we are going to take both the cases anyway. So, whereas, the B is very small you know at film boundary that is x_{B0} we are calling it, this is forming at the catalyst surface and then diffusing into the bulk gas film like this.

So, x_B would be maximum on the catalyst surface and then minimum at hypothetical film boundary that is at $z = 0$, right. So, what is this x_A as function of z we have to find out and then we have to find out N_{Az} also, ok.

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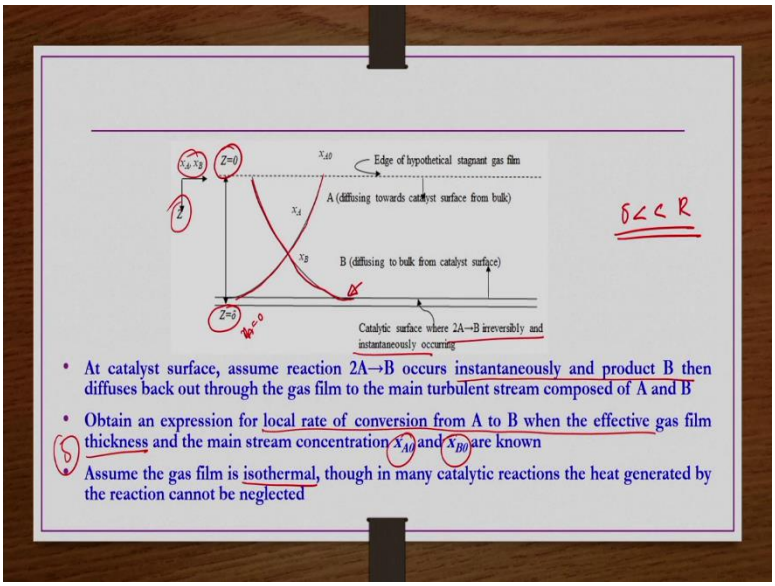

The same thing is pictorially shown here. So, since the film thickness is very very small compared to the radius of the catalyst particle curvature effect we are not taking into consideration. So, hypothetical stagnant gas film edge whatever is there that we are calling $z = 0$, catalyst surface whatever is there that we calling $z = \delta$ right and then the diffusion is taking place in the z direction.

In this representation x axis is concentration x_A or x_B and then y axis and the reverse direction is Z and the vertical direction in the downward is z axis here. So, now x_A is x_{A0} at hypothetical film which is almost pure and then it is reacting and then its concentration dropping and becoming 0 at $x = \delta$, if it is irreversible and instantaneously occurring then x_A would be 0 at $x = \delta$ otherwise it has to be represented accordingly and then this B is forming on the catalyst surface.

Because immediately the reaction is taking place immediately A is being converted into the B by dimerization equation. So, then it having profile maximum concentration on the surface $x = \delta$ and then it diffuses back to the bulk from $z = \delta$ to $z = 0$. At catalyst surface assume reaction 2A tends to B occurs instantaneously and product B then diffuses back out through the gas film to the main turbulence stream composed of A and B.

And then obtain expression for local rate of conversion from x_A to from A to B when the effective gas film thickness that is δ is known and then mainstream concentration x_{A0} x_{B0} are not are known. So, that is x_{A0} x_{B0} are known and δ is also known. Assume the gas film

is isothermal that is actually not true most of the catalytic reactions heat is in energy is involved.

So, then there will be temperature difference also, but we are not considering it here in this problem. So, this problem we are taking two cases first case we take the instantaneous reaction instantaneous irreversible reaction as stated in the problem, second case we are taking a reaction of certain order of reaction and then see the solution.

But the procedure of getting the solution is same only boundary condition would be changing. Because, why only boundary conditions be changing? Because it is a heterogeneous reaction. So, whatever the changes are there they are occurring at boundaries only specified locations only.

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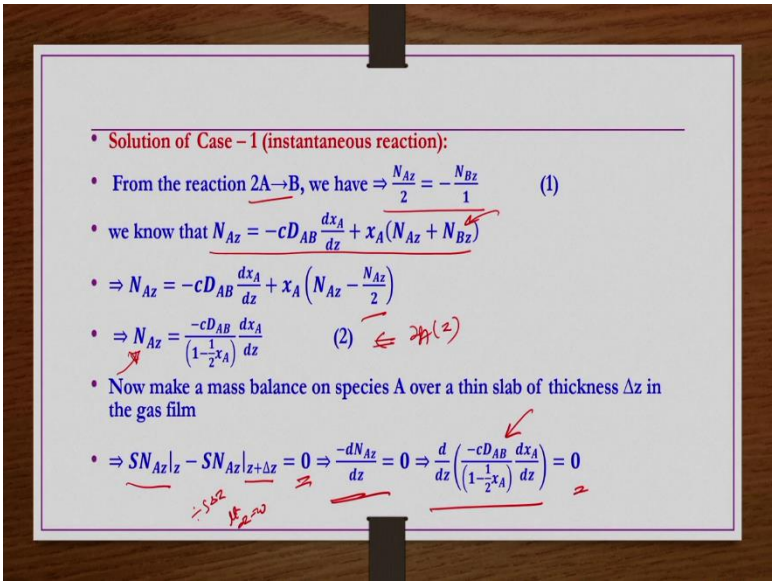

The case 1 instantaneous reaction from the reaction 2A tends to B stoichiometry of the reaction we can get $\frac{N_{Az}}{2} = -\frac{N_{Bz}}{1}$; that means, $N_{Bz} = \frac{-1}{2}N_{Az}$. So, in the combined flux in place of N_{Bz} we can write $\frac{-1}{2}N_{Az}$ like this, then you take all the N_{Az} terms one side then we have $N_{Az} = -\frac{cD_{AB}}{1-\frac{1}{2}x_A}\frac{dx_A}{dz}$.

So, now if you have an expression for N_{Az}, you can find out what is x_A as function of z. For that what we do? In order to get N_{Az} we write a balance equation, then we write $SN_{Az}|_z - SN_{Az}|_{z+\Delta z} = 0$ because reaction part will not come here in the source as a source

term in the balance equation, it will come as a boundary condition because it is a heterogeneous reaction it is occurring only at specified location of the catalyst surface.

It is not occurring in the film; in the film only, diffusion is occurring. Diffusion of species are occurring so that they reach from the bulk to the catalyst surface or catalyst surface to the bulk, ok. So, now, this equation you divide both sides by S Δ z and then take limit Δ z tends to 0 then you have $-\frac{dN_{Az}}{dz} = 0$; that means, $\frac{dN_{Az}}{dz}$ is this one from equation 2 = 0.

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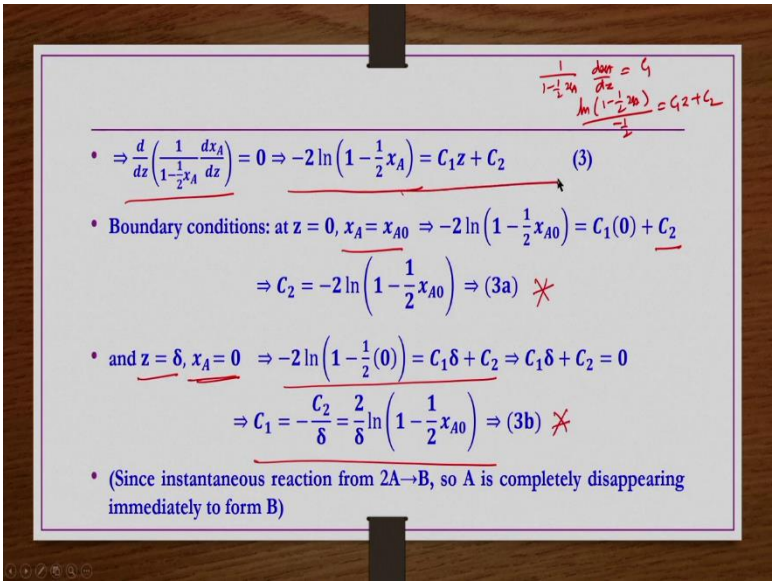

Now, cD_{AB} we can take constant here then what we have? $\frac{d}{dz}\left(\frac{1}{1-\frac{1}{2}x_A}\frac{dx_A}{dz}\right) = 0$ when you integrate first time what you get? $\frac{1}{1-\frac{1}{2}x_A}\frac{dx_A}{dz} = C_1$. Now, this dz you take to the other side and then integrate both sides one side with respect to x another side with respect to z then what we have?

$\frac{ln\left(1-\frac{1}{2}x_A\right)}{-\frac{1}{2}} = C_1 z + C_2$, this $-\frac{1}{2}$ I can write here as a $-2ln\left(1-\frac{1}{2}x_A\right) = C_1 z + C_2$. Two boundary conditions we have at z = 0 x_A is x_{A0}. So, C_2 is $-2ln\left(1-\frac{1}{2}x_{A0}\right)$ that is what we get. The other boundary condition that is at = δ, x_A = 0, why it is x_A = 0? Because the reaction is very instantaneous; the reaction is very instantaneous.

That is moment the molecules of A comes and reaches the surface of the catalyst immediately they are reacting and forming B, right. So, there is no A present at any given time immediately they are converting into the B because of the reaction. So, that is the reason x_A is 0 at z = δ, z = δ is nothing but catalyst surface location.

So, when you apply this boundary condition you get this one that is and this ln of 1 is 0 and then $C_1 \delta + C_2 = 0$ that is C_1 you get $-\frac{C_2}{\delta}$. So, that is this one. So, now, you have C_2 and then C_1 these two if you substitute in an equation number 3 you will get the concentration profile.

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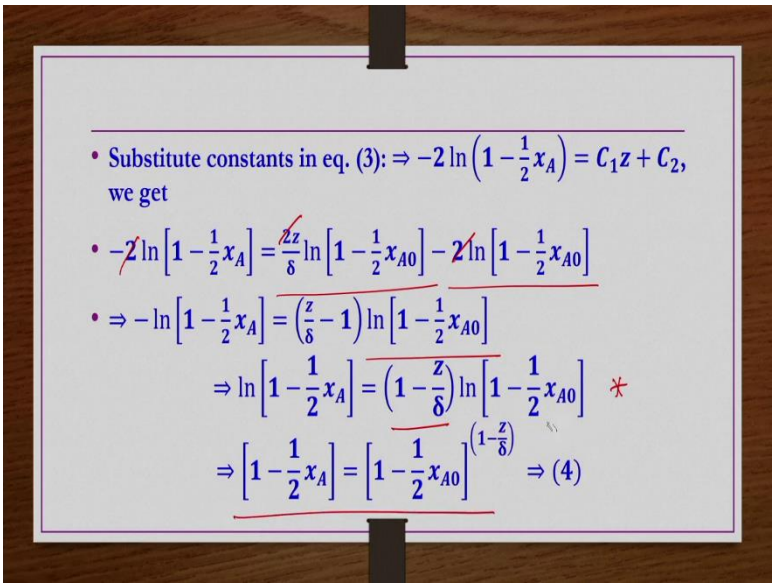

So, this $\frac{2}{\delta}\ln\left[1-\frac{1}{2}x_{A0}\right]$ is nothing but C_1 and then $-2ln\left[1-\frac{1}{2}x_{A0}\right]$ this is nothing but C_2. These two we can cancel out right. Then what we are doing? From these two terms $ln\left[1-\frac{1}{2}x_{A0}\right]$ if we take common we get $\frac{z}{\delta}-1$ right or these both sides if you multiplied by minus, so, $ln\left[1-\frac{1}{2}x_A\right] = \left(1-\frac{z}{\delta}\right) ln\left[1-\frac{1}{2}x_{A0}\right]$ this is the concentration profile.

The same thing you can write this way also right, so, but we need to find out the flux that is the question.

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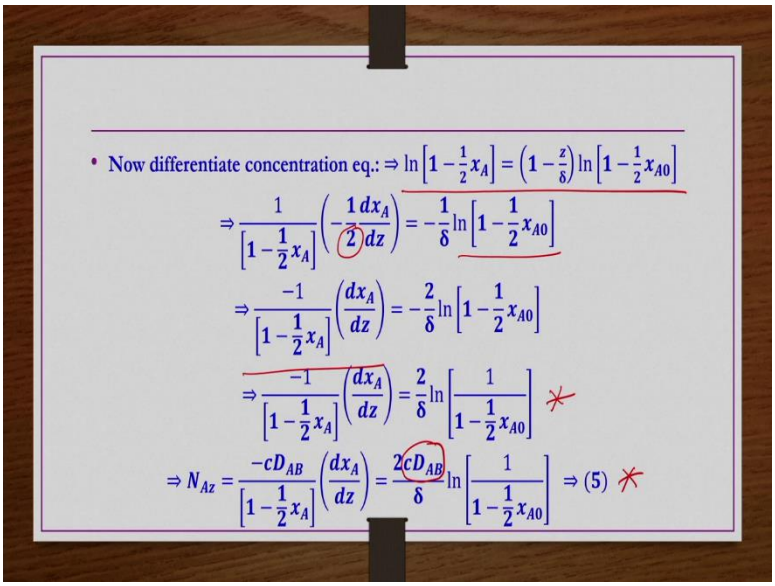

So, then what we have to do? This concentration profile this equation we have to differentiate with respect to z then we get $\frac{1}{\left[1-\frac{1}{2}x_A\right]}\left(-\frac{1}{2}\frac{dx_A}{dz}\right) = 0 - \frac{1}{\delta}ln$ of this is constant anyway right. So; that means, these 2 also if I take to the right hand side $-\frac{2}{\delta}ln\left[1-\frac{1}{2}x_{A0}\right]$ I will get in the right hand side.

Left hand side I am not doing any simplification and keeping as it is because flux equation we derived it as you know $N_{Az} = \frac{-cD_{AB}}{\left[1-\frac{1}{2}x_A\right]}\frac{dx_A}{dz}$; that means, this equation whatever is there if I simply multiply by c D_{AB} I will get the flux.

That is flux is $\frac{2cD_{AB}}{\delta}ln\left[\frac{1}{\left[1-\frac{1}{2}x_{A0}\right]}\right]$. This is the case where you know reaction is instantaneously occurring, an irreversible instantaneous reaction, but in general always we may not have such kind of reaction. So, what we do now?

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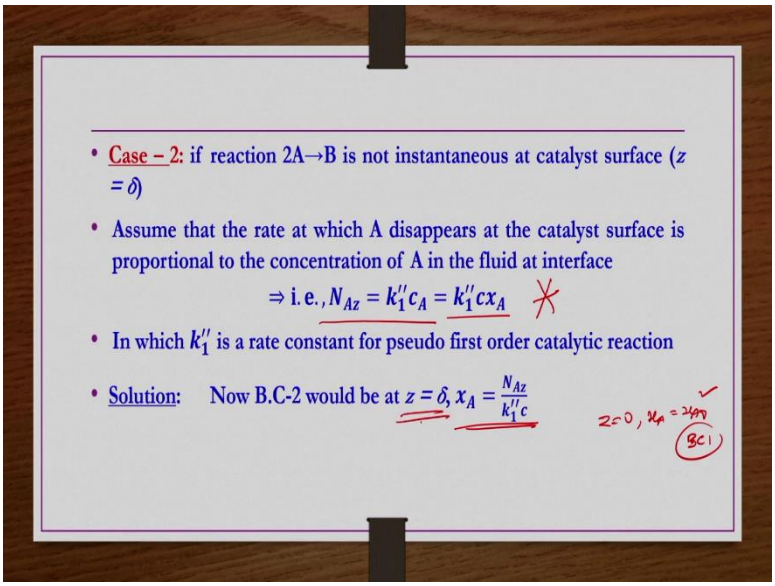

We take a case 2A tends to B is not instantaneous at catalyst surface at z is equals to δ. For that assume that the rate at which A disappears at the catalyst surface is proportional to the concentration of A in the fluid at interface. So, that is $N_{Az} = k_1^{"} c_A$ that I can write $k_1^{"} c x_A$. This is one of the boundary conditions that we have seen in the first lecture on mass transfer, right.

So, here $k_1^{"}$ is rate constant for pseudo first order catalytic reaction right. So, now, from this equation what we get? This equation is actually valid at z = δ right. So, z = δ at z = δ $N_{Az} = k_1^{"} c x_A$; that means, $x_A = \frac{N_{Az}}{k_1^{"} c}$ this is the boundary condition.

Whereas, the other boundary condition at z = 0 $x_A = x_{A0}$ that is remaining same B.C. 1 that is not changing right. Only at the catalyst surface which is at z = δ. Now the reaction is not instantaneous it is a first order reaction as per that one the flux N_{Az} is $k_1^{"} c x_A$ and then $x_A = \frac{N_{Az}}{k_1^{"} c}$.

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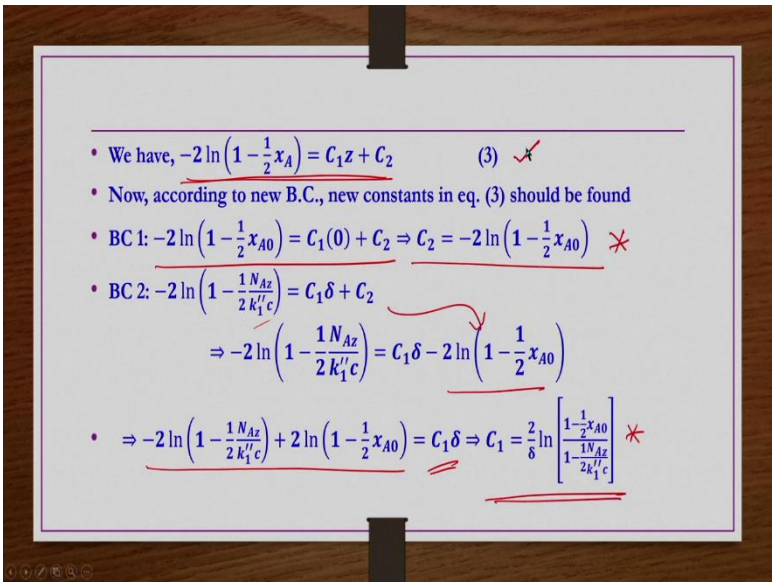

So, in the previous problem whatever this you know solving the you know combined flux equation etcetera integrating up to that part it is same; it is same, the changes whatever is there compared to the previous part of the problem that is occurring in this problem only from boundary conditions point of view.

So, B.C. 1 is same. So, that is at x = 0, $x_A = x_{A0}$. So, C_2 you are going to get the same constant like previous case and then B.C. 1 at z = δ x_A is nothing but $\frac{N_{Az}}{k_1'' c}$. So, that is we have written here. So, from here C_1 you get this particular term. The C_2 we have substituted here and then this ln term also we have taken to the left hand side.

So, that all the ln terms are together and then right hand side we have only C_1 δ. So, that δ also if you take to the left hand side you have $\frac{2}{\delta} ln \left[\frac{1-\frac{1}{2}x_{A0}}{1-\frac{1}{2}\frac{N_{Az}}{k_1'' c}}\right]$. So, this C_1 this C_2 you got it that if substitute in the equation number 3 you will get concentration profile.

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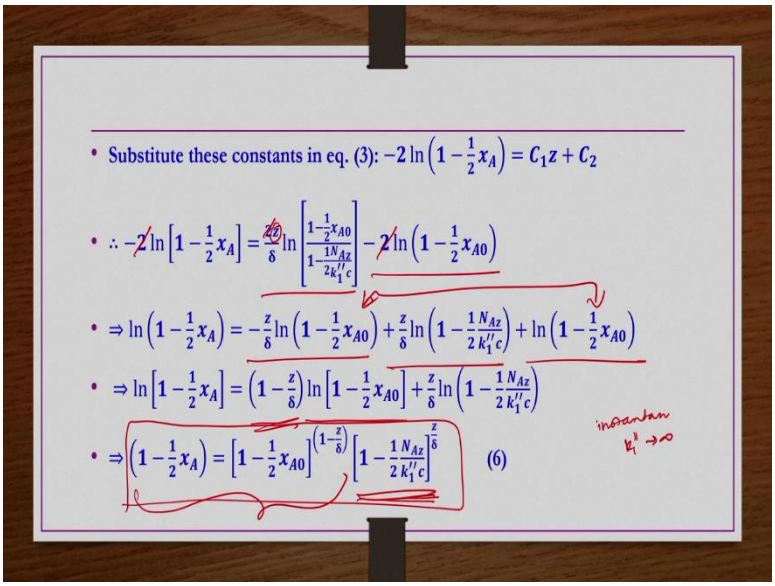

So, that is this other than z here in the first term in the right hand side is C_1, the second term $-2ln\left[1-\frac{1}{2}x_{A0}\right]$ is nothing but C_2. Now, here also 2 you can take off from both sides, then this particular term you expand it as two terms. Last term you keep it as it is and then both sides you are multiplying by minus 1. So, that right hand side we have plus term and then left hand side all the terms are multiplied by minus. So, this is what you are getting right.

Now these two terms are same actually similar right. So, $ln\left[1-\frac{1}{2}x_{A0}\right]$ if you take common, you get $\left(1-\frac{z}{\delta}\right)$ as a multiplication factor, the other term is remaining same right. So, now, all the terms are having ln. So, then these multiplication factors whatever are there, you can take as a power terms and then you get concentration profile expression this one.

Now, if the reaction instantaneous; that means, $k_1^{"}$ tends to infinity very large. So, then if $k_1^{"}$ is very large $\frac{1}{k_1^{"}}$ would be 0. So, under such condition this term would be you know this entire parenthesis term would be $1-0$ power something. So, it will be 1 power something. So, it will be 1 only.

So, only this part would be there. So, which is the same as a solution of part a where the reaction is instantaneous. So, if the reaction is not instantaneous and then reaction is

pseudo first order reaction. So, then this additional correction term will appear in the right hand side.

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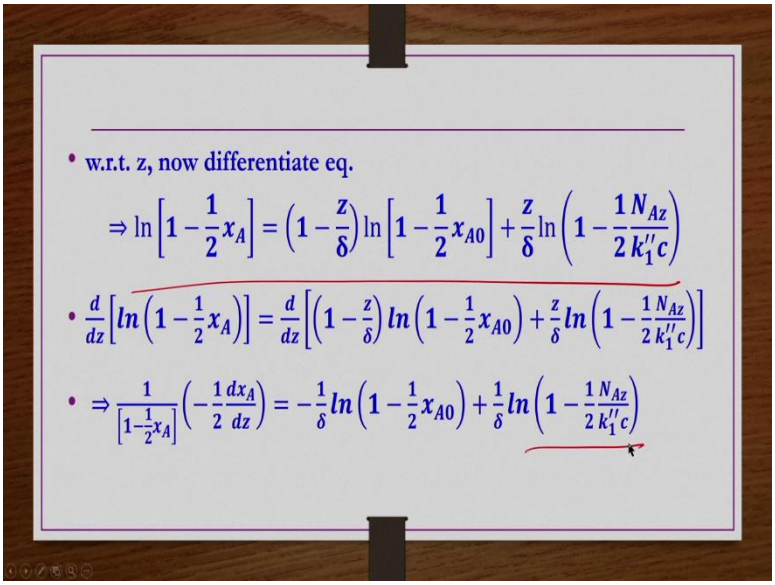

Now, we have to differentiate that concentration profile in order to get the flux equation because in the flux equation what we have? We have $-\frac{dx_A}{dz}$ terms. So, this is the equation that we got. This if you differentiate both sides with respect to z, what you get? Left hand side $\frac{1}{\left[1-\frac{1}{2}x_A\right]}\left(-\frac{1}{2}\frac{dx_A}{dz}\right) =$ here $-\frac{1}{\delta}ln\left(1-\frac{1}{2}x_{A0}\right) + \frac{1}{\delta}ln\left(1-\frac{1}{2}\frac{N_{Az}}{k_1'' c}\right)$.

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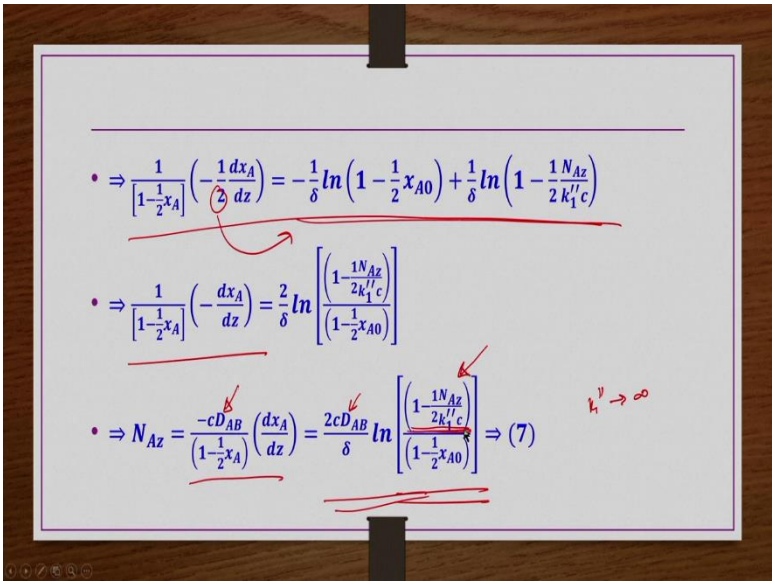

So, the same equation is written here again and then what we have done? We combined these two terms by you know after taking these 2 also to the right hand side. So, $\frac{2}{\delta}$ we are having an $ln\left[\frac{\left(1-\frac{1N_{Az}}{2k_1^{"}c}\right)}{\left(1-\frac{1}{2}x_{A0}\right)}\right]$ right.

So, now this equation left hand side we are keeping as it is because this left hand side if you multiply by c D_{AB}, then that would be same as flux. So, the same thing c D_{AB} if you have to; you have to multiply in the right hand side. So, $\frac{2cD_{AB}}{\delta} ln\left[\frac{\left(1-\frac{1N_{Az}}{2k_1^{"}c}\right)}{\left(1-\frac{1}{2}x_{A0}\right)}\right]$.

Now, here also, if the $k_1^{"}$ the reaction is instantaneous. So, then infinity then this term again will go to 1 and then solution would be same as a previous case; previous case of party where the reaction is instantaneous only thing that $k_1^{"}$ would be infinity. So, then $1-0$ in the numerator you will be having 1 here if the reaction is instantaneous which is the solution like a previous one.

So, if the reaction is not instantaneous, then this correction factor of this particular term is coming into the picture. So, for large values of $k_1^{"}$instantaneous conversion case whatever $ln\left(1-\frac{1}{2}\frac{N_{Az}}{k_1^{"}c}\right)$ if $k_1^{"}$ is very large. So, then this term would be very small.

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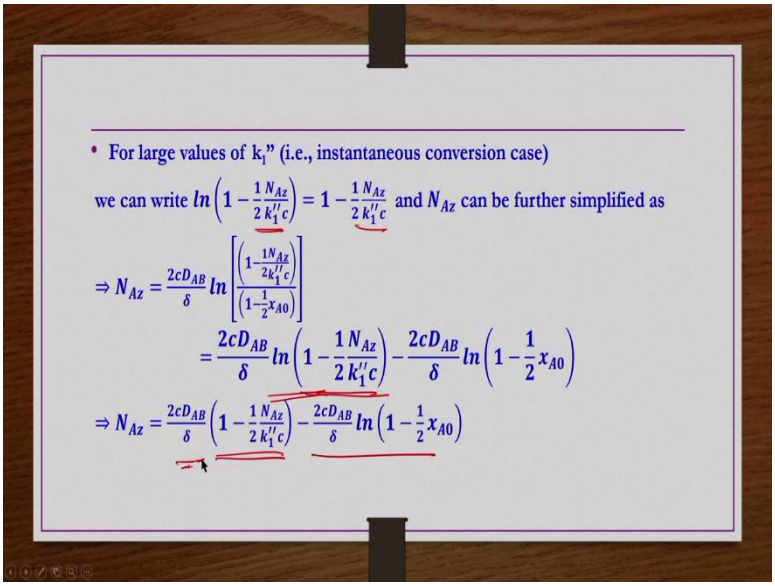

So, ln of 1 minus this term is we can write 1 minus of that term right. So, N_{Az} we can further simplify like this, then we have this particular term. So, in this particular term after expanding in place of ln of this term, I can directly write 1 minus of that particular term. Then after that I what I can do? I can combine these $\frac{2cD_{AB}}{\delta}$ terms together and then we see what we can get.

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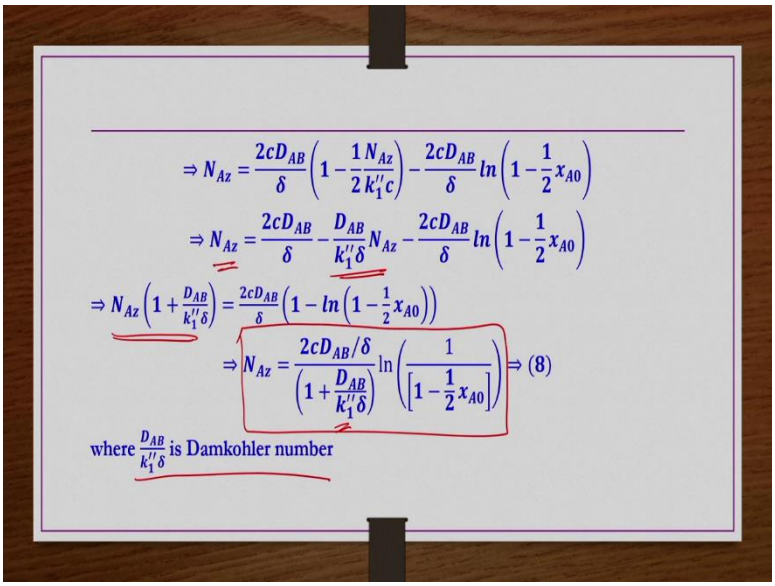

We can get this particular term on expansion and then all the terms whatever N_{Az} terms are there have been taken one side and then remaining terms are taken other side. So, here again, what we can get? $N_{Az} = \frac{2cD_{AB}/\delta}{\left(1+\frac{D_{AB}}{k_1''\delta}\right)} ln\left(\frac{1}{\left(1-\frac{1}{2}x_{A0}\right)}\right)$.

If k_1'' is large so, then this would be you know 0. So, in the denominator we will be having only 1. So, that is $\frac{2cD_{AB}}{\delta} ln\left(\frac{1}{\left(1-\frac{1}{2}x_{A0}\right)}\right)$ is the same as a previous part a problem when the reaction is instantaneous and this $\frac{D_{AB}}{k_1''\delta}$ is nothing but Damkohler number. Now, we take a homogeneous reaction diffusion with a homogeneous chemical reaction, ok.

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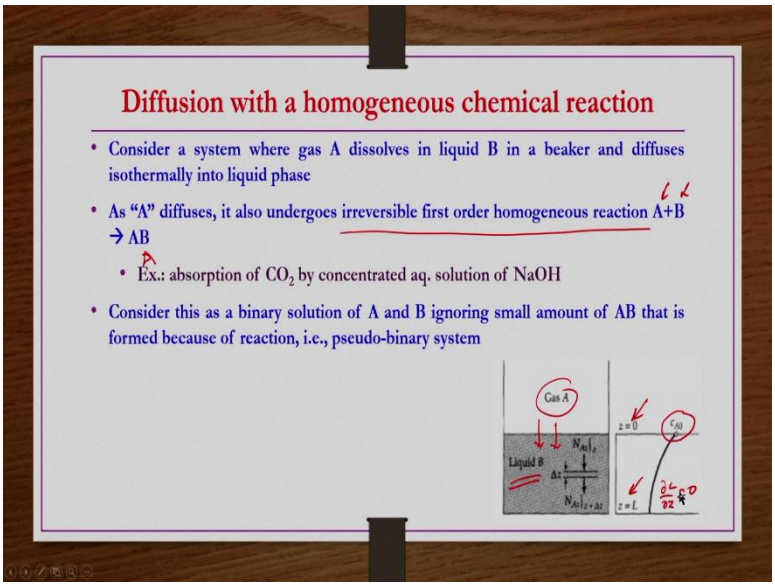

Here also we are not taking the first convective mass transfer, we are taking only diffusive mass transfer because if you take the first convective mass transfer, the velocity part would come into the picture and then the equation will become more complicated you have to go for you know numerical solution.

So, in order to have the complexities you know under detectable conditions, we are taking only diffusive mass transfer which is affected by the homogeneous chemical reaction that is simultaneously occurring while the diffusion is occurring, ok. Consider a system where gas A dissolves in liquid B in a beaker and diffuses isothermally into liquid phase.

So, as A diffuses it also undergoes irreversible first order homogeneous reaction A + B giving rise to AB. So, a reversible first order homogeneous reaction is occurring. Since it is a homogeneous reaction whatever the contribution in the flux is there, that should come as a source term in the balance equation species conservation balance equation whatever we write.

So, the contribution from the reaction has to come as a source term in the right hand side or the third term and then here A, B and then AB are there. So, it is not a binary system three components are there. So, what we do? We assume this AB is present in very small component so, that we can have a pseudo binary system.

The examples where we can have such kind of situations are absorption of CO_2 by concentrated aqueous solution of NaOH. Consider this as a binary solution of A and B ignoring small amount of AB that is formed because of the reaction that is pseudo binary system. Pictorially if you see we have a beaker in which liquid B is there.

Now, gas A is coming in and then dissolving here. So, while it is dissolving because of the diffusion, the reaction is also taking place that reaction is irreversible first order reaction homogeneous reaction right. So, now, the liquid whatever height is there that height we are taking L.

So, the top location we are taking z = 0, bottom location of the beaker we are taking z = L. So, at the top only pure gas A is there. So, at the gas liquid interface whatever the concentration of A is there that is c_{A0} that is the solubility of A in B whatever is there that was right and then at z = L we do not know how much it is. So, usually that is $\frac{\partial c}{\partial z} = 0$.

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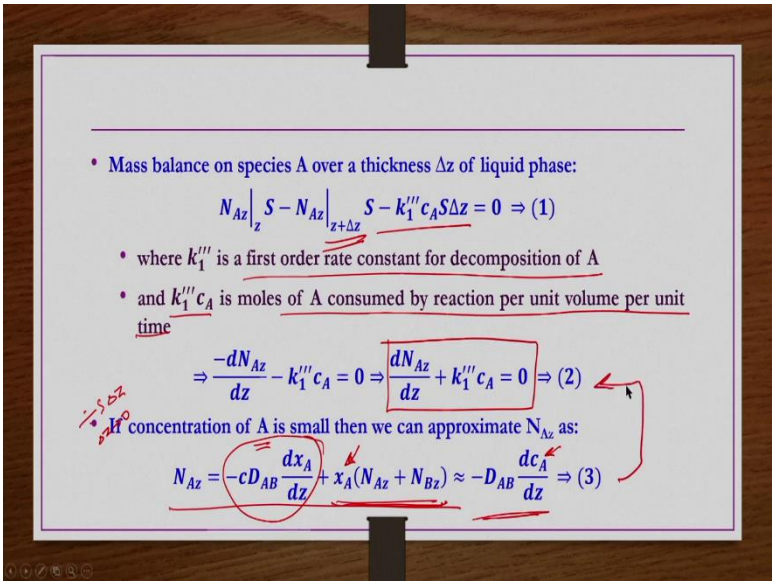

Now, mass balance and species A over thickness Δz of liquid face if you right $N_{Az}|_z S - N_{Az}|_{z+\Delta z} S -$ rate of disappearance of A that is $k_1''' c_A S \Delta z = 0$. Now, this equation here again if you see k_1''' is first order rate constant for decompression of A whereas, $k_1''' c_A$ is the moles of A consumed by the reaction per unit volume per unit time right.

Now, here also what we do? Divided by $S\Delta z$ and Δz tends to 0 limiting conditions you take $\frac{-dN_{Az}}{dz} - k_1''' c_A = 0$ that is $\frac{dN_{Az}}{dz} + k_1''' c_A = 0$ right. This we can solve to get the concentration profile, but that we can do only when you know the N_{Az} expression.

So, if concentration A is small then we can approximate this combined flux N_{Az} as simply $-D_{AB}\frac{dc_A}{dz}$. If x_A is very small assuming the concentration of A is very small right within the defusing a within the liquid system because we are saying that only a small amount is diffusing.

So, then this contribution would be very small compared to this contribution. So, we are not considering this part second part in the right hand side we are taking only this part. So, c x_A we can write as c_A. So, $-D_{AB}\frac{dc_A}{dz}$ that is nothing but N_{Az}.

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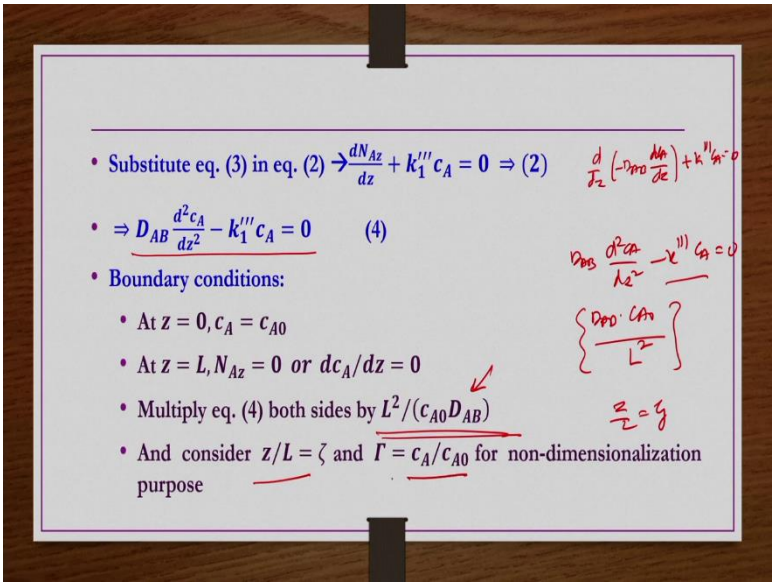

So, that if you substitute here in this equation, what we have? $\frac{d}{dz}\left(-D_{AB}\frac{dc_A}{dz}\right) + k_1''' c_A = 0$. So, that $-D_{AB}\frac{d^2c_A}{dz^2} + k_1''' c_A = 0$. So, that we can write $D_{AB}\frac{d^2c_A}{dz^2} - k_1''' c_A = 0$. This equation we have to solve to get the concentration profile.

Boundary conditions we have already seen at z = 0 c_A is nothing but c_{A0} at z = L $\frac{dc_A}{dz} = 0$ or if $\frac{dc_A}{dz} = 0$; that means, N_{Az} is 0. So, now this equation we are multiplying both sides by

$L^2/(c_{A0}\, D_{AB})$ why are we doing? We are trying to non-dimensionalize equation. So, how this $L^2/(c_{A0}\, D_{AB})$ is coming? So, because this equation $D_{AB}\frac{d^2c_A}{dz^2} - k_1'''c_A = 0$.

So, D_{AB} this c_A in place of c_A we can have c_{A0} and then in place of z we can have L^2. So, this particular term is having the units of this one right. So, then inverse of this one if you multiply, then this particular first term would become dimensionless same would be true for this one also right.

So, then if you have that one what we will have? Non-dimensionalized parameters we have to find. z/L we will be writing as ζ and then $\frac{c_A}{c_{A0}}$ we will be writing Γ alright. So, that is what we are doing now.

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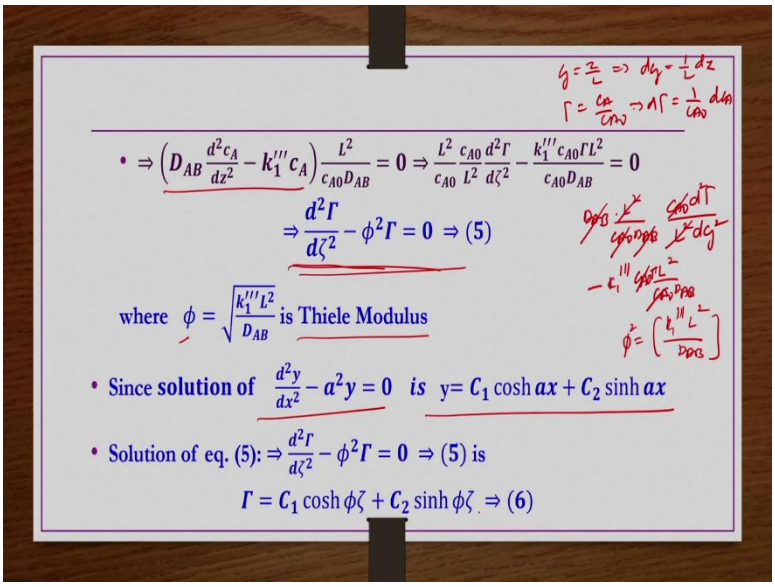

So, this is the equation, we are multiplying by $\frac{L^2}{c_{A0}D_{AB}}$. So, this what we do? We just do it once again so, that you can understand clearly. So, this $\zeta = \frac{z}{L}$ so; that means, $d\zeta = \frac{1}{L}dz$ and then $\Gamma = \frac{c_A}{c_{A0}}$; that means, $d\Gamma = \frac{1}{c_{A0}}dc_A$ that we are applying.

So, now, here first we have first term D_{AB} we are having and then multiplying by $\frac{L^2}{c_{A0}D_{AB}}$ and then in place of this d^2 is c_A we have $c_{A0}\, d^2\Gamma$ divided by in place of dz^2 we have L^2

$d\zeta^2 - k_1''' c_A \frac{L^2}{c_{A0} D_{AB}}$ this is the other time other term. So, then what we have? This c_A what we can write? In place of c_A we can write $c_{A0}\Gamma$.

So, there is this c_{A0} this c_{A0} is cancelled out ok. So, $\frac{\Gamma L^2}{D_{AB}}$ is here, now here this D_{AB} this D_{AB} is cancelled out, this c_{A0} this c_{A0} cancelled out, $L^2 L^2$ cancelled out. So, then what we have? We have only $d\Gamma$ we have only $\frac{d^2\Gamma}{d\zeta^2} -$ this term what is having? $\frac{k_1''' L^2}{D_{AB}}$ is there.

So, this term we are calling ϕ^2. So, ϕ^2 Γ we are having. So, this ϕ is nothing, but $\sqrt{\frac{k_1''' L^2}{D_{AB}}}$ under root or $\phi^2 =$ this one. So, this is nothing but Thiele modulus right. So, this equation we have to get the solution, this is this equation 5 is in this form; $\frac{d^2y}{dx^2} - a^2 y = 0$ whose solution is $y = C_1 \cosh ax + C_2 \sinh ax$. So, for this equation 5 similarly solution would be $\Gamma = C_1 \cosh \phi\zeta + C_2 \sinh \phi\zeta$ right.

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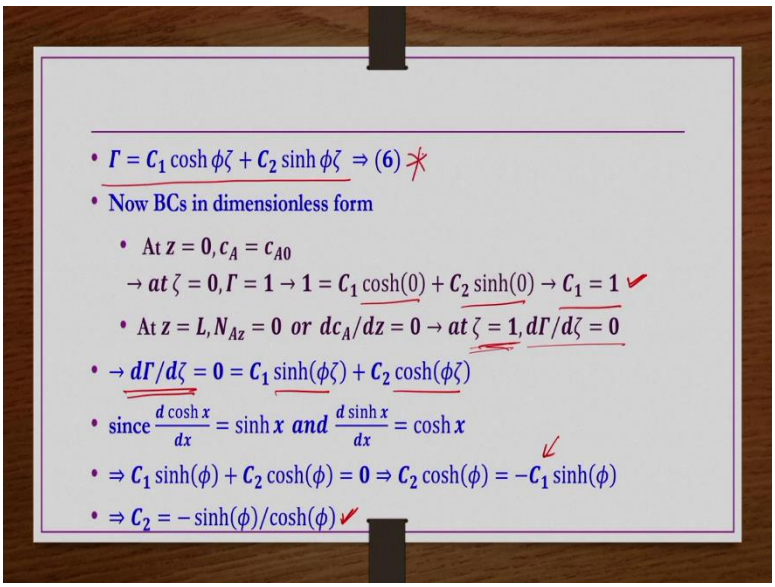

The same solution is written here. So, now, actually this is nothing but the concentration profile in dimensionless coordinates, then some boundary conditions also we have to write in dimensionless form. So, at z = 0 $c_A = c_{A0}$, if z = 0 then $\zeta = 0$ and then if $c_A = c_{A0}$; that means, $\Gamma = 1$ so; that means, $1 = C_1 \cosh 0 + C_2 \sinh 0$; that means, $\sinh 0$ is 0 and then $\cosh 0$ is 1.

So, C_1 is 1 right and then at z = L, $N_{Az} = 0$ or $\frac{dc_A}{dz} = 0$. If n is if z = l; that means, ζ = 1 and then if $\frac{dc_A}{dz} = 0$; that means, $\frac{d\Gamma}{d\zeta} = 0$ and dimensionless form; that means you have to get what is this? $\frac{d\Gamma}{d\zeta}$.

So, $\frac{d}{dx}\cosh x$ is $\sinh x$ similarly $\frac{d}{dx}\sinh x$ is $\cosh x$. So, $C_1 \sinh \phi\zeta + C_2 \cosh \phi\zeta$. Now ζ = 1 you have to substitute here because it is true with ζ = 1 right. So, that is $C_1 \sinh \phi + \cosh \phi = 0$; that means, $C_2 = \cosh \phi = -C_1 \sinh \phi$, C_1 is 1.

So, that is C_2 is nothing but $-\tanh \phi$ ok. So, now, you have C_2 and then C_1 those things if you substitute in this equation we have the final solution.

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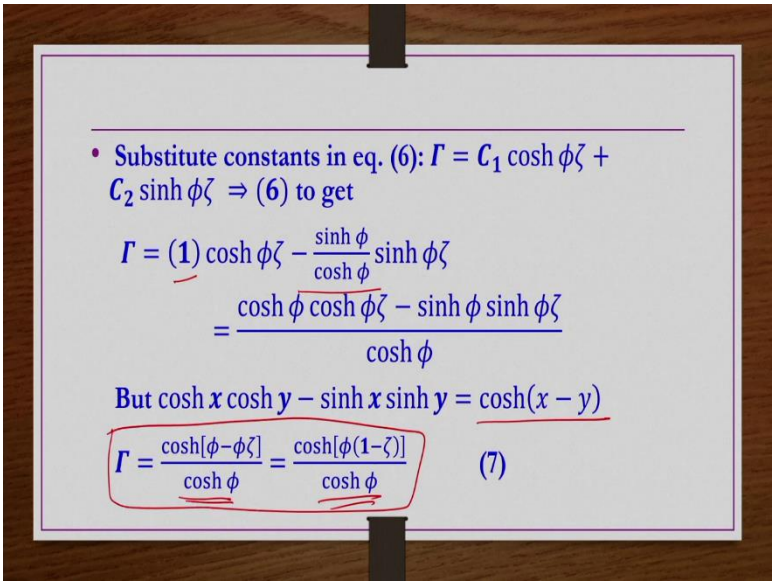

So, C_1 is 1, C_2 is $-\frac{\sinh \phi}{\cosh \phi}$, we are not writing as a tan hyperbolic of ϕ for some reason. So, then what we do? Now you do the LCM. So, then you have this one. So, now, this is $\cosh x \cosh y - \sinh x \sinh y$cos x is nothing but $\cosh(x - y)$ form right.

That means, you get Γ = $\frac{\cosh[\phi - \phi\zeta]}{\cosh \phi}$ is as it is. So, that is $\frac{\cosh[\phi(1-\zeta)]}{\cosh \phi}$ is the concentration profile in non-dimensional form right.

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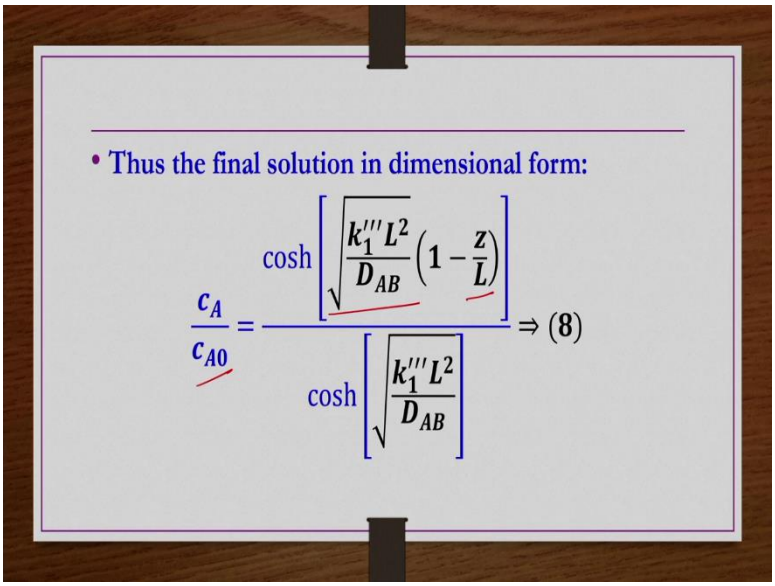

Thus the final solution in dimensional form would be in place of phi, you write $\sqrt{\frac{k_1''' L^2}{D_{AB}}}$ in place of ζ you write z/L and in place of Γyou write $\frac{c_A}{c_{A0}}$. So, $\frac{c_A}{c_{A0}} = \frac{cosh\left[\sqrt{\frac{k_1''' L^2}{D_{AB}}}\left(1-\frac{z}{L}\right)\right]}{\cosh\left[\sqrt{\frac{k_1''' L^2}{D_{AB}}}\right]}$.

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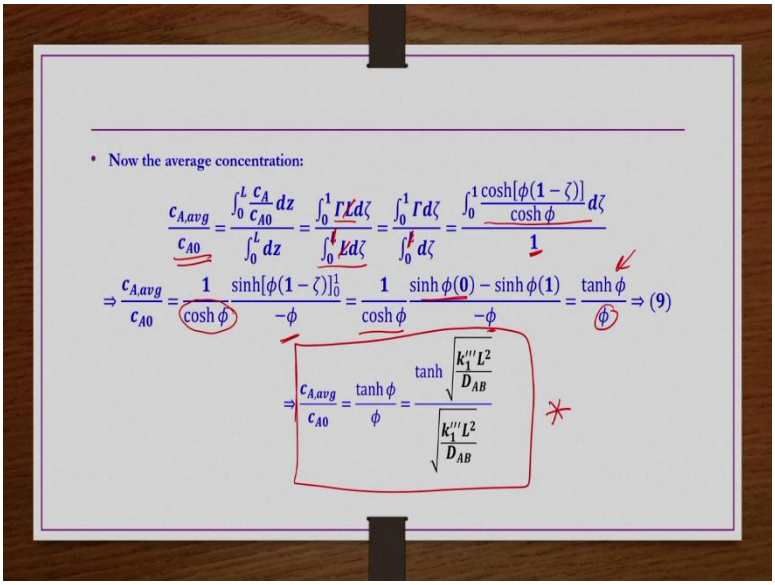

So, now, we wanted to find out the average concentration. So, this average concentration if you do $\frac{c_{A,avg}}{c_{A0}}$ it will be $\frac{\int_0^L \frac{c_A}{c_{A0}} dz}{\int_0^L dz}$. If the same thing if you write dimensionless form $\frac{c_A}{c_{A0}}$ is Γdz is nothing but L dζ integral limits would be 0 to L. If L = 1; that means, ζ = 1 right similarly here also we can write the same thing the limit is 1 here also the limit is 1. So, this L this L you can cancel out right.

So, now you do the integration of this particular term divided by integration of the d ζ is one limits 1 to 0. So, it is 1 and then integration of cosh x is nothing sinh x. So, that is $\frac{\sinh[\phi(1-\zeta)]_0^1}{-\phi}$ whatever divided by cosh ϕ is the constant as it is.

So, limits 0 to 1 if you substitute $\sinh\phi(0) - \sinh\phi(1)$. So, this term is 0 and then the $\frac{\sinh\phi}{\cosh\phi}$ is $\tanh\phi$ and then this divided by ϕ is as it is. So, then $\frac{c_{A,avg}}{c_{A0}}$ is nothing but $\frac{\tanh\phi}{\phi}$. So, if you replace $\frac{\phi}{\sqrt{\frac{k_1''' L^2}{D_{AB}}}}$ this is what you have the equation, this is the equation for the average you know concentration right.

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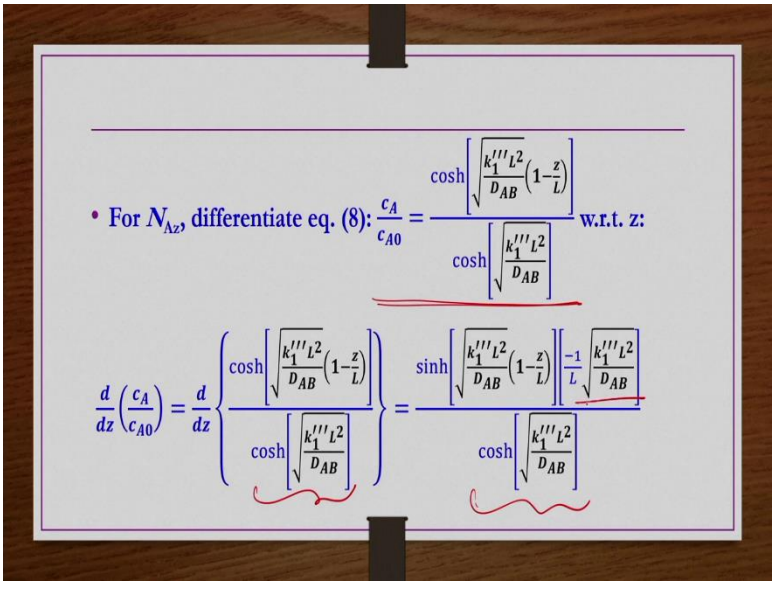

Now, the flux if you wanted to find out this whatever the $\frac{c_A}{c_{A0}}$ expression that we have in the previous slide. So, that we have to you know differentiate with respect to z. So, then

$\frac{d}{dz}\frac{c_A}{c_{A0}}$ is $\frac{1}{c_{A0}}\frac{dc_A}{dz}$ and then $\frac{d}{dz}$ of this entire thing this is anyway constant. So, this is as it is and then $\frac{d}{dx}\cosh x$ is nothing but $\sinh x$.

So, $\sin h\left[\sqrt{\frac{k_1''' L^2}{D_{AB}}}\left(1-\frac{z}{L}\right)\right]$ and then differentiation of a $-\frac{z}{L}\sqrt{\frac{k_1''' L^2}{D_{AB}}}$ is nothing but $-\frac{1}{L}\sqrt{\frac{k_1''' L^2}{D_{AB}}}$.

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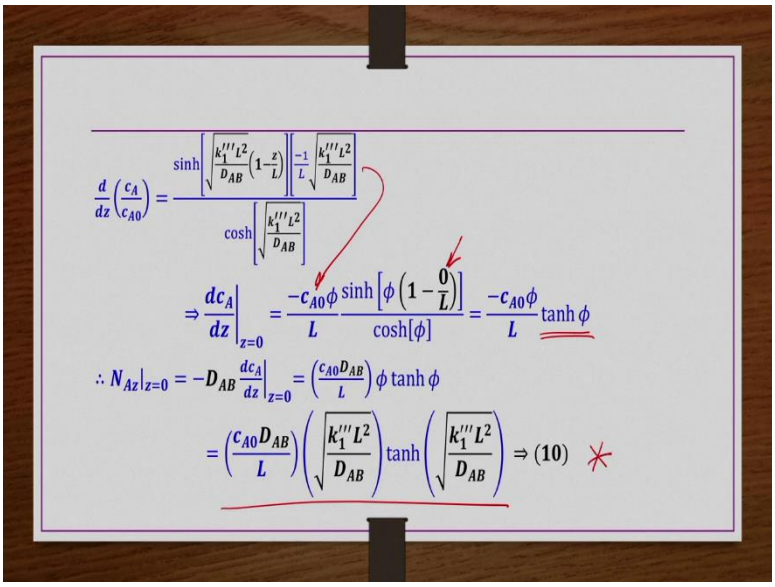

So, the c_{A0} in the left hand side that we take to the right hand side and then wherever this $\sqrt{\frac{k_1''' L^2}{D_{AB}}}$ is there that we are writing phi. So, $\frac{-c_{A0}\phi}{L}$ is this term,$\sinh\left[\phi\left(1-\frac{0}{L}\right)\right]$ because this we are now doing it z = 0, the flux we are calculating at z = 0 location that is at the interface gas liquid interface.

So, if you substitute z = 0 this sinh ϕ you get. So, $\frac{sinh\,\phi}{\cosh\phi}$ is $tanh\,\phi$ right. So, this term both sides if you multiply by $-D_{AB}$, you will be getting the flux because $N_{Az} = D_{AB}\frac{dc_A}{dz}$ at z = 0 $= \left(\frac{c_{A0}D_{AB}}{L}\right)\phi\tanh\phi$.

That if you write in terms of you know dimensional coordinates, so, then we have $\left(\frac{c_{A0}D_{AB}}{L^2}\right)\sqrt{\frac{k_1''' L^2}{D_{AB}}}\tanh\sqrt{\frac{k_1''' L^2}{D_{AB}}}$ as the flux that we suppose to find out.

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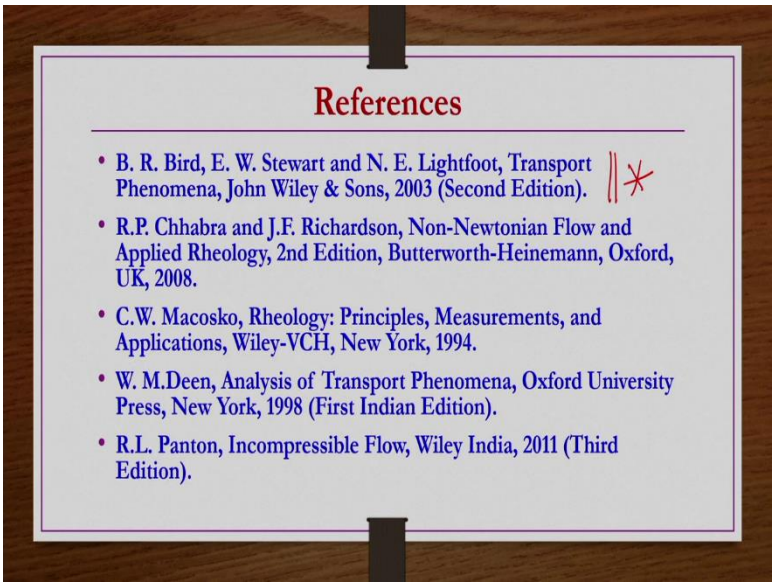

The references for this lecture are provided here. Of course, both the problems you can find out in this book Transport Phenomena by Bird, Stewart and Lightfoot.

Thank you.