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Structural Dynamics
Week3: Module 04

Resonant Frequency
&
Half Power Band Width

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Welcome back to structural dynamics class, so in this module we will study.

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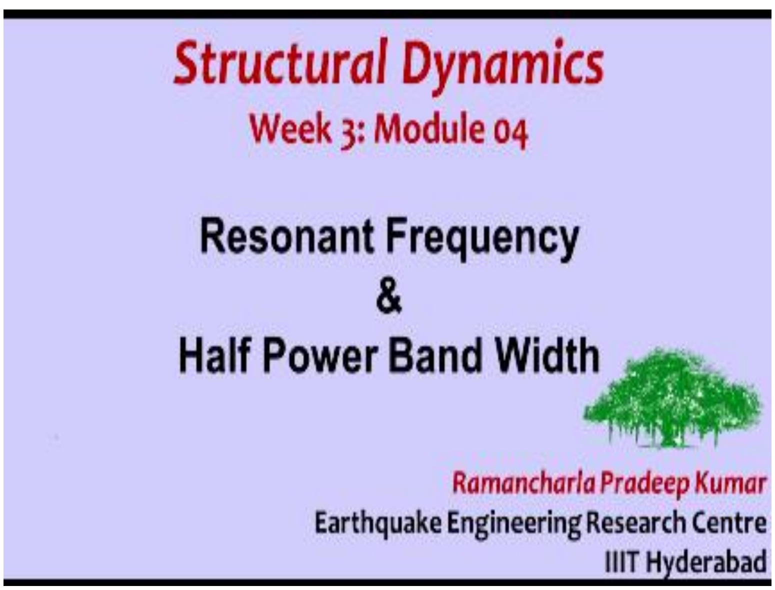

Resonant frequency and half power band width, so first of all let us discuss what is resonant frequency. So resonant frequency is like when resonant takes place so if we apply a static load on any system or any member it deflects but slowly if we increase the frequency of the load application of the load then the deformation increases so if the frequency of the load application that is frequency of the load are you called it as forcing frequency.

If it is matching with the natural frequency of the system large amplitude of displacement velocity accelerations take place. But like what all the special conditions in this one, when is the acceleration maximum, when is velocity maximum, when is displacement maximum that we are going to study in this class and also like how this maximum accelerations can be controlled displacements and velocities can be control that we are going discuss in the class.

In addition to it like if we have a R_d curve, R_d curve means dynamic amplification factor that curve R_d is a function of R that is frequency ratio. What are the properties in that curve that is what that is also we are going to study.

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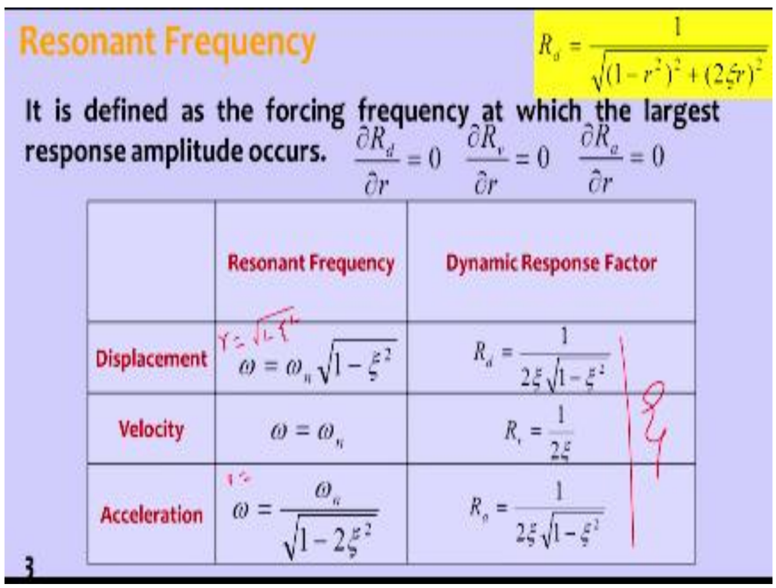

Now if you a R_d curve it looks like this so R_d formula that is $1/\sqrt{(1-r^2)^2+(2\xi r)^2}$ this is R_d they represents R_d this is a function of R and ξ. Now it is defined as the forcing frequency at which the largest response amplitude occurs, resonant frequency. So largest response amplitude, so we have three responses displacement has a response, velocity has a response and acceleration has response. Now let us look at displacement responses when it is becoming maximum so when it is going to resonating frequency.

So for that we need to differentiate this with R because it is a function of R we need to differentiate it with R, so $\delta R_d/\delta r=0$ so if we maximize or minimize at first derivative should be 0 to get the maximum value, so we are differentiating that and if we differentiated and equated to 0 so what we get out of it is $\omega=\sqrt{1-\xi^2}$ in a way $r=\sqrt{1-\xi^2}$ so this is what we will get from the derivative of that and if we equate it to 0 we get $r=\sqrt{1-\xi^2}$ so that means $\omega=\omega_n\sqrt{1-\xi^2}$ so that is a resonating frequency for displacement.

Now let us similarly, let us look at for velocity so what is that resonating frequency for velocity. So if we differentiate it with R it r so $\delta R_v/\delta r$ so $\delta R_v/\delta r$ and equate it to 0, so if we equate it to 0 ω becomes equal to ω_n so that means r=1 value we will get, r=1 so usually we believe that resonating condition occurs when $\omega=\omega_n$ so that resonating condition occurs for velocity not for displacement or for acceleration so usually when we talk resonating condition when frequency matches that is $\omega=\omega_n$ for velocity that is.

And in some special cases, special case means when no damping is there all three becomes equal, so as you can see the third one $\delta R_a/\delta r$ if it is equal to 0 so that means what we have first derivative of R_a function if we are making it equal to 0 what relation we will get so that is $\omega=1/\sqrt{\omega}/\omega r$ or $r=\sqrt{1-\xi^2}$ as you can see $\omega=\omega_n\sqrt{1-\xi^2}$ so in displacement resonating condition ξ is there in acceleration resonating condition also ξ is there and velocity resonating condition ξ is like insensitive.

So that means $\omega=\omega_n$ that means r=1, now if we substitute these values in the original equation so what we will get we will look at it but before that let us look at when $\xi=1$ that means when 0 damping case $\omega=\omega_n$ condition will get velocity anyway $\omega=\omega_n$ and here if it is 0 again $\omega=\omega_n$ so that means under no damping condition all three are same. Now what is the value of this dynamic amplification factor at resonating frequency as you can see $R_d=1/2\xi\sqrt{1-\xi^2}$ that means R_d can be controlled with damping at resonating condition.

And here, at velocity can be controlled at resonating condition by damping and acceleration can be controlled by $1/2\xi\sqrt{1-\xi^2}$ so that means what at resonating conditions damping controls the amplitude of vibration. So in real systems damping will be present, damping 0 condition is a ideal condition.

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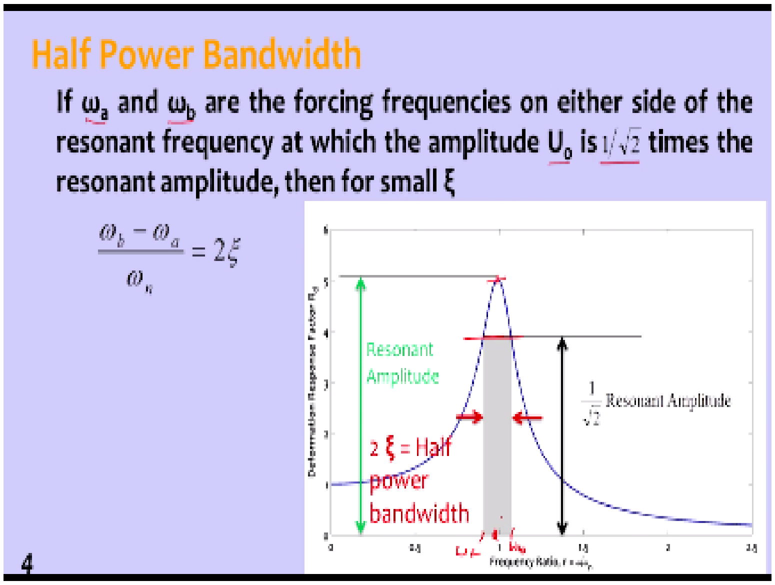

Now let us look at the R_d curve and what properties it has, so let us look at this statement that is when ω_a and ω_b are the forcing frequencies on either side of the resonating frequency at which amplitude $U_0=1/\sqrt{2}$ times of resonating amplitude, then for all small values of ξ there is a relationship we will let discuss that, so this is a R_d curve so dynamic amplification factor for R_d if you look at this one the curve, okay.

The definition says that, ω_a and ω_b frequencies on either sides so this is ω_a and this is ω_b frequencies on either side of resonating frequency, resonating frequency is this one so somewhere here, okay somewhere in the middle. At which amplitude U_0 so this is U_0 and that is $1/\sqrt{2}$ times of this resonating amplitude, okay resonating amplitude is $1/\sqrt{2}$ times of resonating amplitude then the relationship will be $\omega_b-\omega_a/\omega_n=2\xi$ that means what it is this area you can look at it, okay. So this is like $\omega_b-\omega_a/\omega_n.2\xi$ so how this equation we are getting let us look at it.

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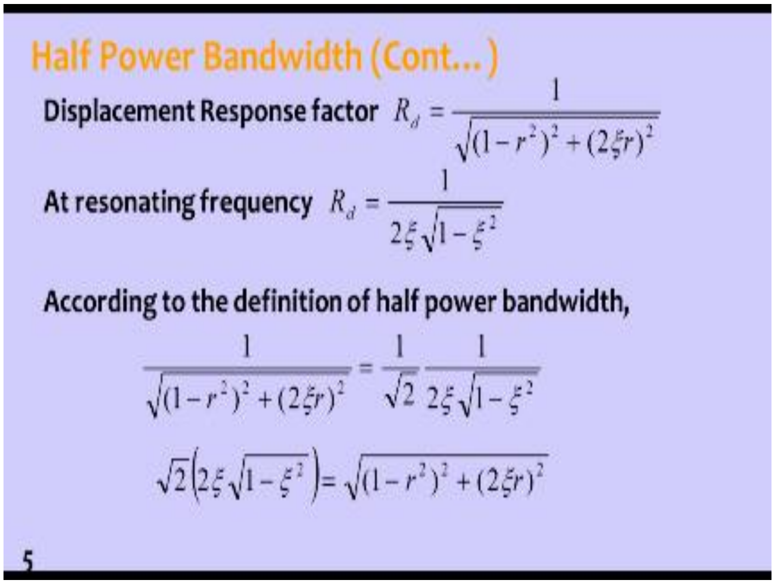

So displacement response factor R_d if we already know that $1/\sqrt{(1-r^2)^2+(2\xi r)^2}$ now at resonating condition for displacement is $R_d=1/\sqrt{}$ sorry, $1/2\xi\sqrt{1-\xi^2}$ so now according to the definition this R_d should be equal to $1/\sqrt{2}$ times of resonating amplitude, so this is resonating amplitude $1/\sqrt{2}=R_d$ so that is what is given R_d is this function.

So if you look at take this point so this point is equal to $1/\sqrt{2}$ times resonating amplitude so we know resonating amplitude, this is resonating amplitude. So $1/\sqrt{2}$ times of resonating amplitude is equal to this function, so the function value we know. So if we equate this one and work it out and simplify.

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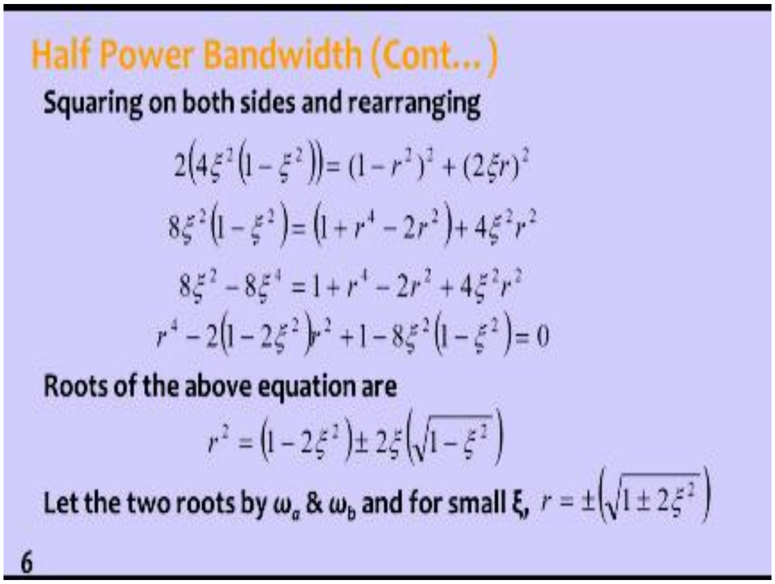

So what happens is finally we will get r^2 as $1-2\xi^2 \pm 2\xi(\sqrt{1-\xi^2})$ so if we take the roots of this one we will get that $r= \pm\sqrt{1 \pm 2\xi^2}$ so by expanding in Taylor series this one because bracket to the power of 1/2 that is square root, so if we expand it, it will becomes r approximately equal to $1\pm\xi$ r means ω/ω_n is approximately equal to $1\pm\xi$, so if we expand it further if we simply it then two roots are there $\omega_b-\omega_a/\omega_n=2$ times of ξ so this is what is the definition of that as you can you see that.

If ω_a and ω_b are the forcing frequencies on the either side of the resonating frequency at which the amplitude is $U_0=1/\sqrt{2}$ times of resonating amplitude, so at two sides this one and this one so this frequencies are ω_a and ω_b so they are related to this curve or related to damping in this form, so if we know this R_d curve we can find out the damping in the system so how we can do is we know the resonating amplitude and then just mark the value of $1/\sqrt{2}$ of that peak value you take and $1/\sqrt{2}$ of that you mark the value here so two things so let me explain once again here.

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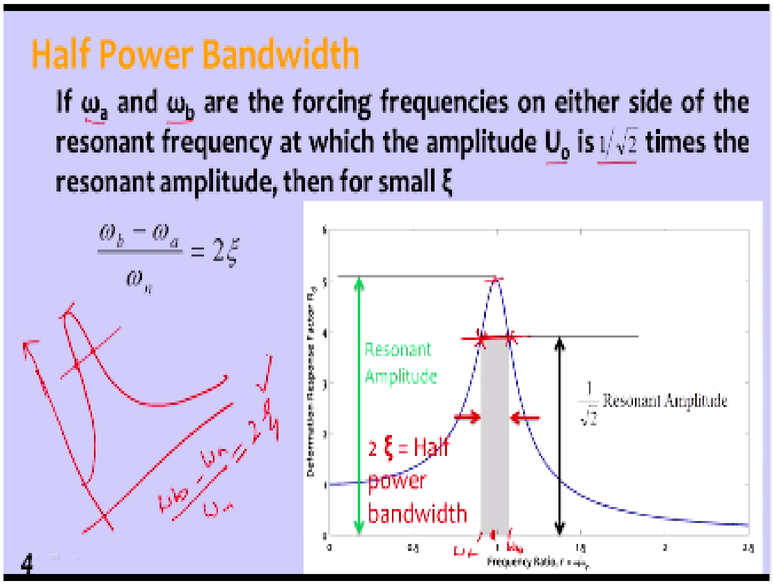

So we have this resonant response curve, so this is peak okay, so take that peak value and divide by 1/√2 so then 1/√2 you mark a line this value is the, this is value is a, so 1 will be ω_a and ω_b is this one ω_a is this one so ω_{b-} ω_a/ ω_n is known value that is equal to 2 times ξ, so with this curve also we can find out what is the damping present in the system.

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So in this class we have studied about two things one is what is the resonating condition for displacement resonating condition for velocity resonating condition for acceleration, so this if we usually we will feel ω forcing frequency when it is equal to natural frequency resonating condition occurs that is true but it is only for velocity under any given damping. But for displacement and for acceleration it is slightly away from the natural frequency because of the presence of damping.

Damping is not present then all three resonating conditions are same, $\omega=\omega_n$ it resonating condition occurs for velocity displacement as well as acceleration then we have derived what is the maximum value of R_d R_a and R_v at the resonating condition that we have worked out, and next is the half power band width we have worked out. So the relationship between damping and the frequency so if given R_d curve how can we find out the damping present in the system so it is very simple first we have to locate the peak value, peak amplitude in that R_d value and then divide that by $\sqrt{2}$ then we get two points on the R_d curve.

So corresponding frequency if we measure that will be ω_b and ω_a so ω_b- ω_a/ ω_n=2 times of damping, so like using this equation we can measure the damping present in the system.

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