Introduction to LASER

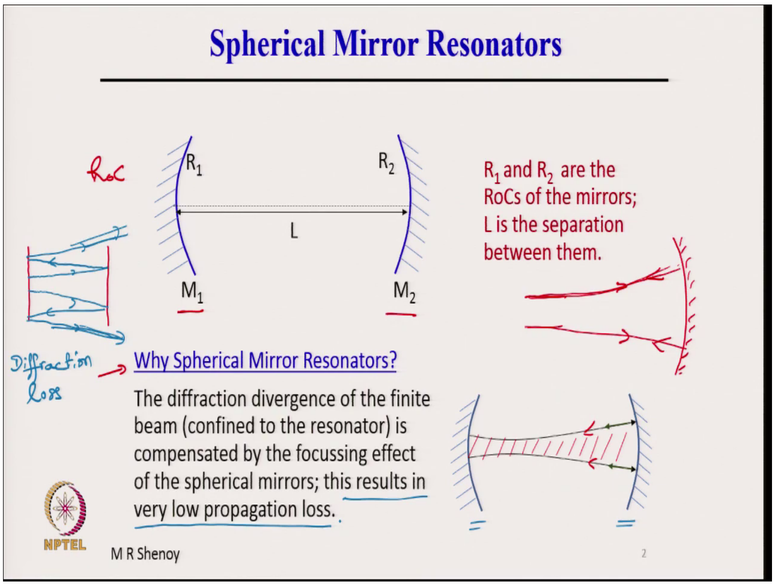
Prof. M. R. Shenoy
Department of Physics
Indian Institute of Technology, Delhi

Lecture - 16
Spherical Mirror Resonators

Welcome to this MOOC on LASERS. In the last couple of lectures, we discussed the basic properties of an optical resonator namely, the resonance frequencies, free spectral range and the spectral response of the resonator.

We also looked at various parameters which characterize the loss in the resonator because loss in the resonator is very important. It determines, as we have seen, the line width of the cavity resonances. So, today we will take up Spherical Mirror Resonators, spherical mirror resonators.

(Refer Slide Time: 01:01)



A spherical mirror resonator comprises of two spherical mirrors here M 1 and M 2 are two spherical mirrors of Radius of Curvature RoC, RoC radius of curvature R 1 and R 2 and the mirror separated by a distance L. So, R 1 and R 2 are the RoCs of the mirrors, L is the separation between them.

Now, why spherical mirror resonators? The diffraction divergence of a finite beam which is confined to the resonator is always there is a finite diffraction divergence which is compensated by the focusing effect of the spherical mirrors. So, what it means is whenever you have a finite beam of a finite width, then as the beam propagates there is a inevitable diffraction.

Every finite beam diffracts and unless you have a mirror for example, if we have a concave mirror here which focuses; so, this is diffracting now if the concave mirror focuses it back, then it compensates for the diffraction divergence. So, that is what is shown here in the spherical mirror resonator.

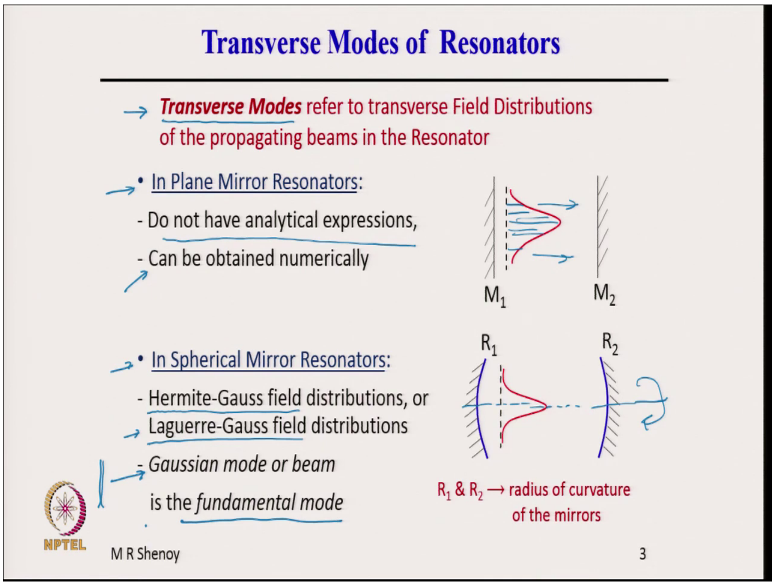
A finite beam which is confined to the resonator as it propagates from one mirror to the other mirror it spreads, it diverges. But, the other end being a concave mirror it has a focusing effect and then it sends back it focuses it back and this going back and forth of the beam continues with a certain fixed beam dimensions.

This is not the case to appreciate this; this is not the case if we consider plane mirror resonators. As we know in plane mirror resonators if a finite beam starts from this end of the mirror let me just change the color alright. If the finite beam starts from this end of the mirror then it will diffract as it goes to the other end it widens.

Now, when it propagates back then it further widens. So, it further widens like this and then when it propagates back it further widens and now you see that a part of the beam is outside the mirror. So, this is the mirror extent of the mirror outside the extent of the mirror and, this part forms loss from the resonator and this is called the diffraction loss.

In a plane mirror resonator, there is diffraction divergence and therefore, there is diffraction loss. So, diffraction loss this is called diffraction loss. In a spherical mirror resonator because of the focusing effect of the concave mirrors, we have compensation for the diffraction divergence and therefore, it results in very low propagation loss. This is why we go for spherical mirror resonators.

(Refer Slide Time: 04:29)



Further the transverse modes of a resonator. We will discuss this transverse modes in detail, but right now transverse modes refer to transverse field distributions. We have seen so far longitudinal modes, which are the resonance frequencies, which are determined by the length of the resonate.

Now, transverse modes represent the field distributions of the propagation beam, the allowed field distributions of the propagation propagating beam. As the beam goes back and forth there are certain specific field distributions which will propagate as they go back and forth without change in the field distribution, they are called transverse modes. We will see this in more detail a little later.

So, transverse modes referred to transverse field distributions of the propagating beams in the resonator. In a plane mirror resonator because there are finite diffraction losses, we do not have an analytical expression for the field distribution, which is going back and forth. I have qualitatively shown a field distribution here. So, this is the field distribution.

So, what is plotted is amplitude of the electric field of the beam in the transverse direction. So, the beam propagates back and forth. So, the beam goes back and forth, there are certain field distributions which then subsequently evolve and remain steady, but we do not have any analytical expressions or the field distributions inside a plane mirror resonator, although we can obtain them numerically. So, numerical fields can be obtained for plane mirror resonators.

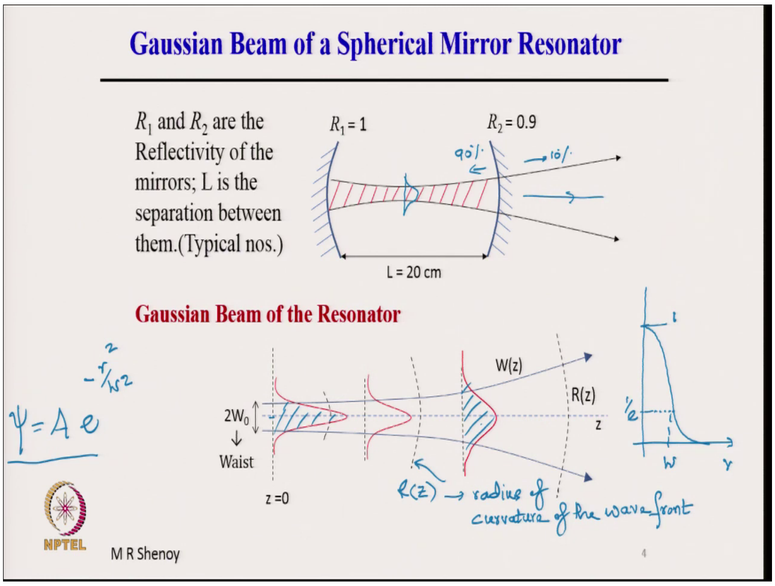
In the case of spherical mirror resonator the Hermite-Gauss field distributions form modes of the resonator and if the resonator has a cylindrical symmetry that is symmetric about a cylindrical symmetry then Laguerre-Gauss field distributions also form modes of the resonator. So, these are family of modes, we will discuss this a little later.

But, the main point is both this family Hermite-Gauss which comprises of Hermite polynomials and Gaussian field; a product of Hermite polynomials and Gaussian field, a product of Laguerre polynomials and Gaussian field gives the field distributions. The fundamental mode in both these families is the Gaussian mode or the Gaussian beam of the resonator.

The Gaussian mode of the resonator is the fundamental member of these two families and in many applications we would like to have the Gaussian mode of the resonator and we will discuss this Gaussian mode subsequently with little bit more detail. Now, the two primary reasons is one the diffraction loss is very high and in the case of plane mirror resonators whereas, in spherical mirror resonators the diffraction loss is low.

Therefore, low propagation loss we are interested in low loss because we would like to have very sharp cavity resonances. The second reason is that the field distributions have analytical expressions and the fundamental mode is a Gaussian mode in a spherical mirror resonator.

(Refer Slide Time: 08:21)



So, these are the two primary reasons why we go for spherical mirror resonators. The Gaussian beam of a spherical mirror resonator is again illustrated here. So, what is shown is a spherical mirror resonator one of the mirrors is assumed 100 percent reflecting which means the reflectivity R 1. So, R 1 here is the reflectivity; R 1 is equal to 1, which means 100 percent reflection and R 2 is 0.9 which means 90 percent reflection which means 10 percent is the transmission.

So, 10 percent transmission and 90 percent back reflection. So, that is the meaning of R 2 is equal to 0.9. So, the beam is building up inside and every time 10 percent of the energy goes out in the form of an output beam. The field distribution the transverse field distribution across this mode is Gaussian. So, the Gaussian beam of a resonator is illustrated here it has a

Gaussian field distribution Gaussian as suppose you are familiar e to the power minus r square by W square.

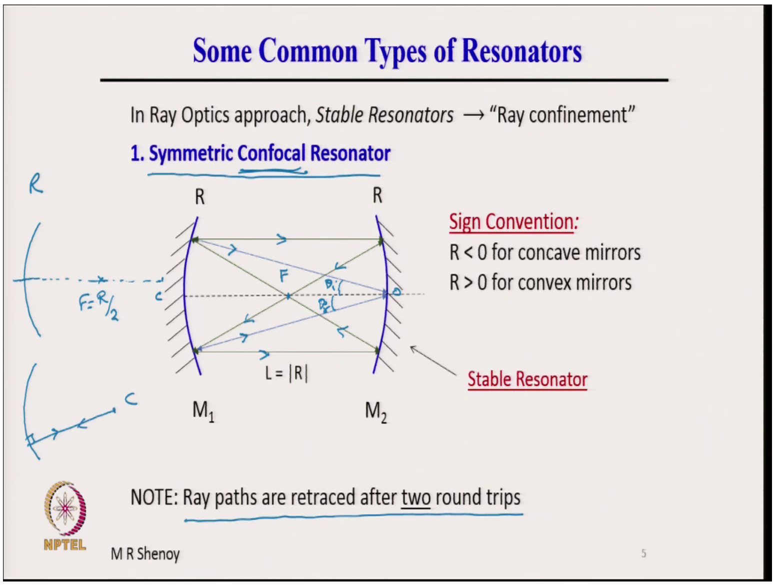
So, this is and there can be an amplitude factor A. So, if you have a field psi is equal to A into e to the power of minus r square by W square, this is representation of the Gaussian. It is a bell shaped curve because at r is equal to 0 this function has maximum. So, as you can see r is equal to 0 is here. So, it is maximum on the axis and the beam is propagating.

As the beam propagates there is a finite spread of course, and that is why the beam is spreading and therefore, the peak amplitude is decreasing, but the energy content which is given by the area under this curve here is the same whether it is this one or you consider this the total area under the curve remains same, but the beam spreads W of z represents the width of the beam.

The width is defined. So, if we show in terms of plot the function as a function of R, then it is maximum on the axis and let say this is 1, then where it drops down to 1 by e. So, at R is equal to W, please see at R is equal to W this term becomes e to the power minus 1 or 1 by e. So, where the field drops down to 1 by e of it is maximum value is called the spot size, this is W. So, that is W of z because the beam is spreading W is changing with z, z is the propagation direction.

The second important characteristic of the Gaussian beam is the radius of curvature R of z of the wave front. So, this is radius of curvature of the wave front. So, we will discuss these in more detail radius of curvature of the wave front, wave front is surface of constant phase wave front. So, both R of z and W of z evolve as the beam propagates in the z-direction we will discuss more details about the Gaussian beam as we go further.

(Refer Slide Time: 12:07)



Now, some of the common types of spherical mirror resonator. So, we will follow in this ray optics approach. So, here the beam is confined means when a ray is confined to the resonator, then we call that the resonator is stable or the beam is confined to the resonator. So, this will again become clear.

So, what is shown I have taken some common resonators: 1st – Symmetric Confocal Resonator. What it means is it comprises of two identical mirrors of radius of curvature R mirror M 1 and M 2 separated by a distance which is also equal to radius of curvature. Now, if we have a mirror of radius of curvature R, then the focal point is at R by 2. So, this is the focal point. It is F is equal to R by 2.

The focus, the distance from the pole to the focal point F is equal to R by 2 and therefore, if the total separation is R. And, if this is the midpoint, then both the mirrors have their focus over lapping or that is why it is called confocal; confocal means focal point coinciding, hence the name confocal resonator.

So, this is F actually and therefore, in this type of a resonator if you have a parallel beam of light which travels like this so, let me show a parallel ray travelling towards the other mirror then by definition of focal point the ray will get reflected and pass through the focus here.

And the ray which is coming from the focus to the second mirror will be rendered parallel here. Now, again the parallel ray which reaches here passes through the focus and then it reaches the other mirror. In other words, we see that in two round trips ray paths are retraced after two round trips retraced. Please see we started a ray from here this parallel ray to the other mirror.

After one round trip; round trip means from this mirror back to this mirror it comes here and then from here again it goes parallel because it has come from the focal point. So, it has come from the focus therefore, it is rendered parallel. So, after one round trip the ray has reached here. Now, again it goes back and reaches the original point. In other words, in two round trips the ray retraces it is original point.

Another type of ray is also shown here for example, a ray which goes from here to the point O here; what is the distance? This is the radius of curvature and therefore, the ray which reaches here will be reflected satisfying the law of reflection that is theta 1 is equal so, theta i here is equal to theta r reflected angle. So, it comes here.

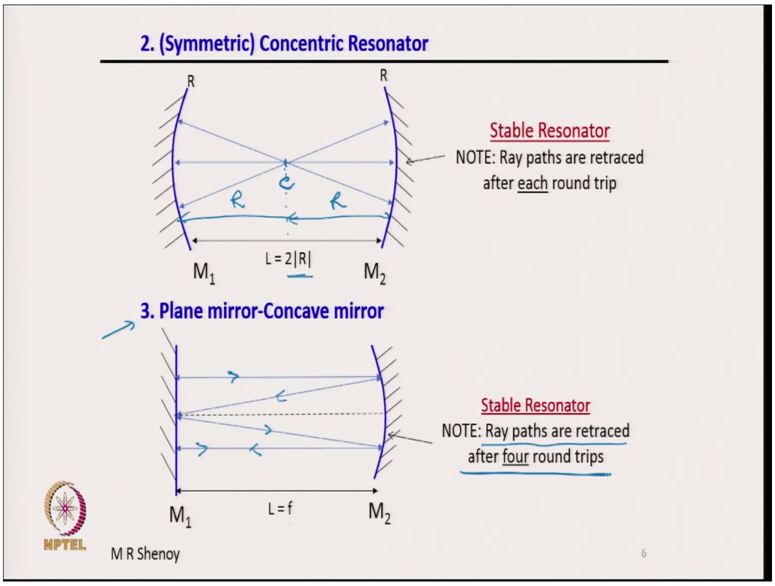
Now, any ray which is coming from the radius of curvature please see this is the focal point of this mirror, this is the point corresponding to the radius of curvature that is center of curvature C. So, this is F and there is a C, which is here center of curvature and a ray which comes from the center of curvature is always normal, it is normal to the mirror.

So, a ray which comes; so, if this is C, the center of curvature, then any ray which comes to the mirror will be at 90 degrees and therefore, it will be reflected back along the same path.

So, the ray which comes from here is reflected back along the same path here it satisfies the law of reflection and then it comes back here.

Again, note that after two round trips the ray has come back. So, if the ray has started from here, then it reaches here then comes down to this point, from here it retraces back. So, after two round trips the ray reaches, its original point. So, in both the ray paths which I have shown after two round trips the ray reproduces or retraces it is path. So, you can draw several types of rays. So, this is for a symmetric confocal resonator.

(Refer Slide Time: 17:09)



The 2nd resonator a common type of resonator shown is Symmetric Concentric Resonator. Now, as the name indicates here is C that is the center of curvature of this mirror as well as this mirror. They are coinciding, hence the name concentric. So, two spherical mirrors with a separation equal to twice the radius of curvature.

So, this is center of curvature therefore, up to this it is R. So, this is R and this is also R. So, the total separation is 2R. So, that is a concentric resonator. So, you can see the ray path. So, if a ray path starts from here passing through the center, it will be incident normally on the second mirror and reflected back.

So, any ray that you take you will see that in one round trip, the ray comes back to its original path original point and that is what it is written here. Ray paths are retraced after each round trip. Right now I am explaining this qualitatively from the physics that we know the school level physics that we know and school level optics that we know, ray optics that we know.

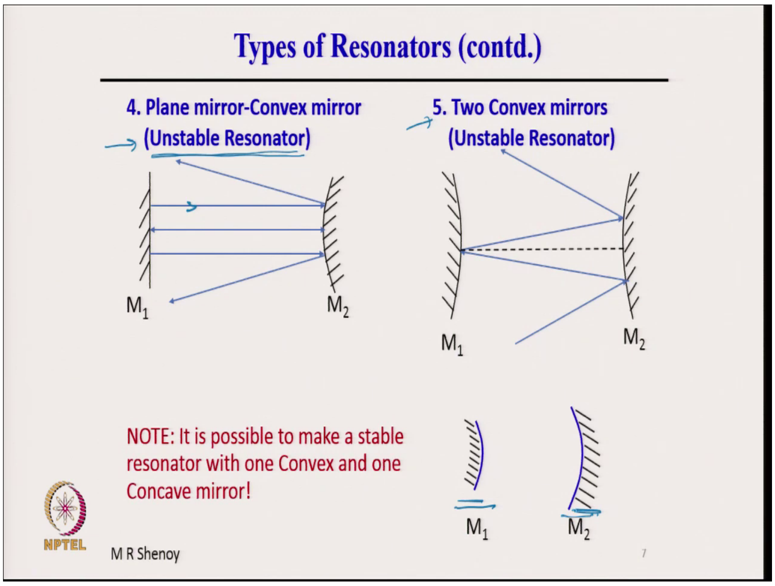
But, we will see, we will write mathematical expressions and the mathematical expressions will show that the rays will retrace their path after two round trips, or four round trips, or one round trip in different types of resonator ok. So next what is shown is, a plane mirror-concave mirror resonator.

We have one plane mirror and one concave mirror. So, note that the ray which travels parallel from the plane mirror at the separation here is equal to the focal length. This separation is focal length and therefore, the ray goes to the focus where it undergoes reflection by the plane mirror and then comes to the spherical mirror again.

Now, anything which comes from the focal point will be rendered parallel and what is incident parallel on the plane mirror will be reflected back. So, please see the ray starts from here a typical ray, it after one round trip it reaches here. It continues from here for the second round trip, it comes here and then it reaches here after two round trips. Round trip refers to from this mirror back to this mirror. The path need not be straight line path, the path can be anything.

Now, after the second round trip, now it again is reflected normally back to the second mirror and from there it is focused to the focal point, from there again it undergoes reflection and then rendered parallel. So, the ray which starts from here in this case you see that after four round trips it comes back to it is original point. So, the ray paths are retraced after four round trips. So, I have shown three different types of resonators.

(Refer Slide Time: 20:29)



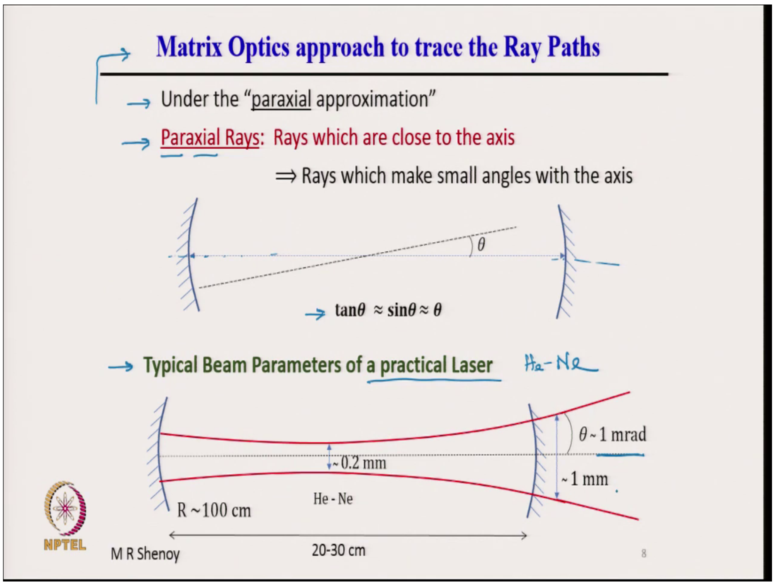
So, let us look at some other types of resonators where the ray does not come back. So, for example, if we have one of the mirrors as the plane mirror and then the other one as a convex mirror you will see that whatever ray that you consider it will not retrace. It will go because of the divergence given by the convex mirror any ray which is incident will finally, eventually it will go out of the resonator.

And, therefore, we say that such resonators are unstable resonators; unstable resonator here means we will work out the mathematical expressions for this as I said. But, right now unstable resonator means where rays cannot be confined optical rays are not confined to the resonator. And, stable resonators in simple term are those resonators where rays can get trapped. Once they are trapped they can build up inside the resonator, alright.

One more resonator which is shown here comprising of two convex mirrors and obviously, it is an unstable resonator because both the mirrors are diverging mirrors. So, naturally any ray will subsequently go out of the resonator. We will see later on that it is possible to have stable resonator with the one convex and one concave mirror.

With two concave mirrors obviously, we have discussed several concentric, confocal all are two concave mirror resonators. We have seen that two convex mirrors form unstable resonator, but if appropriate radius of curvatures are chosen we will show later that having one convex mirror and one concave mirror we can form stable resonators or the rays can remain confined for a long time, alright.

(Refer Slide Time: 22:45)



In this part to study the spherical mirror resonators, we will use matrix optics. So, matrix optics approach to trace ray paths. Now, matrix optics is valid under paraxial approximation. What is this paraxial approximation? Now, paraxial rays means rays, so, para-axial. So, it is close to the axis rays which are close to the axis that is the literal meaning of paraxial rays; that means, rays which make small angles with the axis.

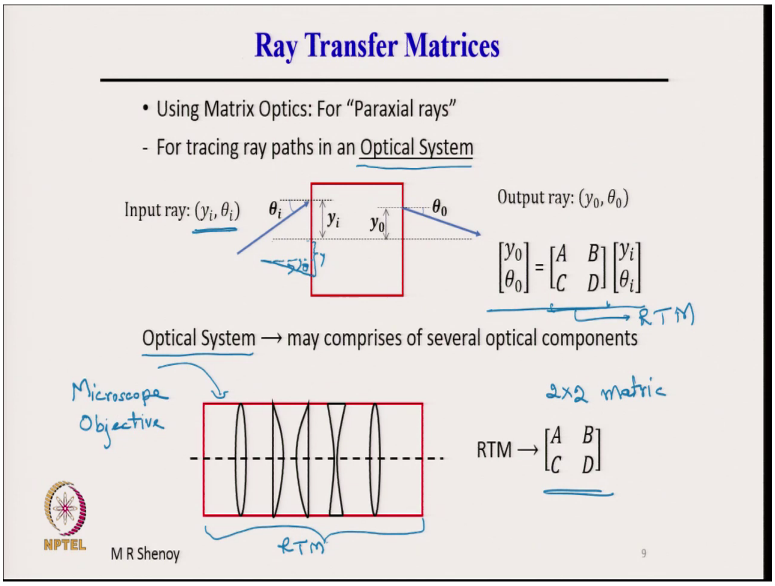
When the rays are travelling, so, this is the axis of the resonator then if the rays make very small angles, then these can be considered as paraxial rays. Such rays will travel back and forth close to the axis. Now, if the ray angle is very small then we can use such an approximation as we will see tan theta nearly equal to sin theta equal to theta. So, this is the approximation that will be used by assuming that we are treating paraxial rays.

Now, how good is this assumption? For example, what is shown here is typical beam parameters of a practical laser. If we take a, helium neon laser a practical He-Ne laser helium-neon laser, then the length of the laser tube is generally 20 to 30 centimeters and the beam diameter at the width is of the order of 0.2 millimeter we will take actual numbers and see that indeed this is the kind of numbers that we will get.

And, the output beam divergence typically is 1 milli radian and the beam diameter here is approximately 1 mm. The divergence angle is typically 1 milli radian for a practical laser.

You can see the data sheets of practical commercially available lasers and you will see that the divergence angle is of the order of 1 milli radian, which means 0.057 degrees. So, indeed the theta is very small and therefore, the use of paraxial approximation is perfectly valid in such situations.

(Refer Slide Time: 25:23)



So, the ray transfer matrices we have to determine the ray transfer matrices of an optical system. An optical system here we refer to it shown as a black box here. So, you have an optical system which may comprise of many many optical component for example, the optical system here shown may comprise of several optical components.

For example, if you take a microscope objective, microscope objective typically this may have two, three or four lenses and those zoom lenses and if you have seen those zoom lenses which are fitted to a camera that may have several components optical component inside the zoom lens and that is what is shown. So, we call it as optical system.

Now, tracing rays through the optical system can be done using the ray transfer matrix and if the ray transfer matrix for each component can be determined, then the product of these matrices will give you the ray transfer matrix for the whole system. That is the way that rays can be traced if I get the ray transfer matrix I call this as RTM for the whole system, which comprises of several optical components because all these matrices are 2 by 2 matrices.

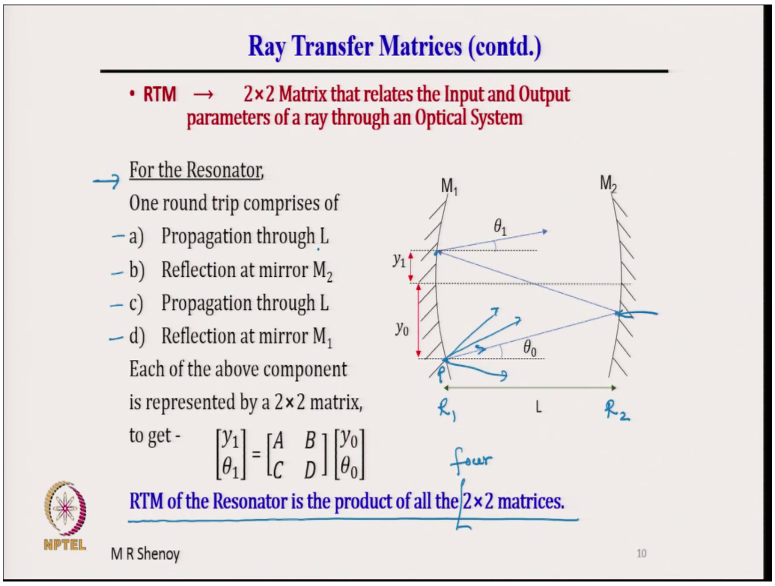
Therefore, the product of these matrices will also be a 2 by 2 matrix. So, these are 2 by 2 matrices. The matrix links the input parameters of the ray to the output parameters. A ray is characterized by two coordinates input and output. At the input it is characterized by a displacement y i is a displacement from the axis of the optical system and theta i is the angle that it makes with the horizontal.

So, if you have a ray which is like this for example, then this will be the displacement y i and the angle that it would make would be this theta. So, this is y and this is theta. Now, we will see the sign convention because y which is a displacement which is below the axis will be negative and the displacements above the axis will be positive, we will discuss about this.

But, right now every ray which is entering the optical system is characterized by a displacement y i and theta i. And accordingly at the output there will be a corresponding y o and theta o. And, the y o, theta o is linked or related to y i theta i through a 2 by 2 matrix which is the A B C D matrix.

And this A B C D matrix is called the ray transfer matrix, ray transfer matrix. It actually gives you the coordinates when the ray is transferred through the optical system, hence the name ray transfer matrix. This will become very clear as we take some examples of the ray transfer matrices.

(Refer Slide Time: 29:13)



Now, if we consider straight go to the optical resonator our problem is optical resonator not zoom length. So, the optical resonator comprises of two mirrors separated by a distance L. Now, if a ray starts from here, let us say the ray starts from a point P here. It has a displacement initial displacement y 0 and an angle that it makes with the horizontal theta 0. The ray is allowed to go at a particular direction. We can also of course, consider a ray which travels like this.

You could also propagate rays like this. So, from this from every given point on the mirror you could propagate rays at different angles some of them will eventually go out of the resonators some of them will remain trapped inside. If you can find some rays which are trapped inside the resonator or confined to the resonator, then we call the resonator a stable resonator.

So, coming back to the initial ray here with a displacement of y 0 and theta 0, it moves to the other mirror therefore, the ray has propagated in free space here or in a medium if there is a medium through a distance L. It undergoes reflection here at this point it undergoes reflection at the spherical mirror of radius of curvature R 2. Let us say R 1 is the radius of curvature of this mirror and R 2 is the radius of curvature of the second mirror.

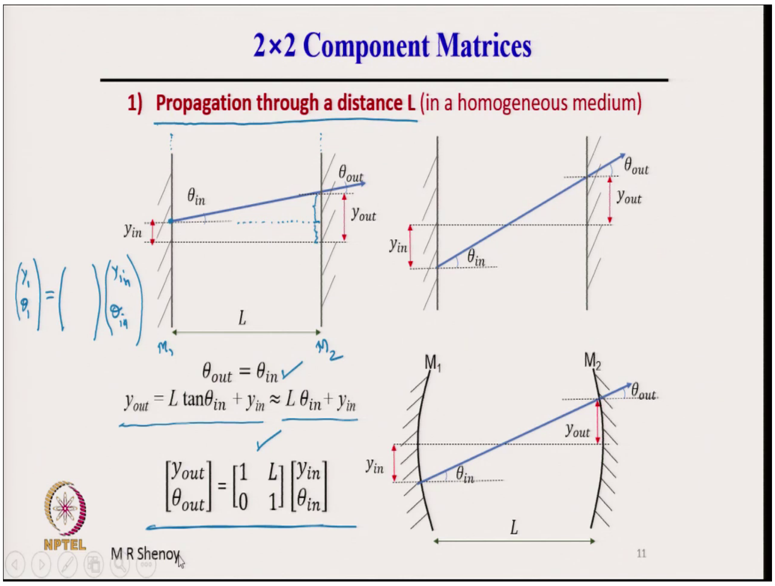
And, from here again it propagates to the second mirror that is propagating through a distance L, horizontal distance L. This distance is of course, higher than the horizontal separation and it undergoes a reflection at mirror M 1 and then again propagates back. Now, in one round trip therefore, we had one length propagation in free space, one reflection at a spherical mirror, then propagating back in free space and then reflection at another spherical mirror and then propagating.

So, one round trip comprises of two propagation lens and two reflections at the spherical mirrors. So, that is what I have written here that for the resonator one round trip comprises of propagation through L, reflection at mirror M 2 here reflection, propagation through L back to the mirror M 1, reflection at mirror M 1.

And, each of the above component these are called four component of propagation can be represented by a 2 by 2 matrix and to get the final coordinates y 1 and theta 1. The y 1 and theta 1 is related to y 0 into theta 0 through the ray transfer matrix A B C D. Once you know A B C D you can calculate y 1 theta 1.

If you know y 1 theta 1 you can calculate y 2 theta 2 and so on. In other words, you can trace a given ray a starting ray through the resonator any number of times back and forth and find out whether the ray always remains confined or does it eventually leave after a few round trips. So, the RTM of the resonator is the product of all the four. So, they are all the 2 by 2 means all the in this case four four 2 by 2 matrices corresponding to these four operations 1, 2, 3.

(Refer Slide Time: 33:41)



So, let us look at the 2 by 2 component matrices first propagation through a distance L. So, this is the first one, propagation through if we look back the first one is propagation through a distance L. Please see, it is not propagating through the mirror, it is from mirror M 1 to M 2, the mirror could be spherical mirror or plane mirror.

So, the first component says propagation from this plane this is actually plane of the mirror M 1 and plane of the mirror M 2. This is not plane mirror but, plane of the mirror M 1 and M 2. So, from M 1 to M 2 it propagates through free space. Let us say y in is the initial displacement the starting point from here is y in, is the height from the axis of the resonator and it is propagating at a angle by making an angle theta in.

Then, when it reaches the other end, we know that if this is a homogenous medium it will travel in a straight line with a new displacement y out which is different from y in because it is travelling at an angle. If it were travelling parallel to the axis then we would have had y in is equal to y out, but in general if it is travelling with theta not equal to 0 then y out will be different from y in. What is the relation?

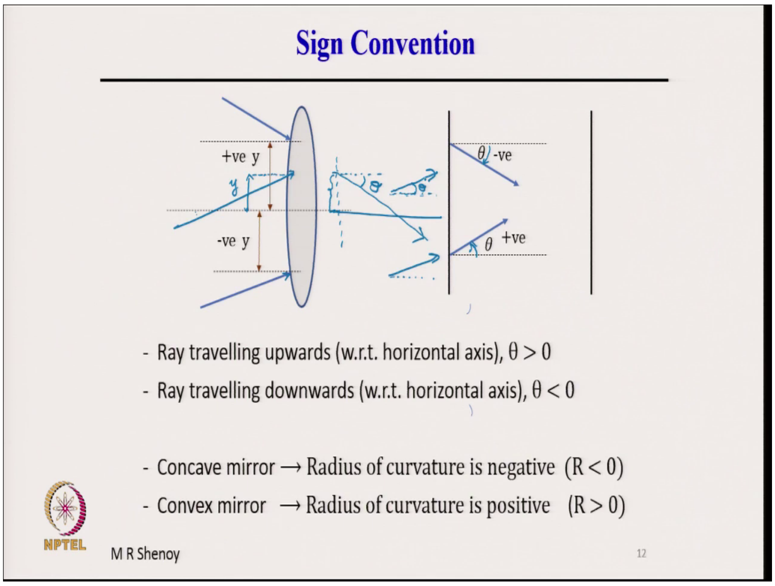
So, we what is our objective? Our objective is to determine y out or after one propagation y out and theta 1 to the input which is y in and theta in. So, this is our objective. We want to find this. Now, first point is theta out is equal to theta in when it comes from here up to this point there is no change in the angle with respect to the horizontal. So, first point is theta out is equal to theta in.

Now, y out is more, now how much is y out? So, you can find out y out. So, y out is equal to if this is L, then L tan theta in. So, L into tan theta in is this height. So, this height is L into tan theta in plus the original height which is y in. So, y out is equal to L tan theta in plus y in, but we are considering paraxial approximation. Therefore, tan theta in is nearly equal to theta in and therefore, we write this as L theta in plus y in.

And, immediately now, we have both the equations relating theta out and y out to theta in and y in and the matrix is here y out is equal to 1 L and 0 1 theta in. So, theta out is equal to this row into this column product which is simply theta in, y out is equal to product of this row into this column which is y in plus L into theta in. So, this is true.

So, this is not for just plane mirrors this is only propagation through a distance L. So, it could be spherical mirrors with a ray starting from below the axis and going above the axis, everywhere you will see that this matrix will represent will relate propagation through the change in coordinates after propagating through a distance L.

(Refer Slide Time: 37:15)



The sign convention is shown here. If the ray as I already mentioned, so, this is a lens which is shown for example. A ray which is incident in the lower half, so, this is the position of the ray will have a negative displacement. So, the displacement is considered or measured from the axis of the system.

So, this ray as a, is negative and if the ray is incident from top, then the displacement is positive. It could be a ray which goes like this then also the displacement will be this. So, this is positive. So, where it hits the optical component or the ray transfer matrix of the component that you need you consider that as your displacement and if it happens to be in the upper half then it is positive and if it happens to be in the lower half, then it is negative.

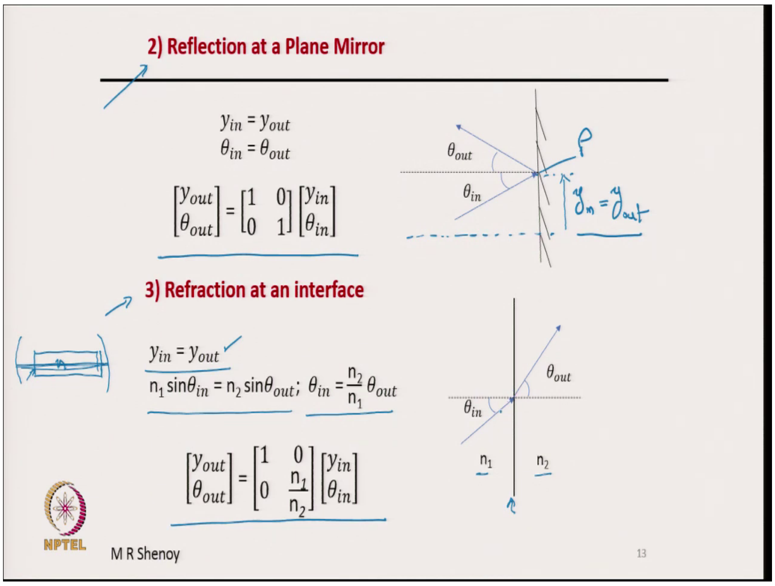
The second point is for concave mirror the radius of curvature is negative R is less than 0 and for convex mirrors the radius of curvature R is greater than 0, one other point. So, this is about the displacement. The second point about the angles if the angle is upward if a ray is moving upward, which means it makes an angle which is positive with respect to the horizontal then that angle is positive.

If a ray is travelling downward then it makes an angle which is negative with respect to the horizontal and then we have to designate that angle as negative. So, let me show it here for example. So, if we are considering a ray which is travelling downward like this, then of course, if it was at this point then this is the height, but the angle this angle theta will be negative because if the ray is moving downward with respect to the axis.

And, if any ray which is whether it need not cross the axis even if the ray is travelling like this for example, here in the upper half, then the angle here will be positive because now the ray is moving upward whether it is in the upper half or lower half that does not matter as far as the angle is concerned.

It is only whether the ray is moving upward or downward. Whether it is in the upper half or in the lower half then the displacement y will be different. If the displacement that we are considering is in the upper half that is above the axis of the system optical axis of the system then the displacement is positive and if it is below as shown here, then it is negative.

(Refer Slide Time: 40:27)



Now, let us go to the second component namely reflection at a plane mirror. So, the 1st one that we have seen is here for propagation through a distance L we are looking at propagation between mirrors. Now, let us consider reflection by a mirror. So, first plane mirror. It is illustrated here. So, this may not be the axis, axis may be here.

So, at any point the ray is incident at a point P here, let us say on the mirror. Then the displacement is here. So, this is the displacement which is positive. After reflection angle of incidence is equal to angle of reflection and therefore, theta in is equal to theta out. Please see that both the angles are positive. This ray is also going up and this ray is also going up and at the point of reflection theta in y in is equal to y out.

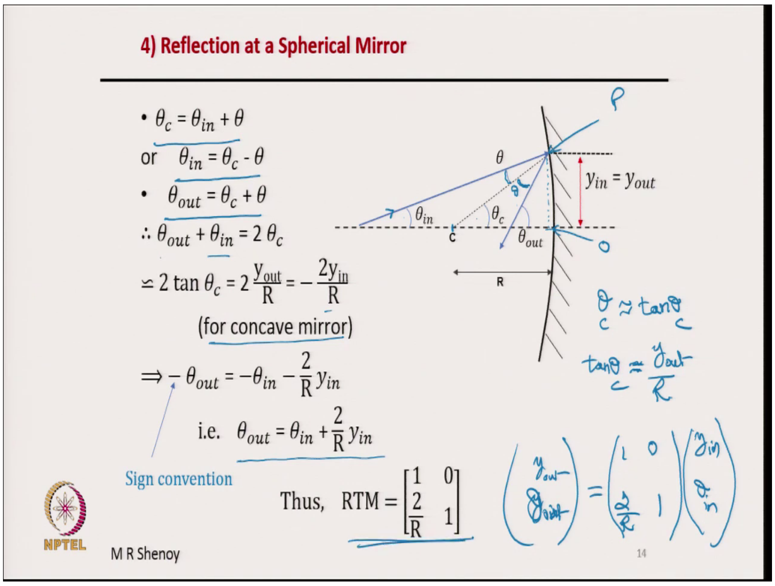
We are finding the ray transfer matrix for the operation of reflection. Therefore, we are considering y only at the point of reflection. Therefore, at that point y in is equal to y out and theta in is equal to theta out and therefore, immediately the relation is an identity matrix y out is equal to y in and theta out is equal to theta in ok.

Next, refraction at an interface: so, this is sometimes important because we will see later on that there are laser resonators, where you have the laser rod or active medium, which is a rod of a certain refractive index n. Then, when the ray goes back and forth or beam goes back and forth, then it will encounter a interface and that is why we are looking at interface between two medium n 1 and n 2.

So, if you have an interface between a medium of refractive index n 1 and n 2, then what is the relation what is the ray transfer matrix corresponding to the transfer of ray across this interface. So, at this point y in is equal to y out because we are finding the ray transfer matrix for the operation at the interface only. Therefore, y in at that point is equal to y out.

However, theta in and theta out are related through the Snell's law which is n 1 sin theta in is equal to n 2 sin theta out and because sin theta is nearly equal to theta, we write theta in is equal to n 2 by n 1 into theta out. So, y in is equal to y out, theta in is equal to n 2 by n 1 into theta out. And, therefore, immediately we can write the ray transfer matrix for this operation; the operation is refraction at an interface. We are not looking at propagation only at the point where the refraction takes place.

(Refer Slide Time: 43:53)



Now, let us go further reflection at a spherical mirror. So, again the geometry of the problem is shown here. So, a ray which is coming from some arbitrary ray which is incident at an angle theta in; so, theta in is the angle the ray makes when it is incident on the mirror at some point P, let us say some point P. And, after reflection the ray goes here making angle of incident is equal to angle of reflection this angle theta will be equal to the this angle theta.

Now, when it makes the angle this is with respect to the line joining the center of curvatures; C is the center of curvature because with respect to center of curvature this angle theta will be equal to the angle theta here. So, we call theta c, let theta c be the angle the line here connecting the center of curvature makes theta c.

Then in this geometry we can see that theta c will be equal to sum of these two angles, that is theta in plus theta, theta c is equal to theta in plus and therefore, theta in is equal to theta c minus theta. Similarly, theta out; so, theta out here will be equal to this angle plus this angle. So, this angle is also theta, this is also theta, this is also theta.

And, therefore, theta out is equal to theta c plus theta these two angles and therefore, if we add theta in and theta out so, theta in plus theta out, then this theta cancels and we have theta out plus theta in is equal to twice theta c. Now, twice theta c here this is the angle.

So, theta c is nearly equal to tan theta c because here of course, we have shown the angles as big, but please remember all angles considered are very small angles. So, tan theta c and tan theta c will be equal to the distance R and this height here and because the angle is very small this distance is the same as this distance and therefore, we can write tan theta c is equal to y out that is this height divided by R. So, y out by R.

R is actually the separation from C to O, O is the pole here O, but the separation here is very very small because theta c is very small. And, therefore, theta c is approximately equal to y out by R and therefore, we can write 2 theta c is equal to 2 tan theta c is equal to 2 into y out by R and y out is equal to y in and therefore, we can write minus twice y in by R.

Now, this minus sign has come because we have considered a concave mirror. So, the negative sign has been introduced because R is negative and therefore, we have minus theta out. Please note again that theta out is the angle that the ray makes when it goes down and therefore, this is also negative.

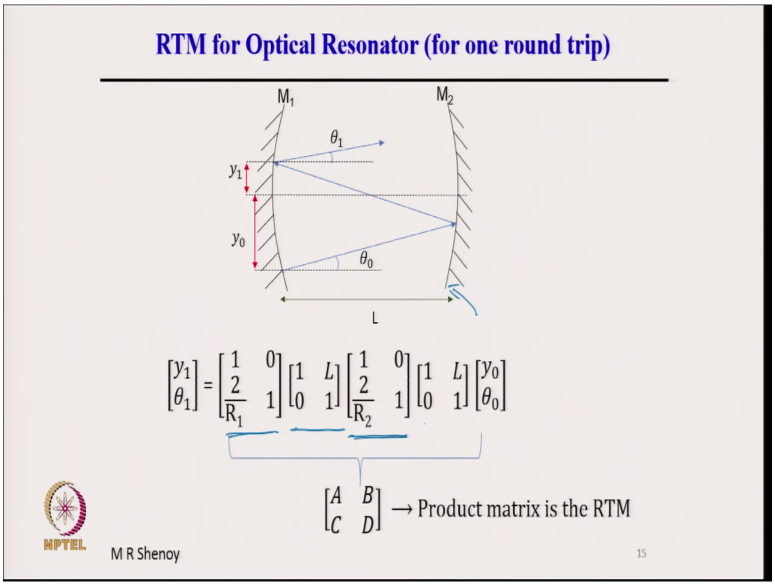
Here the ray was going up, therefore, theta in is positive now the ray is going down therefore, theta out is negative. And, therefore, we have written minus theta out. So, we are writing this expression theta in to be taken to the other side. Now, minus theta out is equal to minus theta in; this minus is because we have taken theta in to the other side.

And minus twice y in by R or all the minus signs will go and we have theta out is equal to theta in plus twice 2 by R into y in. Thus therefore, the RTM - Ray Transfer Matrix for reflection at a spherical mirror is 1 0 1 0 because y out is equal to y in.

So, y in is equal to y out. Therefore, it is the identity row. So, we have y out by y in sorry, theta out, theta out is equal to the ray transfer matrix into y in theta in. So, y out is equal to y in which means this is 1 0 and theta out is equal to theta in. So, theta in therefore, this must be 1, 2 by R. So, y 2 by R into y in so, this row multiplied by this column 2 by R into y in theta in.

So, this is the RTM for reflection at a spherical mirror this is very important. So, now, we have got in a spherical mirror resonator. Reflection at two spherical mirrors, so, we know the RTMs propagation through a distance L we know the RTMs and therefore, we can determine the ray transfer matrix for one complete round trip.

(Refer Slide Time: 49:59)

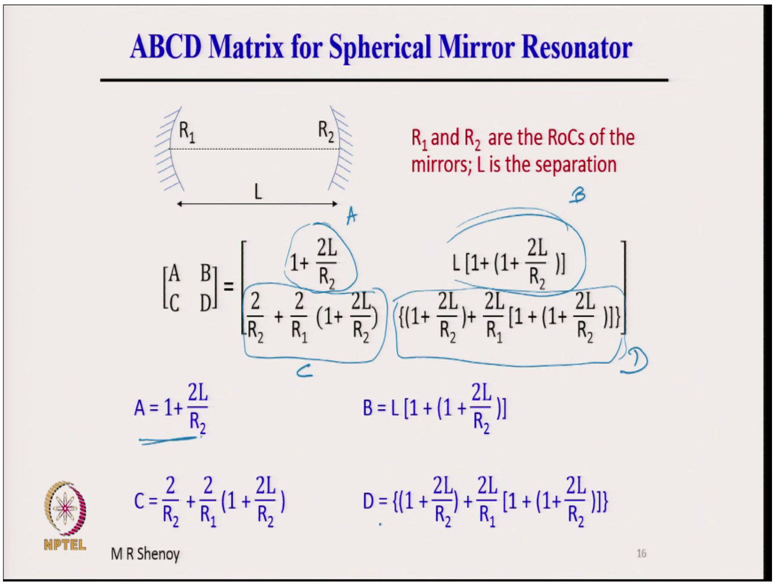


And, therefore, the RTM for the optical resonator for one complete round trip is given here that y 1 theta 1 is equal to. So, we start with y 0, theta 0 this is our starting point y 0 theta 0.

This first it gets multiplied on the left side by the matrix for this propagation and then it reaches the other end gets multiplied by this matrix for reflection at mirror M 2 of radius of curvature R 2.

It propagates back, gets multiplied by this matrix and then gets reflected here and gets multiplied by this matrix. And, the product matrix is called the A B C D matrix for the optical resonator. This A B C D matrix is the ray transfer matrix for the given spherical mirror resonator.

(Refer Slide Time: 51:03)



So, if we multiply this then here is the product matrix. So, this component so, this element is A. So, it is written here this is identified as A, the second element which is here is identified as B. So, that is what is written B and this is C what you have is C. So, this is C and the last element is D.

So, this is designated that A B C D matrix for this spherical mirror resonator has components A, B, C and D. Given a resonator, which means you know the radius of curvature R 1 and R 2 and the separation L. So, given a resonator you can determine the A B C D matrix for the resonator because all the components A, B, C, D contain only R 1, R 2 and L.

So, given a resonator the ray transfer matrix can be determined. Now, with this, in the next lecture we will consider the Ray Transfer Matrices of a spherical mirror resonator and we will find out under what conditions a ray will be confined mathematically not by qualitative explanations, ok.

Thank you.