Nuclear and Particle Physics
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Module – 07
Elementary Particles
Lecture – 04
Quark Model-continued

We will continue our discussion on quark model. In the last discussion we discussed how the meson, mesons can be grouped into a group of 8 and 1 singlet considering the flavor symmetry, which is basically associated with what is known as SU 3 group.

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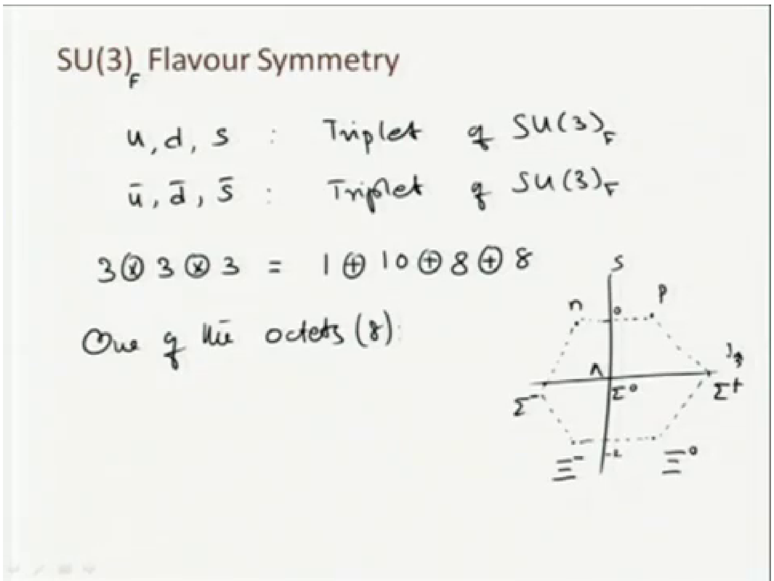

We will continue the discussion on the flavour symmetry and the grouping under grouping of hydrogen's under this.

So, today we will look at the baryonic resonances or baryonic composite particles called like proton, neutron etcetera to recap we said u quark, d quark, and s quark can be considered as a triplet under s u 2, s u 3 flavor. Similarly u bar d bar the anti s bar the anti quark terms of this uds quark also from a triplet of s u 3. So, in the case of meson's we needed to take a product combination of the uds and u bar d bar s bar, to form masonic bound states.

But in the case of baryons what we will take as, a group of triplet of the quarks and another triplet of quarks and another triplet of quarks, one quark each from one group to form baryonic bound states, which consists of 3 quarks. As per the su 3 group such product can be split into a singlet a group of 10 had d couplet to groups of 8 octets. As we mentioned in the last class, this is very similar to the spin angular momentum or normal angular momentum algebra, where when we actually take 2 spin half particles and consider them as a combined system.

Then the combined system will have or can have spins either 0 or 1. One spin one case is actually a triplet with 3 different projections spin 3 projections, and the spin 0 case is a singlet with only one spin projection possible in that. So, the spin one has can be thought of as having 3 members a triplet. Similarly if you take other spins we can combine them as well. So, in a similar fashion we can actually think about 3 triplets forming and a 2 octets and d couplet and a singlet.

One of the this one of the octets is formed by the proton, the neutron, the sigma 3 of them, sigma plus sigma minus and sigma 0 and 2 other spin sorry 2 other iso spin half resonances or particles called cascade particles; one is cascade 0, the other is cascade minus. So, in the I 3 s plane we can put them in an arrange them in a fashion, very similar to what we had done in the case of meson. This gives us a group of 7 along with this the lambda particle which is again neutral isospin 0 strangeness minus 1.

So, here strangeness is 0 here, minus 1 for the model point particles sigma's and lambda and minus 2 for cascade particles.

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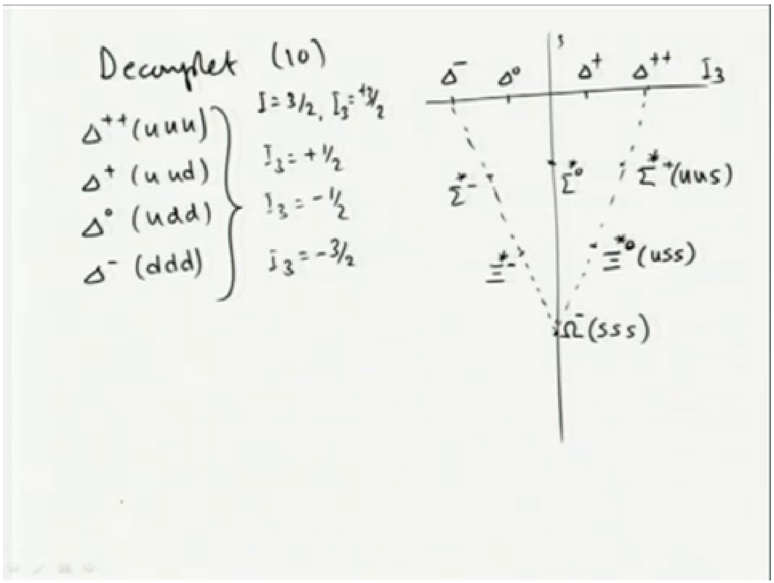

The d couplet group of 10 can also be arranged in a similar fashion, in the s I 3 plane. There is the a resonance delta plus plus which can be thought of having a u having 3 u quarks inside it, and delta plus which is a u u d which consists of u u d. Delta 0 u d d and delta minus d d d. This for form an isospin 3 by 2, I equal to 3 by 2, and I 3 equal to plus 3 by 2, I 3 equal to plus 1 by 2, I 3 equal to minus 1 by 2 for delta 0 I 3 equal to minus 3 by 2 for delta minus.

All of them are strangeness zero particle. So, we can put them in I 3 is equal to 0 I 3 line as follows. So, delta plus plus with I 3 3 by 2, delta plus I 3 half, delta 0 by 3 minus half and delta minus I 3 minus 3 by 2, then a rest of them actually will form an inverted pyramid rest of the members of the De couplet. So, when we consider the next isospin group that consists of another particle with quantum quark combination u u s and sigma 0 with u d s and sigma minus with d d s.

Notice that this sigma's have exactly the same quark content as the earlier sigma in the octet, thus we had earlier mentioned what are the quark contents of the sigma's there. So, what is the difference here? The difference is that they are more massive; all the 3 of these sigma's which are members of this have a different mass higher mass compared to the sigma's in the octet.

So, this can be thought of as some kind of an excited state I mean in a vague sense of the u u s system. So, you have a quarks 3 quark u u s system say, the ground state of that will give you the sigma plus which is part of the octet, and the higher excited state one of the higher excited states will give you sigma star plus or sigma plus star, which is a member of the d couplets similarly, sigma 0 and sigma minus.

Then it was observed that there are 2 other resonances which otherwise look similar to cascade, quark content can be thought of as u s s strangeness minus 2, and a cascade minus with quark content d s s to strange quarks therefore, strangeness minus 2, but their masses are larger compared to the cascades coming appearing in the octet. So, we will say think about the must be excited states of the same quark combination.

At the time this ideas or this such grouping in super multiplets bigger. Super multiplets i said because we already had earlier multiplets according to the isospin grouping that we had discussed. Now in thus this octets or d couplets what we are doing essentially if you look at this is that, considering different such I 3 groups and put them in a bigger group here.

So, for example, the cascades the delta minus delta 0 or delta plus, delta plus plus all the deltas belong to a particular isospin group, and all the sigma's belong to another isospin group I is equal to 1. At the time of such a development when these sub multiplets were discussed those s u 3 flavor multiplets were discussed or developed, we had already observed or the these resonances delta's sigma's sigma stars cascades, cascade star, neutrons and protons of course. But then there was a natural point in this decoupling grouping. There is one missing particle which was not observed at that time and as per the s u 3 grouping this should have a quark content of 3 strange quarks.

So, that its strangeness is minus 3 isospin 0, isospin 3 0, and this is basically the last member of that D couplet group and it was named omega minus it is predicted that a particle such an with such and such isospin such and such strangeness should be there for axis. We will come to the mass relations in a minute, and we will see that mass can also be predicted to some magnetic accuracy. And indeed such a particle was discovered later on in a couple of years in 61, and this was a triumph of such grouping s u 3 flavor grouping, which gave big boost to the quark model of the hydrants.

So, far we had not observed any free quark. So, the question still was whether quark is actually a real particle or just kind of a mathematical entity. So, we had to wait for a little more while to actually establish the quarks, inside the quadrille hydrons also of course, physical particles existing inside the quark and later on of course, we had discovered the top quark also. So, the unambiguously that unamicgous, at the moment we have no doubt in the existence of quarks dust particles.

So, now although we have been talking about these words, let us summarize this properties of the quarks associated with the or the different properties associated with the quark.

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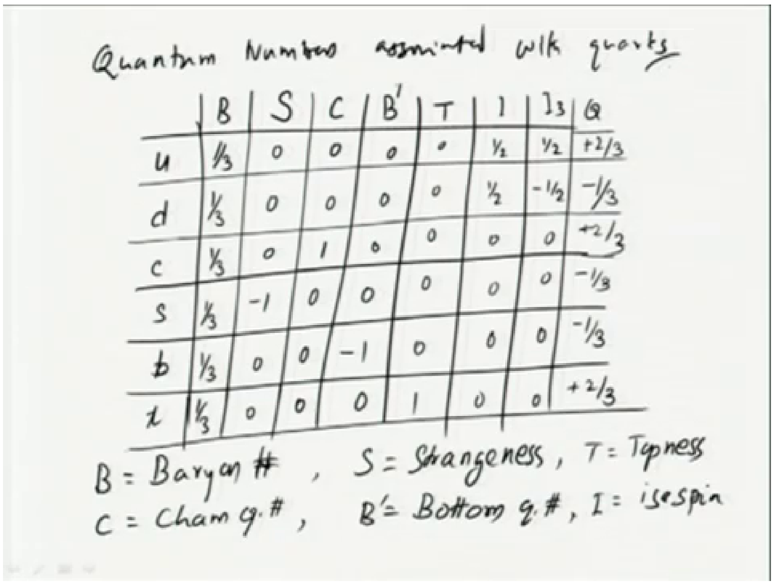

Let me call the s as the quantum numbers associated with the quarks, just a summary of this. We have u quark and d quark, and charm quark and b quark of course, there is also the top quark although they do not really form the bound states they are there, and since 3 of them form bound states called baryons like protons and neutrons lambda particle, sigma particle and delta particles.

We have to associate the baryon quantum number with the quarks baryon quantum number of u quark or each of these quark should be 1 by 3 so that 3 quarks together add the baryon number 2 1. So, we will put 1 by 3 as the baryon number, which we denote by the letter b as 1 by 3 for each quark. Then we said there is this strangeness that we had to consider. So, the strangeness quantum number u quark has no strangeness oh I have to put one more quack there this strangeness quark. So, this is charm strangeness top and bottom let me put bottom here and then top below this thing.

So, strangeness quantum number for historic reasons for strange quark was associated with a strangeness equal to minus 1. This is because the strangeness initially was introduced by looking at particles like jameson's, which used to be produced strong in strong interactions of proton proton collision for example, but d k weakly. In fact, this was the reason why these particles were given such an additional quantum number, which is called strangeness.

So, these particles were given a strangeness plus 1 and later on in the quark model we saw that, the strangeness the quark content of the k means 1 k plus and k 0 or actually s bar note s inside that k plus is associated or the core content of k plus is u and s bar similarly quark and I know d k 0 is d and s bar. So, since strangeness was associated or plus 1 strangeness was associated to k plus and k 0 historically as they were discovered, and then found that they behaved strangely in the sense that they are produced strongly, but d k weakly.

So, s bar quark naturally got a plus one as the strangeness and s quark therefore, got as strangeness minus 1. B in top bottom and top will not have anything. So, in a same fashion we actually have charmness we can also talk about bottomless since bottom, B letter is already taken up by the baryon number we will put B prime as the bottomness and you can also talk about topness. So, you can see that C quark will have charmness 1 and B quark s quark all others quarks 0 charmness.

Similarly, b bottom quark will have m b minus 1 similar to the strangeness quark and then top quark in half a plus one top ness. And isospin this is half and half for the u quarks and 0 for the other quarks. We can also talk about I 3 plus half here and minus half here and of course, 0 0 for the rest of this thing. So, let me write this although we have said this clearly B is B denotes Baryon number, C denotes charm quantum number or chamness, S is strangeness quantum number B prime is bottomless or bottom quantum number, T is top ness or top quantum number I is the isospin ok.

Now, not all of these are independent quantum numbers, there are some relations which these quantum numbers together satisfy along with the; I should mention the quark the charge of these particles as well maybe the electric charge of electric charge Q is the electric charge, electric charge of u quark is plus 2 by 3 electric quark charge of d quark is minus 1 by 3, cham quark plus 2 by 3 bottom quark sorry strange quark minus 1 by 3 bottom quark minus 1 by 3, and top quark plus 2 by 3.

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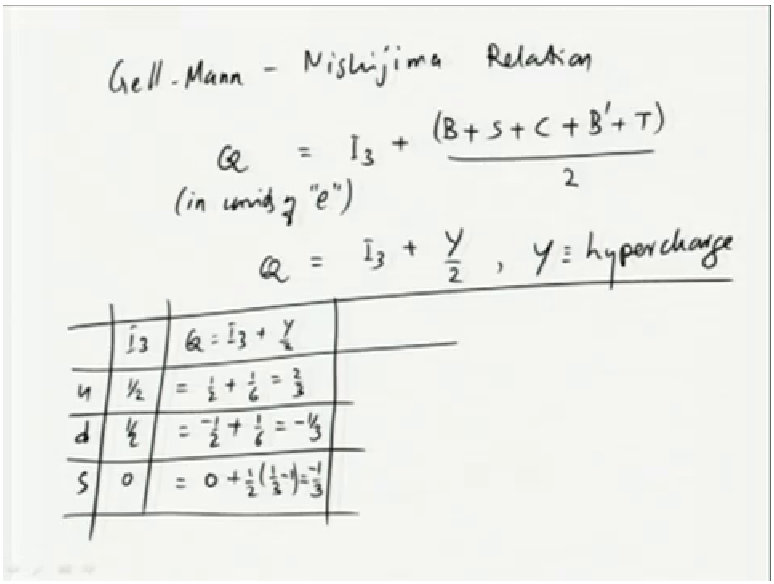

As per Gell-Mann and independently in Nishijima, there is a relation between these quantum numbers Q in units of electronic charge e that is what we had done in the earlier slide. In fact, is equal to I 3 plus baryon number plus strangeness plus chamess plus bottom ness plus topness divided by 2. In fact, later on we will associate in fact, this whole thing in the bracket above B plus S plus C plus B prime plus T as what is known as hypercharge and denote this by y. So, the Gell-Mann Nishijima relation becomes Q equal to I 3 plus y by 2 ok.

So, let us see some of the quarks, the I 3 of this quark Q of this quark as per the relation. So, u quark I 3 is plus half, baryon number is 1 by three. So, Q should be equal to I 3 plus y by 2. So, baryon number is 1 by 3 therefore, y is 1 by 3. So, you have Q equal to 1 by 2 plus 1 by 3 by 2, which is 1 by 6 which adds up to 2 by 3, which is the correct charge that we have for the u quark. And similarly for the d quark this is equal to minus 1 by 2 I 3 plus 1 by 6 same hypercharge equal to minus 1 by 3.

Similarly, for strange quark I 3 is equal to 0. So, therefore, this is equal to 0, plus y is now baryon number 1 by 3 and strangeness minus 1. So, you have 1 by 3. So, there is an overall 1 by 2, 1 by 3 minus 1 should add up to minus 1 by 3. And similarly for other quarks you can work this out and then check that all this new Gell-Mann Nishijima relation holds good for all the quarks.

Now, we can even extend this beyond the quarks to other multiplets like other bound states as well.

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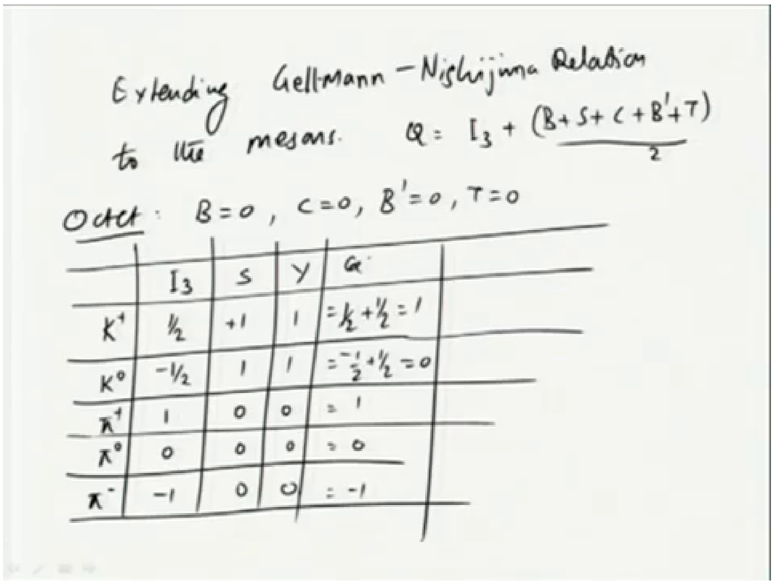

So, extending to the massan's extending Gell-Mann Nishijima relation to the mesons, what we have is the same relation let me write the relation here Q equal to I 3 plus baryon number B plus S plus C plus B prime plus top divided by 2. So, let us take the octet that we had considered in mesonic octet.

So, b baryon number equal to 0 for mesons, they are made of strange quarks u quarks and d quads, but no charm quark. So, C is equal to 0, B prime is equal to 0 and T equal to 0. When we consider the mesons including the charm quark number quarks or made of charm quarks, we had to consider the champ the chamness quantum number as well.

Similarly, for bottomless, but baryon number is always 0, for the mesons because very own number of a u quark case 1 by 3, and anti quark and an antiquark is minus 1 by 3. So, they add up to 0. So, let us consider the octet. So, one member is. So, this is basically I 3 right and let me write it here I 3 and considers start with the K plus, I 3 is half and strange baryon number is 0 what is a strangeness. Strangeness is plus one therefore, hypercharge is 1 and qQ equal to I 3 plus y by 2 Q is equal to half plus 1 by 2 is one that is fine ok let me write it as half plus half equal to 1.

When we consider K 0 hypercharge is the sam, s is the same, but I 3 is minus 1 by 2. So, therefore, this adds up to 0 and pi 1 pi plus I 3 is plus 1 s is 0 y is 0 charge is one pi 0 it is again 0 0 0 0; pi minus is minus 1 0 0 minus 1 similarly for the other mesons K 0 bar and K minus. So, we can see that this Gell-Mann Nishijima relation holds good for all the quarks as well as their bound states.

In a similar fashion we can also think about the baryon updates, and see that the same relation holds good there as well. Now let us come to another aspect masses of particles in the su 3.

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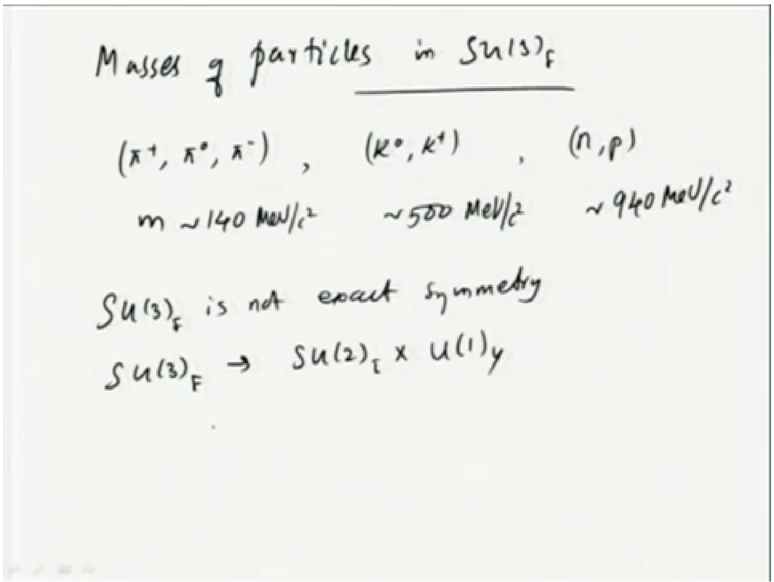

We saw that. In fact, in the beginning when we discussed the isospin grouping, we said take pi plus, pi 0 and pi minus. We wanted to group them together because they are or having similar mass about 140 MeV per c square, and similarly for k 0 and k plus we said we could group and them under the same isospin grouping, because they have similar mass by about 500 MeV per c square.

Similarly, proton and neutron also have mass about 940 MeV per c square both of them can be grouped with it. So, the masses were same more or less, but when we consider the octet of s u 3, let us say the masonic octet, we see that we want to group is these ions along with k mesons and the antiparticles k 0 bar and k minus, they also have similarly about 500 MeV per c square mass.

So, when we group these all these particles in the octet, did not have similar mass. You can see that here that the pi ons have mass and for 140 maybe and k ion's 500 something like 3 times plus more than 3 times (Refer Time: 32:47). So, there is this kind of thing as per the symmetry goes can be accommodated this difference in the masses can be accommodated by symmetry breaking what is not similar. So, the symmetry then we will say s u 3 symmetry is not an exact symmetry, but it is broken down to s u 3 is not exact symmetry, but it is broken down to s u 2 isospin and u one y u 1 y. Again if you look at y we are defined by the hypercharge in the earlier slides, if you look at that you will see that each of this isospin grouping group will have a particular y value.

The s u 3 F is not an exact symmetry and it is broken and the breaking is so that the groups with similar masses are grouped under s u 2 or can we are symmetric under the s u 2 grouping and what can be, but done is a kind of mass relation that can be worked out for su 2 su 3 multiplets.

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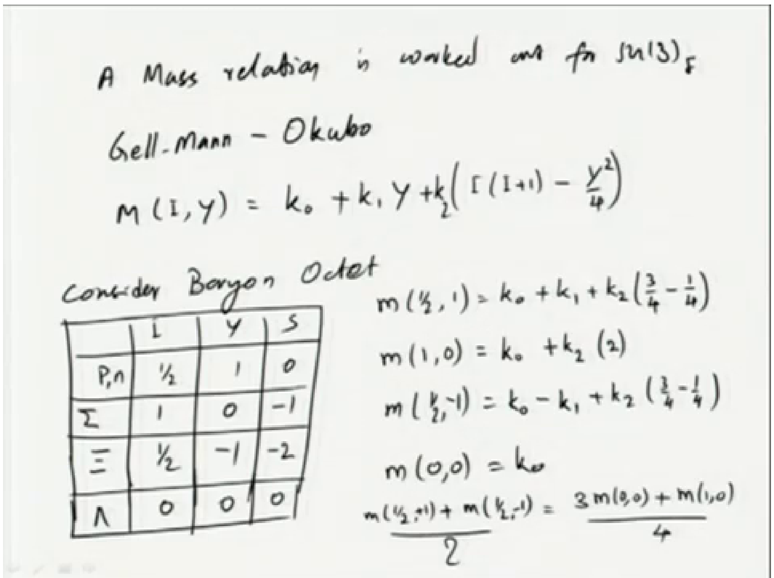

This is due to again Gell-Mann and independently by Okubo.

So, this relation is known as mu Gell-Mann Okubo mass relation mass of any member with I isospin I and hypercharge y is equal to some k 0 a constant and a k 1 times the hyper charge plus I times I plus 1 minus y square by 4 or y into y by 2 into y by 2. This is true for each super s u 3 multiplet. So, for each s u 3 multiplet, k 0 k 1. So, there is another constant here k 0 k 1 and k 2 need to be found out, and once you fix that then other members masses can be all the members masses can be found out.

For example let us consider the baryon octet one of the baryon octets. So, we have proton and neutron with i half, y equal to 1 because the baryon number is 1 let me make it a proper table. Now and there is this sigma 0 plus minus 1 belonging to i equal to 1 y equal to 0 cascade i equal to half y equal to minus 1 and there is this lambda particle with i equal to 0, y equal to 0.

Let me also write down the strangeness baryon number for all of this is equal to 1 we know, and strangeness is 0 for this therefore, y is equal to b plus s equal to 1strangeness is minus 1 for sigma therefore, y be equal to b plus s is equal to 0 and minus 2 for cascade. So, it is equal to minus y equal to minus 1 for this, and lambda also has s equal to minus 1 b equal to 1 therefore, this is equal to 0 y equal to 0.

So, now the mass relation will tell us that m half y 1 that is one group is equal to k 0 plus y equal to 1. So, k 1 plus k 2 into i equal to half. So, i times i plus 1 is 3 by 4 minus let me write it in that fashion 3 by 4 minus y is 1. So, y square is 1, y square by 4 is minus 1 by 4 together in that bracket it is 2 by 4 or 1 by 2. And the other is i equal to one hypercharge 0 group, which is equal to k 0 y equal to 0. So, k 1 y equal to 0, but plus k 2 into 3 by 4 sorry this is not 3 by 4 isospin is equal to 1. So, i into i plus 1 is 2 and hypercharge y is equal to 0 it is 2 new half, m half and hypercharge minus 1, this is k 0 minus k 1 right plus k 2 3 by 4 minus 1 by 4.

And there is an m 0 0 the lambda which has only the mass is equal to k 0. So, you can see that if you know the mass experimentally of the lambda particle then k 0 is fixed, if you know the mass of sigma particles then knowing k 0, k 2 is fixed and the proton and a neutron mass will fix k 2, then there is no more additional more free parameter and therefore, the mass of sigma particle should agree with the relation given in here where k 0 k 1 and k 2 are determined from the masses of protons or neutrons sigma and lambda.

For example we can write down the relation from here as m half plus 1 plus m half minus 1, divided by 2 you can work this out easily is equal to 3 m 0 0 plus m 1 0 divided by 4. It is a simple algebra you can just add this in this fashion and then you will get this mass relation is basically or due to the Gell-Mann Okubo relation and then it is observed that this relation holds good in the case of this baryon update.

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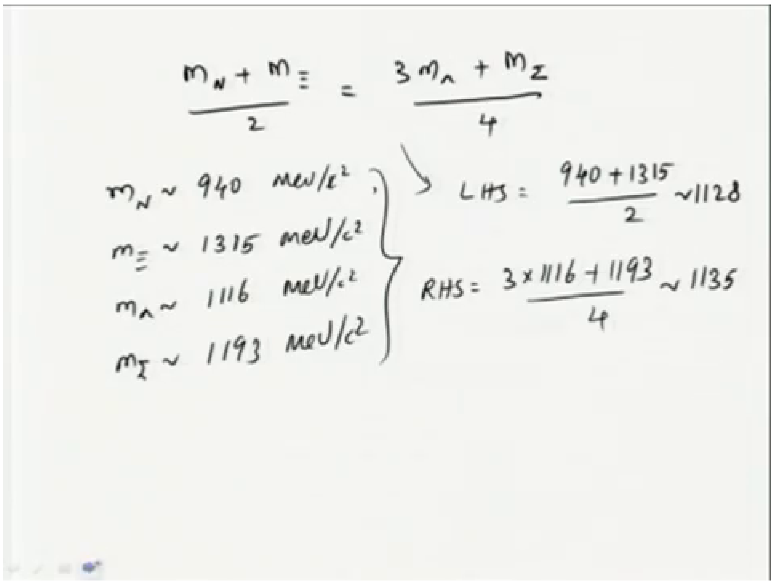

Let us see that m of nucleon half minus 1 is m of nucleon was m of cascade divided by 2 is equal to 3 times sigma mass plus mass of 3 times mass of the lambda particle plus mass of the sigma particle divided by 4 this is the relation predicted observed m an bout 940 MeV per c square and m cascade is about 1315 MeV per c square, m lambda is about 1116 MeV per c square m sigma is about 1193 MeV per c square and that gives LHS of this relation is equal to 940 plus 1315 divided by 2, it is about 1128 right this work that out and RHS is equal to 3 times 1116 plus 1193 divided by 4 is about 1135.

So, to a good accuracy the relation holds good how about the meson's. In the case of meson's the mass relation is the square of the masses with same relation k 0; where k 0 will now below may be different when you consider different group.

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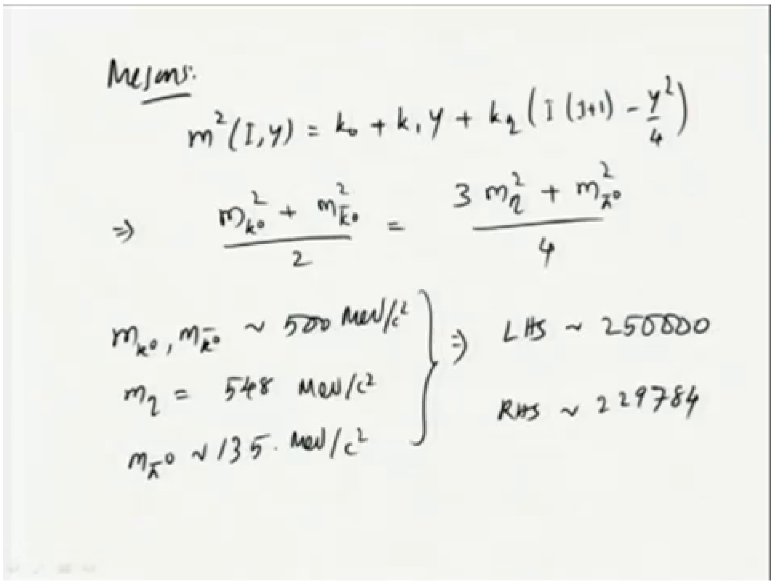

So, k 1 y k 2 I plus I into I plus 1 n minus y square by 4, and that gives for the meson octets m k 0 square plus m k 0 bar square divided by 2, equal to 3 times m eta square plus m pi 0 say pi one squared divided by 4, and experimentally m k 0 is and m k 0 bar are about 500 MeV per c square, m eta is about 548 m pi 0 is about 135 MeV per c square for (Refer Time: 44:26) MeV per c square, and you will see that LHS work it out is about 25 and RHS is about 229784 with these numbers.

These masses are not exact these are just approximate numbers. What is seen is that basically this mass relation holds better for the case of baryon multipliers compared to that of the meson multiplets alright. So, this relation holds good for other multipliers like the d couplets and other octets and octets when we consider other quarks also. So, these are such observations actually gave good faith in the quark model, but as I said earlier we had to wait before actually confirming that there are particles called quarks which are physical particles existing inside this one.

For this we had to do high energy deep inelastic scatterings meaning, you send probes basically the electrons which are neutral as far as the strong interactions are concerned. Send them at high energy inside these bound states like protons, and then the scattering of these particles of the phase ones. Now expected to scatter directly interact with the quarks inside this if the quarks exist there and bring out the information in the scattering distributions etcetera energy distributions etcetera.

So, this is what was observed later on and then that gives us good faith in the quark model later stage. We will come to that a little later in a different discussion today we will close it here.