Modern Optics
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Lecture – 08
Wave propagation in anisotropic media

Now, we will discuss the propagation of electromagnetic waves in anisotropic media. In anisotropic media various interesting phenomena happen, and it is very interesting in the sense that most of the devices that we will be discussing in this course, will be around this anisotropic behavior of the medium through which, the electromagnetic waves will be travelling. We have seen the isotropic medium where the electric field and the displacement vector there in the same direction.

The other aspects of isotropic medium that also we have seen in general the propagation characteristics with that background in mind, we will now look into the various different aspects of the electric field, magnetic field, the displacement vector their relation their orientation in this particular discussion.

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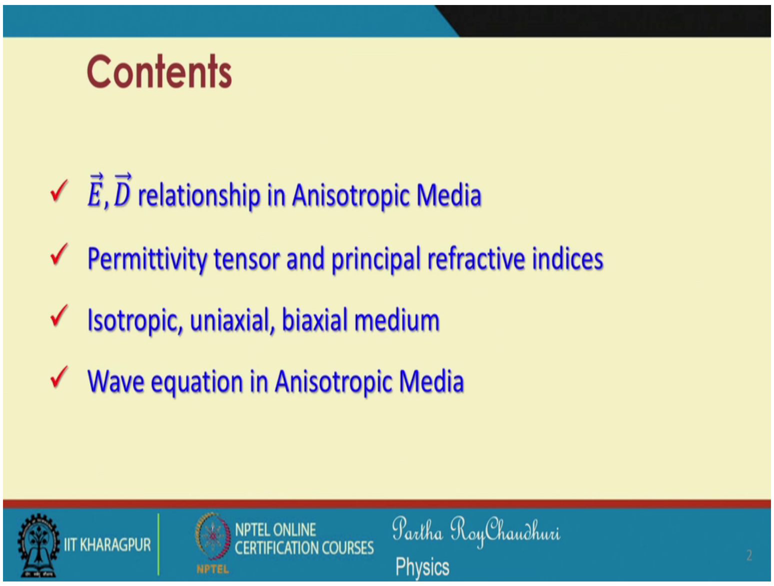

So, the content goes like this, first we look at the E D that is, electrical field and the displacement vector the relationship in a general anisotropic media. And from there we will try to organize the permittivity tensor in a very special case the permittivity tensor will correspond to a tensor, which will which will indicate only the principal refractive indices. Then will make a comparison of the isotropic Uniaxial and Biaxial medium we will defined the various properties of this. Then at the end we will try to bring in the wave equation for anisotropic medium.

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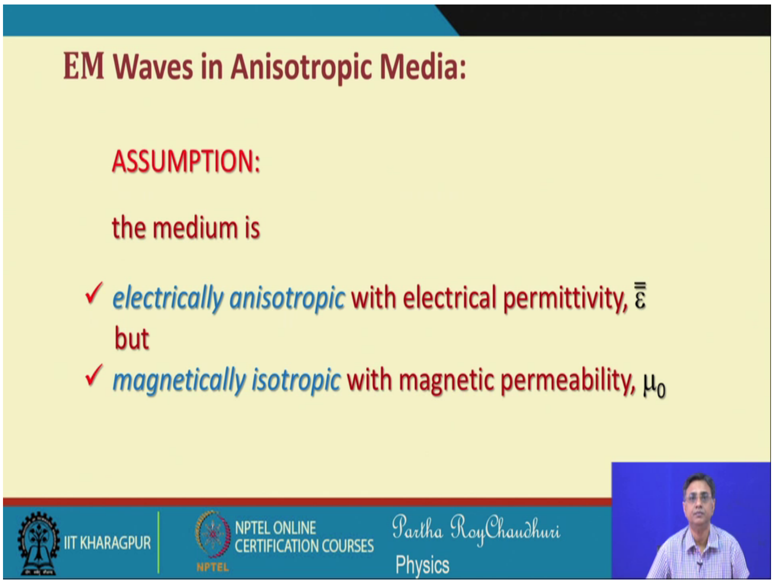

So, in the case of a general anisotropic medium in this discussion we will first make these assumptions that; the medium is electrically isotropic with electrical permittivity given by this epsilon bar that is a tensor, but the medium is magnetically isotropic with magnetic permeability, mu naught which is the free space permeability.

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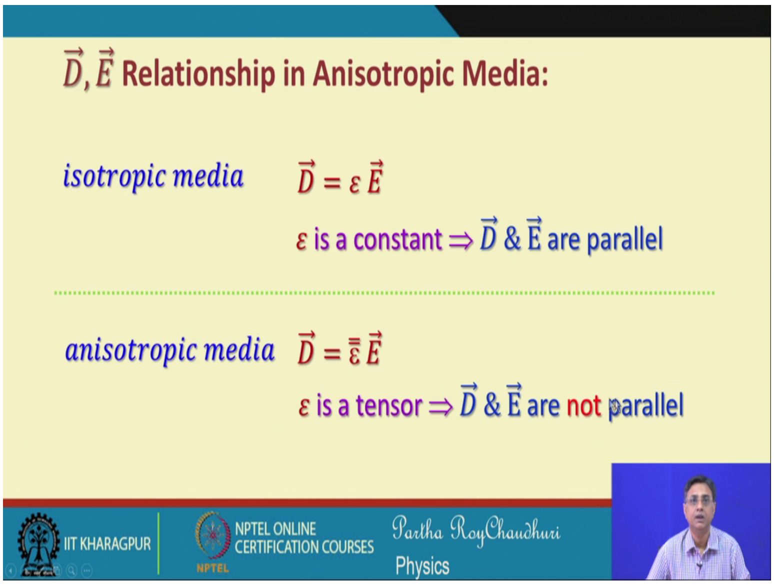

So, looking at the E D relationship in anisotropic medium the constitutive relation in this case is, the displacement vector is equal to the epsilon times the electric field vector, which is for isotropic medium this epsilon is a constant. And as a result it is a scalar being and the D and E are parallel. So, the electric field vector and the displacement vectors will point along the same direction where as in the case of anisotropic media this D and E in general are in different directions.

Epsilon is a tensor in this case which is a 9 by 3 by 3, 9 component tensor and as we have mentioned that D and E are not parallel as a consequence of this tensor property of the permittivity of this of the of the anisotropic material.

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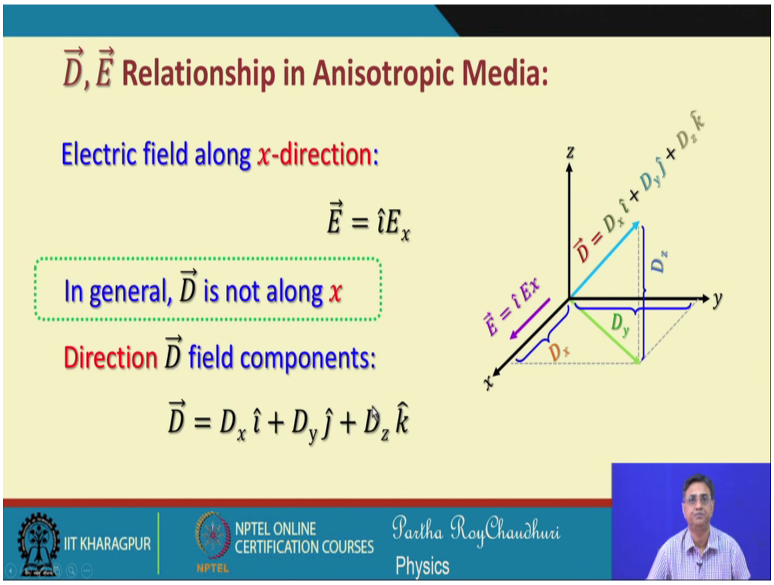

So, first let us look at the E and D behavior in anisotropic medium. Let us suppose that the electric field is incident along x axis. So, that one can represent the electric field vector as E equal to i cap E x and there is no y component or any z component of the electric field. So, electric field is solely along the x axis. In that case, in general the D is not along x axis, it will be somewhere different from the x axis. And this D vector will be represented by D equal to D x i D y j and D z k so; that means, there will be 3 components excited because of this incident electric field, which are in general not along the x axis.

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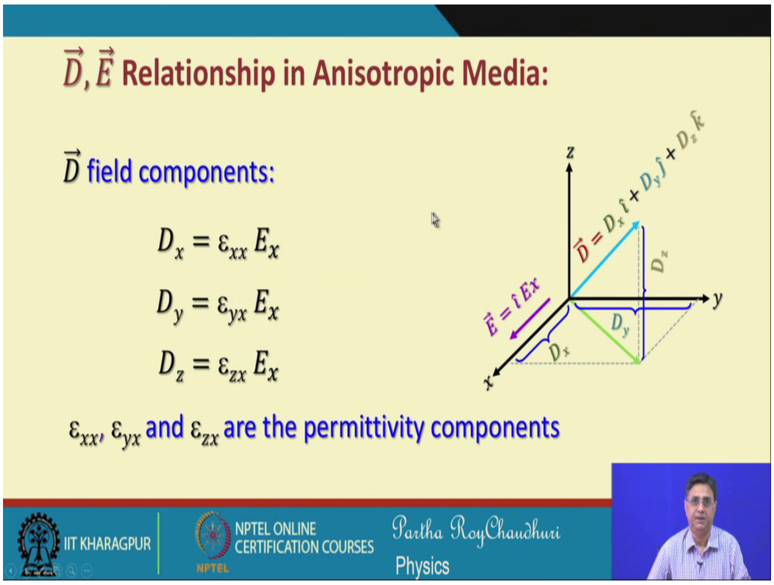

Similarly, we can apply an electric field in the y direction and as well as in the z direction, but let us see what happens when the electric field is applied along the x axis. So, the D component, because the by the very property of the displacement vector that the displacement vector is proportional to the electric field. So, D x that is the component of the electric displacement along the x axis will be proportional to E x x. And this proportionality constant for this particular medium will be represented by epsilon x x. And likewise for D y this will be epsilon y x E x, because this electric field is E x. So, we write in this notation epsilon y x. And similarly for the z component of the displacement we can write the D z equal to epsilon z x into E x.

So, we have been able to decompose the 3 displacement components, arising out of electric field only along the x axis. So, these epsilon x x epsilon y x and epsilon z x are the permittivity components of the medium when the electric field is incident along the x axis.

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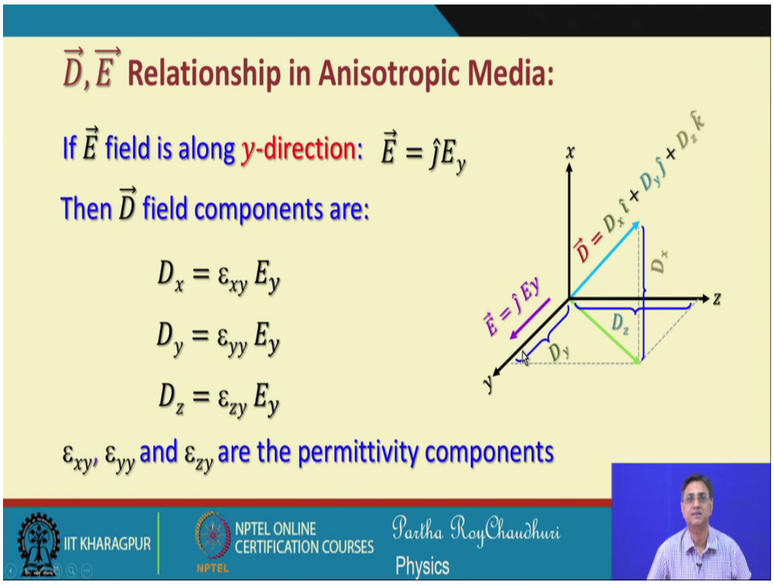

Next, let us consider the behavior when we apply an electric field along the y axis. In the same way the displacement vector will be in general not in the direction of the electric field, but in a in a direction which is different from the direction of the electric field. As a result, in the same way we can write the displacement vector in this case, the x component is equal to epsilon x y E y, for the y component it is epsilon y y E y, and for the z components it is epsilon z y E y.

Here also epsilon x y, epsilon y y and epsilon y z are the respective permittivity components when the electric field is applied along the y axis. We have one more case which will complete the discussion that is if the electric field is incident along the z axis in that case in the same way the displacement vector will be in general along a direction which is not along z axis.

As a result, in the same way we can decompose the displacement vectors along the 3 mutually orthogonal perpend orthogonal axis where D x will be equal to epsilon x z E z, D y will be equal to epsilon y z E z and D z will be equal to epsilon z z E z, where again this epsilon x z, y z and z z are the permittivity components in this case.

So now we will arrange all the 3 electric field vectors to be representing in one arbitrary direction.

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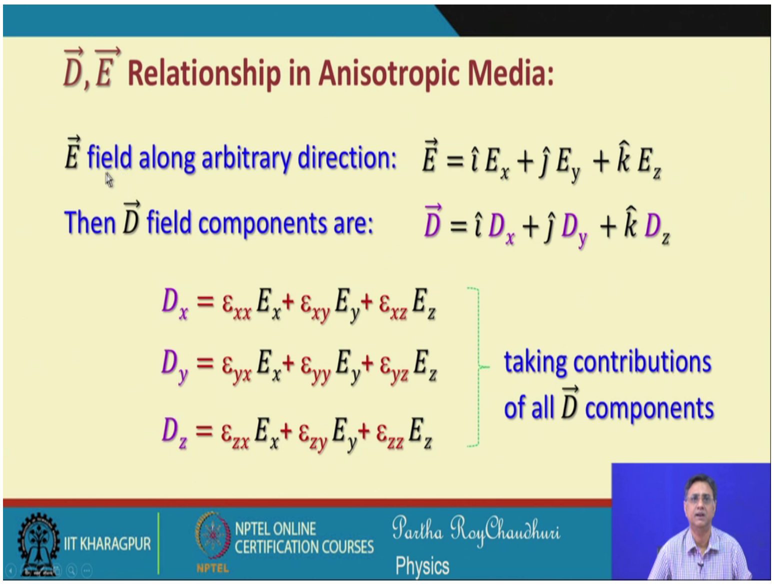

That is, let us consider that the electric field is now along any general direction that is E will have 3 components E x, E y and E z. As a result, the displacement field components will also be represented by D x, D y and D z, but they have a relationship. Because this D x D y and D z all of them will end it is excitation from E x from E y as well as from E z. So, when you apply an electric field E x you get E x x E x E epsilon y z epsilon y x E x and epsilon z x E x. So, these are the 3 components which are due to the electric field E x, these 3 components are due to the electric field E y and these 3 components are the electric are the displacement component which is due to the z component of the electric field. So, these are 3 simultaneous equations.

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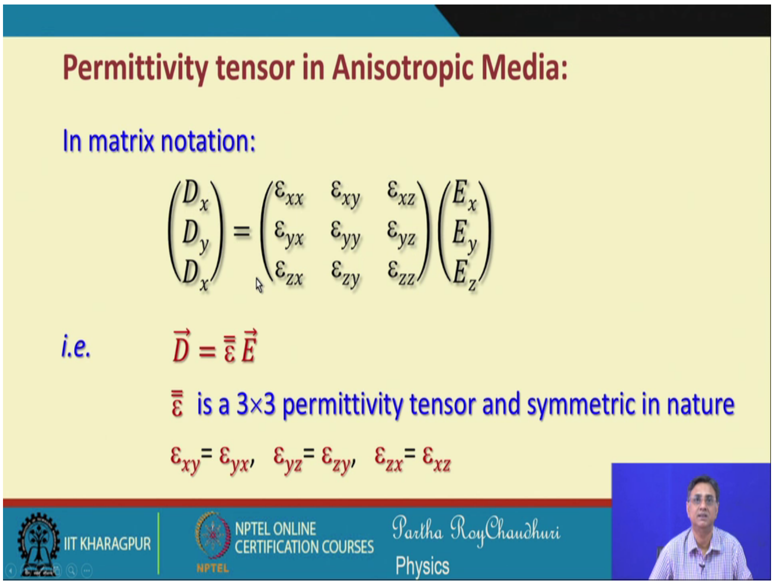

So, which can be which can be written in the in the form of matrix equation that is D x, D y, D z will be equal to this all the all the 9 components 3 by 3 matrix; E x x E x y E x z and so on, multiplied by this E x E y and E z. In a compact notation we can write this matrix equation as D equal to epsilon tensor into the electric field. So, this epsilon tensor as you have seen is a 3 by 3 permittivity tensor. And this tensor you see that it is symmetric in nature; that means, that epsilon x y will be equal to epsilon y x, epsilon y z will be equal to z y and so on, that is epsilon z x will be equal to x z. So, they are symmetric in nature this component will be equal to this component, this one will be equal to this one and this one will be equal to this one. And we will find lot of interesting applications using this particular property.

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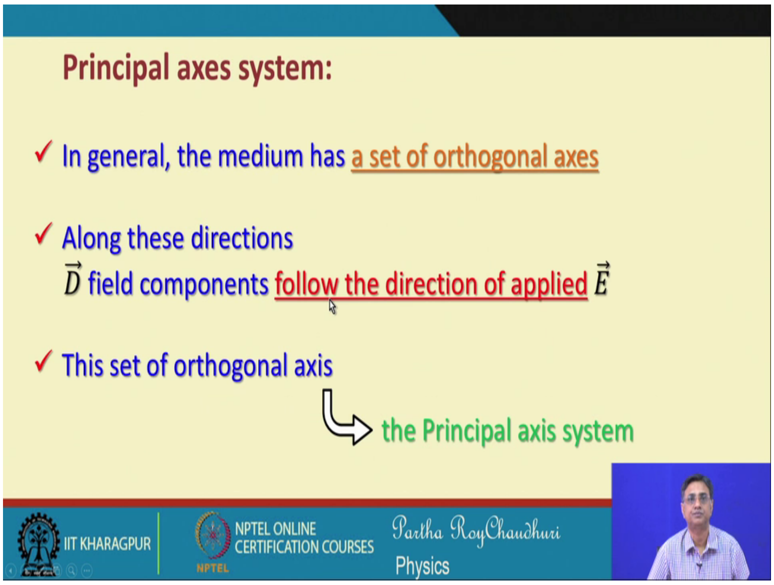

So, what you find so far is that the principal in the principal axes system, in general the medium has a set of set of 3 orthogonal axes. And along these directions D field components follow the direction of the applied E. This is an interesting finding that if we if we search, if we look for a set of orthogonal coordinate axes within the medium, there is one representing there is one which will correspond to the system that in which, the electric field will follow the displacement field will follow the electric field in the same direction. That means, when you apply the electric field along the x direction due will be only along the x direction and so on. Such a set of orthogonal coordinate axis is known as the principle axes system of the medium and that is, the characteristic property of the medium.

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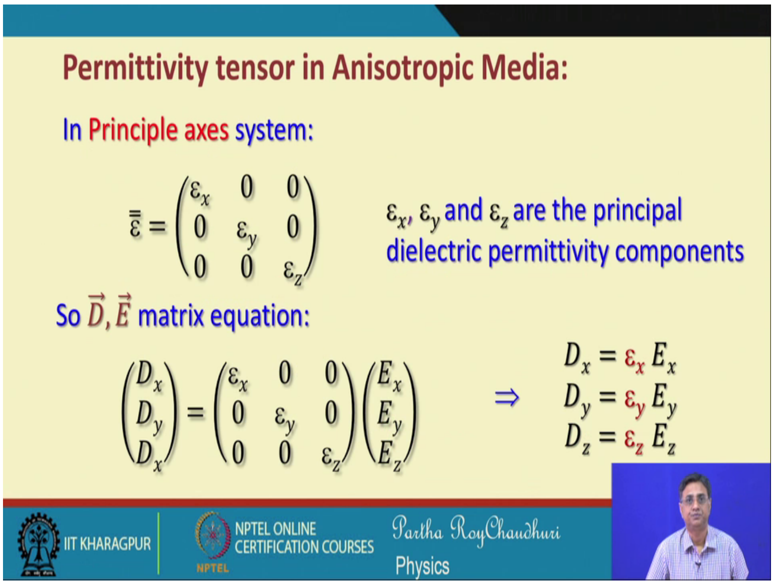

So, in such a principal axes system we can write the epsilon permittivity tensor as epsilon x x, epsilon y y and epsilon z z. If we if we represent with a sort notation then we can write that this is because there is no other components no other cross components x y, y z or z x. So, we can write that epsilon tensor as epsilon x, epsilon y and epsilon z.

These are the 3 principal dielectric permittivity components of the medium so; that means, we could find one orthogonal coordinate system, orthogonal axes within the medium along which the displacement fields will follow the electric field. And if you apply an electric field along the x direction the displacement will be only along the x axes and so on and so forth, for the y and z component z component of the displacement vectors, when you apply electric field along the x and y directions respectively.

So, D and E matrix equation in the case of the principal axes system for the medium can be represented by this equation; that is, D x, D y, D z is now a simple diagonal matrix E x epsilon x, epsilon y and epsilon z multiplied by E x, E y, E z. So, simply there will relate themselves in this way the D x is equal to epsilon x E x D y equal to epsilon y E y and so on.

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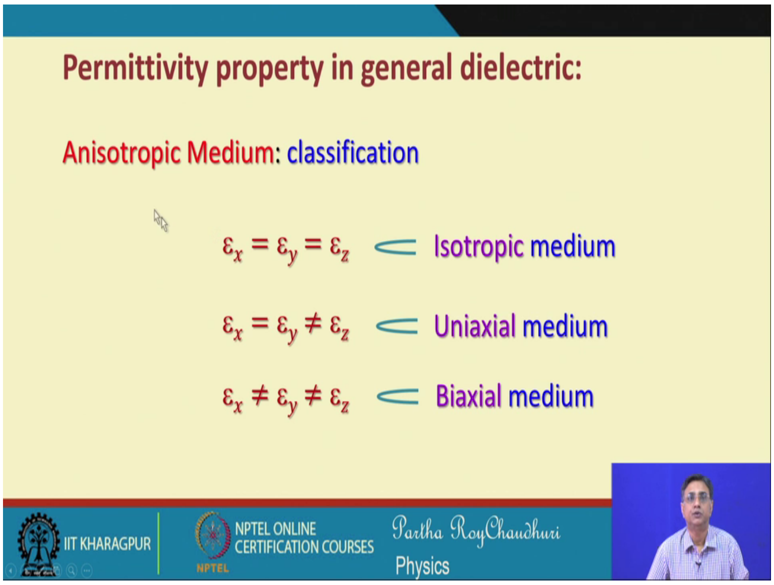

So, for anisotropic medium there are possibilities that E x and E y and E z maybe related in some way or other. For example, if E x E y epsilon x epsilon y and epsilon z all of them are equal; that means, the electromagnetic wave is direction independent. Then you call that the medium is anisotropic medium. So, in anisotropic medium there for all directions, whether it is incident along any electric field is incident along any arbitrary direction or along x direction or along y direction or any other direction.

So, the displacement vector will follow the direction of the electric field, which is anisotropic medium, but there is there is another situation where 2 of them may be equal, but is not equal to the third one. For example, epsilon x is equal to epsilon y, but not equal to epsilon z. In that case this such a medium will be called a Uniaxial medium. Whereas, there is another situation where, where you will find another group of media, where this epsilon x and epsilon y epsilon z all 3 of them are different. In general, such a medium will be called a Biaxial medium; that means, in a Biaxial medium the refractive indices or the permittivity seen by the wave will be all different in the all 3 mutually orthogonal directions.

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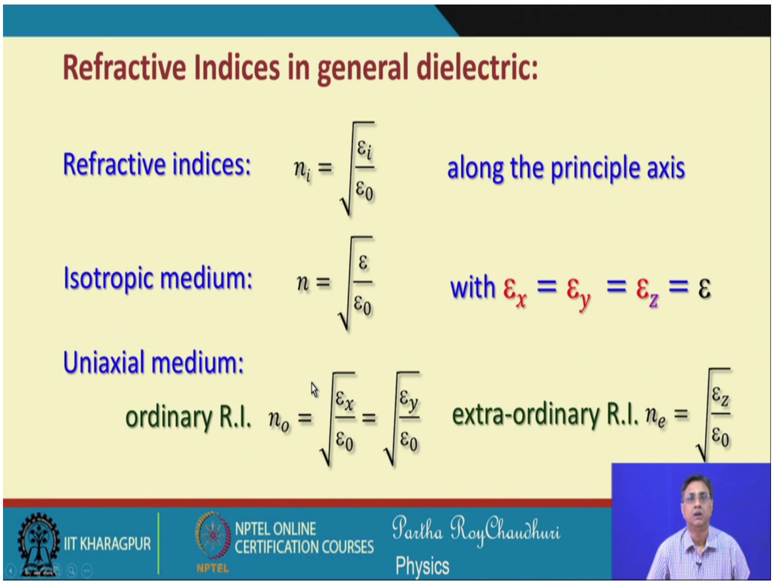

So, the refractive indices we can write in this form that; n i is equal to epsilon i epsilon 0, where this is the refra permittivity. And this is the free space permittivity the ratio and square root of that will represent the refractive index. So, that is along the principal axes. Now, for anisotropic medium this is in general all of them are equal. So, epsilon x equal to epsilon y equal to epsilon z is equal to epsilon; so, which will be represented by only one epsilon for all the directions. For Uniaxial medium; however, there will be one value for which along 2 directions the refractive indices or the permittivity seen will be identical whereas, this quantity will be different for the third direction.

The one along which the refractive index seen by the wave is different is called the extraordinary refractive index. And the wave corresponding to that refractive index will be call the extraordinary wave whereas, the one for which there are 2 directions for which the refractive indices are the same, the permittivity's are the same we call that is the ordinary refractive index. And the wave corresponding to that refractive index direction will be called the ordinary wave.

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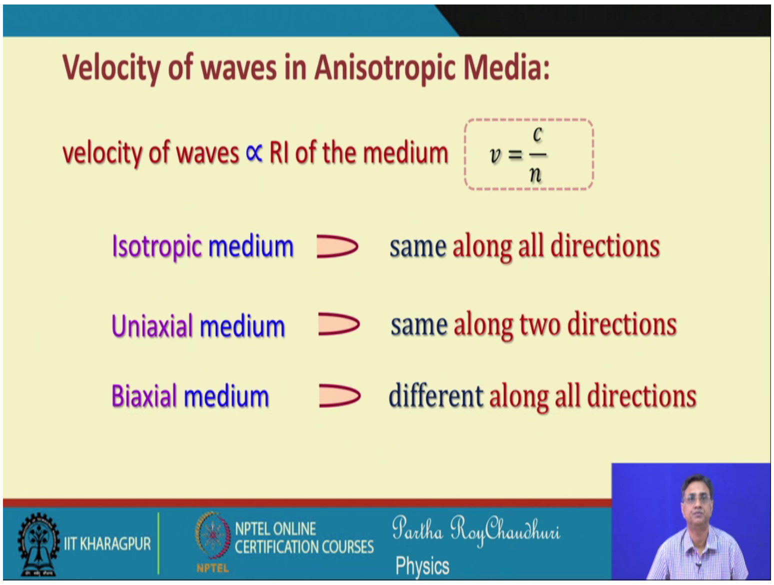

So, to summarize that the velocity of the waves will be proportional to the refractive indices, this is in general true. And for isotropic medium therefore, the velocities of the electromagnetic waves will be same in all directions; whereas, for Uniaxial medium the velocities will be different along 2 different directions, but for Biaxial medium the velocities or different along all the directions.

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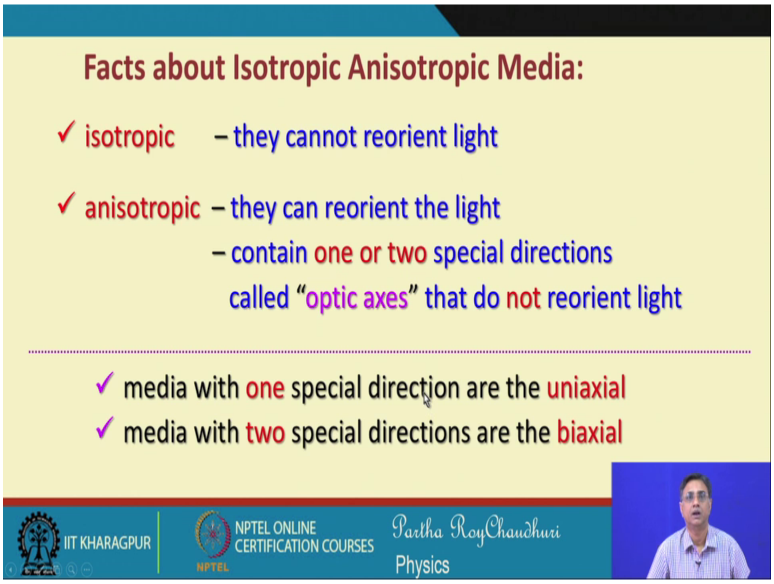

Now, some more facts about the isotropic and anisotropic media, for an isotropic medium there cannot this medium cannot reorient the light, because the light which is incident along a particular direction in such a medium will immerge only along this direction. So, there is no effect, there is no activity of the medium on the travelling electromagnetic waves as regards the direction of propagation. So, the direction of propagation will not be disturbed will not be influenced by the medium; whereas, the situation is different for anisotropic medium. And this property will be extensively used in the case of various modulators and devices, which will be will be discussing later in the later part of this course.

So, for anisotropic medium, the medium can reorient the light that is, the direction of the incident electromagnetic waves can be changed by the anisotropic medium by it is property. And also the velocities can also be also be altered. So in general, such a medium contains one or 2 special directions. And these directions are called the optic axis along which they do not reorient the light. So, in anisotropic medium particularly, in a in a Uniaxial medium there will be directions along reach the electromagnetic waves direction will not be altered. And in the other direction the direction of the electromagnetic waves will be altered.

So, to summarize that a medium with one special direction are called the Uniaxial medium, one special direction means; the direction along which the direction of the propagation of electromagnetic waves will not be altered, such one special direction material or the medium are called the Uniaxial medium, but for a Biaxial medium there will be 2 such special directions along which Biaxial. So there are 2 axes, there are 2 direction along which the electromagnetic waves direction will not be changed, when compared to or with regard to the direction of the incident electromagnetic waves.

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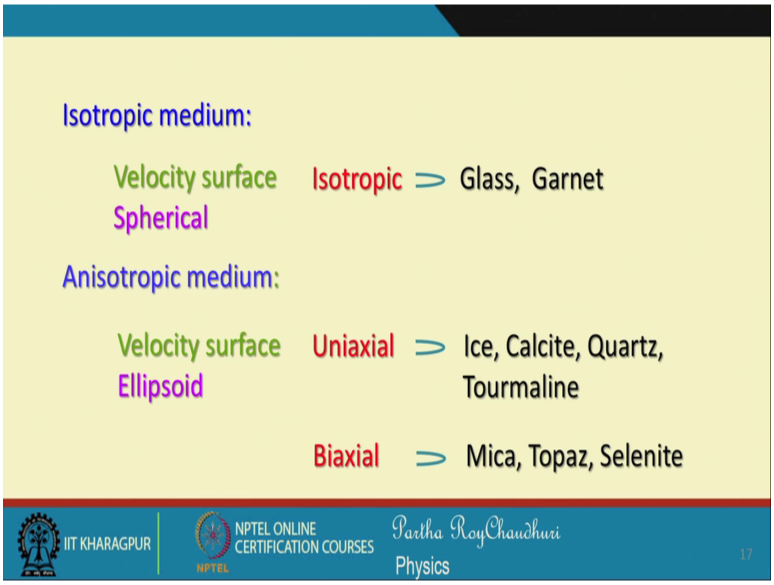

So, for isotropic medium velocity surface will be a spherical that is; if I if I image in the wave front in the all 4 pi solid angle directions, then it will be spherical because the direction the velocity is equal in all directions. The examples of such a medium are ordinary Glass, Garnet and many more. For anisotropic medium the velocity surface will be in general and Ellipsoid. We will discuss more about this Ellipsoid in terms of the index Ellipsoid in the later part of this section. So, for a Uniaxial medium the examples are like Ice, Calcite, Quartz, Tourmaline and these are very widely used in designing an in configuring various optical instruments and devices. Biaxial medium examples are Mica, Topaz, Selenite.

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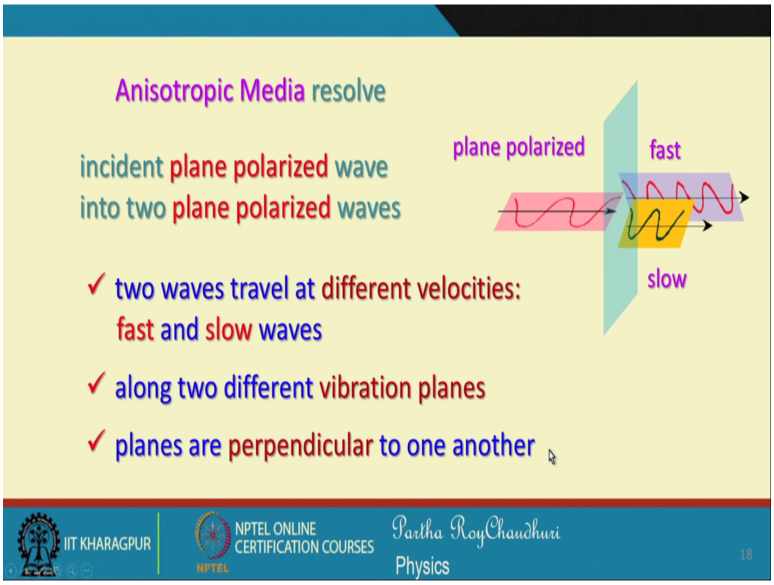

So, in anisotropic medium, incident plane polarized light will decompose into 2 plane polarized light. So, let us suppose that you have an incident plane polarized light this will immerge out from the medium or within the medium, within the medium in 2 mutually orthogonal directions. And in general the 2 waves travel at different velocities these are; the one which is faster will be called the fast waves, and the one which is slower will be the slow waves.

And, in general because they are travelling with different velocities. So, they will develop a different phase difference and this phase difference will be, will be utilized to construct devices. Along the 2 different vibration planes these 2 waves will be travelling; and the planes are perpendicular to each other, the vibration planes are perpendicular to each other.

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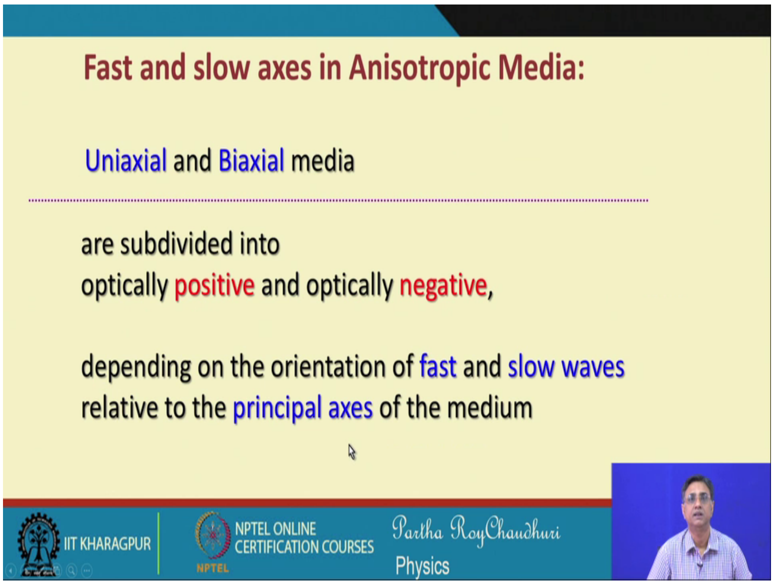

There are some more things so Uniaxial and Biaxial media are against subdivided into positive and negative optically active media depending on the orientation of the fast and slow waves. So, in some medium the one which is fast waves, in some other medium the same will be referred to as a slow waves. So, this is all relative to the principle axes system of the medium.

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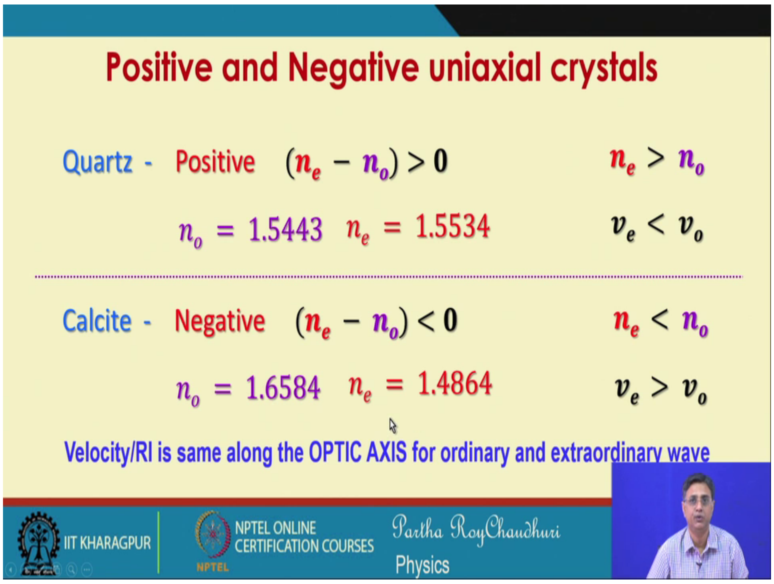

For quartz there is a beautiful example, which is called a positive crystal n e is, is more than n o that is, the extraordinary refractive index is more than ordinary refractive index.

So, the typical values are like this 1.5443, 1.5534. And therefore, the velocity of the of the e, e wave will be less than the velocity of the o wave. For Calcite, which is a negative crystal the situation is just opposite that is n e, n e is less than n o. And the typical values are like this; so you can see that the extraordinary wave travels with a higher velocity then the ordinary wave. So, the velocity or the refractive index is the same along the optic axis for ordinary and extraordinary wave both their same.

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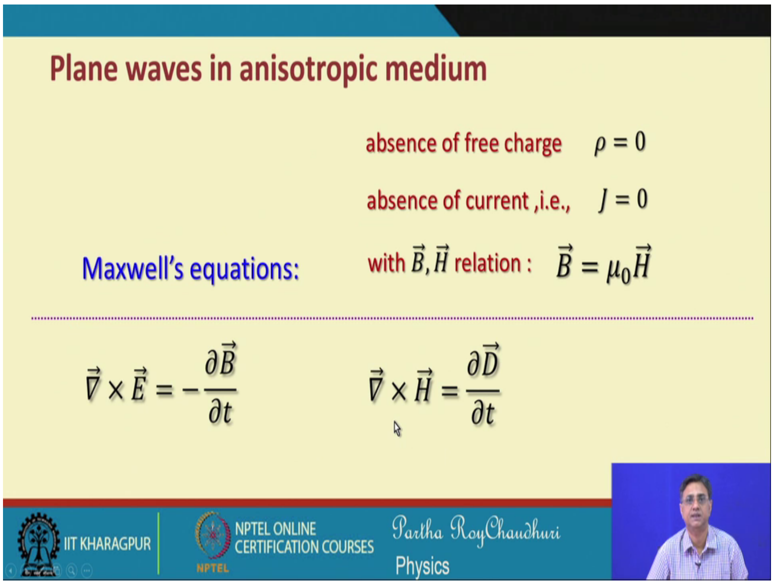

Now, we will we will look at the plane wave equation, solution for this anisotropic medium. And this is the property that we assume that there is no free charge, there is no free current and B and H they are related by this constitutive relation B equal to mu naught H in such a medium. So, these are the 2 Maxwell's curl equation from which; we can we can derive we can organize the wave equation for the anisotropic medium.

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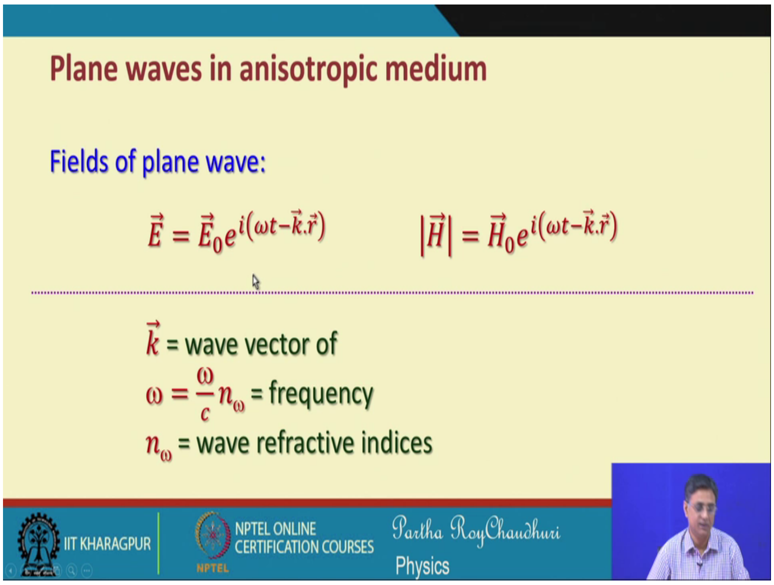

Fields of plane waves are represent we have seen that fields of plane waves are represented by E equal to E naught e to the power of i omega t minus k dot r, and similarly for the h field we can represent in this way.

Where, k is the wave vector and E this is the frequency of the electromagnetic waves and this is the wave refractive indices, we will see that there are 2 refractive indices one is the ray refractive index another is the wave refractive indices.

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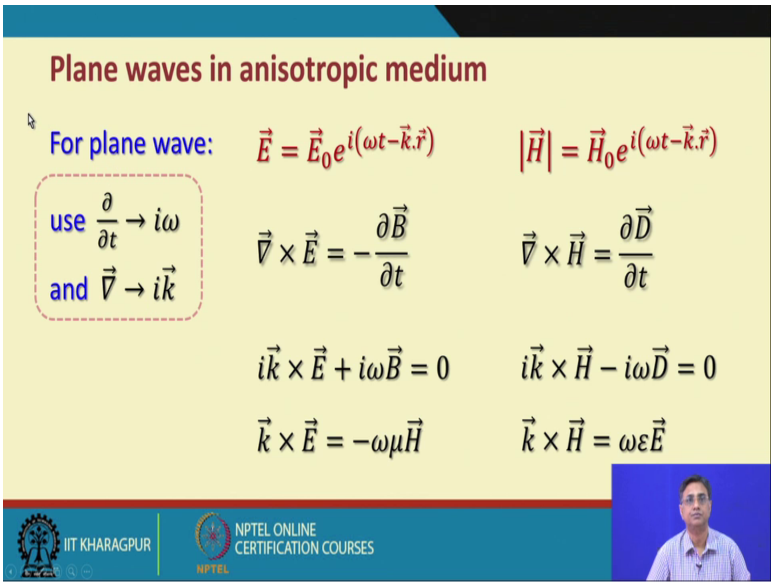

So, if we now use this del del t, as to represent this i omega because this plane wave equation this is true that if you operate this del del t on this equation it will just leave i omega multiplied by this E itself. So, we can write the first equation which is which is which will give you i k cross E. This we have seen earlier again will see in details and i omega B. So, this will give you this equation and from other equation we can write that k cross H equal to omega epsilon E, where epsilon in this case in the case of anisotropic medium is a tensor.

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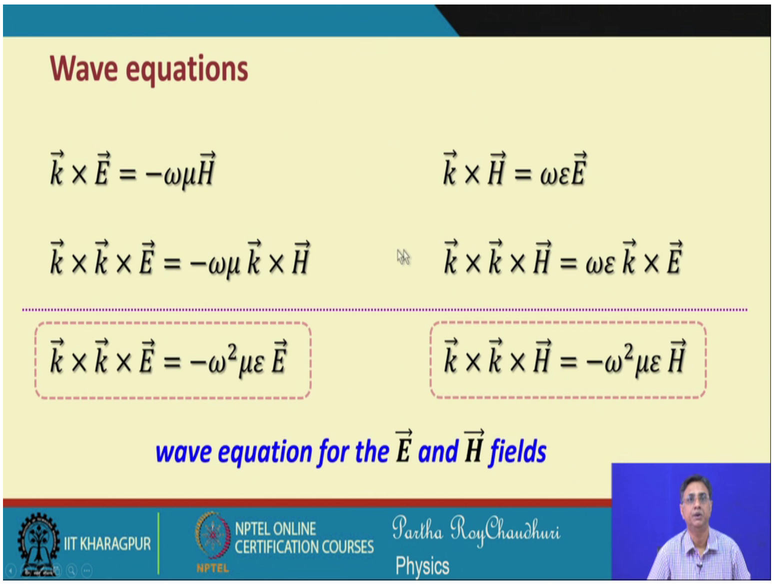

So, if we organize this equation k cross E equal to minus omega mu H then you can take k cross k cross E, which will be equal to this quantity. And this actually represents the wave equation, which is purely in terms of the electric field. And in the same way if we write this equation starting from k cross h equal to omega epsilon E and we replace E by and we replace this by k cross E, then k cross k cross H will be equal to omega square mu epsilon H. So, these 2 are the wave questions pair of a set of wave equation which will represent the electromagnetic waves in an isotropic medium for both the electric field and also for the magnetic field.

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So, in this section what we have discussed is the relation between the displacement field and the electric field incident electric field, there connection in terms of the direction and also in terms of the magnitude, which is actually connected by the permittivity values. Then we discuss the Permittivity tensor for the anisotropic medium. From there we have seen in a very special situation that for a for a for a set of orthogonal coordinate axes characteristic of the medium. We could look at the principal axes system and the values of the permittivity in that Principal axes system that also have seen. Then we have made a small comparison about the Isotropic Uniaxial and Biaxial medium then we talked about the wave equation in Anisotropic medium.

In the next section will continue with the wave equations in the anisotropic medium, and we will see the various aspects of the waves; the relation of the general propagation vector, the electric field, the displacement vector, the pointing vector the magnetic field all these things will be will be connected. And we will see how the propagation of the electromagnetic waves gives rise to various phenomena in general Anisotropic medium.

Thank you.