2.4: Hyperbolic Navigation

Parabola Equations and Graphs

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LORAN is a long-range radio navigation system that uses land-based radio transmitters to aid ships, aircraft and terrestrial navigators. The LORAN (Long Range Aid [to] Navigation) system transmits radio signals at precise time intervals from each of its transmitting stations. LORAN is able to locate objects using these radio signals through the use of hyperbola.

Driving Question: How can we use conic sections to navigate on sea, land or air?

1. In the LORAN system, hyperbolas are used onboard a ship to determine its location. In the figure below, radio stations transmit signals simultaneously for reception by the ship at $P(x, y)$.
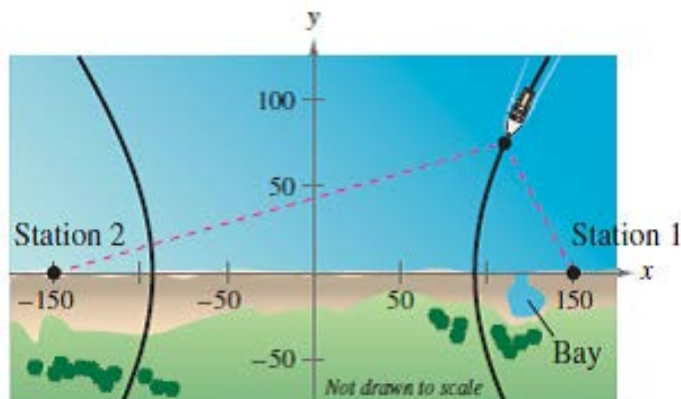

   a. What are the coordinates of the *foci* (the two radio stations)?

2. Describe the shape of a hyperbola. What cut in a conic section results in a parabola?

   Hint: Revisit the **interactive conic section** located at http://ggbtu.be/mqZ8aGDzR if needed.

Definition of Hyperbola

A **hyperbola** is the set of all points in a plane, the difference of whose distances from two distinct fixed points **(foci)** is a positive constant.
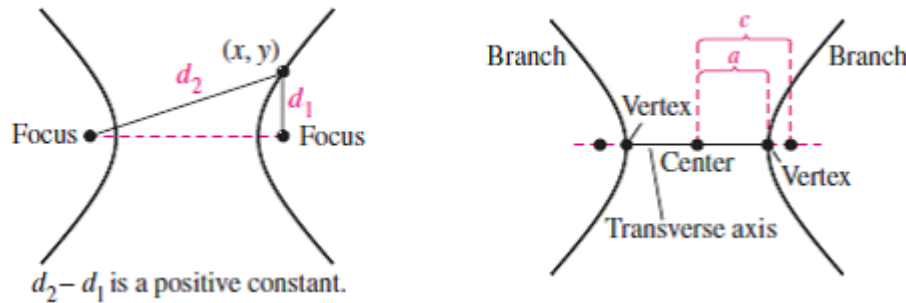

3. How does the definition of a hyperbola differ from that of an ellipse?

4. Why does this difference result in the **two disconnected branches** of a hyperbola?

   Explore the **interactive hyperbola** located http://ggbtu.be/mDYsxjZXS, if needed.

5. Based on the standard form of an ellipse, write the standard form equation of a hyperbola?

   Explore the different applet located http://ggbtu.be/mZNOTugYC, if needed.

6. Write the standard form equation for the ship's location $P(x)$ in the diagram below. Assume that two stations, 300 miles apart, are positioned as pictured.
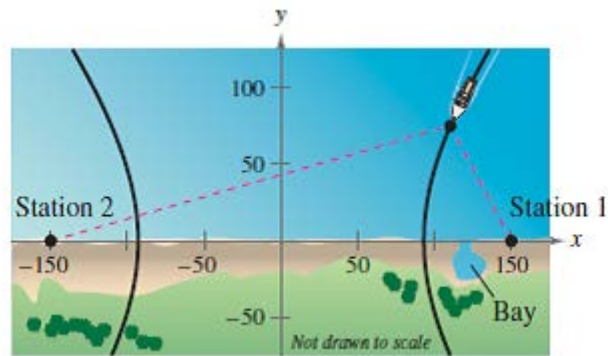
7. Graph the standard form equation for the ship's location $P(x)$ in Desmos and insert below.

8. Zoom out several times.
    a. What does your graph resemble?

    b. Graph the two asymptotes, zoom back in and insert your image below.

9. Switch the signs of the two fractional expressions in your equation from question 7. Do the asymptotes still "fit" your graph? Insert the new graph below and label it with its corresponding equation.

10. What happens to the graph when both fractional expressions are both positive? Explain why this happens.