THEORIE DE LA DETECTION DU SIGNAL

La Théorie de la Détection du Signal (TDS) est à la fois un *instrument de mesure de la performance** et un *cadre théorique dans lequel cette performance est interprétée* du point de vue des mécanismes sous-jacents. En tant que telle, la TDS peut être regardée comme étant à la base de la psychophysique moderne. Le cours développera les bases de la TDS, discutera sa relation avec les concepts de seuil sensoriel et de décision et exemplifiera son application dans des situations expérimentales allant depuis le sensoriel jusqu'au cognitif.

*Signal Detection Theory is a computational framework that describes **how to extract a signal from noise, while accounting for biases and other factors that can influence the extraction process**. It has been used effectively to describe how the brain overcomes noise from both the environment and its own internal processes to perceive sensory signals (from **Gold &Watanabe (2010). Perceptual learning.** *Current Biol., 20*(2) R46-R48.

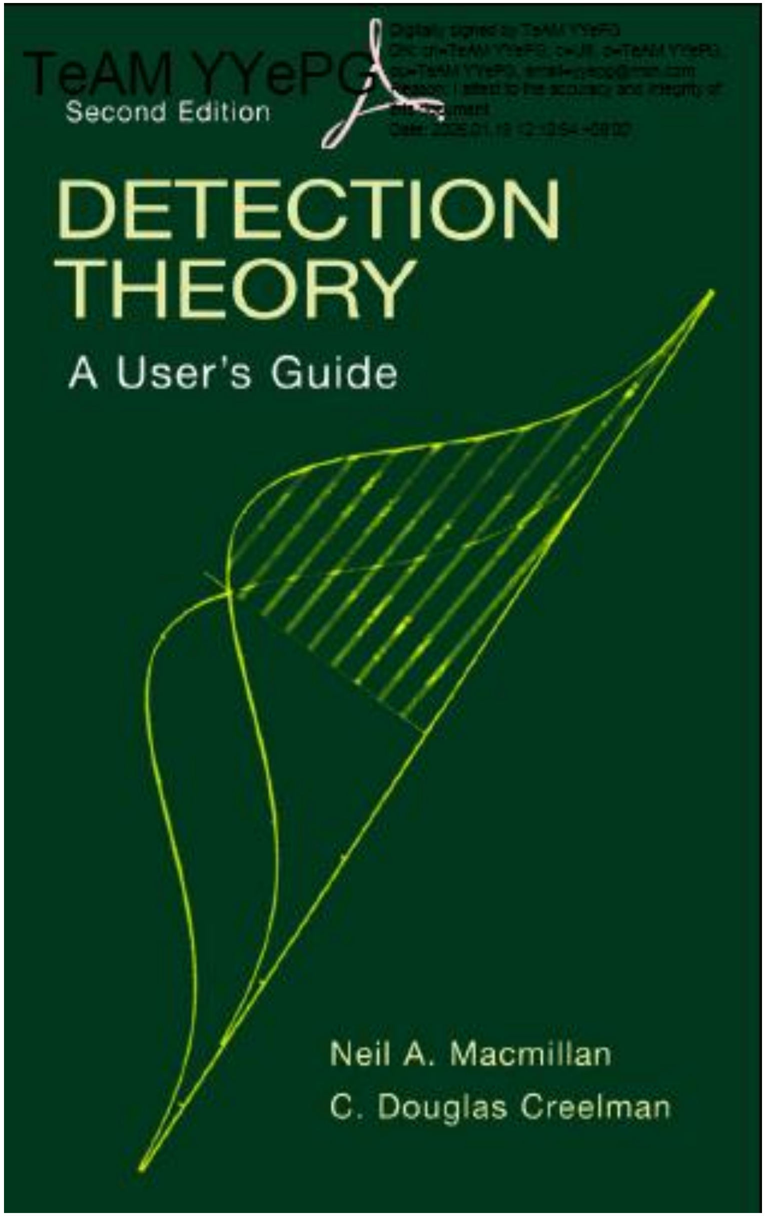
SIGNAL DETECTION THEORY AND PSYCHOPHYSICS

David M. Green & John A. Swets

1st published in 1966 by Wiley & Sons, Inc., New York

Reprint with corrections in 1974

1988 reprint edition published by Peninsula Publishing, Los Altos, CA 94023, USA



Detection Theory: A User's Guide

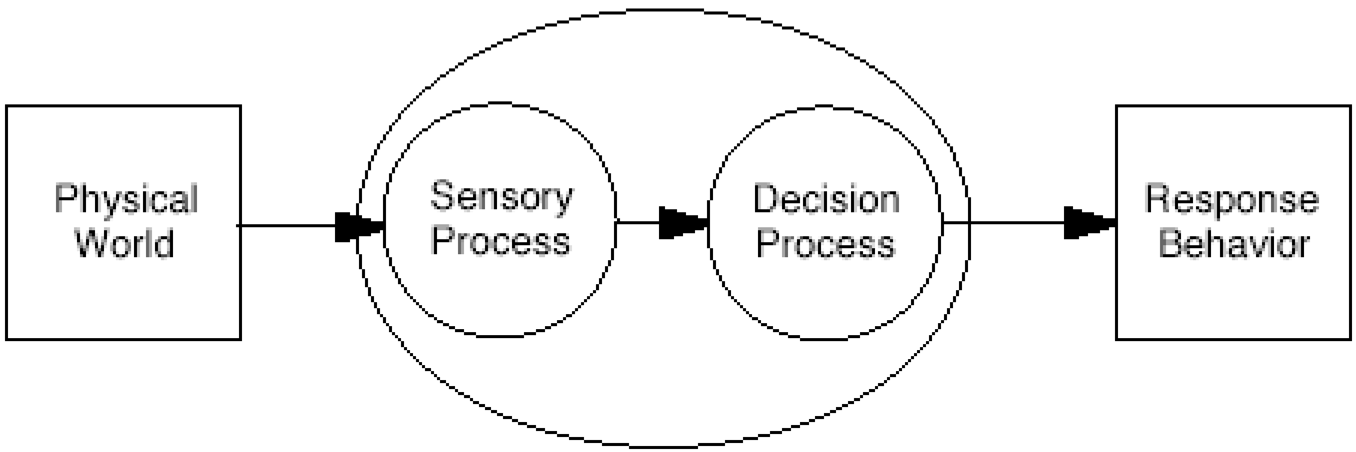
(2nd edition)

NEIL A. MACMILLAN
University of Massachusetts

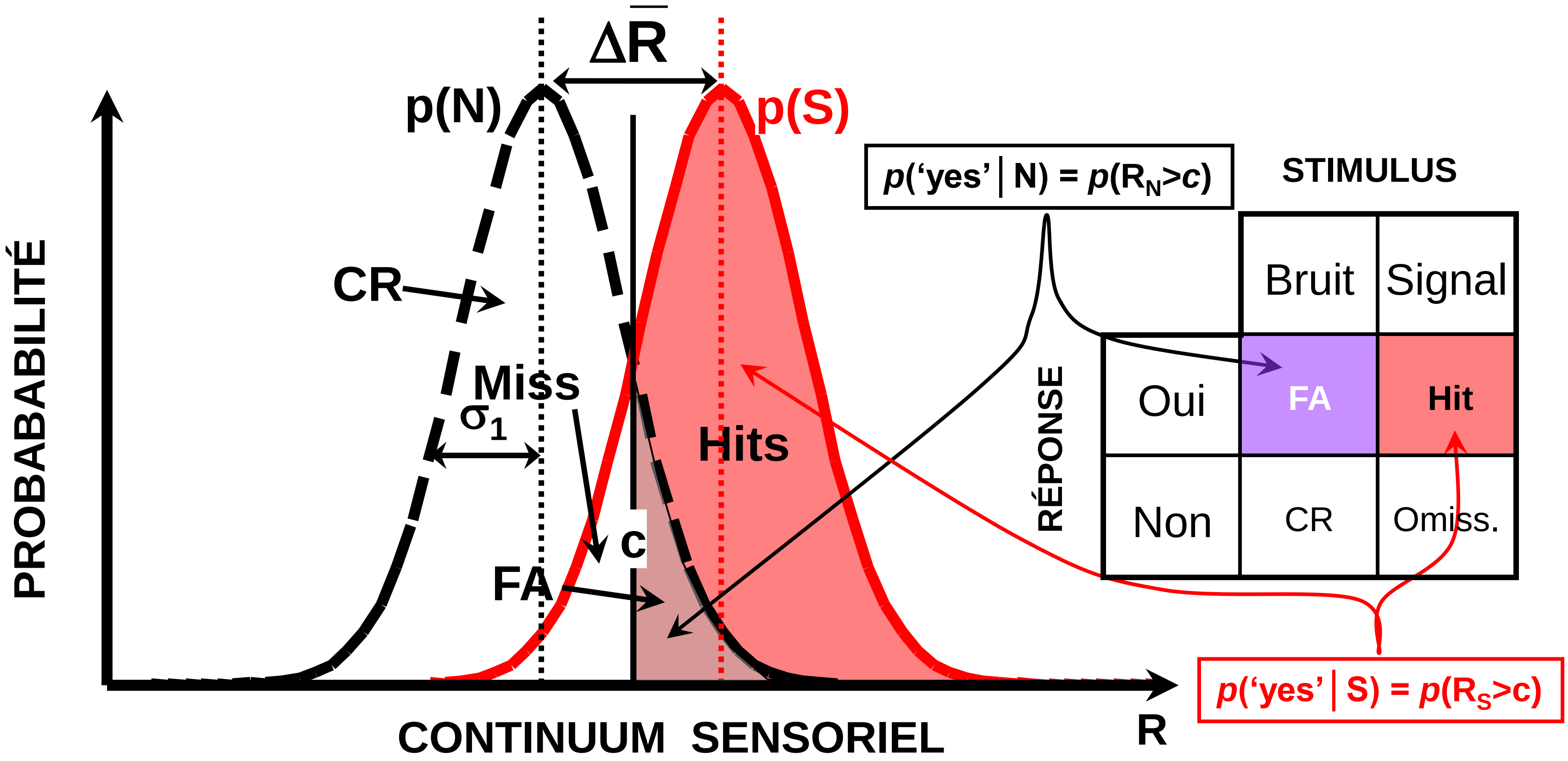
and

C. DOUGLAS CREELMAN
University of Toronto

LEA LAWRENCE ERLBAUM ASSOCIATES, PUBLISHERS
2005 Mahwah, New Jersey London


Physical World
Sensory Process
Decision Process
Response Behavior

LE MODEL STANDARD :
THEORIE de la DETECTION du SIGNAL



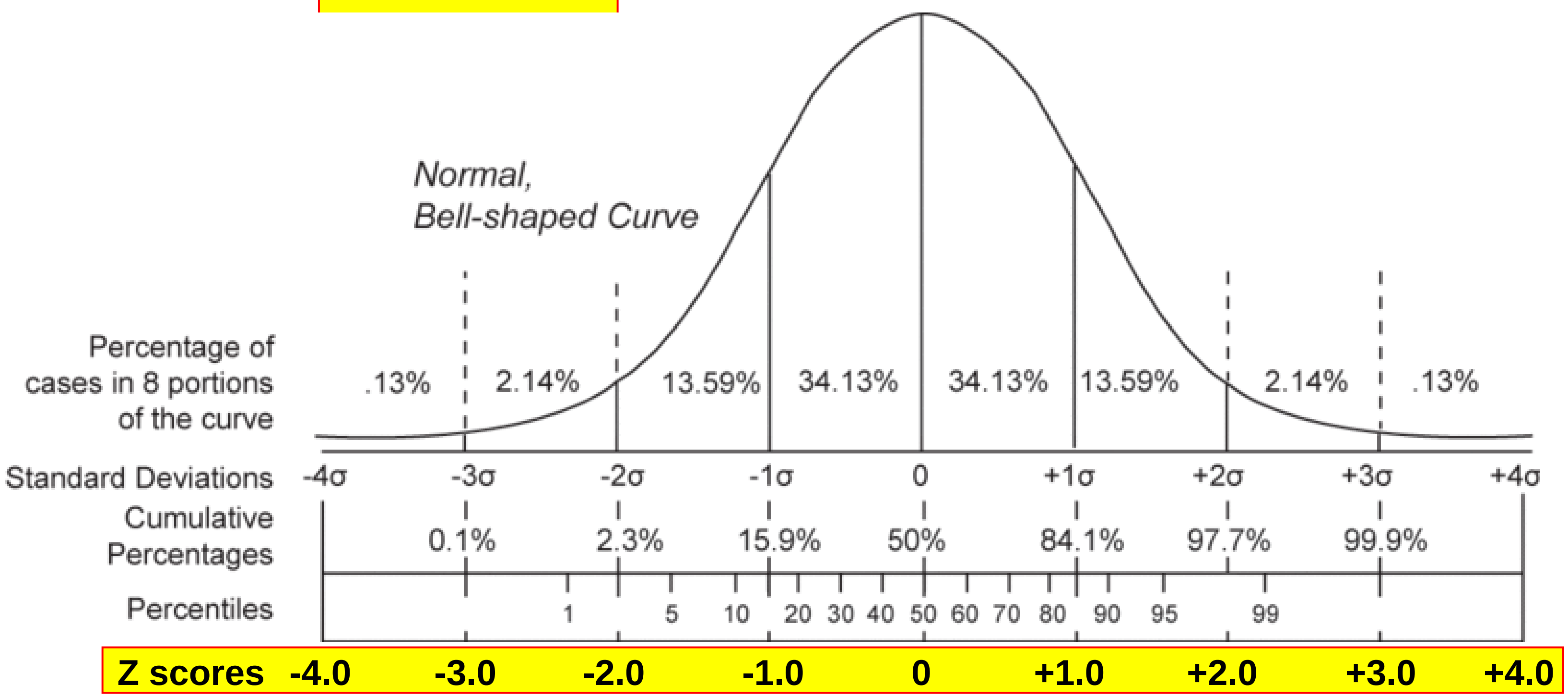
$$d' = \frac{S}{B} = \frac{\Delta\bar{R}}{\sigma} \Rightarrow z(\text{Hits}) - z(\text{FA})$$

$$c_0 = -.5\left[z(H) + z(FA)\right]; \quad c = -z\left(FA\right)$$

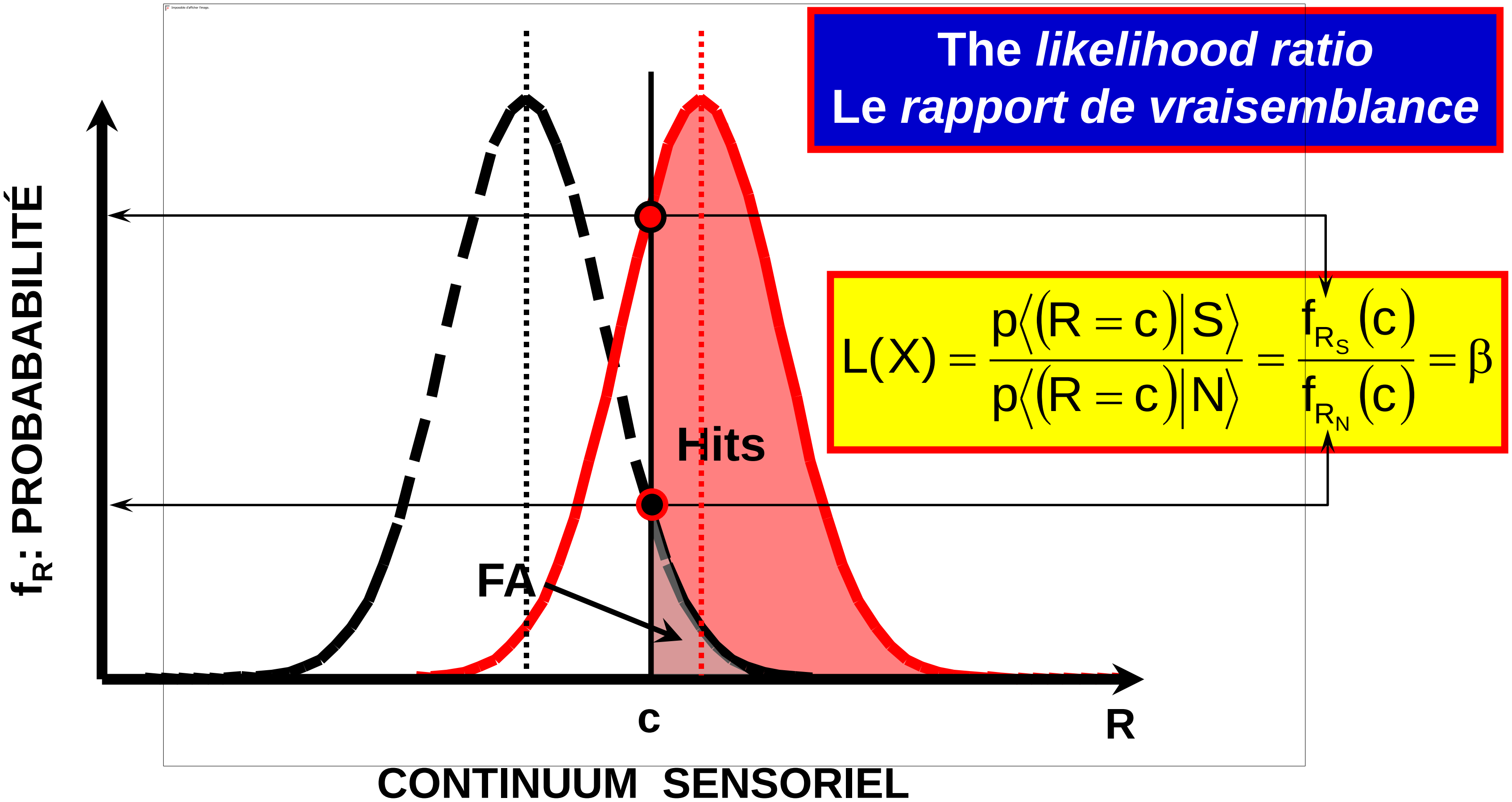
Score z

Ou « score normal » ou « score standard »

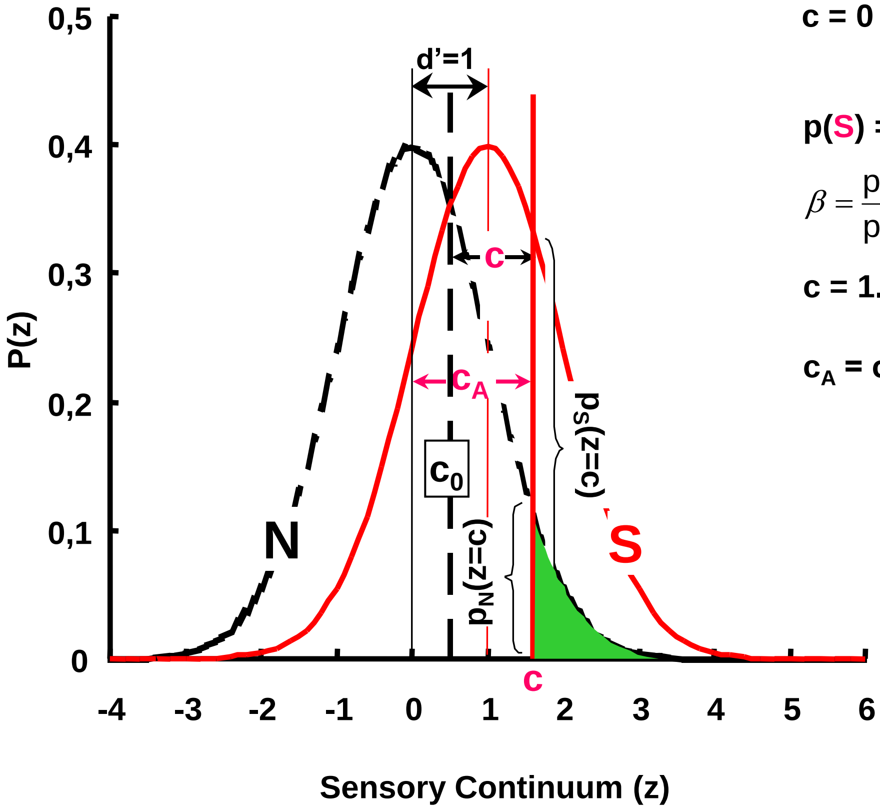
$$z = \frac{X - \mu}{\sigma}$$



LE MODEL STANDARD :
THEORIE de la DETECTION du SIGNAL



LE MODEL STANDARD :
THEORIE de la DETECTION du SIGNAL



p(S) = . 50

β = 1

c = 0

p(S) = .25

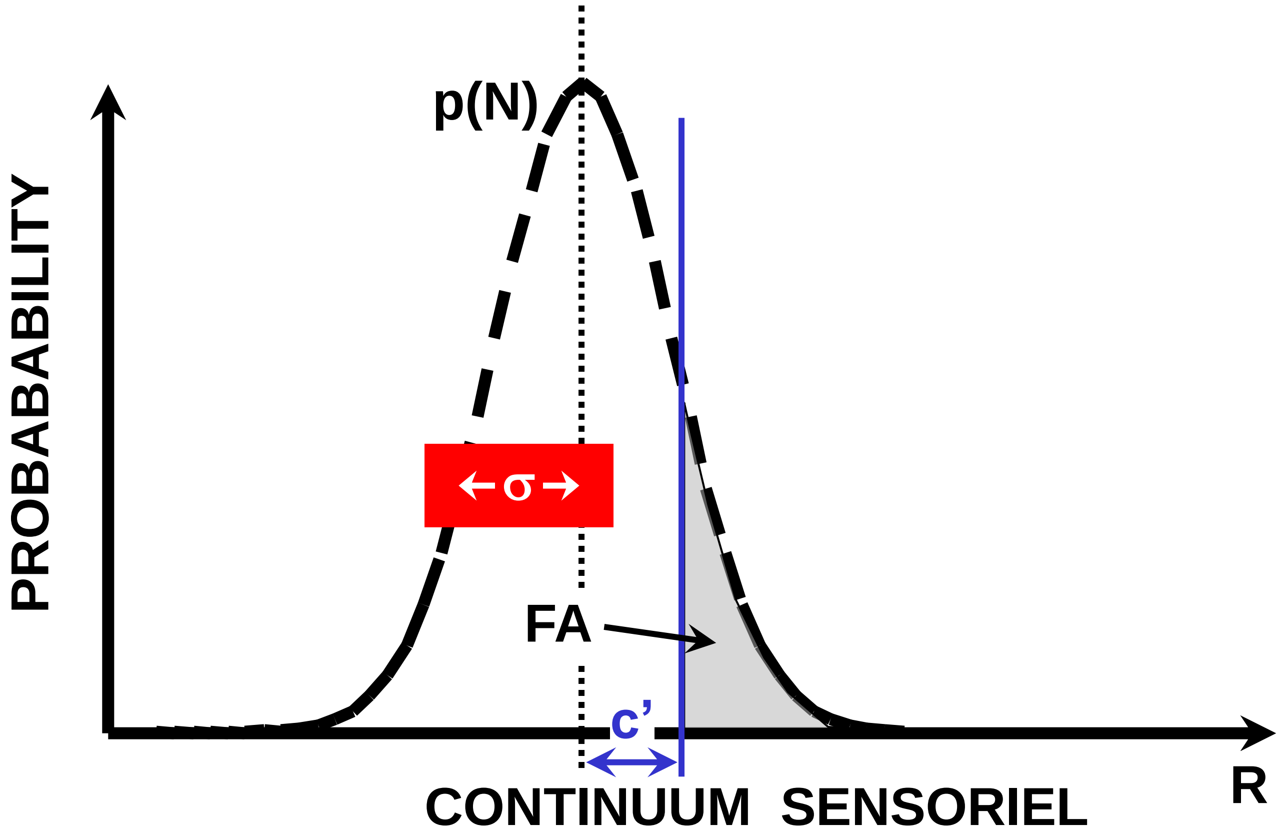
$$\beta = \frac{p_S(z=c)}{p_N(z=c)} = \frac{p_N}{p_S} = 3$$

c = 1.10

c_A = c + d'/2 = 1.60 = -zFA

THE STANDARD MODEL: SIGNAL DETECTION THEORY

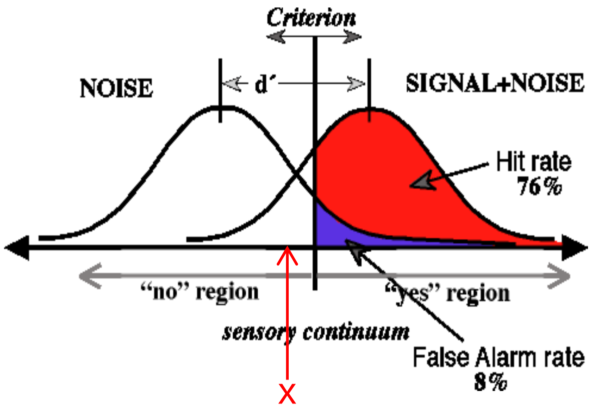
Criterion: the border between seen and not-seen



$c' = -z(FA)$ → c' ⇒ in σ units!

Criterion setting

- **Observers set a criterion that attempts to minimize the total error rate (pMiss + pFA).**



- **For equally probable Signal and Noise, choose S and not N when**

$$p(x|S) > p(x|N)$$

i.e. when the likelihood ratio

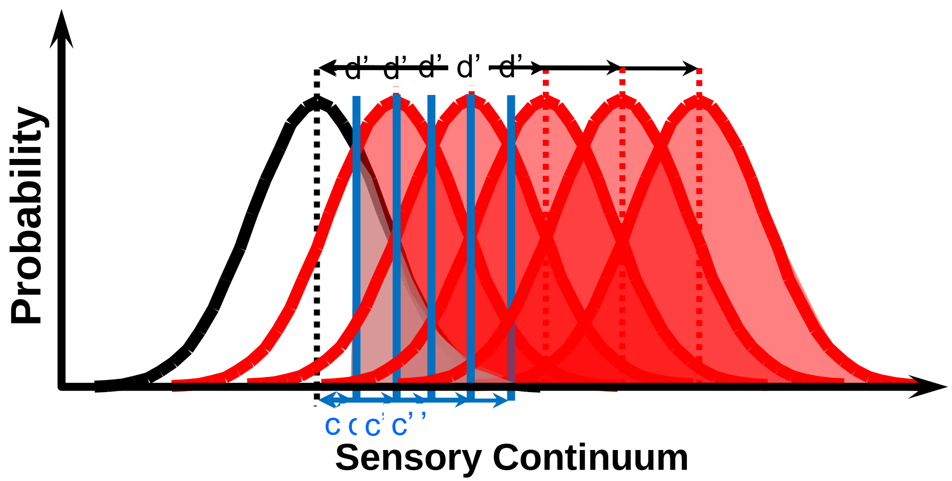
$$\beta = p(x|S) / p(x|N) > 1$$

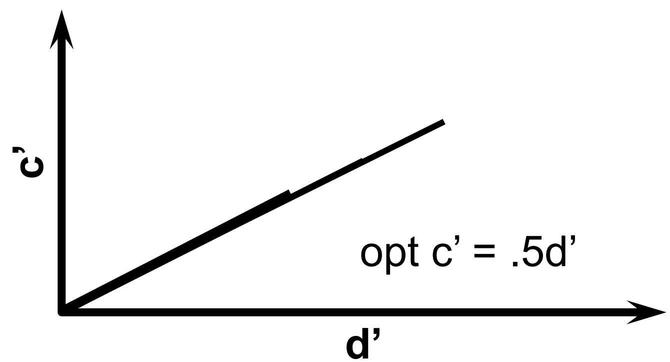
- ***For unequal probabilities of S and N, p(S) and p(N), optimal β***

$$\beta = p(N) / p(S)$$

THE STANDARD MODEL: SIGNAL DETECTION THEORY

Criterion: the border between seen and not-seen





Measured vs. Optimal criterion (β)

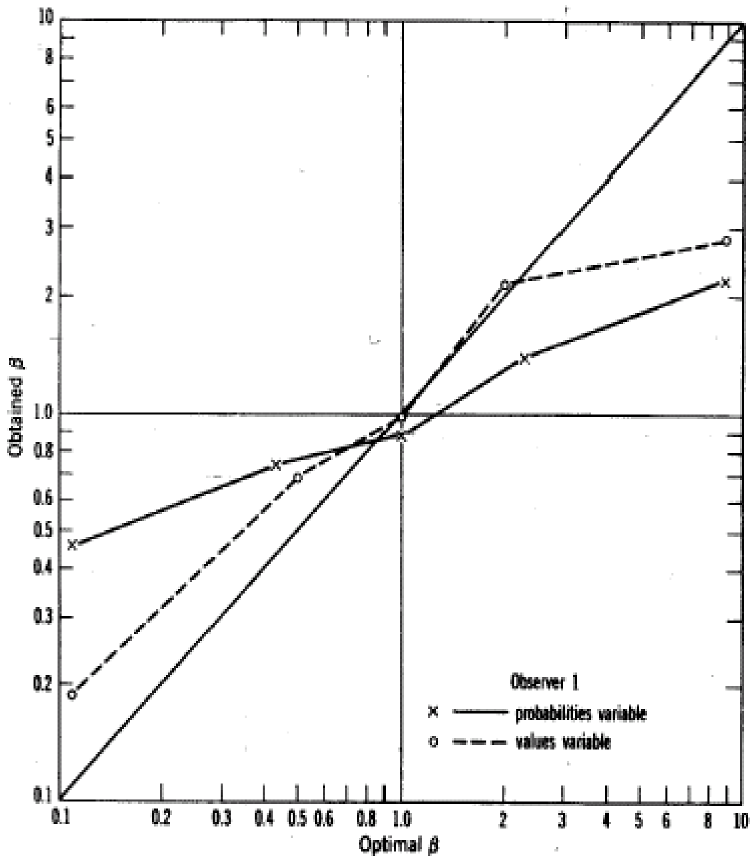
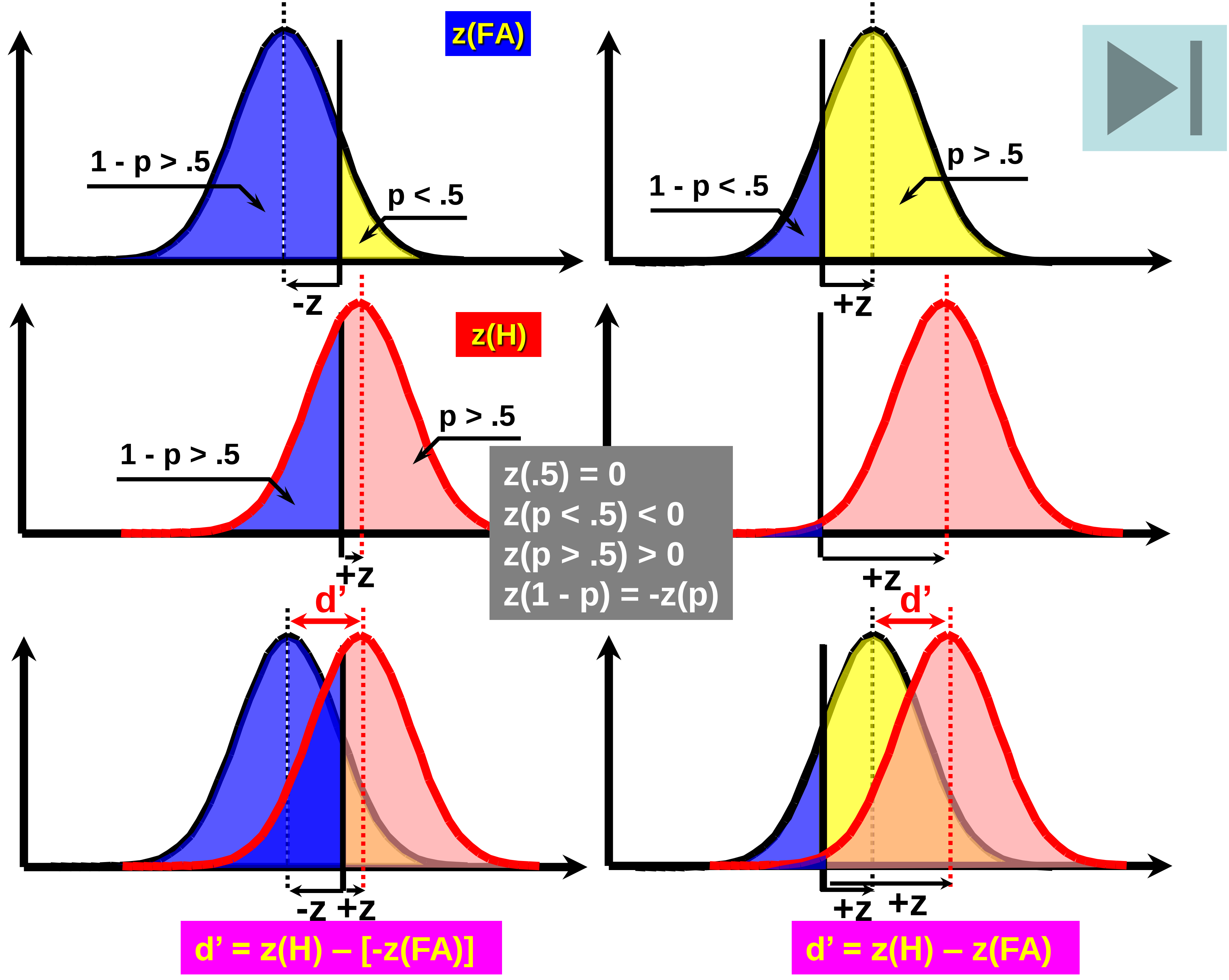


FIG. 4-3 Relationship between obtained and optimal decision criteria.

Observers are close to optimal for **p(S) = p(N)** but become more and more suboptimal with the increasing difference between p(S) & p(N)

Green & Swets (1966)



z(FA)
1 - p > .5
p < .5
-z
1 - p < .5
p > .5
+z
z(H)
1 - p > .5
p > .5
+z
z(.5) = 0
z(p < .5) < 0
z(p > .5) > 0
z(1 - p) = -z(p)
+z
d’
d’
-z +z
+z +z
d’ = z(H) – [-z(FA)]
d’ = z(H) – z(FA)

Signal detection theory: criterion effects

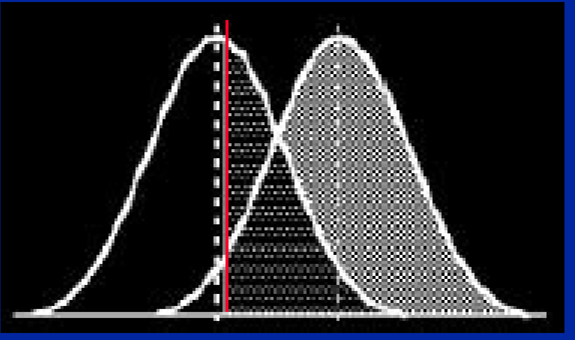
90% Signal Rate

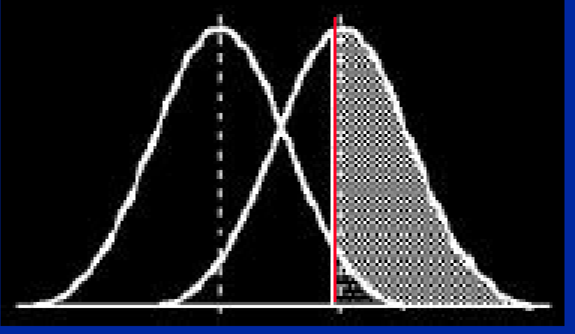
Signal	Response: Yes	Response: No
On	Hits 0.85	Misses 0.15
Off	False Alarms 0.45	Correct Rejects 0.55

10% Signal Rate

Signal	Response: Yes	Response: No
On	Hits 0.52	Misses 0.48
Off	False Alarms 0.14	Correct Rejects 0.86

Measured Threshold





d' computation with Excel

d' = LOI.NORMALE.STANDARD.INVERSE[p(H)] - LOI.NORMALE.STANDARD.INVERSE[p(FA)]

c_0 = -0.5*{LOI.NORMALE.STANDARD.INVERSE[p(H)] + LOI.NORMALE.STANDARD.INVERSE[p(FA)]}

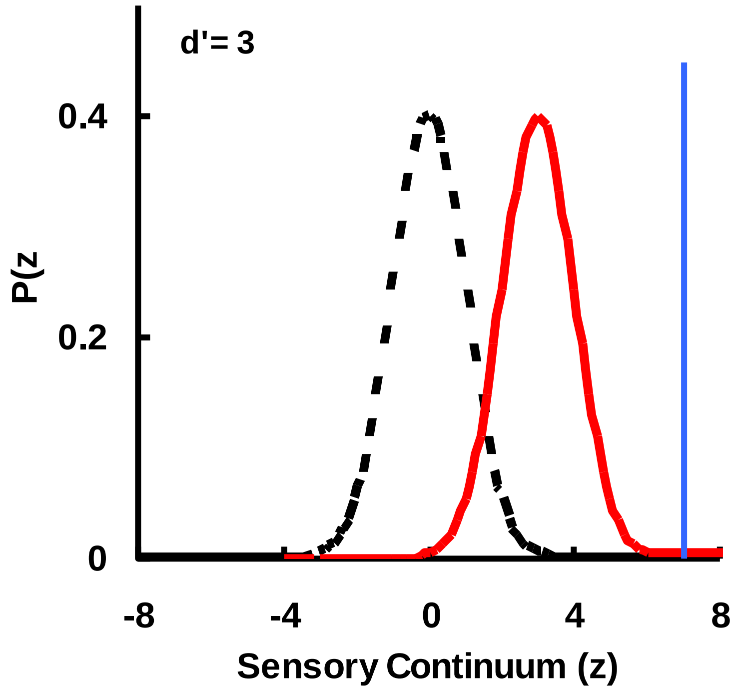
c_A = -LOI.NORMALE.STANDARD.INVERSE[p(FA)]

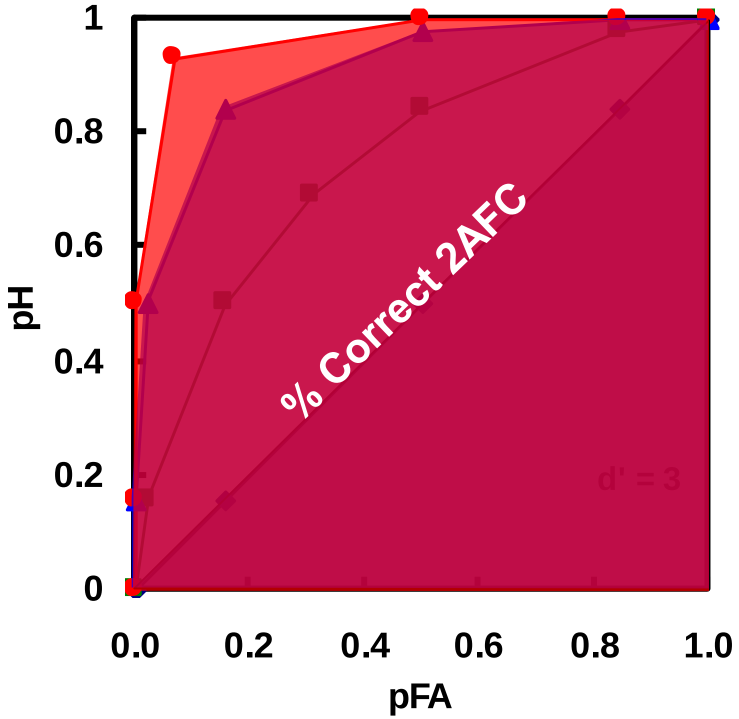
β = LOI.NORMALE(c_A;d';1;FAUX)/LOI.NORMALE(c_A;0;1;FAUX)

	HITS	FA	HITS	FA	HITS	FA	HITS	FA
n	70	30	90	10	83	5	60	20
out of	100	100	100	100	100	100	100	100
N_Tot	200		200		200		200	

d'	1.0488	2.5631	2.5990	1.0950
c0	0.0000	0.0000	0.3453	0.2941
ca	0.5244	1.2816	1.6449	0.8416
β	1.0000	1.0000	2.4536	1.3800

Receiving Operating Characteristics (ROC)





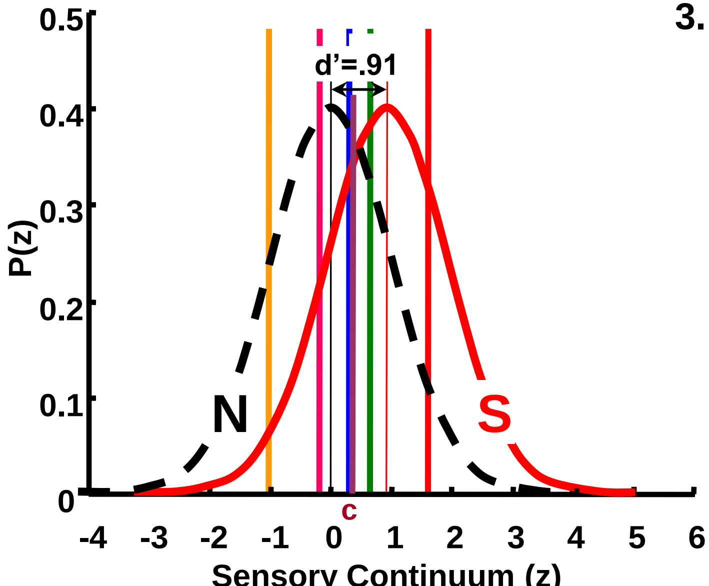
ROC & le Rating experiment

1. Faire le choix entre Oui et Non;

	Oui	**Non**	**Tot.**
S	218	157	375
N	90	285	375

2. Calculer le pH, pFA; puis d’ & c.

pH	**zH**	**pFA**	**zFA**
218/375 = **.58**	**.202**	90/375 = **.24**	**-.706**
d’ zH-zFA	.202-(-.706) = **.908**		
c -.5(zH + zFA)	-.5(.202 - .706) =**.252**		



2. Donner son niveau de certitude de 1 (peu sûr) à (par ex.) 3 (très sûr); l’on calcule les p pour chaque case en divisant sa fréq. par le no. total

	Oui				**Non**			
*****	**“3”**	**“2”**	**“1”**		**“1”**	**“2”**	**“3”**	**Tot.**
S, H	.131	.251	.200		.160	.200	.059	1.00
N, FA	.021	.099	.120		.160	.301	.301	1.00

*Notez que l’échelle « Oui/Non » ci-dessus est équivalente à une échelle «Oui» avec 6 graduations, du plus sûr («6») au moins sûr («1»):

	Oui							
	“6”	**“5”**	**“4”**		**“3”**	**“2”**	**“1”**	**Tot.**

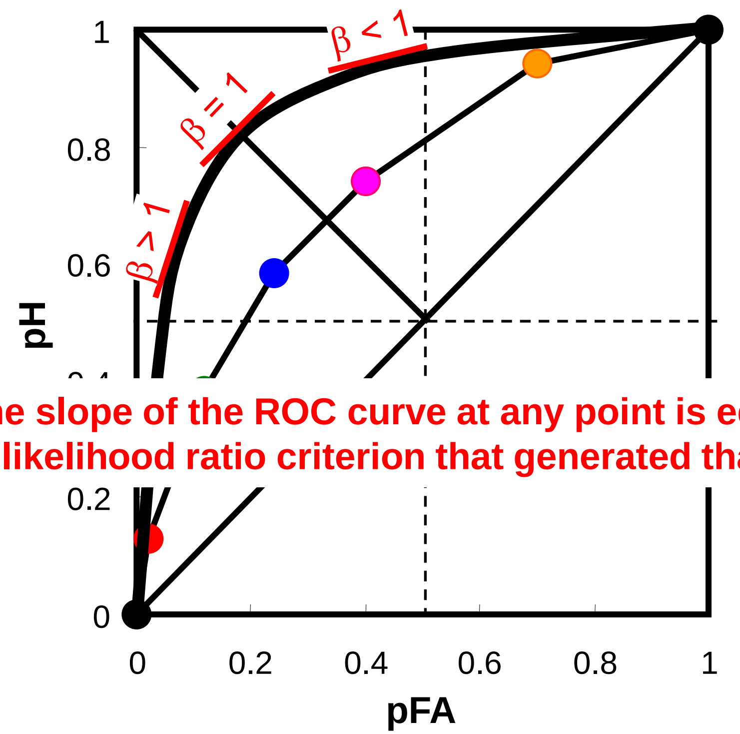
3. Pr chaque case, l’on calcule les ***p cumulés de gauche à droite****, les scores z, les d’ et c*

	Oui						
	“6”	**“5”**	**“4”**		**“3”**	**“2”**	**“1”**
H	.131	.382	.582		.742	.942	1.00
FA	.021	.120	.240		.400	.701	1.00
zH	-1.121	-0.301	0.207		0.649	1.573	
zFA	-2.037	-1.175	-0.706		-0.253	0.527	
d’	0.916	0.874	0.913		0.902	1.046	
c	1.579	0.738	0.250		-0.198	-1.050	

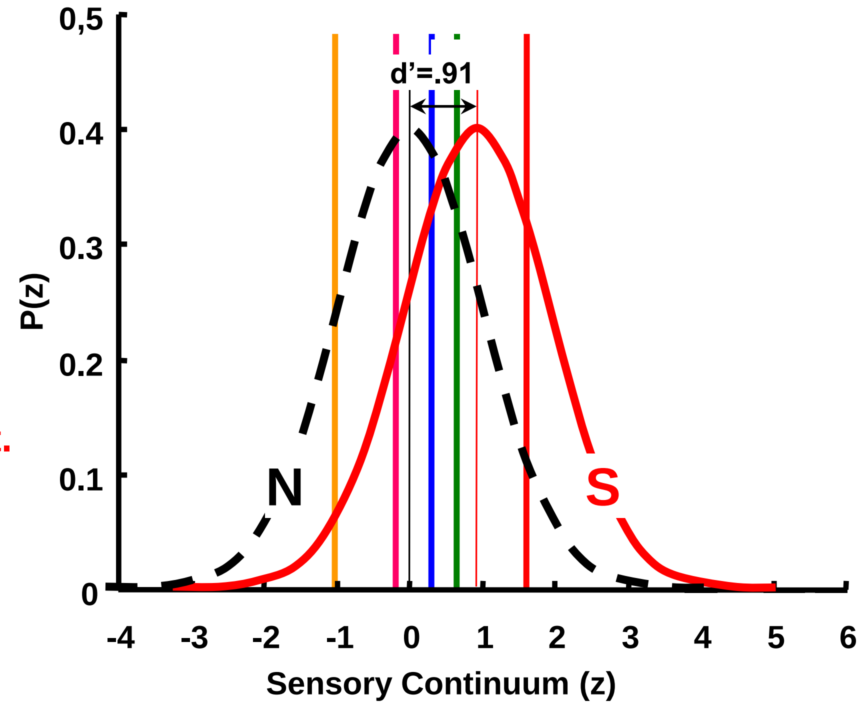
ROC & le Rating experiment

	OUI						NON					
	"3"		"2"		"1"		"1"		"2"		"3"	
	p	z	p	z	p	z	p	z	p	z	p	z
H	0.131	-1.122	0.382	-0.300	0.582	0.207	0.742	0.650	0.942	1.572	1.000	######
FA	0.021	-2.034	0.120	-1.175	0.240	-0.706	0.400	-0.253	0.701	0.527	1.000	######
d'	0.912		0.875		0.913		0.903		1.045		#NOMBRE!	
c0	1.578		0.738		0.250		-0.198		-1.050		#NOMBRE!	

Note that a symmetrical ROC indicates that d' is independent of c_0

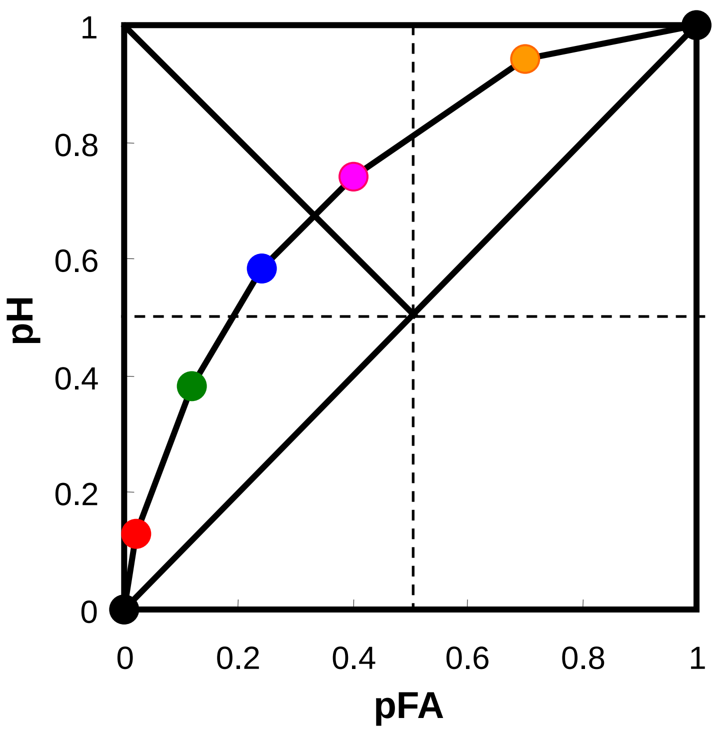


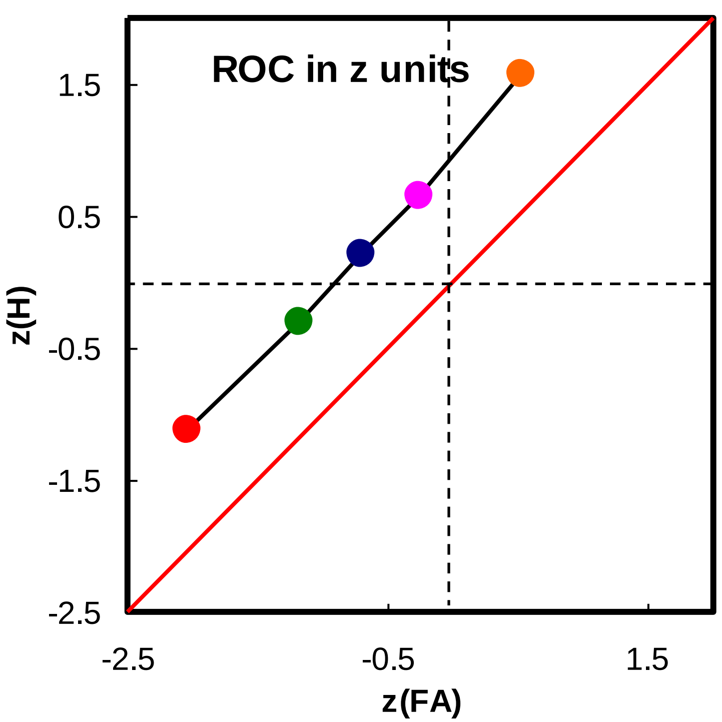
The slope of the ROC curve at any point is equal to the likelihood ratio criterion that generated that point.



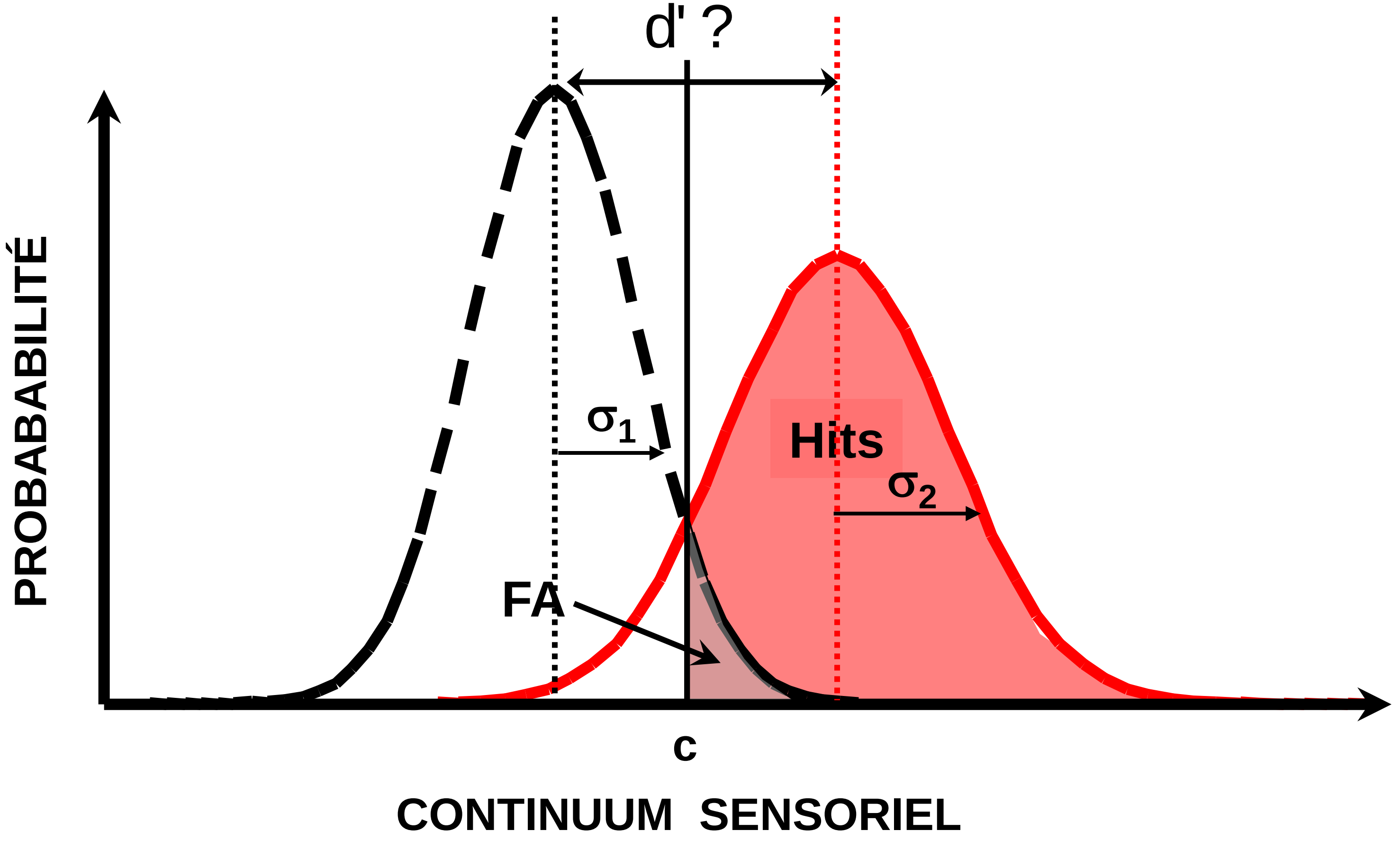
ROC & le Rating experiment

	OUI						NON					
	"3"		"2"		"1"		"1"		"2"		"3"	
	p	z	p	z	p	z	p	z	p	z	p	z
H	0.131	*-1.122*	0.382	*-0.300*	0.582	*0.207*	0.742	*0.650*	0.942	*1.572*	1.000	######
FA	0.021	*-2.034*	0.120	*-1.175*	0.240	*-0.706*	0.400	*-0.253*	0.701	*0.527*	1.000	######
d'	0.912		0.875		0.913		0.903		1.045		#NOMBRE!	
c0	1.578		0.738		0.250		-0.198		-1.050		#NOMBRE!	

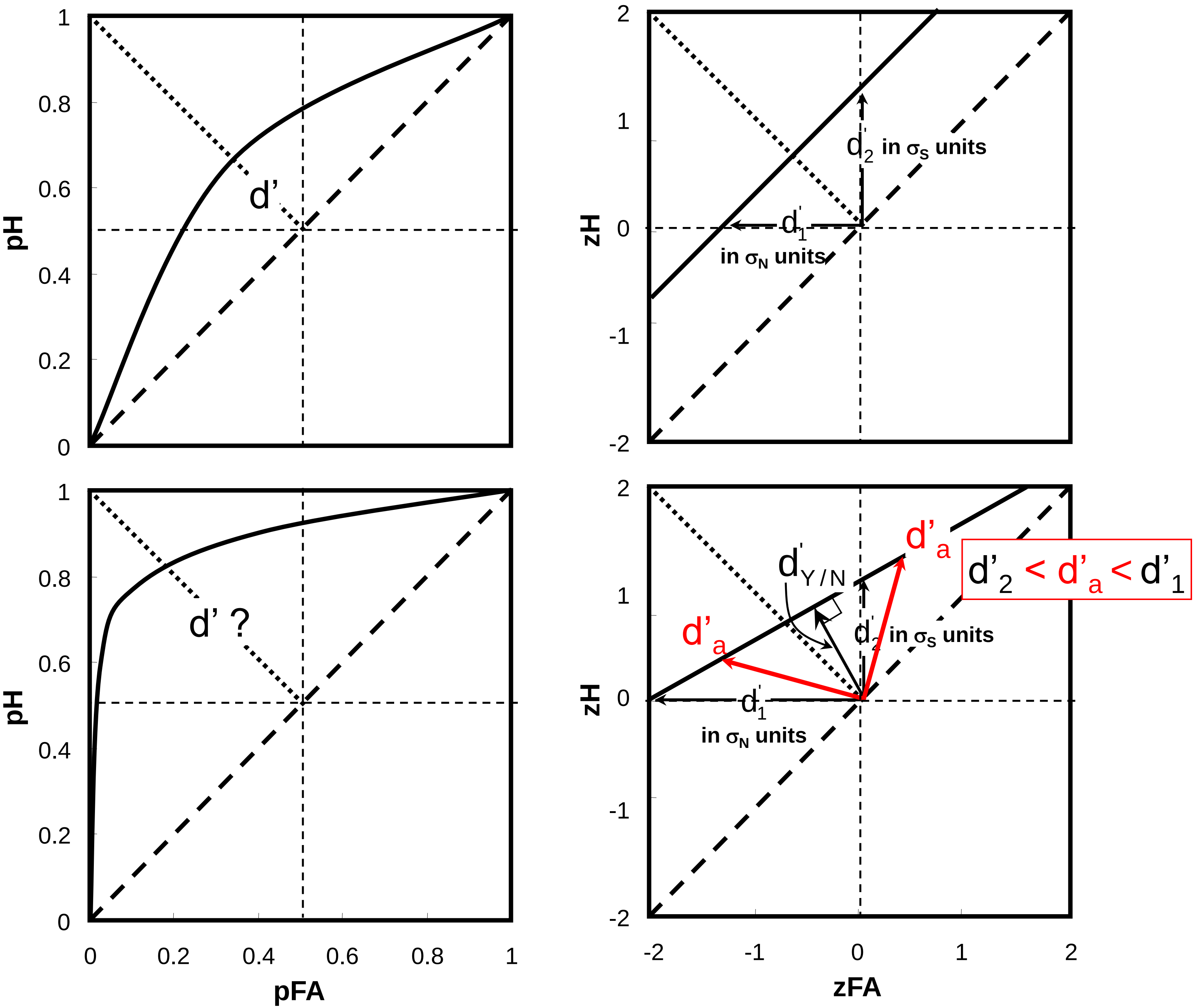




d’ with unequal variances

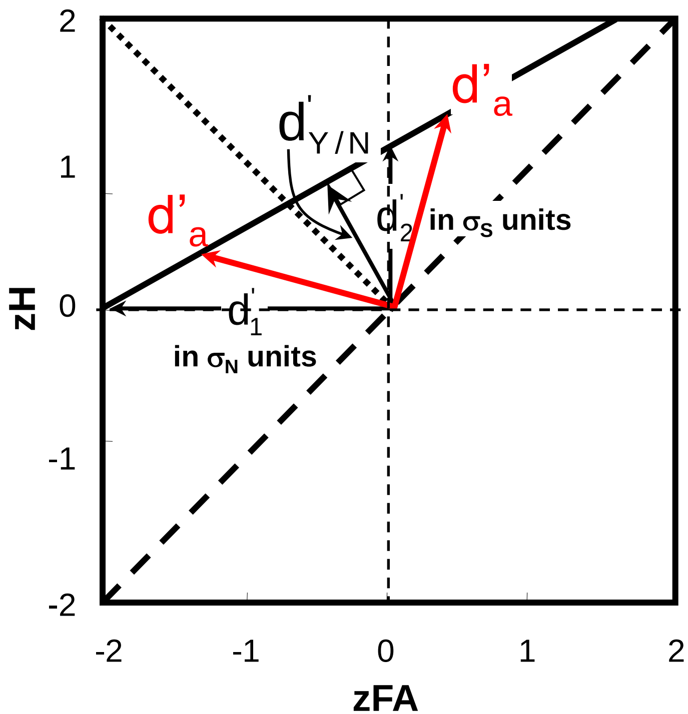


d' with unequal variances



d' with unequal variances

$$d'_a = \sqrt{2}\,d'_{Y/N} = \Delta R / \sqrt{\sigma_1^2 + \sigma_2^2}$$



The index ***d_a*** has the properties we want.

1. It is intermediate in size between d_1 and d'_2.
2. It is equivalent to *d'* when the ROC slope is 1, for then the perpendicular line of length $D_{Y/N}$ coincides with the minor diagonal, and the two lines of length d_a coincide with the d'_1, and d'_2 segments.
3. It turns out to be equivalent to the difference between the means in units of the *root-mean-square (rms)* standard deviation, a kind of average equal to the square root of the mean of the squares of the standard deviations of S1 and *S2.*

To find d_a from an ROC that is linear in *z* coordinates, it is easiest to first estimate d'_1, d'_2 and the slope $s = d'_1/d'_2$. Because the standard deviation of the *S1* distribution is *s* times as large as that of *S2*, we can set the standard deviation of *S1* to *s* and that of *S2* to 1.

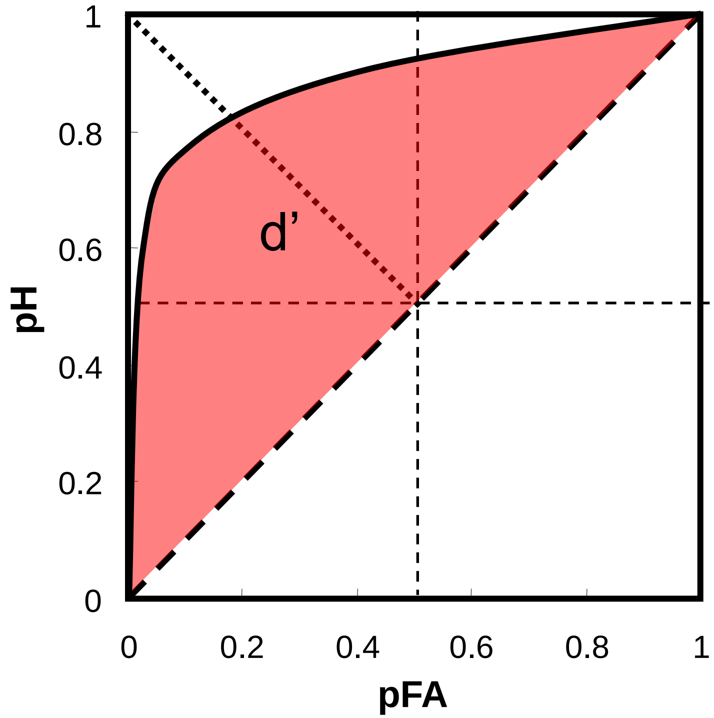
$$d'_a = \frac{d'_2}{[.5(1+s^2)]^{.5}} = d'_2\left(\frac{2}{1+s^2}\right)^{.5}$$

To find d_a directly from one point on the ROC once *s* is known:

$$d'_a = [z(H) - s z(F)]\left(\frac{2}{1+s^2}\right)^{.5}$$

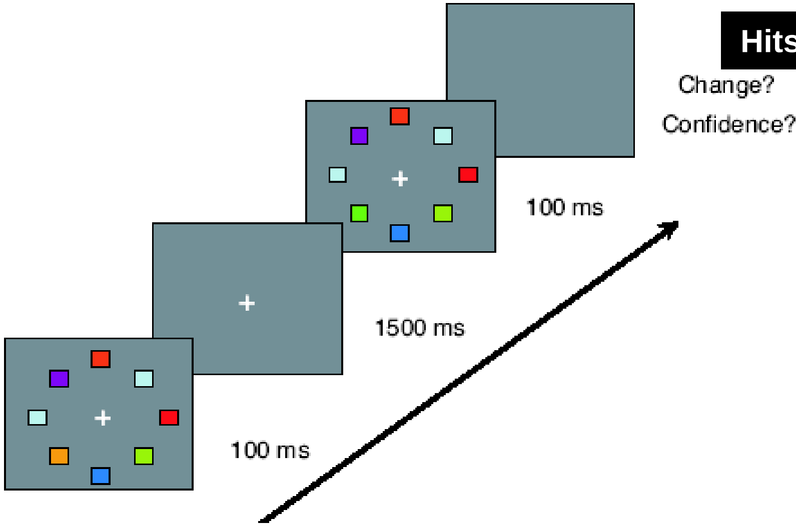
d' with unequal variances

$$\text{Area under ROC,} \quad Az = \Phi(D_{YN}) = \Phi\left(d_a / \sqrt{2} \right)$$

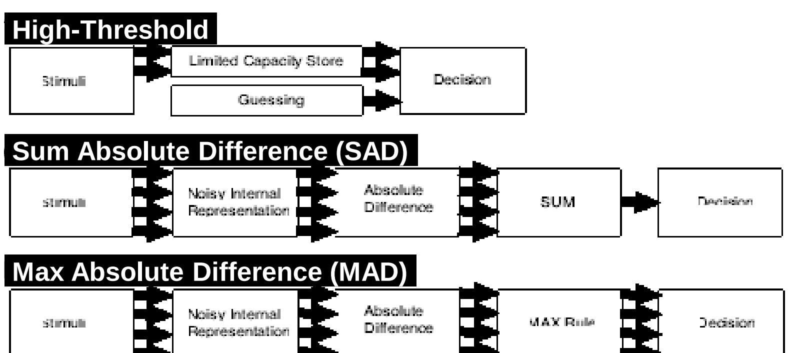


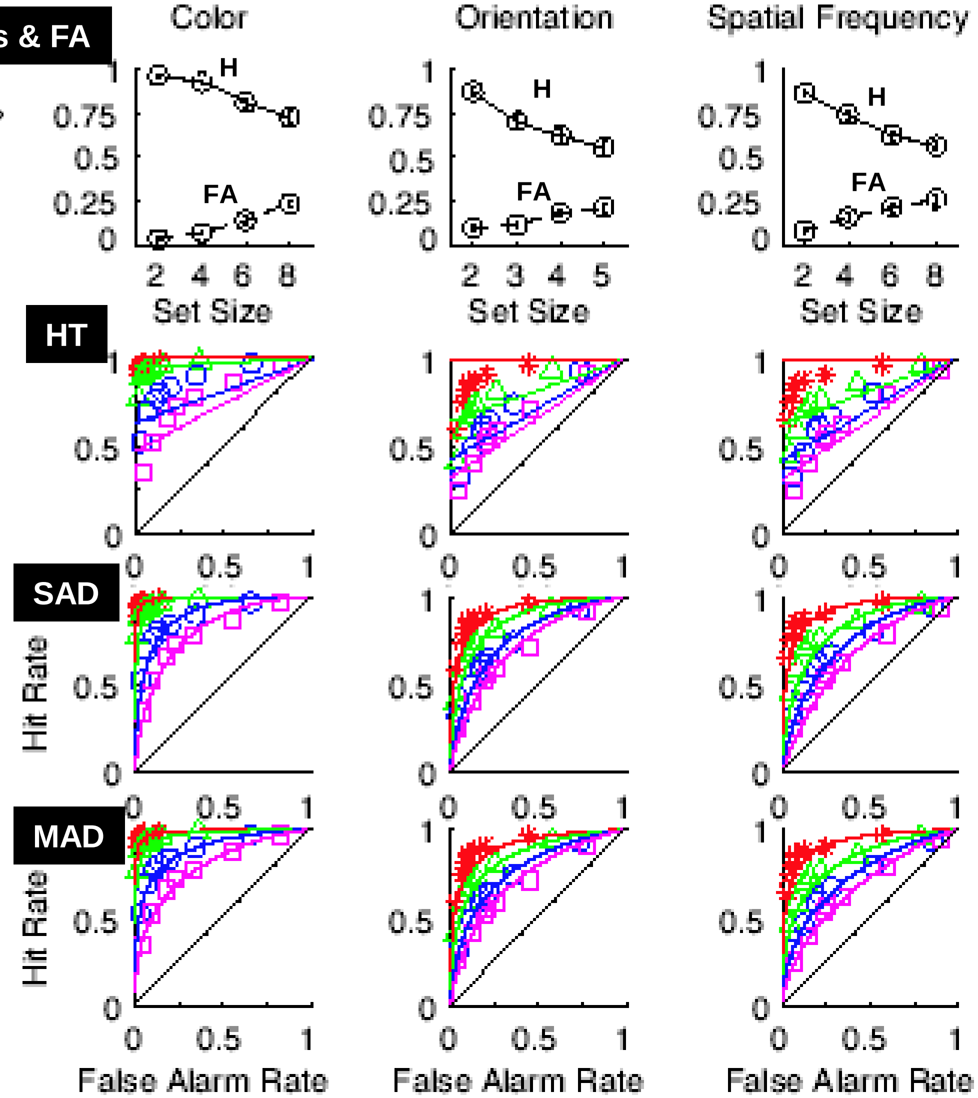
Sometimes a measure of performance *expressed as a proportion* is preferred to one expressed as a distance. The index *Az,* is such a proportion and is simply D_{YN} transformed by the cumulative normal distribution function Φ, i.e. $\Phi(D_{YN})$:

Az is the area under the normal-model ROC curve, which *increases from .5 at zero sensitivity to 1.0 for perfect performance*. It is basically equal to %Correct in a 2AFC experiment. **It is a truly nonparametric index of sensitivity**.



Schematic of a single trial used for the set size and the target number experiments. In the orientation and spatial frequency experiments, the colored squares were replaced with Gabor patches.

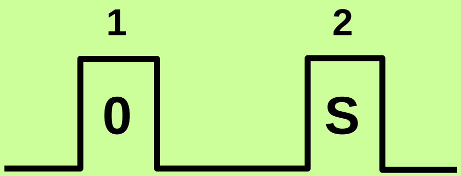




ROCs obtenus avec une méthode de « rating » sur une échelle de 1 à 4. Different symbols represent performance at different set sizes: set-size 2 for color, orientation and SF – red stars; set-size 4 for color and SF, set-size 3 for orientation – green triangles; set-size 6 for color and SF, set-size 4 for orientation – blue circles; set-size 8 for color and SF, set-size 5 for orientation – purple squares.

Wilken, P. & Ma, W.J (2004). A detection theory account of change detection. *J. Vision 4*, 1120-1135.

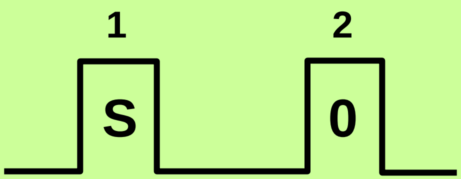
LA RELATION ENTRE Y/N & 2AFC



A = « 1 » - « 2 » ➔ -S

$$\Rightarrow \left| A - B \right| = 2S$$

$$\Rightarrow SD = \sqrt{2\sigma} = \sigma\sqrt{2}$$

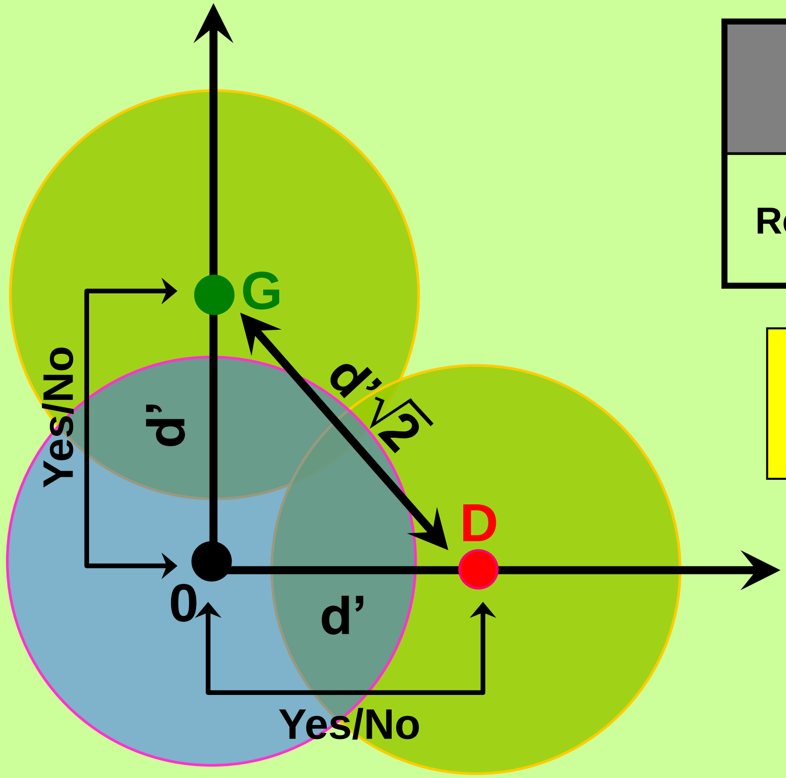


B = « 1 » - « 2 » ➔ S

$$\Rightarrow d'_{2AFC} = \frac{2S}{\sigma\sqrt{2}} = \frac{S\sqrt{2}}{\sigma} \Rightarrow \sqrt{2}d'_{Y/N}$$

See MacMillan & Creelman (2005, 2nd ed.) pp 166-170.

LA RELATION ENTRE Y/N & 2AFC



		Stimulus	
		Gauche	**D**roite
Réponse	**G**auche	Correct	Incorrect
	Droite	Incorrect	Correct

%Correct → Hits
%Incorrect → FA
→ d' 2AFC

$$d'_{2AFC} = \sqrt{2}d'_{Oui/Non}$$

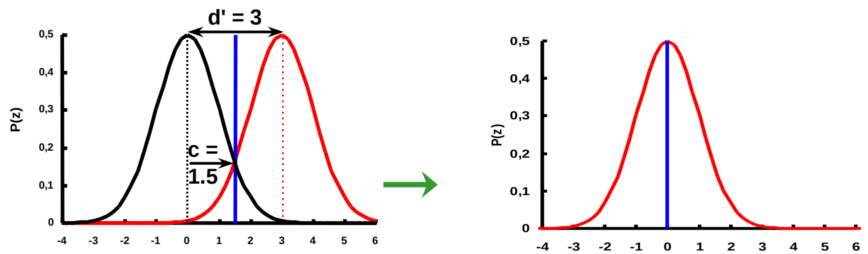
See MacMillan & Creelman (2005, 2nd ed.) pp 166-170.

Reasons for “invisibility”

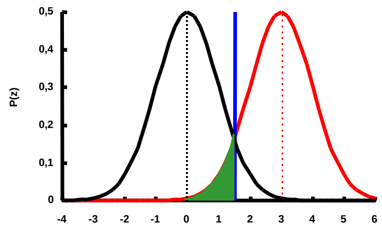
So, objects may be “invisible” for only 2 + 1 reasons;

because the internal activity they yield upon presentation:

- **is undistinguishable from *noise* (null *sensitivity*) → d' = 0,**

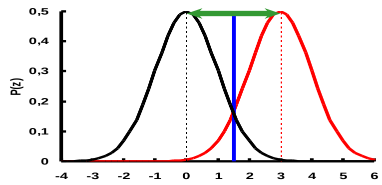


- **is below Sj’s *criterion* → the objet is *ignored*, or**



- **because Sj’s report on the presence/absence of the stimulus is solicited within a time delay beyond his/her *mnemonic capacity* (the case of *change blindness*).**

SENSITIVITY ♣



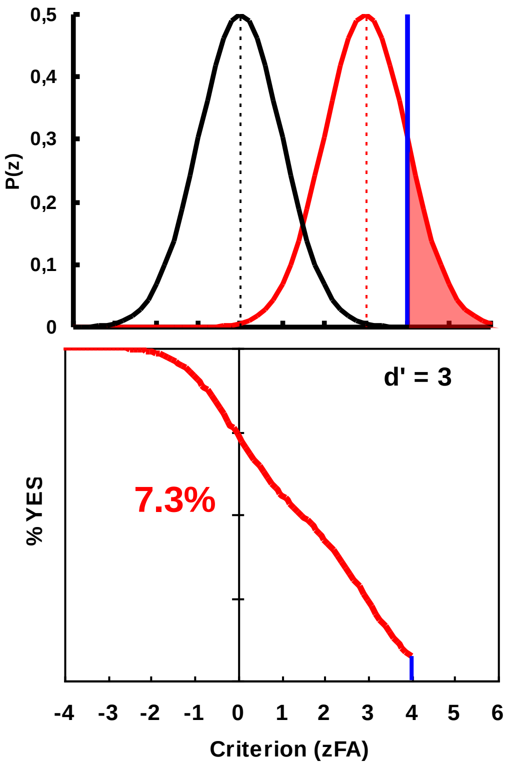
Failure of discriminating Signal from Noise has a triple edge:

- **Stimuli may be *out of the standard* *window of visibility* [♣] (Signal ≤ Internal noise)**
  - ⇒ **e.g. ultraviolet light, SF > 40 c/deg, TF > 50 Hz**
  - ⇒ **reduced window of visibility (experimental or pathological – e.g. blindsight)**
- **Stimuli may be "*masked*" by *external noise* [♣] so that the S/N ratio is decreased by means of:**
  - ⇒ **increasing N (external noise impinges on the detecting mechanism → the classical *pedestal effect*, or increases spatial/temporal *uncertainty* → e.g. the "*mud-splash effect*")**
  - ⇒ **decreasing S (external noise activates an inhibitory mechanism → *lateral masking*/*metacontrast*) [♣]**
- **Change of the *transducer* [♣](experimentally or neurologically induced *gain-control* effects, *normalization failure* …) ⇓•**

CRITERION

The response criterion can be assimilated to response *bias* (and everything that goes with it, i.e. predisposition, context effects, etc.)

- **can be *manipulated* in a number of ways:**
  - ⇒ ***probability***
  - ⇒ ***pay-off***
- **can be affected by *experience* in general**
- **and, possibly, by some *neurological conditions***
  - ⇒ ***blindsight***
  - ⇒ ***neglect / extinction***
  - ⇒ ***attentional deficit syndromes***



AN EXAMPLE: CHANGE BLINDNESS

Dinner

Money

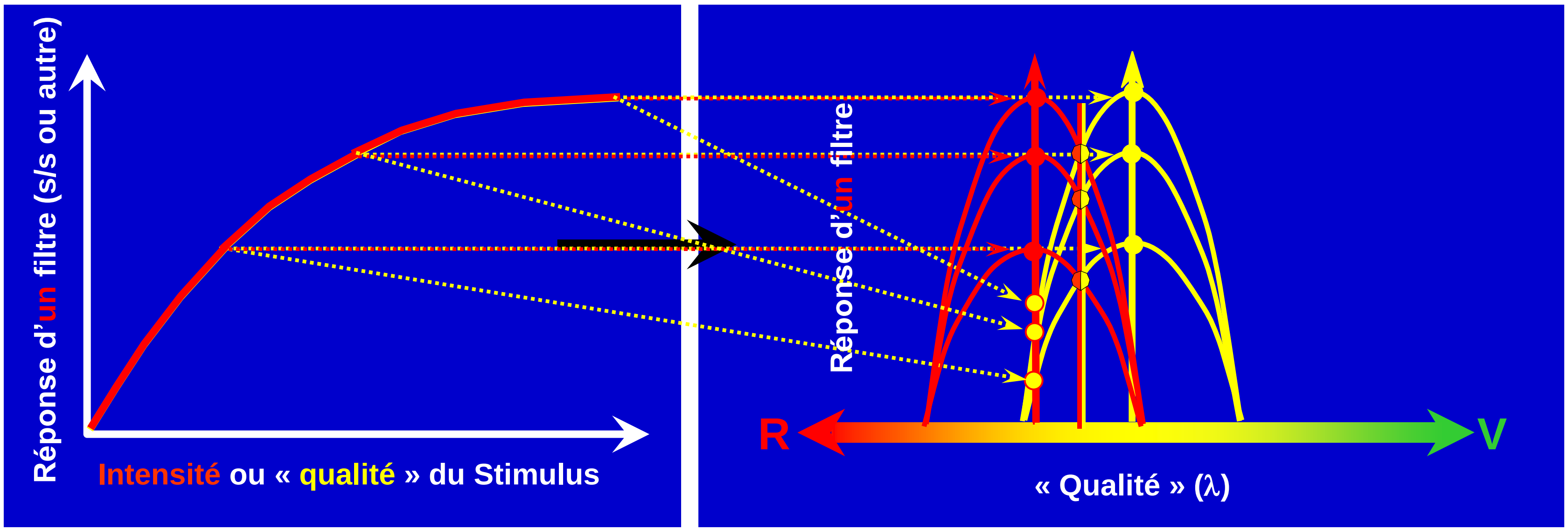
Airplane

CANAUX, BRUIT

et

THÉORIE DE LA DÉTECTION DU SIGNAL

Le principe de l'UNIVARIANCE

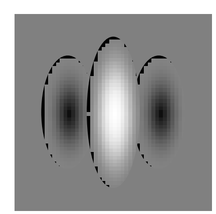


$$R = R_{max}\frac{I^n}{\sigma^n + I^n} + R_S$$

I = stimulus intensity
R = response
R_{max} = max resp.
N = max slope
S = semi-saturation cst.
R_s = spontaneous R

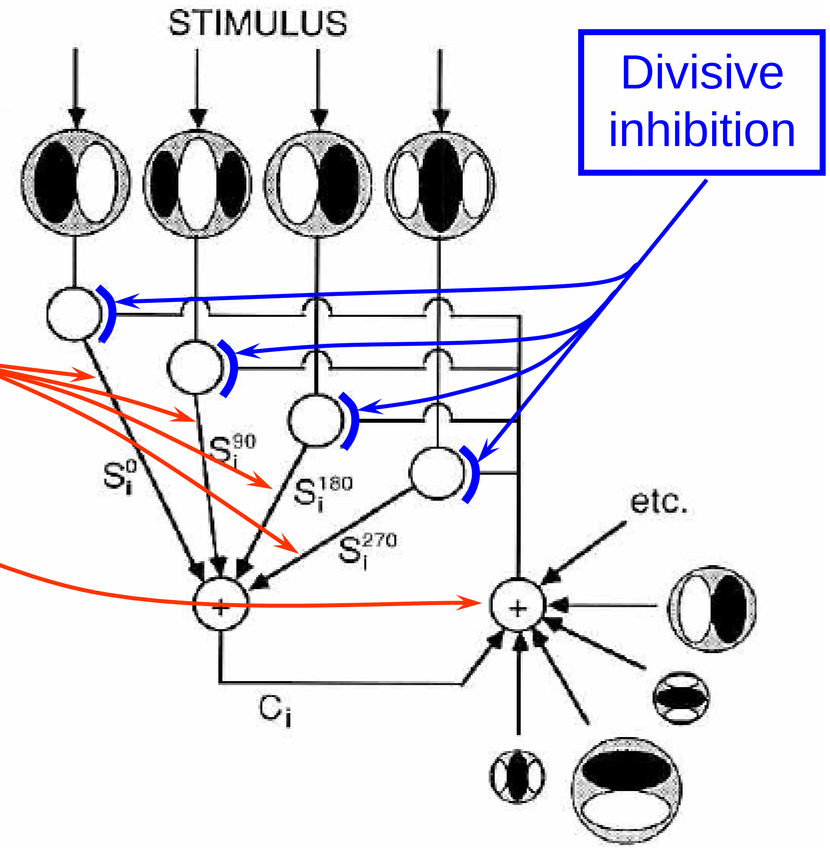
Un autre

Un Canal, Filtre. Chp Récep.



RESPONSE NORMALIZATION

Normalization works (here) by dividing each output by the sum of all outputs.



Divisive normalization

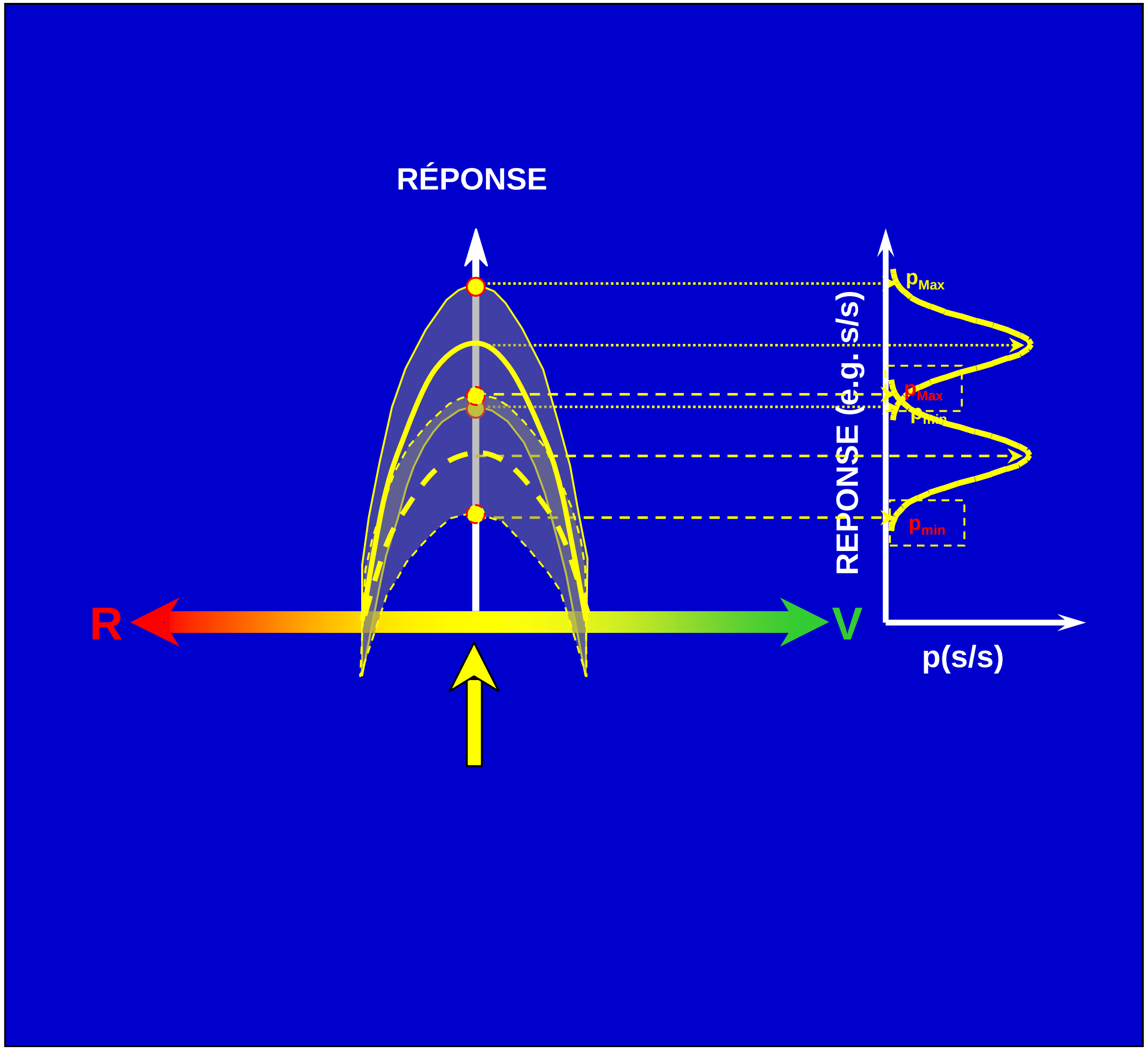
Consider a collection of linear operators and energy mechanisms, with various receptive-field centers (covering the visual field) and with various spatiotemporal frequency tunings. Let $E_i(t)$ be the outputs of each of the energy mechanisms. Normalization, in the model, works by dividing each output by the sum of all of the outputs:

$$\overline{E}_i(t) = \frac{E_i(t)}{\sigma^2 + \sum_i E_i(t)}, \tag{4}$$

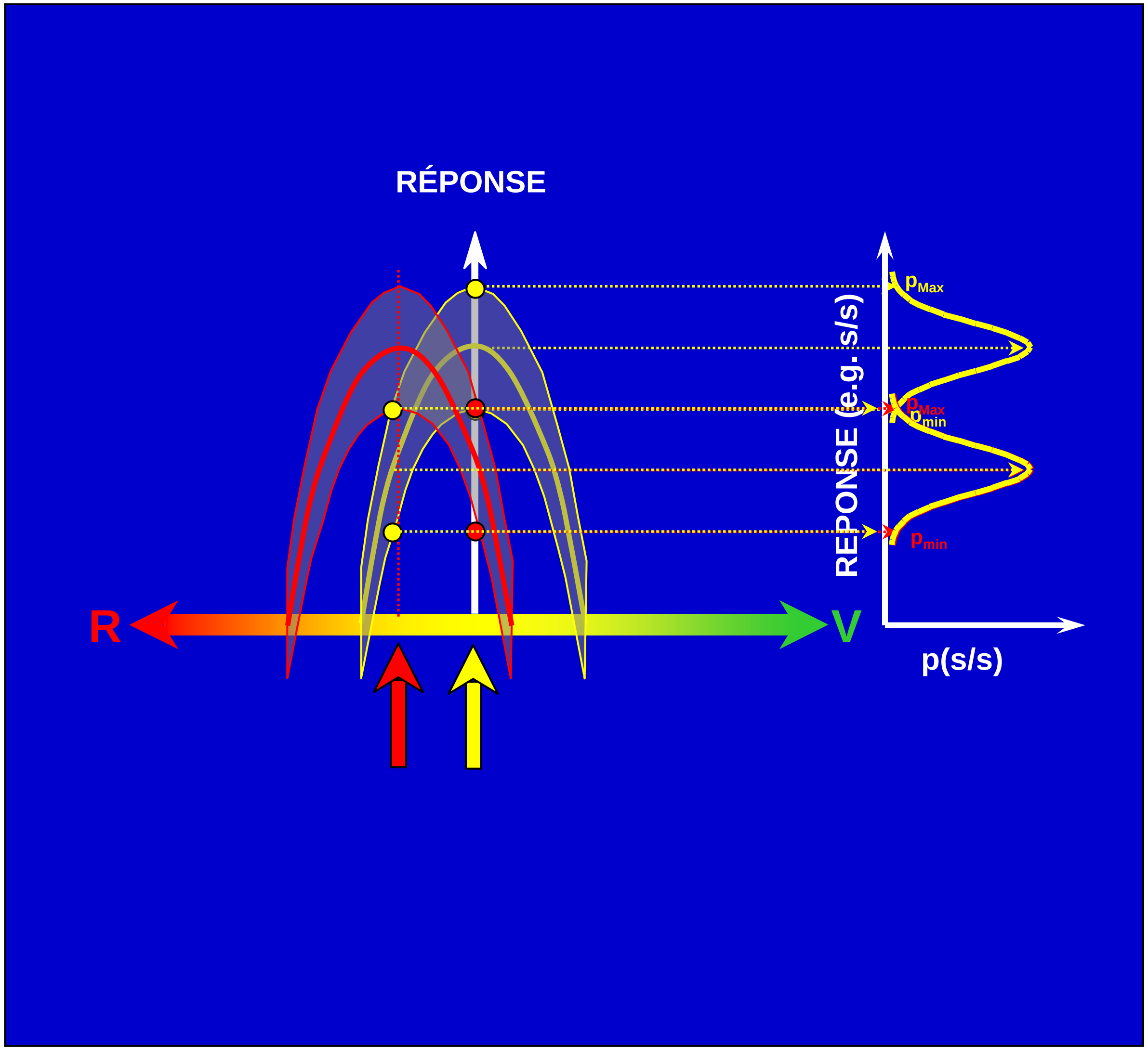
Fig. 1. Diagram of the various stages of the model. Linear weighting functions are depicted as circles, subdivided into excitatory (bright) and inhibitory (dark) subregions. The S_i^ϕ labels represent simple cell outputs, and the C_i label represents a complex cell output. The feedback signal is the combined energy at all orientations and nearby spatial frequencies, averaged over space and time. The feedback signal suppresses the simple cell responses by way of divisive suppression.

Heeger, D.J. (1992). Normalization of cell responses in cat striate cortex. *Visual Neuroscience*, *9*, 181-197.

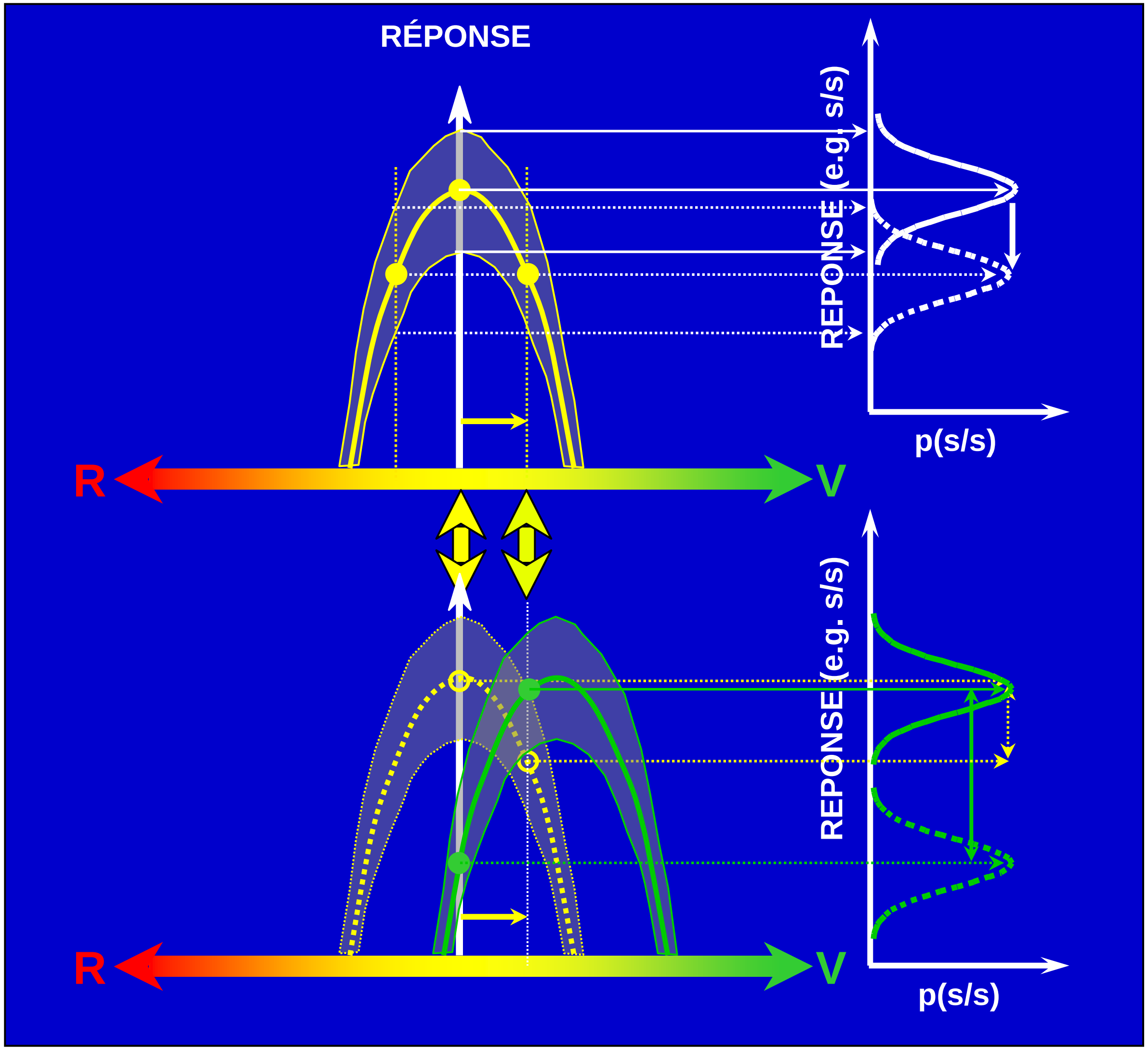
CANAUX & PROBABILITÉ DE RÉPONSE
Intensity Discrimination



CANAUX & PROBABILITÉ DE RÉPONSE
Within & Across Channels Discrimination



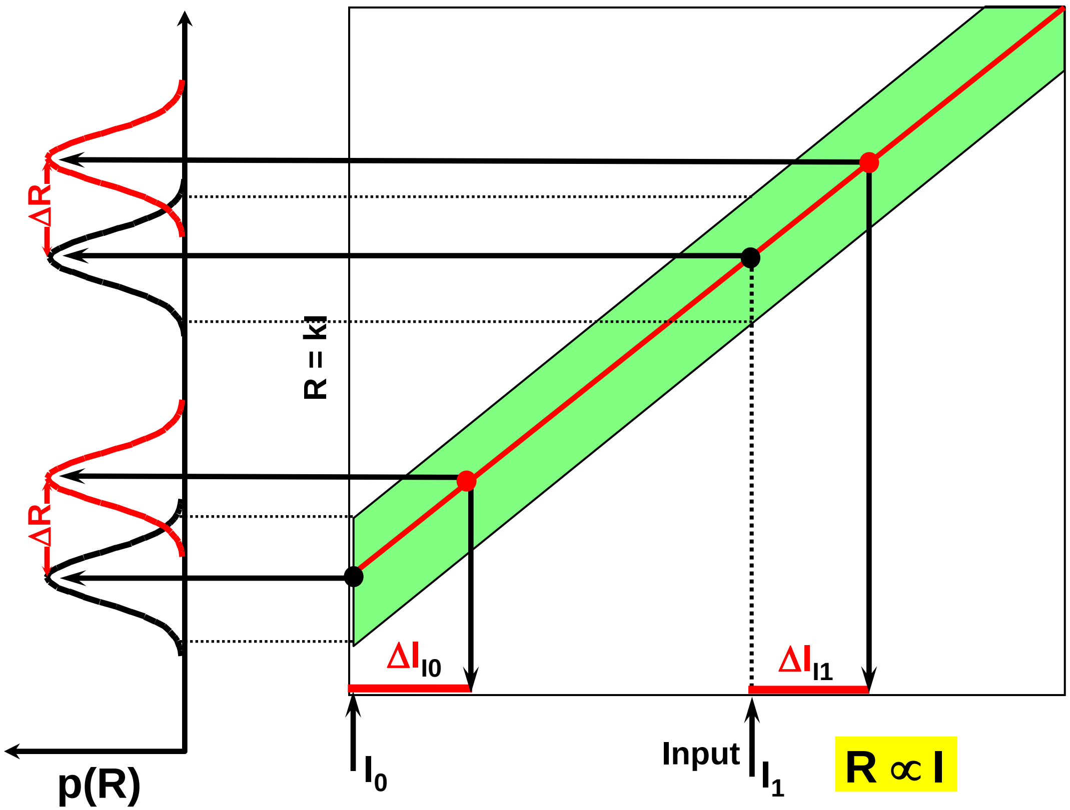
CANAUX & PROBABILITÉ DE RÉPONSE
« OFF-LOOKING » CHANNEL

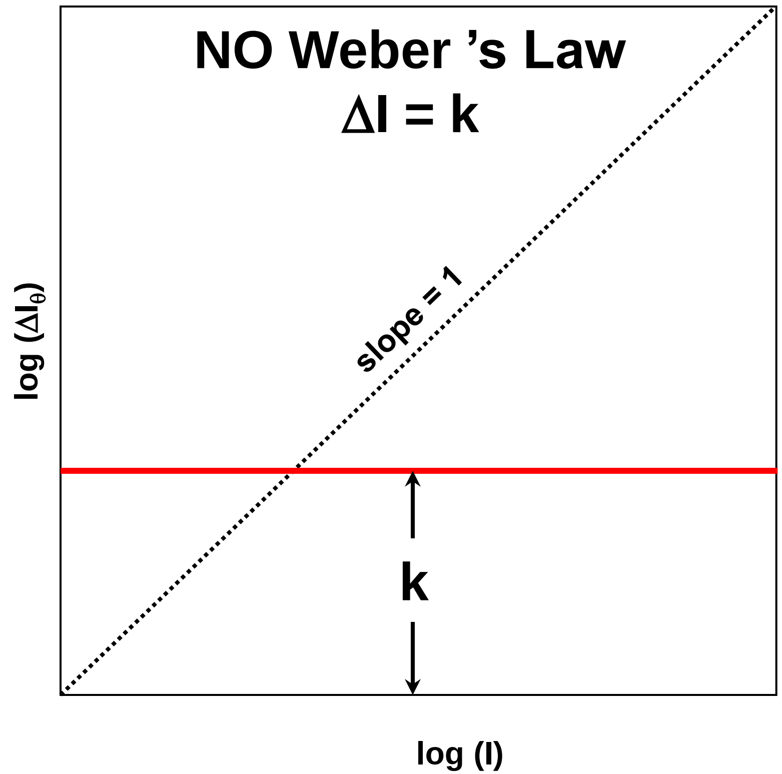


TRANSDUCTION

$R = kI$

$\sigma = cst.$





$R \propto I$

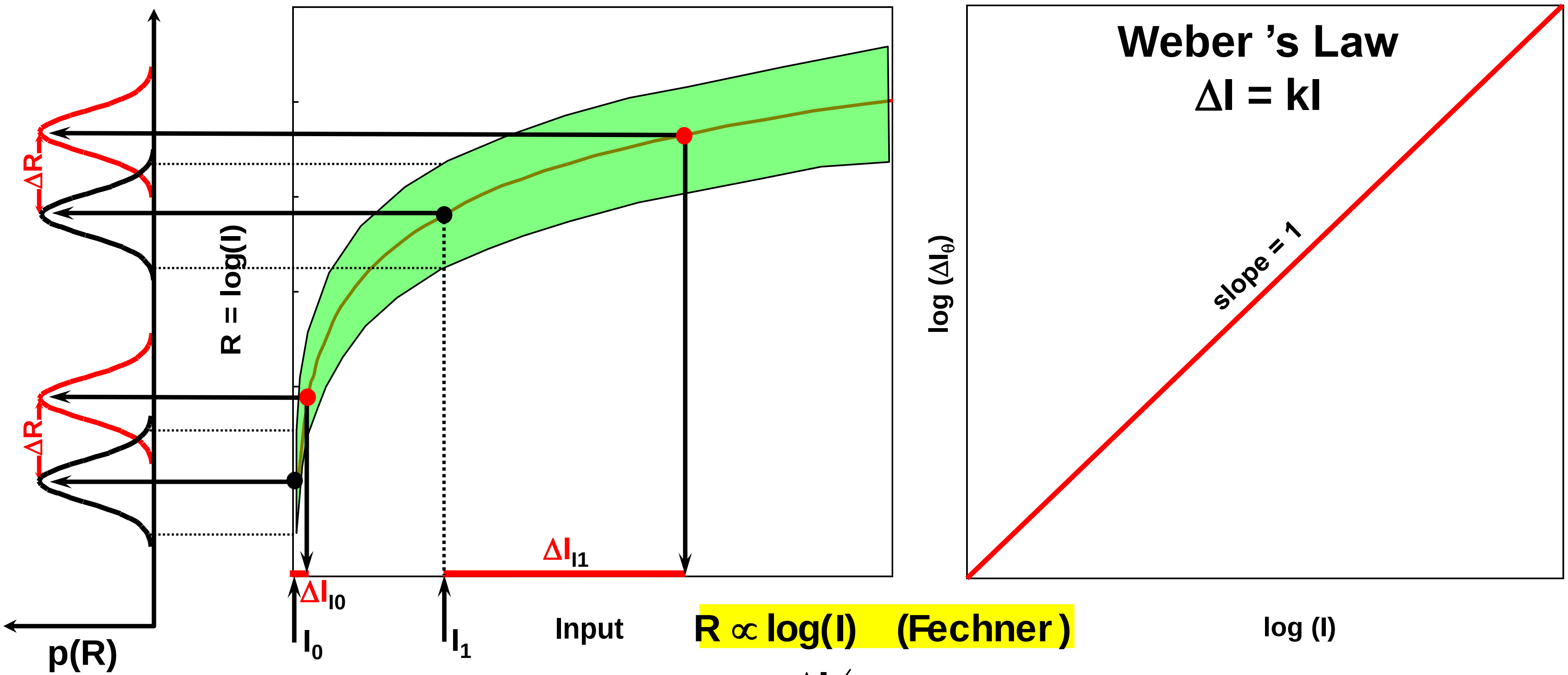
$\Delta R \propto \Delta I$

$\Delta R_\theta = k$ (at θ; iff $\sigma = cst.$)

$\Delta I_\theta = k$

TRANSDUCTION

$R \propto \log(I)$ (Fechner)
$\sigma = cst.$



$R \propto \log(I)$ (Fechner)

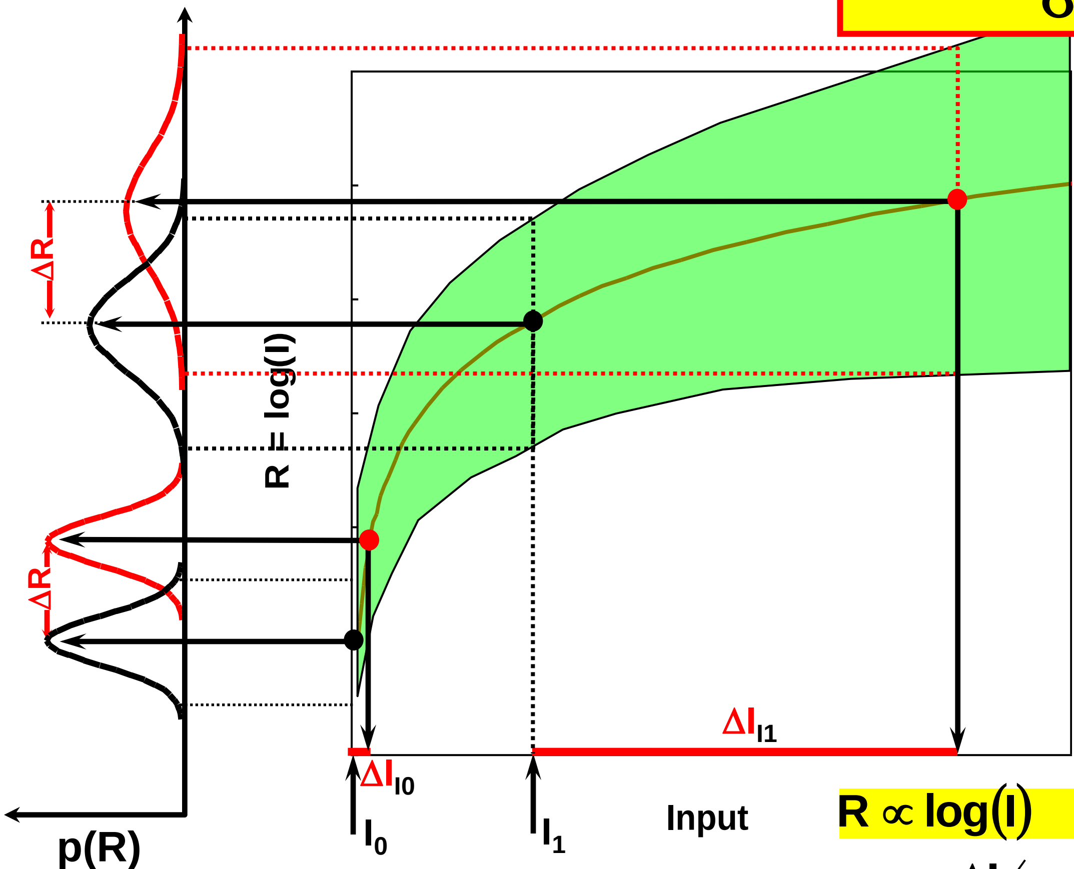
$\Delta R \propto \Delta I / I$

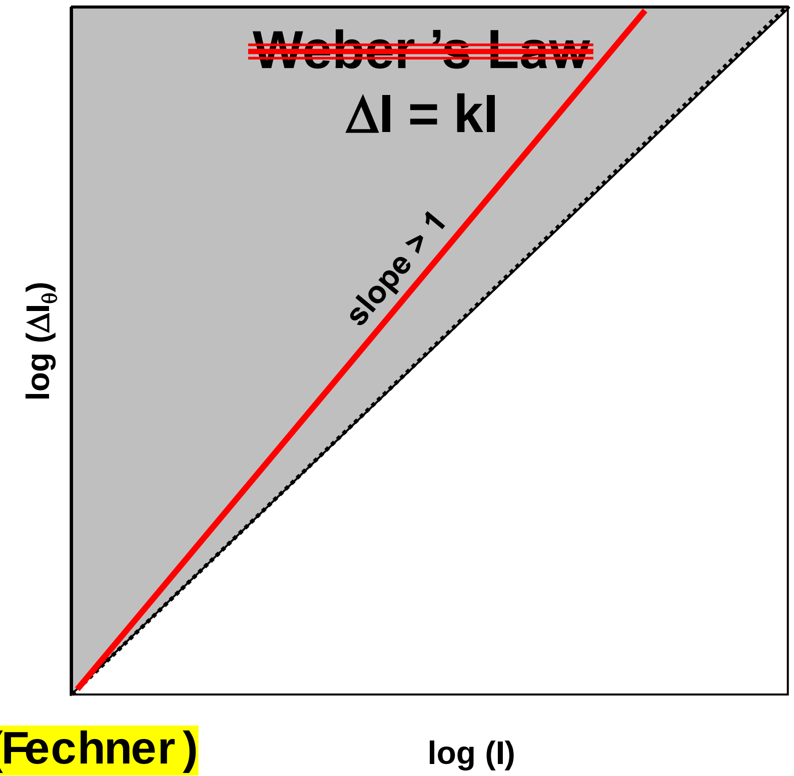
$\Delta R_\theta = k$ (at θ; iff $\sigma = cst.$)

$\Delta I_\theta = k'I$ (Weber's Law)

TRANSDUCTION

$$R \propto \log(I) \quad \text{(Fechner)}$$
$$\sigma \propto R$$





$$R \propto \log(I) \quad \text{(Fechner)}$$

$$\Delta R \propto \Delta I / I$$

$$\Delta R_\theta \propto R^\beta \ (\text{at } \theta;\ \sigma \propto R^\beta)$$

$$\Delta I_\theta = kI(\log I)^\beta \ (\text{Weber's Law iff } \beta = 0)$$

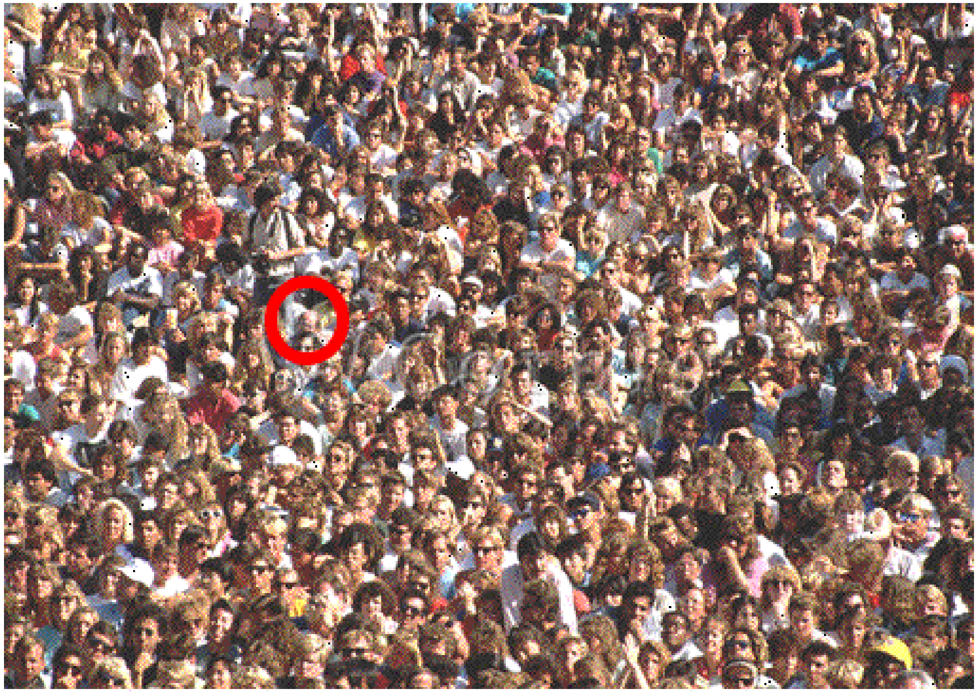
VISUAL SEARCH

and

SIGNAL DETECTION THEORY

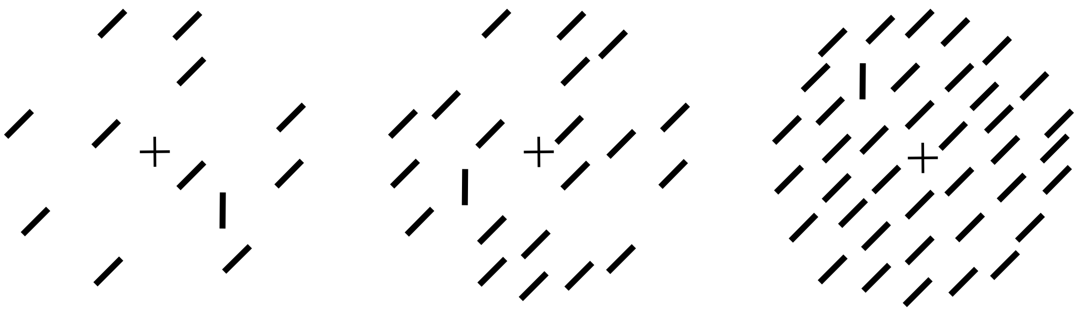
Your friends with the *green* scarves at Pamplona jump out at you

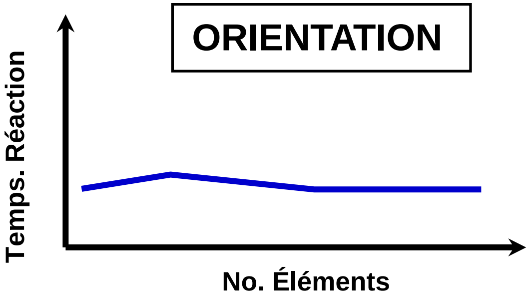
But finding a bald man at a rock concert is hard

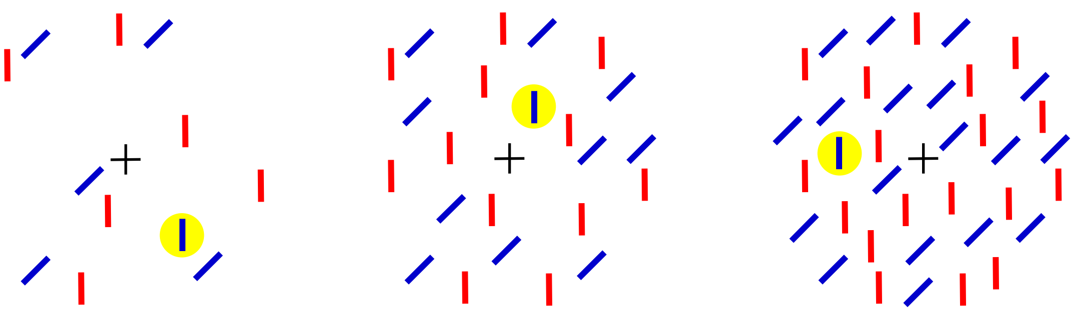


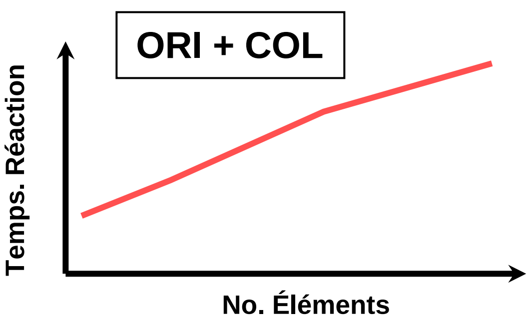
Treisman A.M. & Gelade G. (1980)

Le paradigme type









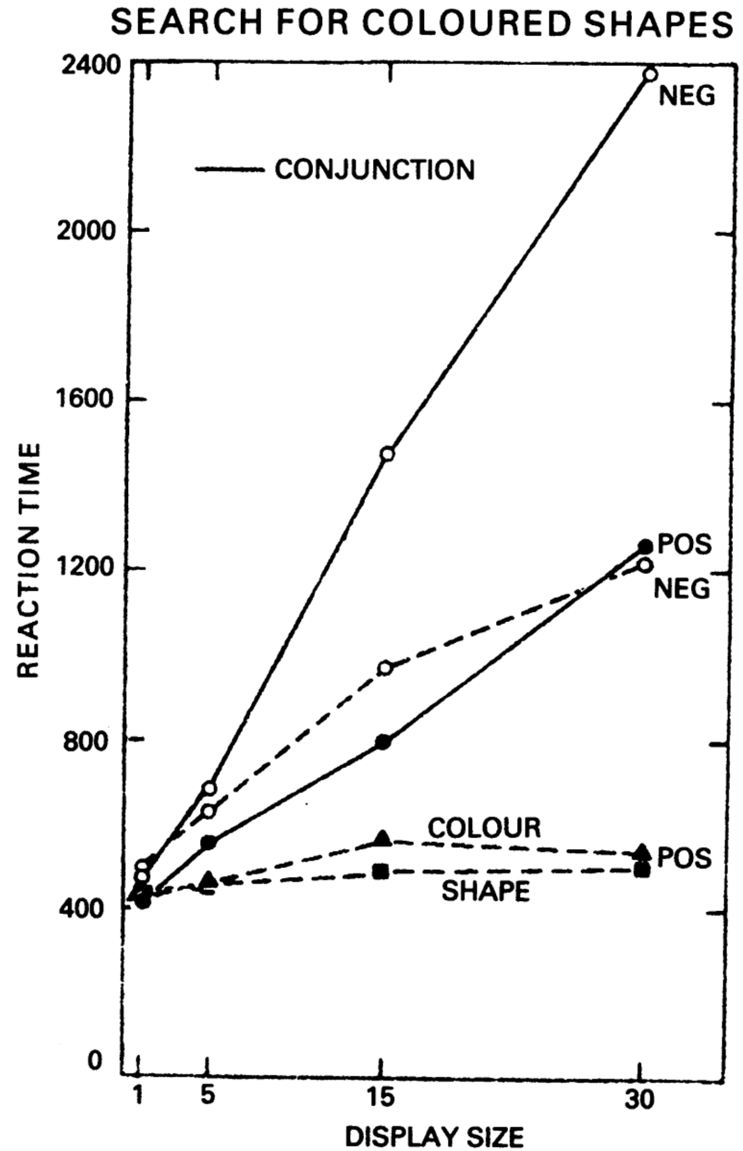
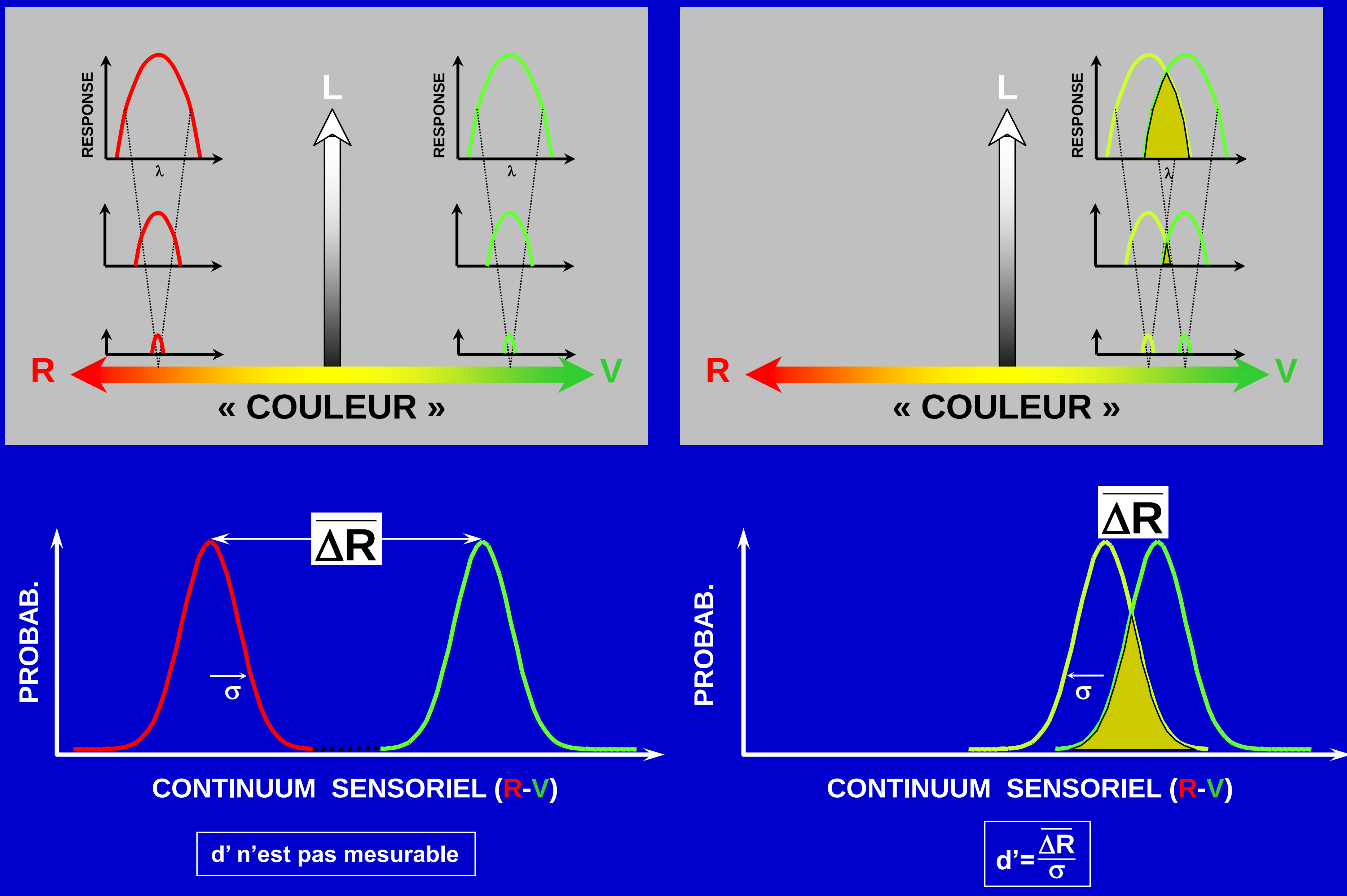
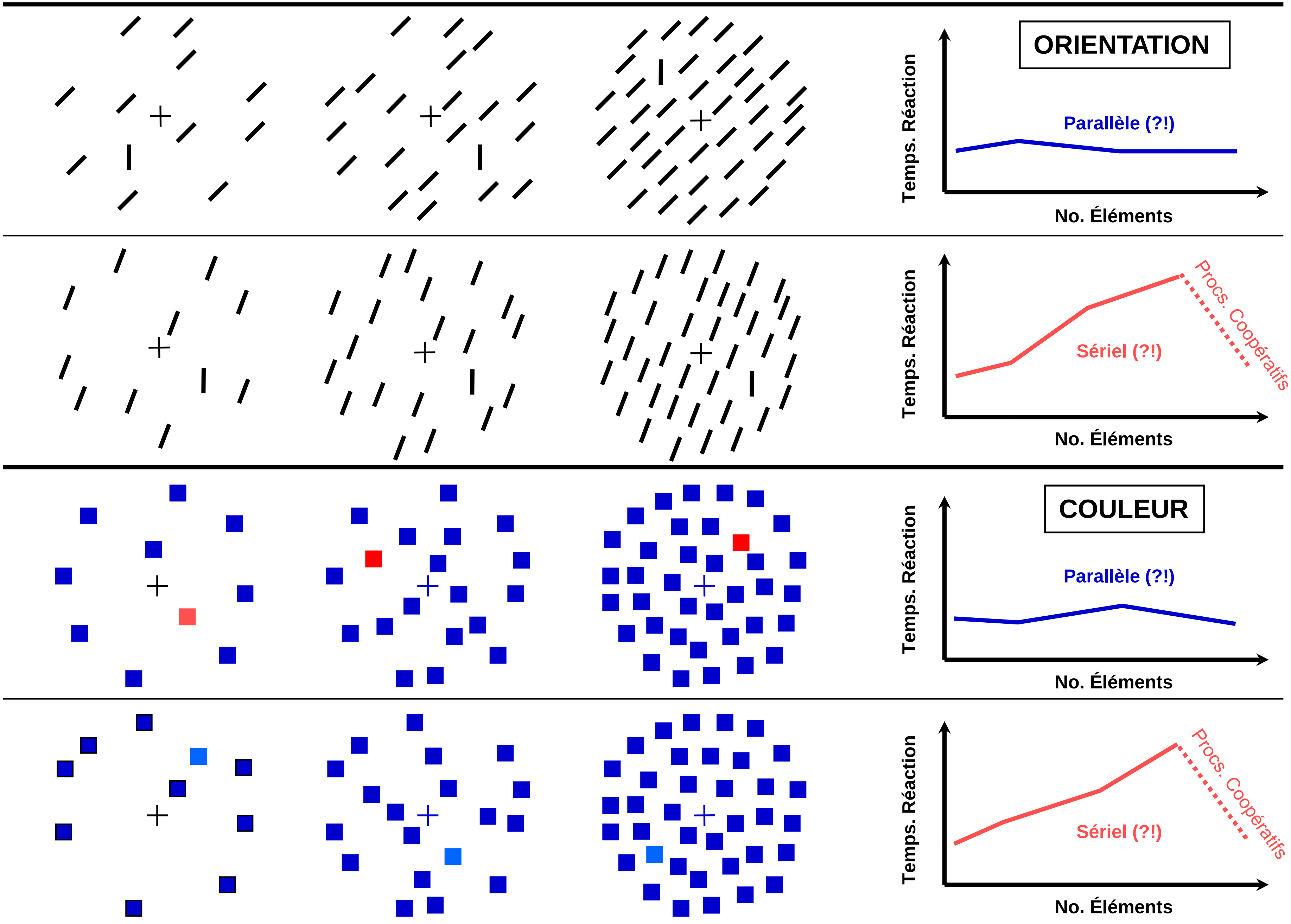


FIG. 5.19. Search for conjunctions and single-feature targets defined by colour and shape. (Reproduced from Treisman & Gelade, 1980, with permission.)

LE MODÉLE STANDARD : LA THÉORIE DE LA DÉTECTION DU SIGNAL




ORIENTATION
Temps. Réaction
Parallèle (?!)
No. Éléments
Temps. Réaction
Sériel (?!)
Procs. Coopératifs
No. Éléments
COULEUR
Temps. Réaction
Parallèle (?!)
No. Éléments
Temps. Réaction
Sériel (?!)
Procs. Coopératifs
No. Éléments