

The p-n Homojunction

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The University of Toledo, Department of Physics and Astronomy
SSARE, PVIC

Principles and Varieties of Solar Energy (PHYS 4400)
and
Fundamentals of Solar Cells (PHYS 6980)

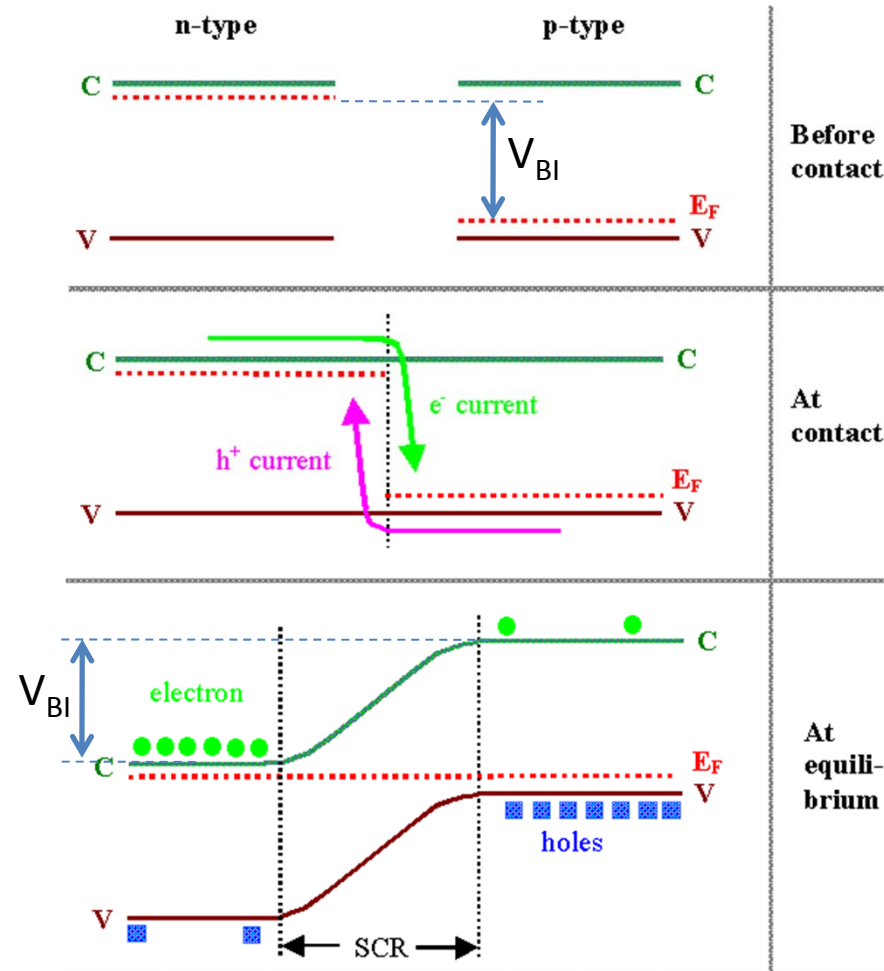
The p-n Homojunction

Consider the the band diagram for a *homojunction*, formed when two bits of the same type of semiconductor (e.g. Si) are doped p and n type and then brought into contact.

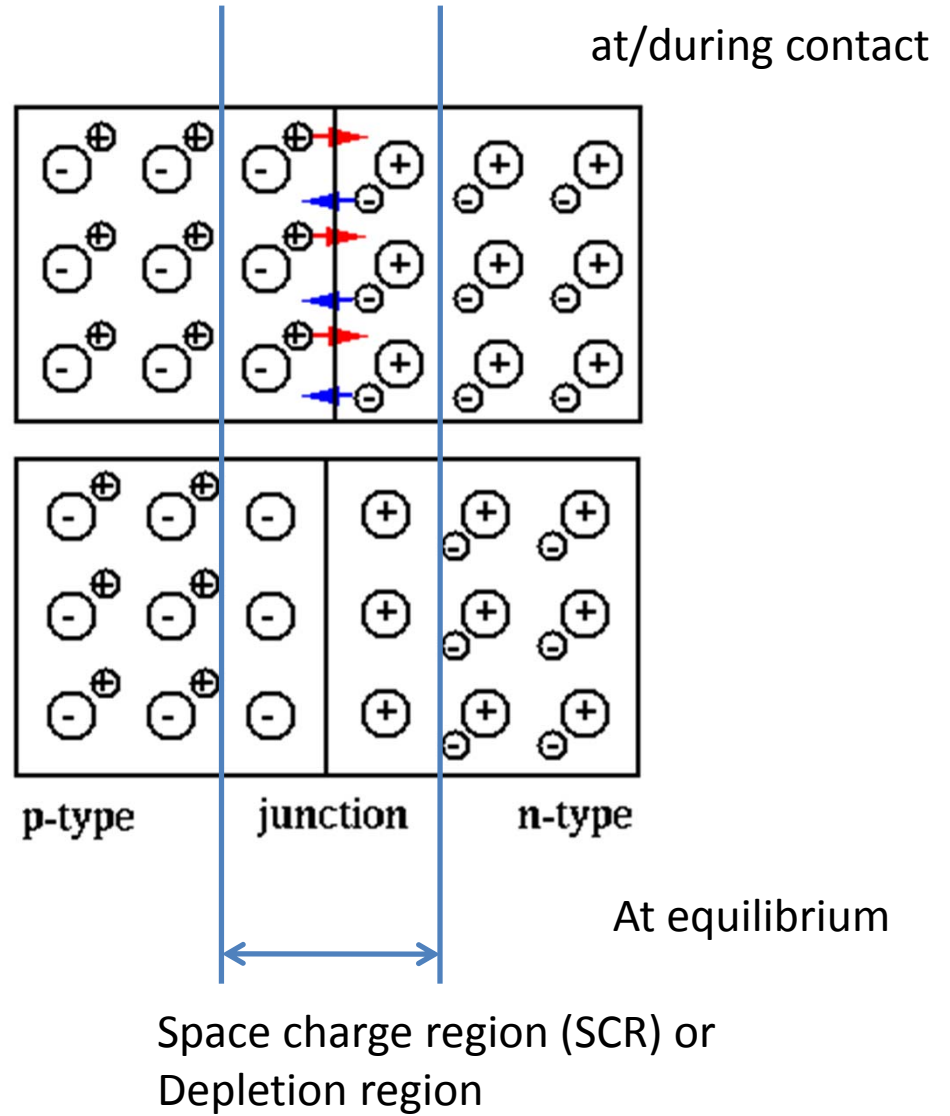
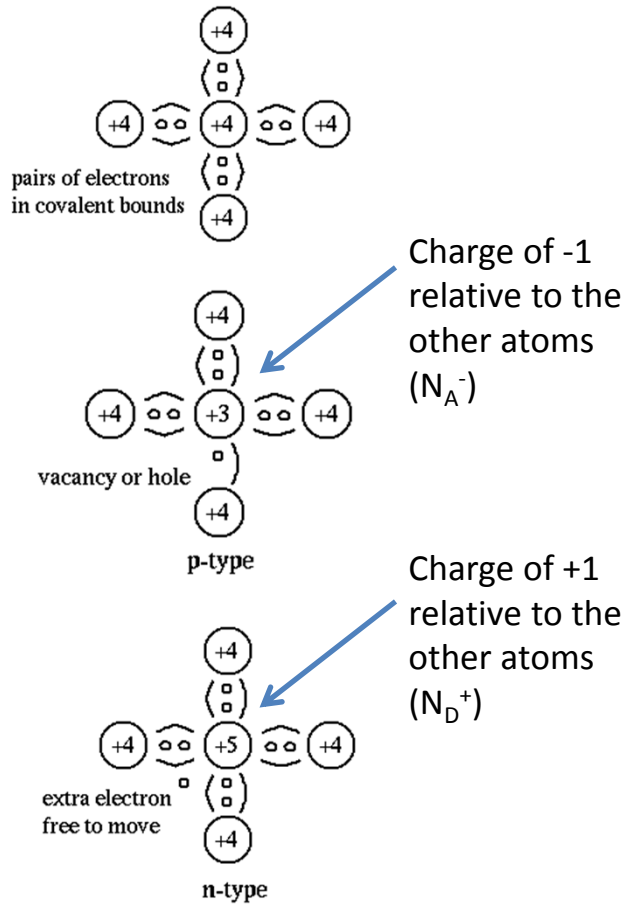
Electrons in the two bits have different electrochemical potentials (i.e. different E_f 's)

Charge transfer occurs at contact (electron go down from the vacuum level, holes go "up")

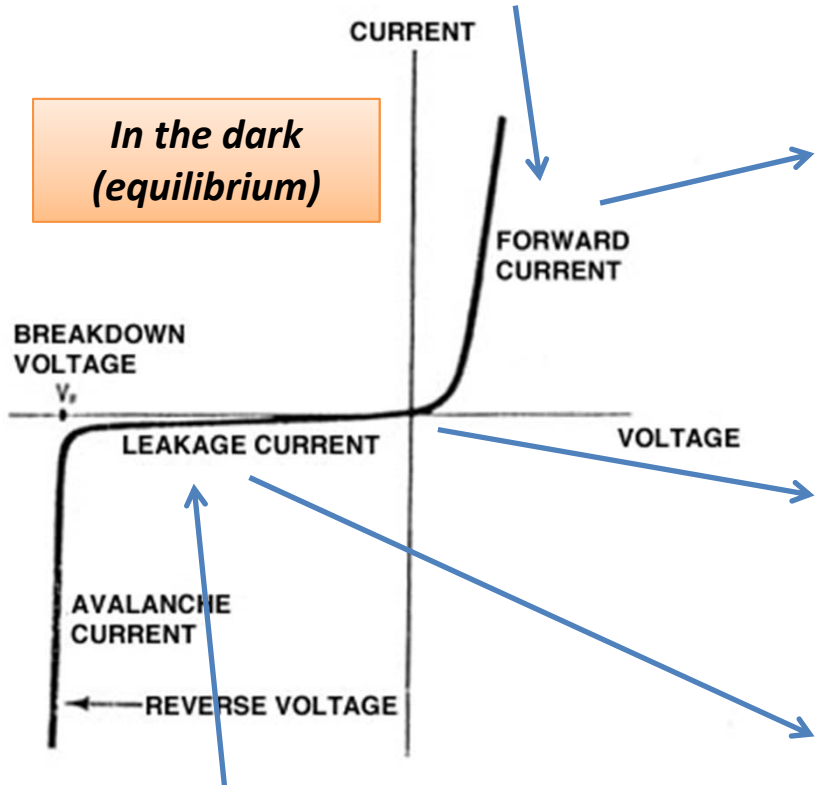
At equilibrium, there is no net transport (E_f is constant throughout the device)



5 B BORON	6 C CARBON	7 N NITROGEN
13 Al ALUMINUM	14 Si SILICON	15 P PHOSPHORUS
31 Ga GALLIUM	32 Ge GERMANIUM	33 As ARSENIC
49 In INDIUM	50 Sn TIN	51 Sb ANTIMONY



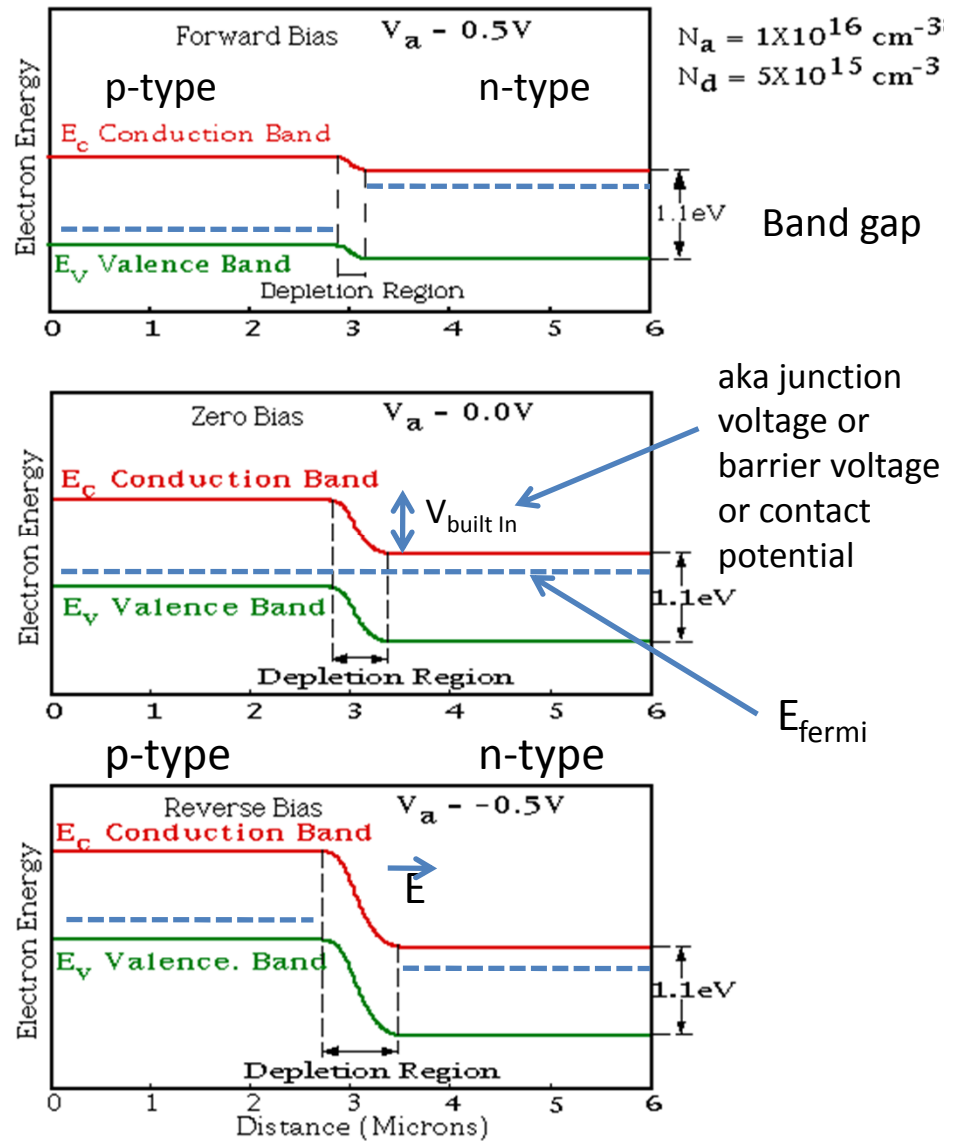
Majority carriers diffuse across the Depletion Region (because the electric field is reduced), where they become minority carriers and recombine (a.k.a. minority carrier injection)



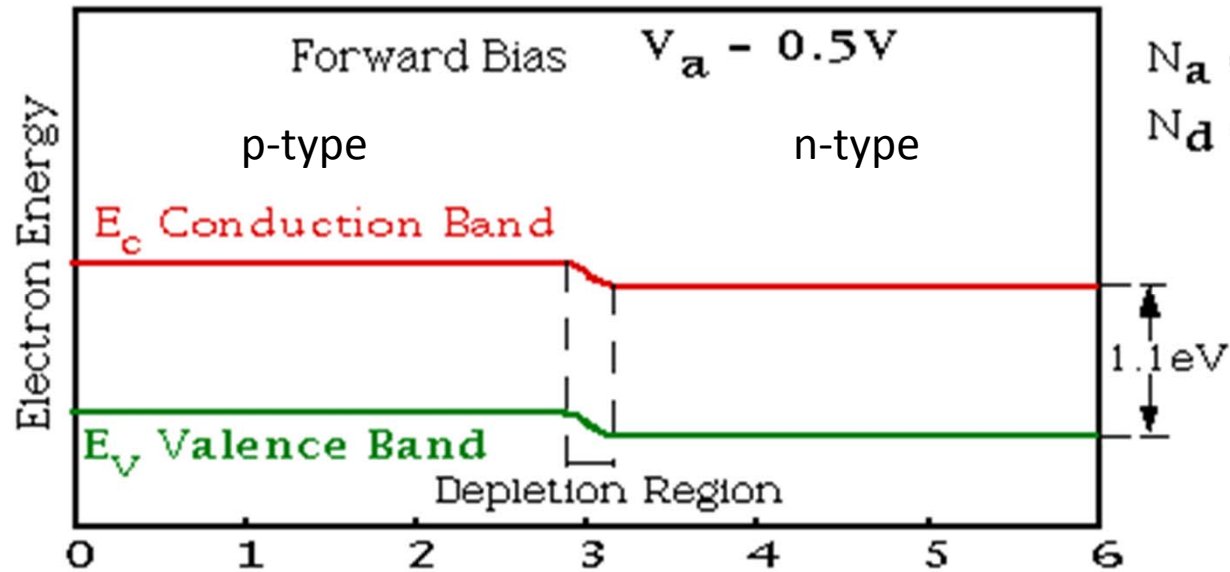
a.k.a. Reverse Saturation Current: **minority carriers drift** across the Depletion Region with the assistance of the larger electric field, where they become manority carriers and recombine



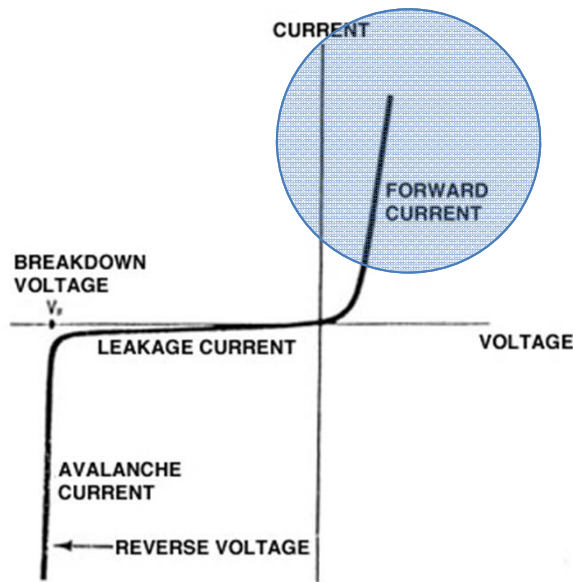
Energy Band Diagram for p-n diode



Forward Biased p-n junction



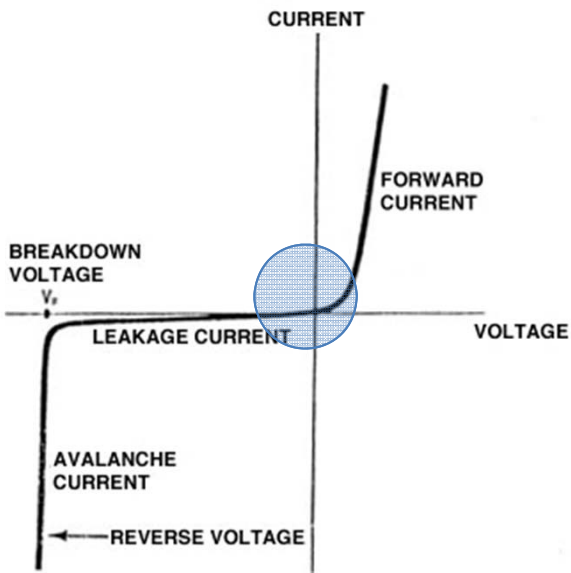
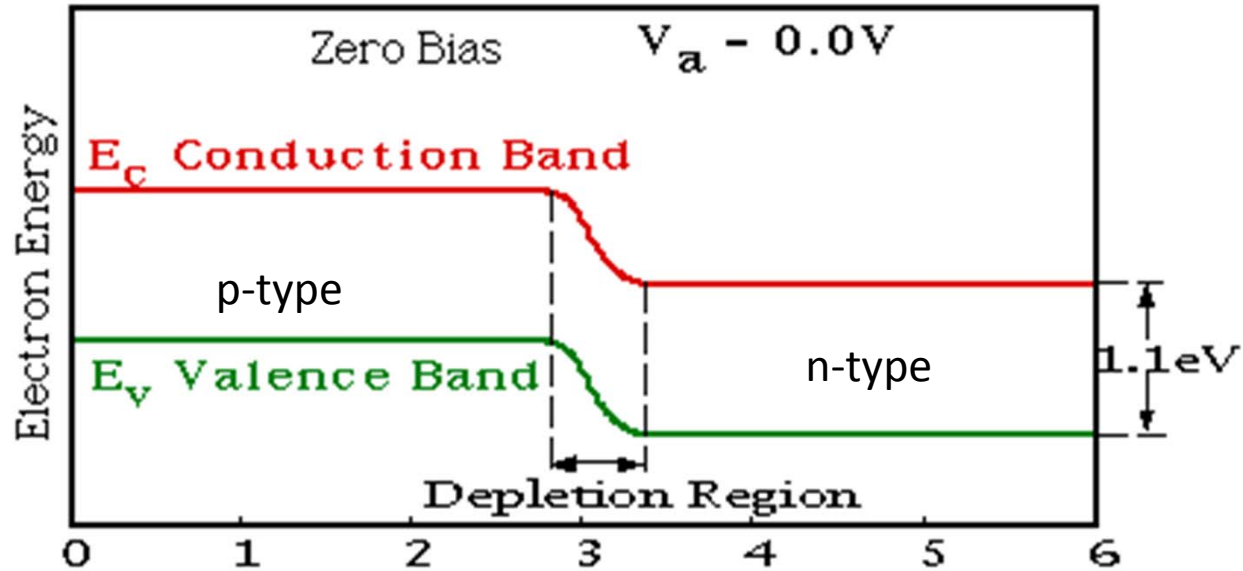
Majority carriers cross the junction, and become minority carriers (e.g., electrons from n-side to p-side), and recombine w/ a majority carrier supplied by external circuit (net current flow).



- Applied voltage reduces E- field across the depletion region
- Positive voltage to the p -type and negative voltage to the n -type
- Resistivity in depletion region high (few free carriers)
- Resulting E-field consists of the built-in field minus the applied field (built-in field > applied field)
- Barrier to diffusion is reduced, and diffusion current increases
- Drift current changes little -- depends on carriers generated near or within depletion region
- Depletion region width decreases very slightly → minority carriers changes little.

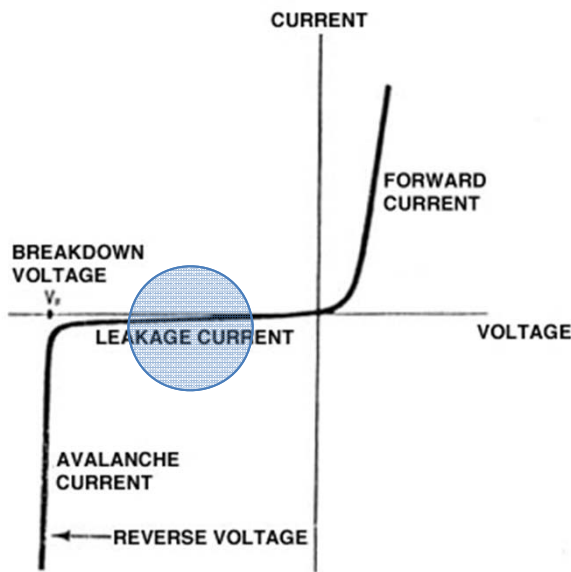
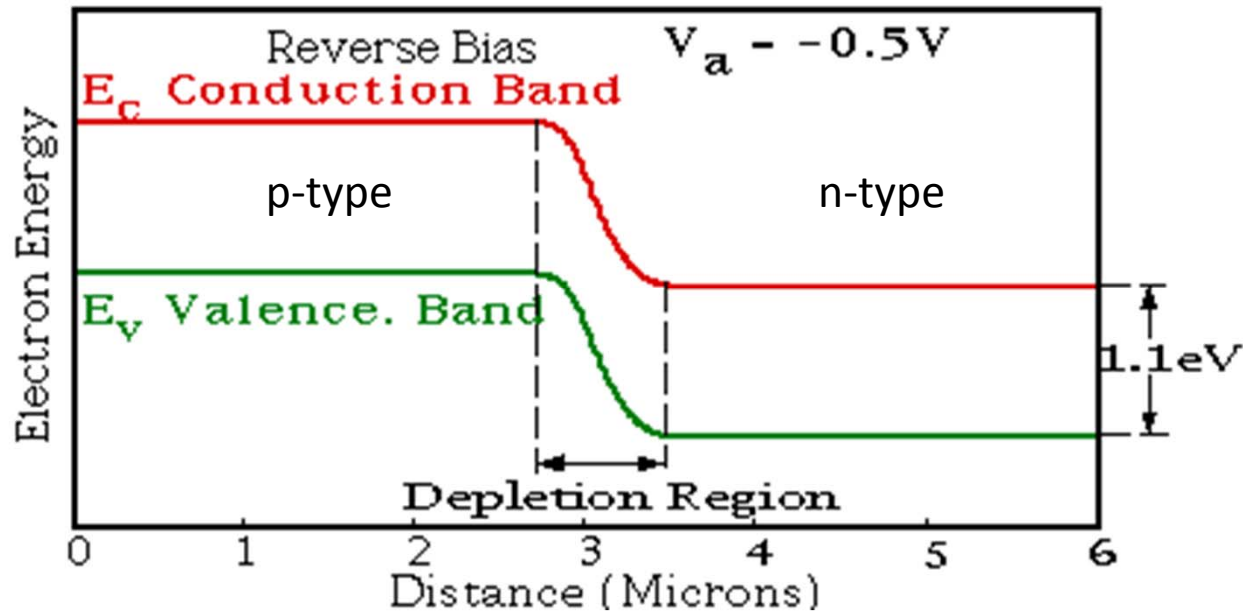
Unbiased p-n junction

<http://www-tcad.stanford.edu/tcad/education/anim.html>



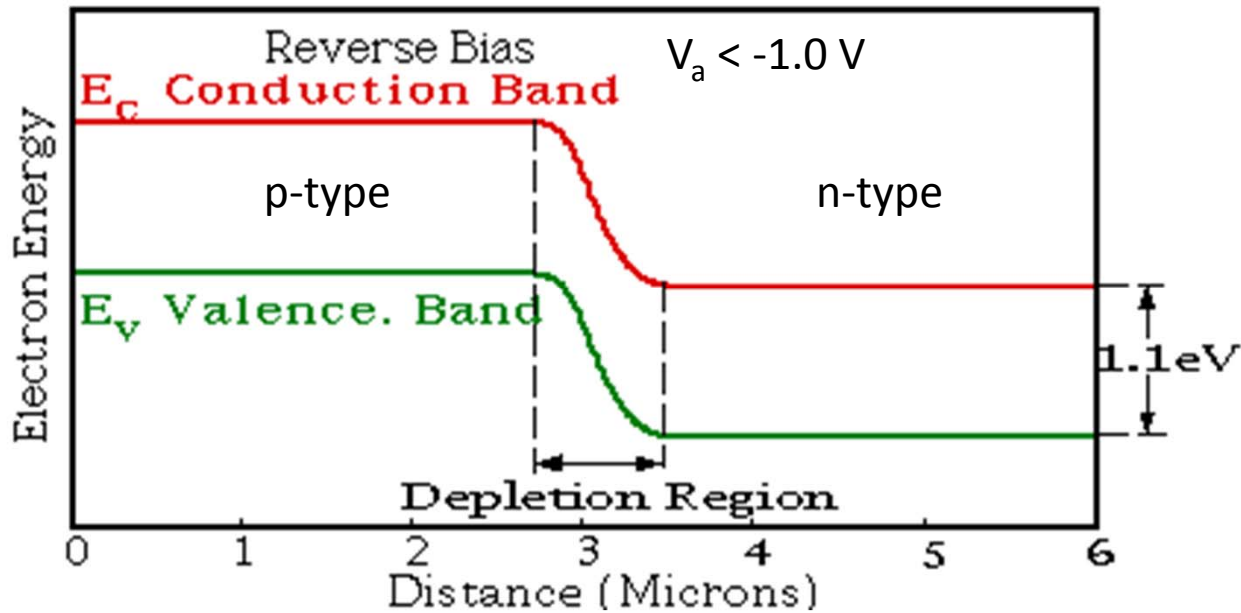
- Diffusion of majority carriers balances the drift of minority carriers across the junction, resulting in no net current (only the case in the dark)
- Diffusion current is “uphill”
- Drift current is “downhill”
- Which carriers in n-type (or p-type) contribute to diffusion current? And which to drift current?
- In n-type (p-type), which carrier type predominates?

Reverse Biased p-n junction

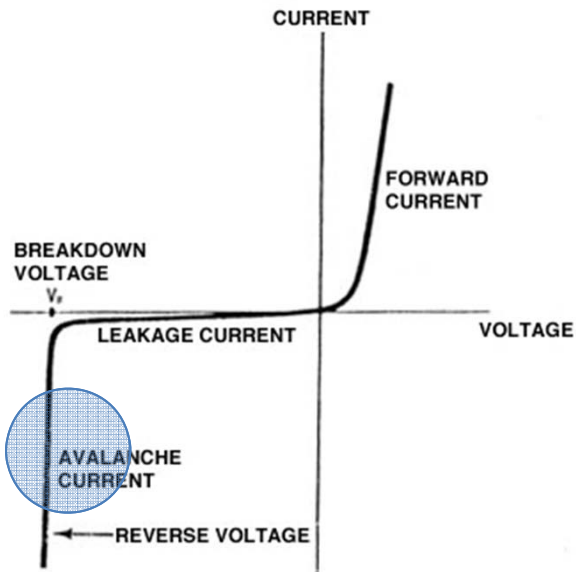


- Voltage applied such that the E-field at the junction increases
- Negative voltage to p-type, and positive voltage to n-type
- Higher E-field in depletion region decreases diffusion across the junction → diffusion current decreases
- Drift current limited by the number of minority carriers, and changes little
- Drift current increases slightly (depletion region width increases slightly) – minor effect in Si solar cells, which are much thicker than the depletion region width
- Why is the reverse saturation current so small?

Reverse Biased p-n junction (breakdown, avalanche current)



In breakdown, electrons can tunnel from the p-type material valence band through the depletion region material to contribute to large negative current flow (electrons to the right through n-type material)



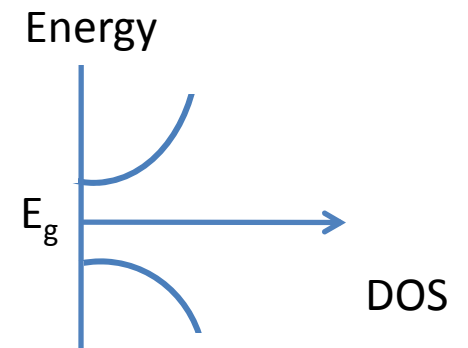
A bit about Density of States....

The DOS in a semiconductor is obtained by solving the Schrödinger equation for the particles in the semiconductor. Rather than solving for the complex potential in the semiconductor, one can use the particle-in-a box model, assuming that the particle is free to move within the material. The boundary conditions which express the fact that the particles can not leave the material, force the density of states in k-space to be constant. The DOS are approximately parabolic in energy near the conduction and valence band edges (i.e. the DOS goes with the square root of the energy distance above (below) the conduction (valence) band minimum (maximum))

Density of states as f(E):

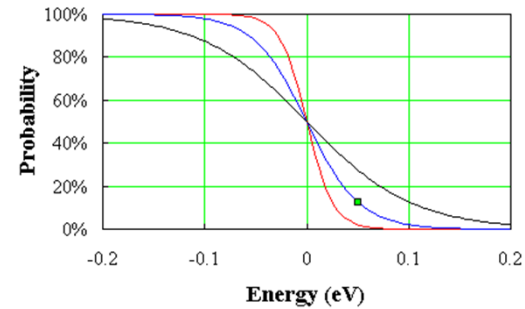
$$g_c(E) = \frac{8\pi\sqrt{2}}{h^3} m_e^{*3/2} \sqrt{E - E_C}, \quad \text{for } E \geq E_C$$

$$g_v(E) = \frac{8\pi\sqrt{2}}{h^3} m_h^{*3/2} \sqrt{E_V - E}, \quad \text{for } E \leq E_V$$



...The Fermi function tells us about the occupation of these states

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

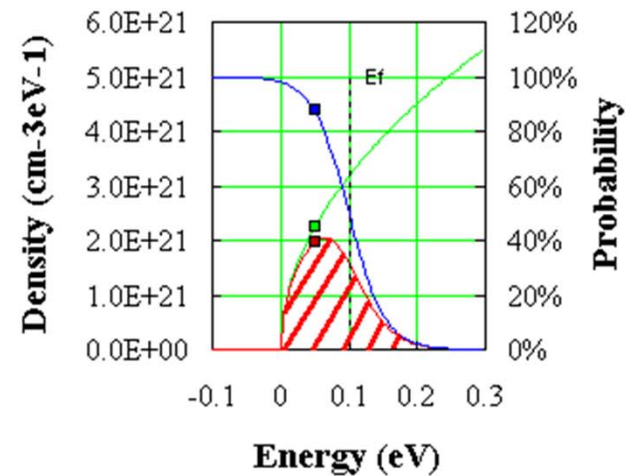


Density of carriers in the band can be obtained (for, e.g., electron in the conduction band):

$$n = \int_{E_C}^{\text{top of the conduction band}} n(E) dE = \int_{E_C}^{\text{top of the conduction band}} g_c(E) f(E) dE$$

$$n = \int_{E_C}^{\infty} \frac{8\pi\sqrt{2}}{h^3} m_e^{*3/2} \sqrt{E - E_C} \frac{1}{1 + e^{\frac{E - E_F}{kT}}} dE$$

(with units of #/cm³)



For non-degenerate semiconductors, we require that the Fermi level be at least $3kT$ (~ 75 meV at room temperature) away from the band edge....

Then, the Fermi function can be replaced by a simple exponential term....

$$n \cong \int_{E_C}^{\infty} \frac{8\pi\sqrt{2}}{h^3} m_e^{*3/2} \sqrt{E - E_C} e^{\frac{E_F - E}{kT}} dE = N_C e^{\frac{E_F - E_C}{kT}}$$

$$p \cong \int_{-\infty}^{E_V} \frac{8\pi\sqrt{2}}{h^3} m_h^{*3/2} \sqrt{E_V - E} e^{\frac{E - E_F}{kT}} dE = N_V e^{\frac{E_V - E_F}{kT}}$$

Where the *effective* Density of States in the Conduction and Valence bands are:

$$N_C = 2 \left[\frac{2\pi m_e^* kT}{h^2} \right]^{3/2} \quad \text{and} \quad N_V = 2 \left[\frac{2\pi m_h^* kT}{h^2} \right]^{3/2} \quad (\text{with units of cm}^{-3})$$

Interestingly,

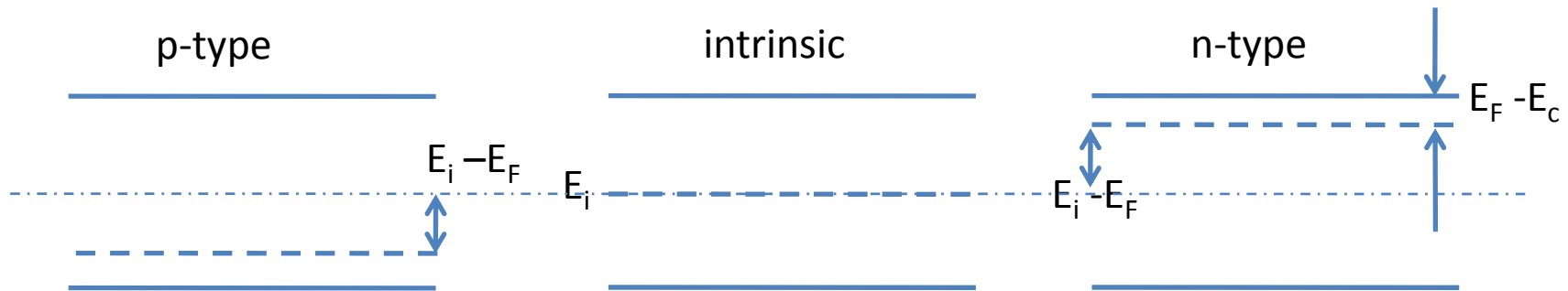
$$np = N_V N_C e^{\frac{E_V - E_F}{kT}} e^{\frac{E_F - E_C}{kT}} = N_V N_C e^{\frac{-E_G}{kT}} = n_i^2$$

Which gives the Law of Mass action:

$$np = n_i^2$$

$$np = N_V N_C e^{\frac{E_V - E_F}{kT}} e^{\frac{E_F - E_C}{kT}} = N_V N_C e^{\frac{-E_G}{kT}} = n_i^2$$

$$n_i = \sqrt{N_V N_C} e^{\frac{-E_G}{2kT}}$$



A negative number

A positive number

$$p = N_V \exp\left(\frac{E_V - E_F}{kT}\right) = n_i \exp\left(\frac{E_i - E_F}{kT}\right) \text{ cm}^{-3}$$

A negative number

A positive number

$$n = N_C \exp\left(\frac{E_F - E_C}{kT}\right) = n_i \exp\left(\frac{E_F - E_i}{kT}\right) \text{ cm}^{-3}$$

n_i ranges from 10^{13} #/cc for Ge ($E_g = 0.66\text{eV}$)
to 10^6 #/cc for GaAs ($E_g = 1.42\text{ eV}$)

for Si: $n_i \sim 1 \times 10^{10} \text{ cm}^{-3}$
 $N_C = 3.2 \times 10^{19} \text{ cm}^{-3}$
 $N_V = 1.8 \times 10^{19} \text{ cm}^{-3}$

Basic Equations for Solving for the Electric Field, Transport, and Carrier Concentrations:
 see <http://www.pveducation.org/pvcdrom/pn-junction/basic-equations>, up through
 “Solving for Region With Electric Field”

1. **Poisson's equation:**

$$\frac{\partial \bar{E}}{dx} = \frac{\rho}{\epsilon} = \frac{q}{\epsilon} (p(x) - n(x) - N_A^- + N_D^+)$$

$$\nabla^2 \phi(x, y, z) = -\frac{\rho(x, y, z)}{\epsilon}$$

$$\frac{d\phi(x)}{dx} = -\mathcal{E}(x)$$

2. **Transport equations:**

$$J_n = q\mu_n n(x)\bar{E} + qD_n \frac{dn(x)}{dx} \quad (\text{first term is drift, second is diffusion})$$

$$J_p = q\mu_p p(x)\bar{E} - qD_p \frac{dp(x)}{dx}$$

3. **Continuity equations:**

General conditions

$$\frac{dn}{dt} = \frac{1}{q} \frac{\partial J_n}{\partial x} - (U - G)$$

$$\frac{dp}{dt} = \frac{1}{q} \frac{\partial J_p}{\partial x} + (U - G)$$

Under thermal equilibrium and steady state conditions

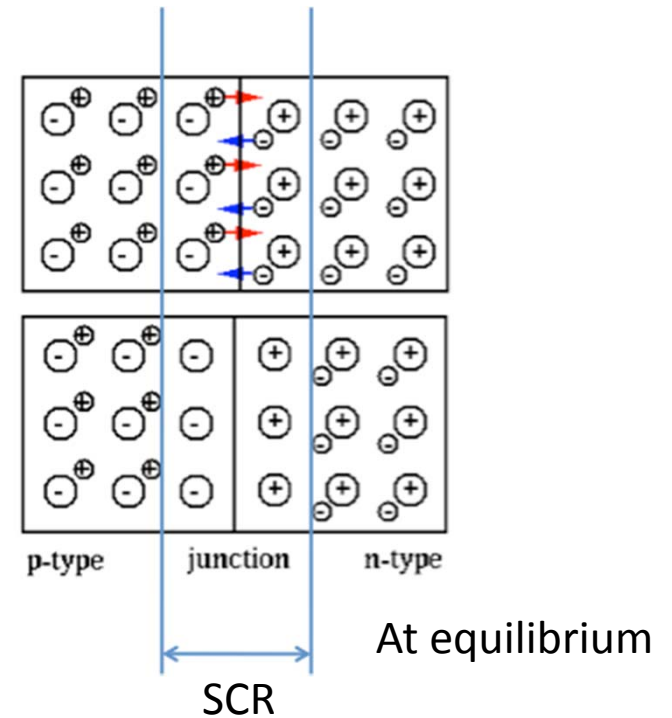
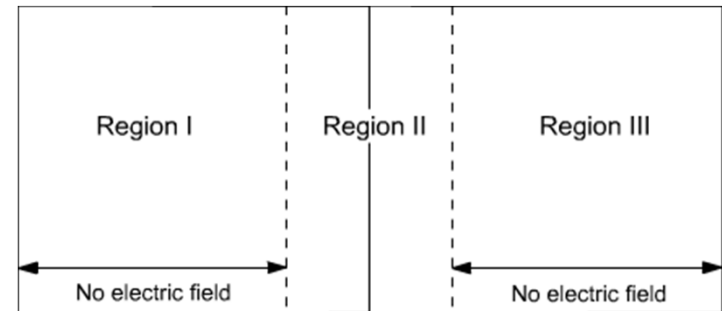
$$\frac{1}{q} \frac{\partial J_n}{\partial x} = (U - G)$$

$$\frac{1}{q} \frac{\partial J_p}{\partial x} = -(U - G)$$

where U and G are the recombination and generation rates in the particular material and depend on the details of the device and may also depend on distance.

General Approach to Solving for the Electric Field, Transport, and Carrier Concentrations

For arbitrary charge distributions, band diagrams, junction types, the equations *may* be solved using numerical approaches, and many device simulators are available. In addition to the assumption of a one-dimensional device, the most valuable simplifying assumption in determining a closed form solution to the above equations is the **depletion approximation**. The **depletion approximation** assumes that the electric field in the device is confined to some region of the device. The device can then be broken up into regions that have an electric field and those that do not. This is shown below for a pn junction, where Regions I and III do not have an electric field (called quasi-neutral regions or QNR) and Region II has an electric field (which is called space-charge or depletion region).

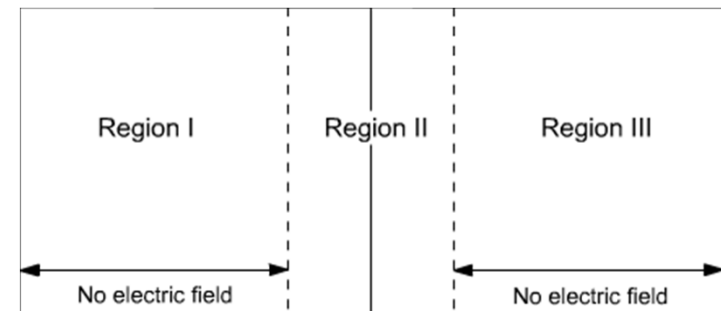


General Procedure using the depletion approximation:

1. Divide the **device into regions** with an electric field and without an electric field.
2. Solve for electrostatic properties in the **depletion region** (Region II on the diagram). This solution depends on the doping profile assumed. Here we will restrict the calculations to constant doping profiles.
3. Solve for the carrier concentration and current in the **quasi-neutral regions** (Regions I and III on the diagram) under steady-state conditions.

The steps in this are:

- (a) Determine the general solution for the particular device. The general solution will depend only on the types of recombination and generation in the device.
 - (b) Find the particular solution, which depends on the surfaces and the conditions at the edges of the depletion region.
4. Find the relationship between the currents on one side of the depletion region and the currents on the other side. This depends on the recombination/generation mechanisms in the depletion region.



Basic Equations for Solving for the Electric Field, Transport, and Carrier Concentrations

1. **Poisson's equation:**

$$\frac{\partial \bar{E}}{\partial x} = \frac{\rho}{\epsilon} = \frac{q}{\epsilon} (p(x) - n(x) - N_A^- + N_D^+)$$

$$\nabla^2 \phi(x, y, z) = -\frac{\rho(x, y, z)}{\epsilon}$$

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(first term is drift, second is diffusion)

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Under thermal equilibrium and steady state conditions

$$\frac{1}{q} \frac{\partial J_n}{\partial x} = (U - G)$$

$$\frac{1}{q} \frac{\partial J_p}{\partial x} = -(U - G)$$

where U and G are the recombination and generation rates in the particular material and depend on the details of the device and may also depend on distance.

Solving for Region With Electric Field

1. Depletion approximation: the electric field is confined to a particular region.
2. No free carriers ($n(x), p(x) = 0$) in depletion region.
3. *We can assume no free carriers since the electric field sweeps them out of the depletion region quickly. No free carriers means (1) transport equations drop out and (2) no recombination or generation, so the continuity equation becomes:*

$$\frac{1}{q} \frac{\partial J_n}{\partial x} = (U - G) = 0$$

This means that J_n is constant across the depletion region. Similarly, J_p is also constant across the depletion region.

4. *Abrupt or step doping profile (N_A^- , N_D^+ are constant).*
5. *All dopants are ionized ($N_A^- = N_A$, $N_D^+ = N_D$).*
6. *One-dimensional device.*

Solution

The only equation left to solve is Poisson's Equation, with $n(x)$ and $p(x) = 0$, abrupt doping profile and ionized dopant atoms. Poisson's equation then becomes:

$$\frac{\partial \hat{E}}{\partial x} = \frac{\rho}{\epsilon} = \frac{q}{\epsilon} (-N_A + N_D)$$
$$\rho = \begin{cases} -qN_A, & \text{when } -x_p \leq x \leq 0 \\ qN_D, & \text{when } 0 \leq x \leq x_n \end{cases}$$

Where $\epsilon = \epsilon_0 \epsilon_s$, ϵ_0 is the permittivity in free space, and ϵ_s is the permittivity in the semiconductor and x_p and x_n are the edges of the depletion region in the p- and n-type side respectively, measured from the physical junction between the two materials. The electric field then becomes

$$E = \begin{cases} \int -\frac{qN_A}{\epsilon} dx = -\frac{qN_A}{\epsilon} x + C_1, & \text{for } -x_p \leq x < 0 \\ \int \frac{qN_D}{\epsilon} dx = \frac{qN_D}{\epsilon} x + C_2, & \text{for } 0 \leq x < x_n \end{cases}$$

Solution (continued...)

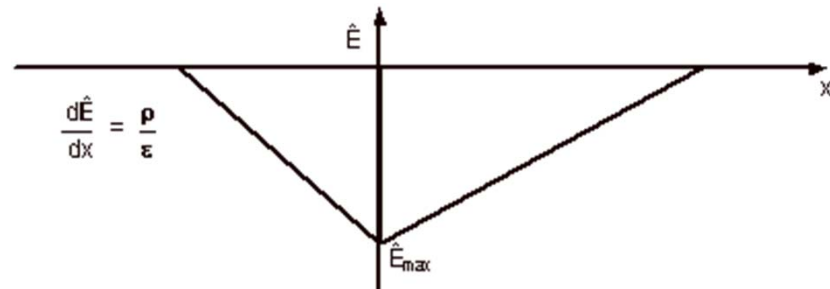
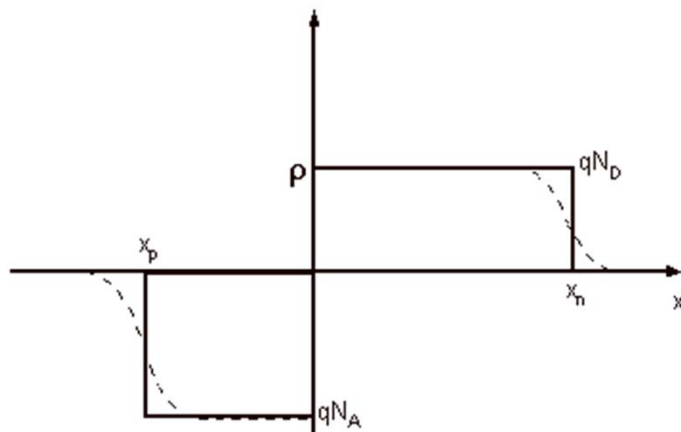
The integration constants C_1 and C_2 can be determined by using the depletion approximation, which states that the electric field must go to zero at the boundary of the depletion regions. This gives:

$$E(x = -x_p) = 0 \quad \Rightarrow \quad C_1 = \frac{-qN_A}{\epsilon} x_p$$

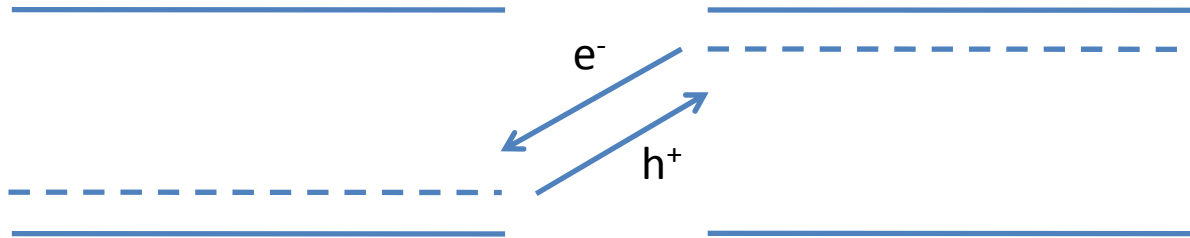
$$E(x = x_n) = 0 \quad \Rightarrow \quad C_2 = -\frac{qN_D}{\epsilon} x_n$$

Solution (Electric field distribution in the space charge)

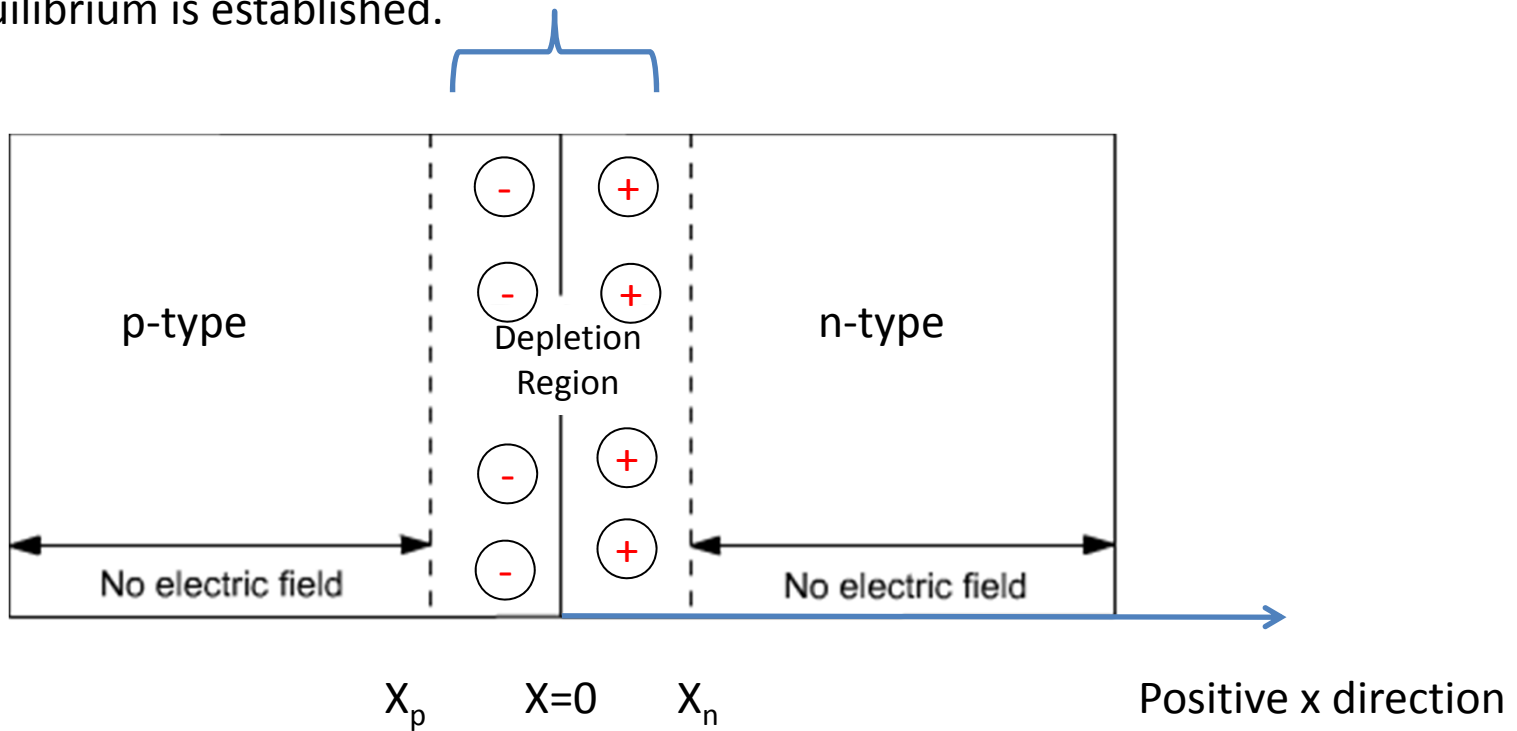
The maximum electric field occurs at the junction between the p- and n-type material. Further, we know that the electric field lines must be continuous across the interface, such that the electric field in the p-type side and the n-type side must equal each other at the interface or when $x = 0$. Putting $x = 0$ in the above equation for electric field and setting the two values of E equal to each other gives: $N_A x_p = N_D x_n$. This equation makes physical sense since it states that the total charge on one side of the junction must be the same as the total charge on the other. In other words, if the electric field is confined to the depletion region, then the net charge in Region II must be zero, and hence the negative charge and the positive charge must be equal. $N_A x_p A$ is the total negative charge, since N_A is the charge density and $x_p A$ is the volume of the depletion region (A is the cross-sectional area and x_p is the depth). Similarly, $N_D x_n A$ is the positive charge. The cross sectional area (A) is the same and cancels out.



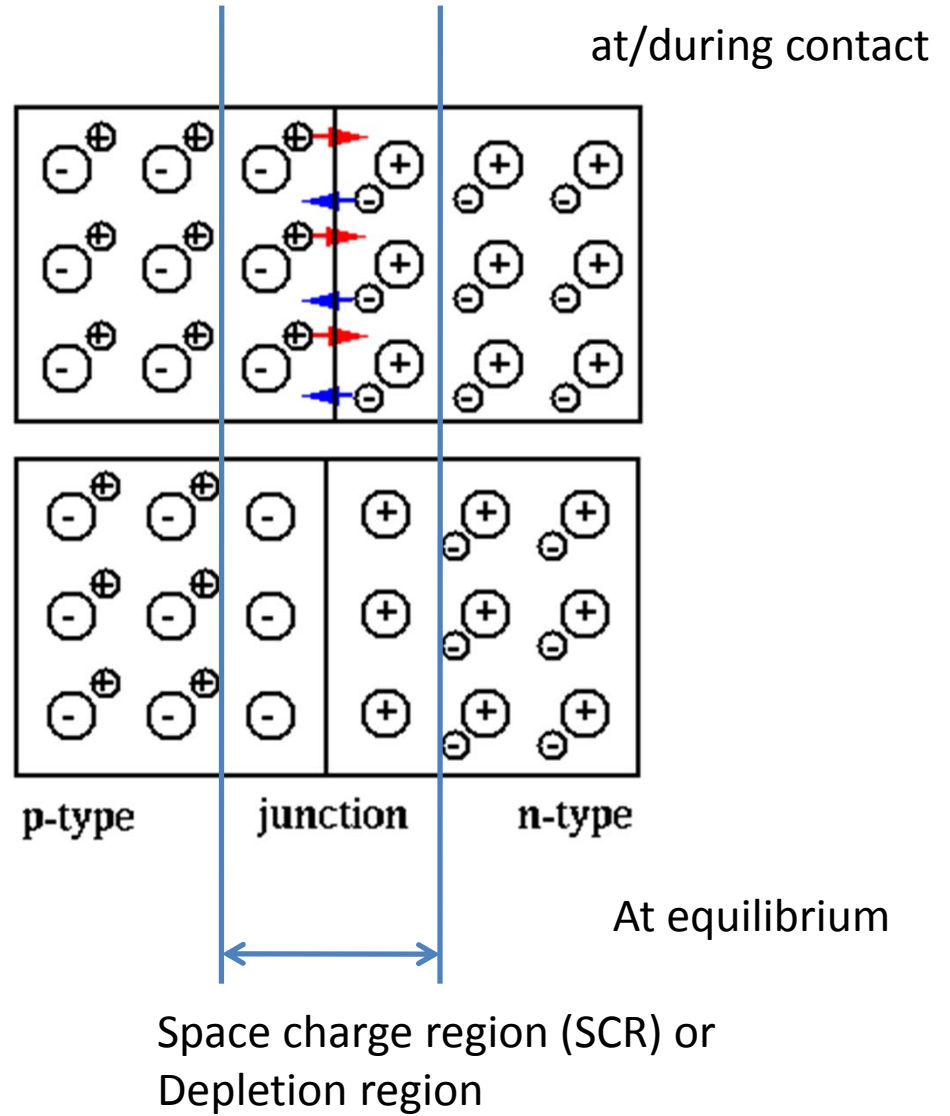
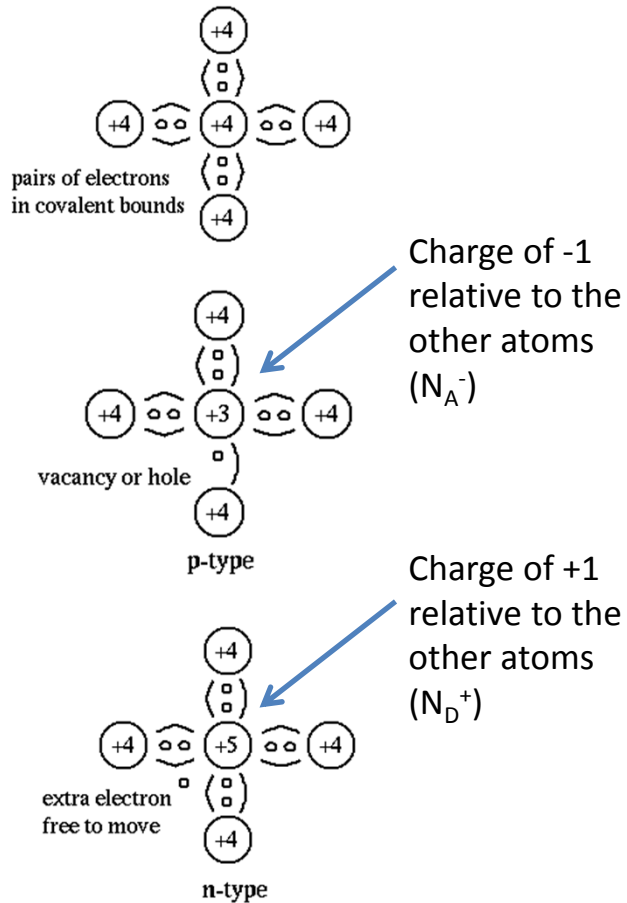
Before/during contact



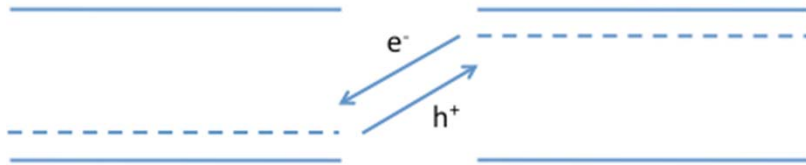
Depletion approximation: electric field is confined to space charge region, fixed charge associated with dopants is left “uncovered” after electronic equilibrium is established.



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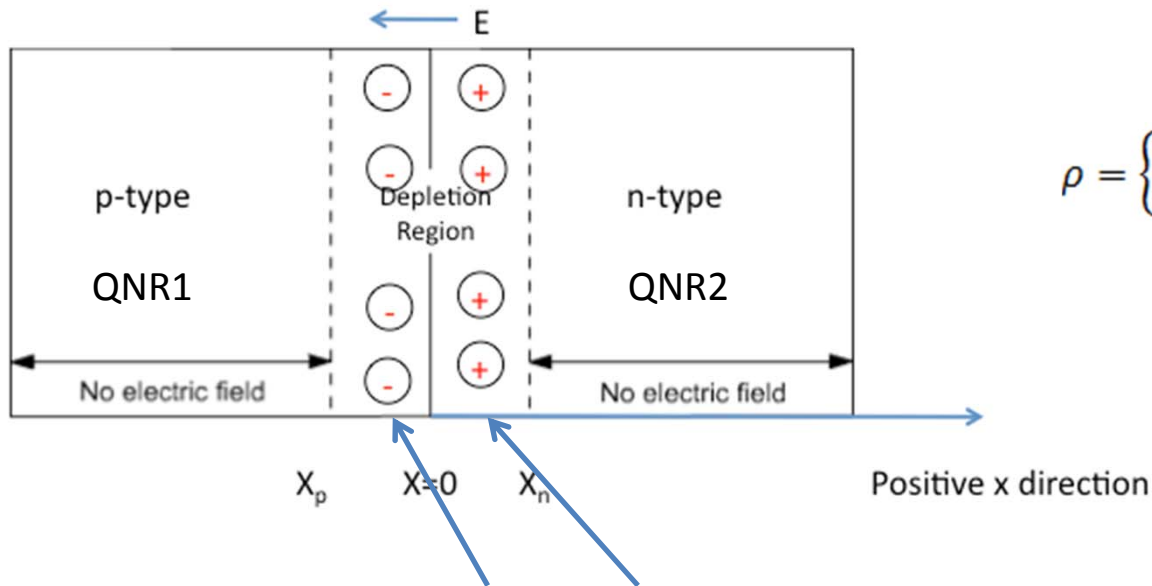


Before/during contact



What is the electric field?

At equilibrium, Electric field develops to oppose further charge



Charge neutrality: $x_p N_A^- = x_n N_D^+$

$$\frac{\partial \hat{E}}{\partial x} = \frac{\rho}{\epsilon} = \frac{q}{\epsilon} (-N_A + N_D)$$

$$\rho = \begin{cases} -qN_A, & \text{when } -x_p \leq x \leq 0 \\ qN_D, & \text{when } 0 \leq x \leq x_n \end{cases}$$

- net charge density is zero outside SCR
- $p(x)$ and $n(x) = 0$ within SCR
- all dopants are ionized, so $N_A^- = N_A$ and $N_D^+ = N_D$

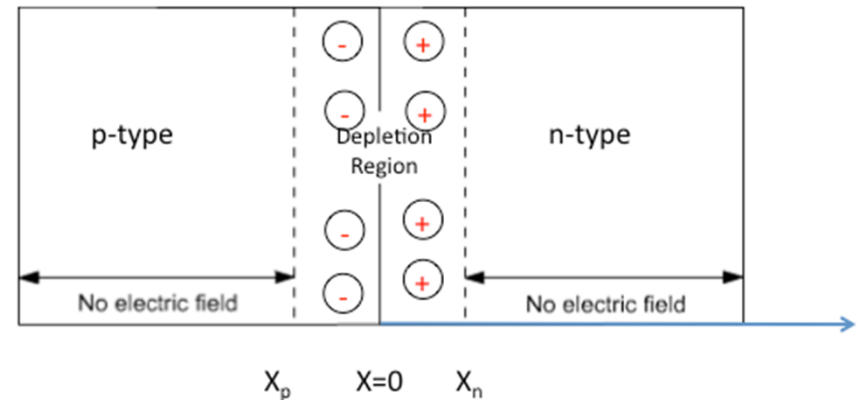
$$\frac{\partial \hat{E}}{\partial x} = \frac{\rho}{\epsilon} = \frac{q}{\epsilon} (-N_A + N_D) \quad \rho = \begin{cases} -qN_A, & \text{when } -x_p \leq x \leq 0 \\ qN_D, & \text{when } 0 \leq x \leq x_n \end{cases}$$

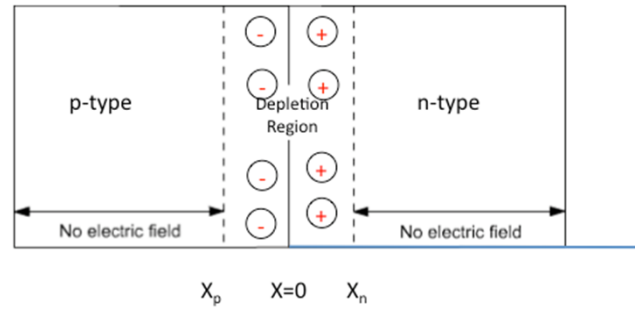
$$E = \begin{cases} \int -\frac{qN_A}{\epsilon} dx = -\frac{qN_A}{\epsilon} x + C_1, & \text{for } -x_p \leq x < 0 \\ \int \frac{qN_D}{\epsilon} dx = \frac{qN_D}{\epsilon} x + C_2, & \text{for } 0 \leq x < x_n \end{cases}$$

Bringing back the Boundary Conditions
(E goes to zero at boundaries of DR)

$$E(x = -x_p) = 0 \Rightarrow C_1 = \frac{-qN_A}{\epsilon} x_p$$

$$E(x = x_n) = 0 \Rightarrow C_2 = -\frac{qN_D}{\epsilon} x_n$$

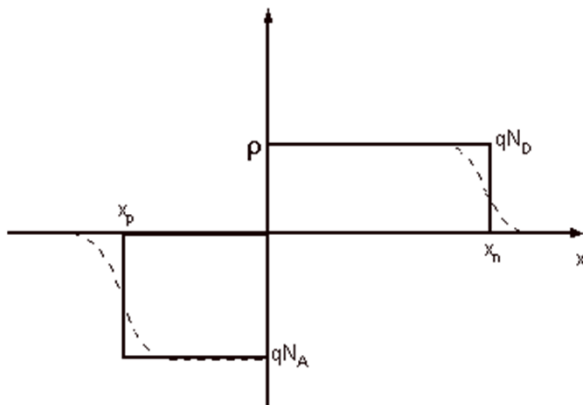




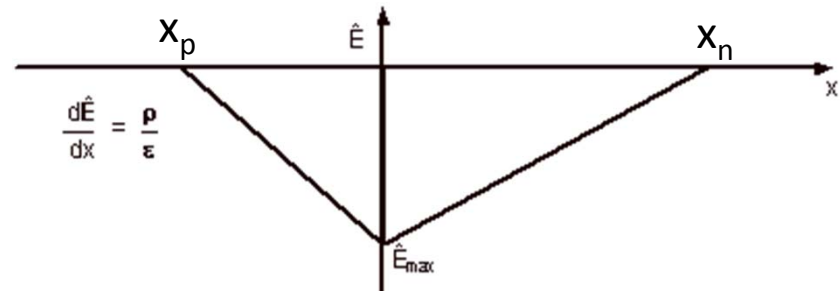
$$E = \begin{cases} \int -\frac{qN_A}{\epsilon} dx = -\frac{qN_A}{\epsilon} x + \frac{-qN_A}{\epsilon} x_p & \text{for } -x_p \leq x < 0 \\ \int \frac{qN_D}{\epsilon} dx = \frac{qN_D}{\epsilon} x - \frac{qN_D}{\epsilon} x_n & \text{for } 0 \leq x < x_n \end{cases}$$

The two parts of the solution must give the same value for E when $x = 0$

so $-N_A x_p = -N_D x_n$, which is the charge neutrality relation written earlier



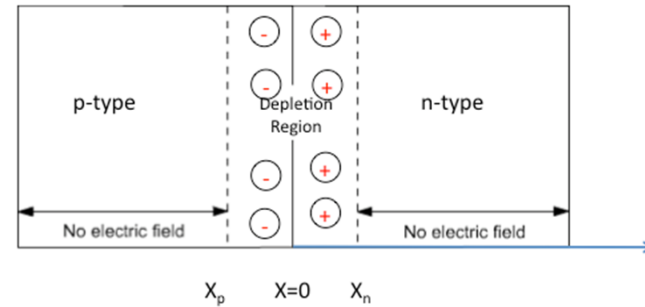
Plots shown for $N_A \neq N_D$



$|E|_{\max}$ at $x = 0$

What is the potential distribution?

- Integrate the electric field over distance.



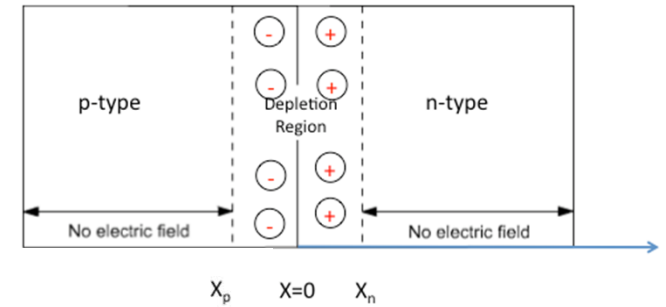
$$E = \begin{cases} \int -\frac{qN_A}{\epsilon} dx = -\frac{qN_A}{\epsilon} x + \frac{-qN_A}{\epsilon} x_p & \text{for } -x_p \leq x < 0 \\ \int \frac{qN_D}{\epsilon} dx = \frac{qN_D}{\epsilon} x - \frac{qN_D}{\epsilon} x_n & \text{for } 0 \leq x < x_n \end{cases}$$

$$V(x) = \begin{cases} \int \int -E(x) dx = \int \frac{qN_A}{\epsilon} (x + x_p) dx = \frac{qN_A}{\epsilon} \left(\frac{x}{2} + x_p \right) x + C_3, & \text{for } -x_p \leq x < 0 \\ \int \int -E(x) dx = \int \frac{qN_D}{\epsilon} (x_n - x) dx = \frac{qN_D}{\epsilon} \left(x_n - \frac{x}{2} \right) x + C_4, & \text{for } 0 \leq x < x_n \end{cases}$$

Since we are interested in the potential difference, we can set the voltage on the p-type side to be zero, such that at $x = -x_p$, $V = 0$.

Thus: $C_3 = \frac{qN_A}{2\epsilon} x_p^2$ and $V(x) = \frac{qN_A}{2\epsilon} (x + x_p)^2$, for $-x_p \leq x < 0$

C_4 may be determined by using the fact that the potential on the n-type side and p-type side are identical at the interface.



$$V(x) = \begin{cases} = \frac{qN_A}{\epsilon} \left(\frac{x}{2} + x_p \right) x + \frac{qN_A}{2\epsilon} x_p^2 & \text{for } -x_p \leq x < 0 \\ = \frac{qN_D}{\epsilon} \left(x_n - \frac{x}{2} \right) x + C_4 & \text{for } 0 \leq x < x_n \end{cases}$$

Thus, at $x = 0$:

$$V_p(x = 0) = \frac{qN_A}{2\epsilon} x_p^2 = V_n(x = 0) = \frac{qN_D}{2\epsilon} \left(x_n - \frac{x}{2} \right) x + C_4$$

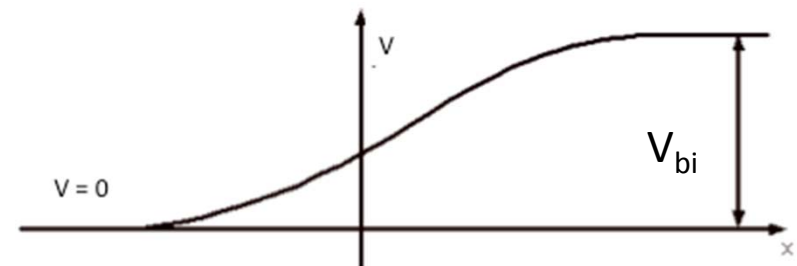
So: $C_4 = \frac{qN_A}{2\epsilon} x_p^2$

Overall:

$$V(x) = \begin{cases} \frac{qN_A}{2\epsilon} (x + x_p)^2, & \text{for } -x_p \leq x < 0 \\ \frac{qN_D}{\epsilon} \left(x_n - \frac{x}{2} \right) x + \frac{qN_A}{2\epsilon} x_p^2, & \text{for } 0 \leq x < x_n \end{cases}$$

Voltage is a max at $x = x_n$:

$$V(x = x_n) = \frac{q}{2\epsilon} (N_D x_n^2 + N_A x_p^2) = V_{bi} \text{ (aka } V_o)$$

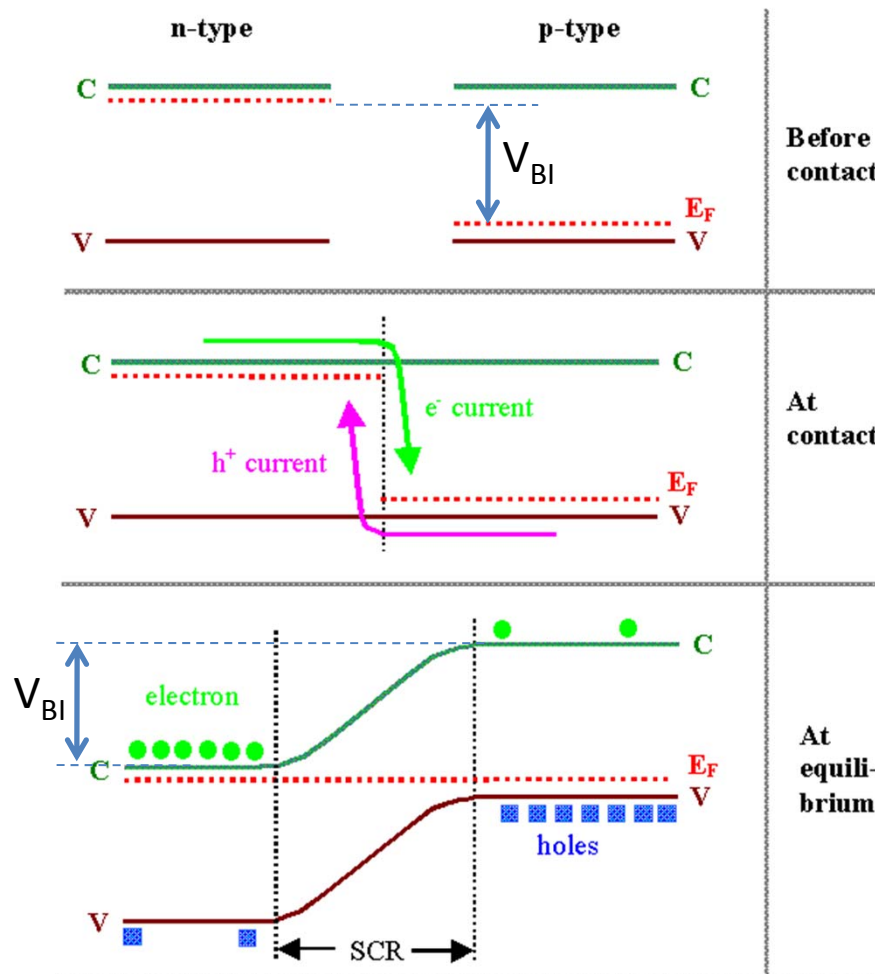


V_o (aka V_{BI}) is the difference between the Fermi Levels in the two contacting “bits”

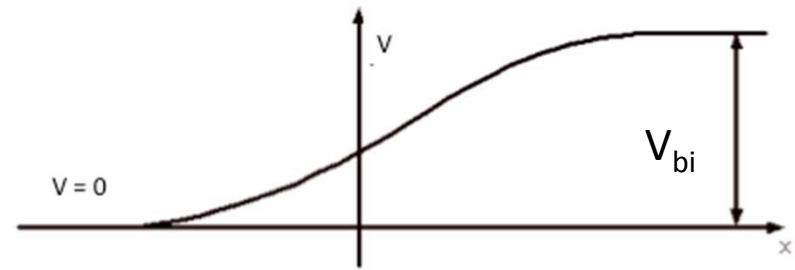
Electrons in the two bits have different electrochemical potentials (i.e. different E_f 's)

Charge transfer occurs at contact (electron go down from the vacuum level, holes go “up”)

At equilibrium, there is no net transport (E_f is constant throughout the device)



$$V_{bi} = \frac{q}{2\epsilon} (N_D x_n^2 + N_A x_p^2)$$



Using the condition for charge neutrality:

$$N_A x_p = N_D x_n$$

We can determine the edges of the DR in terms of V_0 :

$$x_n = \left[\frac{2\epsilon V_0}{q} \frac{N_A}{N_D(N_A + N_D)} \right]^{\frac{1}{2}} \quad \text{and} \quad x_p = \left[\frac{2\epsilon V_0}{q} \frac{N_D}{N_A(N_A + N_D)} \right]^{\frac{1}{2}}$$

Which gives the Depletion Width, W :

$$W = x_n + x_p = \sqrt{\frac{2q}{\epsilon} \frac{V_0}{\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}}$$

Or, x_p and x_n in terms of W :

$$W = \left[\frac{2K_s\epsilon_0}{q} \left(\frac{N_A + N_D}{N_A N_D} \right) (V_{bi} - V) \right]^{\frac{1}{2}}$$

From wiki

$$x_p = W \frac{N_D}{N_A + N_D} \quad x_n = W \frac{N_A}{N_A + N_D}$$