Syllabus: *An introduction to quantum symmetries*

1: *Hopf Algebra Fundamentals*

- Origin of the Lie bracket in differential geometry, definition of a Lie algebra; tensor products, universal enveloping algebras; definition of a coalgebra and the enveloping algebra as a coalgebra; Sweedler notation; definition of a bialgebra; the quantised enveloping algebra$U_q(\mathfrak{sl}_2)$.
- Antipode on a Lie algebra and $U_q(\mathfrak{sl}_2)$, definition of a Hopf algebra; Radford's S^4 formula; finite groups and Hopf algebras.
- The quantised coordinate algebra$C_q[SU_2]$ and the dual pairing with $U_q(\mathfrak{sl}_2)$; classical vector field on functions; general definition of a dual pairing; restricted duals
- Hilbert Nullstellensatz; algebraic group and Hopf structure; Hopf Alg Structures correspond to alg group structures on affine varieties.

2: *Corepresentation, Categories, and Quantum Homogeneous Spaces*

- Definition of a comodule, classification of comodules for $\mathbb{C}_q[SU_2]$; category theory basics, equivalence of the comodule category of $\mathbb{C}_q[SU_2]$; with the classical category; brief discussion of the general $\mathbb{C}_q[G]$ case.
- Yetter–Drinfeld modules and the Woronowicz algebra; monoidal categories, braidings; the Tannaka–Krein philosophy.
- Rigid categories, quantum dimension, applications to knots.
- Quantum homogeneous spaces, faithful flatness, Takeuchi's equivalence

3: *Semisimplicity, Peter–Weyl, and Compact Quantum Groups*

- Coemisimplicity, Haar functionals, algebraic compact quantum groups
- C^*-algebras, the Gelfand–Naimark theorem; compact quantum groups
- Peter–Weyl and the algebra–analysis equivalence
- Woronowicz functionals, cyclic cohomology, twisted cyclic cohomology

4: *Noncommutative Geometry and Quantum Groups*

- Definition of a spectral triple, differential calculi, classification on quantum homogeneous spaces

- Hopf–Galois extensions, quantum principal bundles, and connections; the monpole connection for the Podleś sphere, Chern–Galois theory and index calculations
- The complex geometry of the Podleś sphere and the q-Dirac–Dolbeault operator.
- Index of an operator, the q-Riemann–Roch theorem for the Podleś sphere; the Dabrowski–Sitarz spectral triple

References:

1. C. Kassel, Quantum Groups, Springer; 1995
2. A. Klimyk, K. Schmüdgen, Quantum Groups and their Representations, *Springer–Verlag*, 1997
3. S. Majid, A Quantum Groups Primer, London Mathematical Society Lecture Note Series, *Cambridge University Press*, 2002