- $s(t) > V_c$:
  - diode conducts (forward biased)
  - C charges quickly to the max value of $s(t)$
- $s(t) < V_c$:
  - diode does not conduct (reverse biased)
  - C discharges over R_L until $s(t) > V_c$

- The output of the ED is then lowpass filtered to eliminate the ripple, followed by blocking out the DC component.

Single Sideband Modulation (SSB)

Standard AM and DSB-SC techniques are wasteful of bandwidth because they both require transmission bandwidth of $2B$ Hz, where B is the bandwidth of the baseband modulating signal $m(t)$.

In both cases the transmission bandwidth (B_T) is occupied by the upper sideband (USB) and lower sideband (LSB).

Observations

- USB and LSB are uniquely related to each other, as they are symmetric wrt f_c. Therefore, to transmit information contained within $m(t)$ we used to transmit only one side band.
- As far as demodulation is concerned, we can coherently demodulate SSB (as we did the DSB-SC signal) by multiplying SSB with $\cos(\omega_c t)$ followed by LPF.

Frequency domain representation of SSB signals

Given the baseband signal $m(t)$ with spectrum $M(\omega)$, the spectrum of DSB-SC and SSB are shown below (textbook, p174):

In either of the USB/LSB cases we can demodulate and extract $m(t)$ from SSB_+ or SSB_-, by regular coherent demodulation, shown as below.

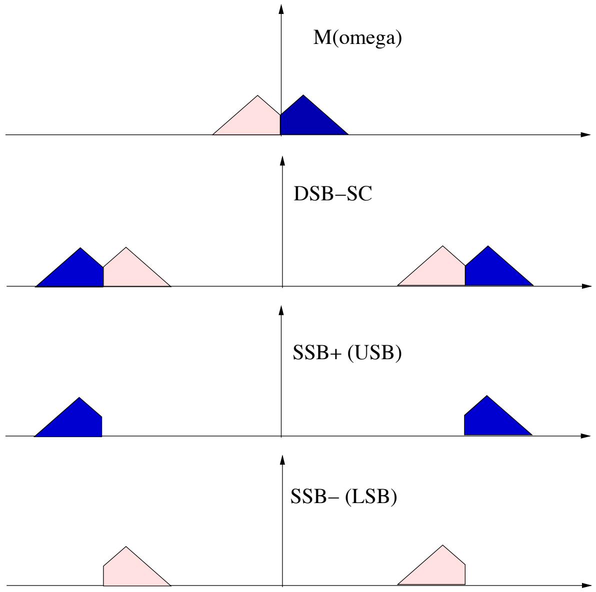


Figure 6: SSB - frequency domain.

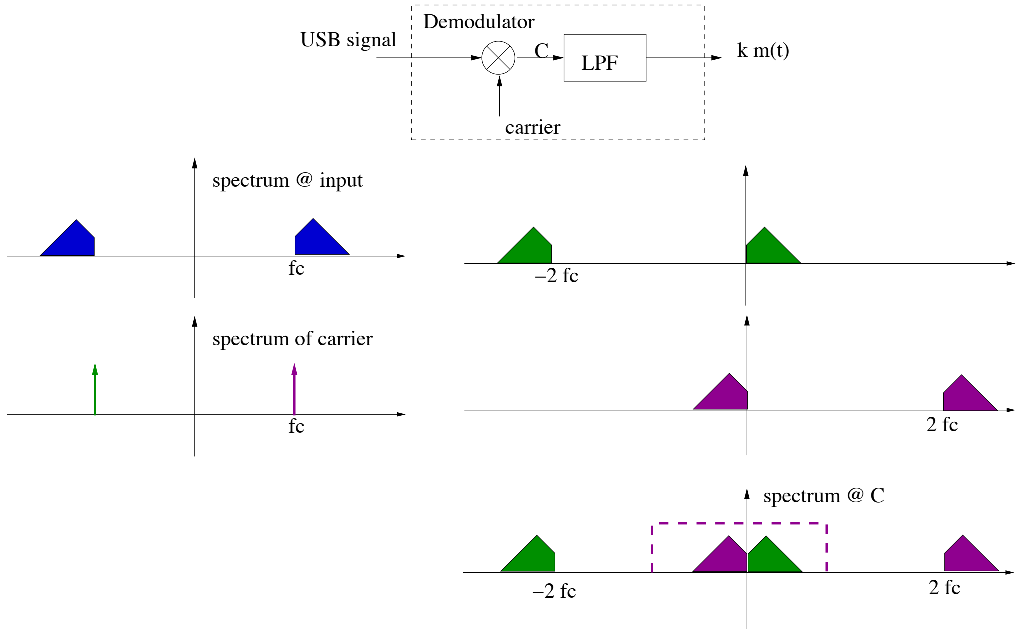
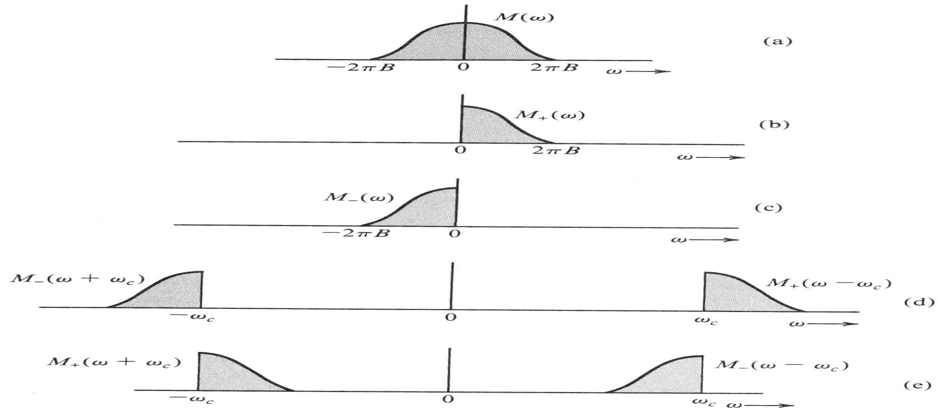


Figure 7: SSB demodulation, frequency domain.

Time domain representation of SSB signals – Hilbert Transform

First, define some notations:

- $M(f), m(t)$ baseband modulating signal (real)
- $M_+(f), m_+(t)$ upper sideband (USB) signal (cannot be real)
- $M_-(f), m_-(t)$ lower sideband (LSB) signal (cannot be real)



Time domain representation of SSB signals - Hilbert Transform

From the spectrum relationship, we have

$$\begin{aligned} M_+(f) &= M(f)u(f) = M(f)\frac{1}{2}[1 + \text{sgn}(f)] = \frac{1}{2}[M(f) + jM_h(f)] \quad (11) \\ M_-(f) &= M(f)u(-f) = M(f)\frac{1}{2}[1 - \text{sgn}(f)] = \frac{1}{2}[M(f) - jM_h(f)] \quad (12) \end{aligned}$$

where

$$\frac{1}{2}jM_h(f) = \frac{1}{2}M(f)\text{sgn}(f) \quad (13)$$

which implies

$$M_h(f) = M(f) \cdot [-j\text{sgn}(f)] \quad (14)$$

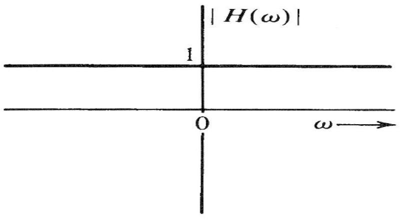
Considering the Fourier transform pair $1/(\pi t) \to -jsgn(f)$, taking inverse Fourier transform then we have

$$m_h(t) = m(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\alpha)}{t - \alpha} d\alpha \quad (15)$$

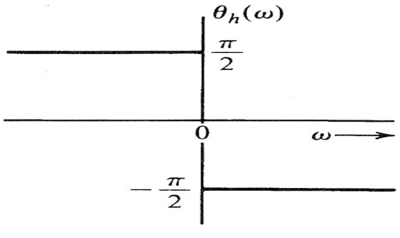
On the other hand, the transfer function can be written as

$$H(f) = -j\mathrm{sgn}(f) = \begin{cases} -j & f \geq 0 \\ j & f < 0 \end{cases}$$

$H(f)$: wideband phase shifter (Hilber Transform).



(a)



(b)

Thus, if we delay the phase of every component of $m(t)$ by $\pi/2$ (without changing its amplitude), the resulting signal is $m_h(t)$, the Hilbert transform of $m(t)$. Therefore, a Hilbert transformer is an ideal phase shifter that shifts the phase of every spectral component by $-\pi/2$.

Based on the Hilbert transform, we have

$$\begin{aligned} m_+(t) &= \frac{1}{2}[m(t) + jm_h(t)] \quad (16) \\ m_-(t) &= \frac{1}{2}[m(t) - jm_h(t)] \quad (17) \end{aligned}$$

where $m_h(t)$ is called Hilbert Transform of $m(t)$.

Time domain representation of SSB signals using Hilbert Transform

The USB spectrum is

$$\begin{aligned}\Phi_{USB}(f) &= M_+(f-f_c)+M_-(f+f_c) \qquad (18)\\ &= \frac{1}{2}[M(f-f_c)+M(f+f_c)]-\frac{1}{2j}[M_h(f-f_c)-M_h(f+f_c)]\end{aligned}$$

The inverse Fourier transform is then

$$s_{USB}(t) = m(t)\cos(\omega_c t) - m_h(t)\sin(\omega_c t) \qquad (19)$$

Similarly, we can show that

$$s_{LSB}(t) = m(t)\cos(\omega_c t) + m_h(t)\sin(\omega_c t) \qquad (20)$$

Hence, a general SSB signal can be expressed as

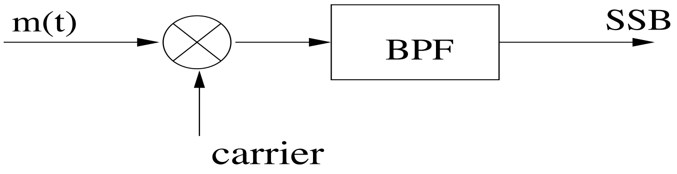
$$s_{SSB}(t) = m(t)\cos(\omega_c t) \mp m_h(t)\sin(\omega_c t) \qquad \text{(USB and LSB)} \qquad (21)$$

Generation of SSB Signals

A. Selective Filtering Method

It is the most common method of generation SSB. The basic idea is the following

- Using $m(t)$ to generate DSB-SC ($m(t)\cos\omega_c t$)
- DSB-SC goes through a BPF



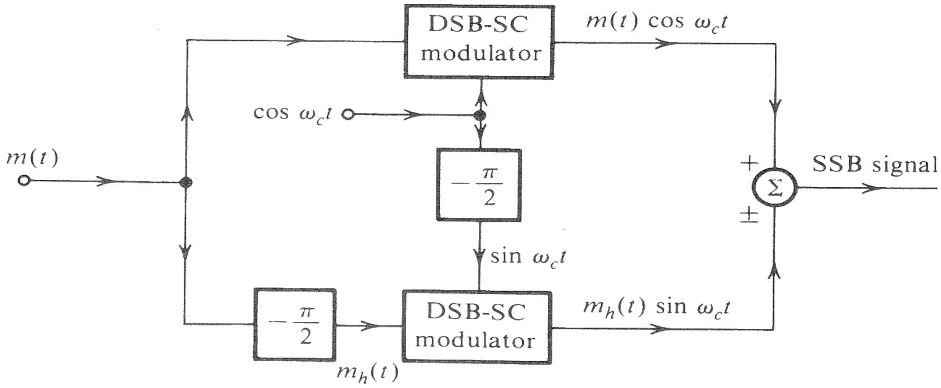
For successful implementation of this method, we must have

- $B << f_c$
- $m(t)$ must have little or no low-frequency content, *i.e.*, $M(\omega)$ has a “hole” at zero-frequency. For example, voice grade speech signal [0.3 3.4] khz.
- Why do we need “frequency hole”? If not, low frequency component cannot be kept.

Ideal filter (not realizable).

B. Phase-shift Method

$$\phi_{SSB}(t) = m(t)\cos(\omega_c t) \mp m_h(t)\sin(\omega_c t)$$



Demodulation of SSB Signals

A. Coherent demodulation

Observe that

$$\begin{aligned} s_{SSB}(t)\cos\omega_c t &= m(t)\cos^2(\omega_c t) \mp m_h(t)\sin(\omega_c t)\cos(\omega_c t) \\ &= \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos 2\omega_c t \pm \frac{1}{2}m_h(t)\sin 2\omega_c t \end{aligned}$$

If we filter $\phi_{SSB}\cos\omega_c t$ with a LPF, we can eliminate the components centered at $2f_c$ and the filter output will be $\sim m(t)$

Hence, any of the coherent demodulation techniques applicable for DSB-SC signals can be used.

B. Envelope Detection with a Carrier (SSB+C)

As a variation to the basic SSB case, we can add a carrier to the SSB signal and attempt to use envelope detector

$$\begin{aligned} s_{SSB+C}(t) &= A\cos(\omega_c t) + m(t)\cos(\omega_c t) \mp m_h(t)\sin(\omega_c t) \\ &= [A + m(t)]\cos(\omega_c t) \mp m_h(t)\sin(\omega_c t) \\ &= E(t)\cos(\omega_c t + \Theta(t)) \end{aligned}$$

where $E(t)$ is envelope, given as

$$E(t) = \sqrt{[A + m(t)]^2 + [m_h]^2}$$

and the phase is given as

$$\Theta(t) = tan^{-1}\left(\frac{m_h}{A + m(t)}\right)$$

At the receiver, a properly designed envelope detector will extract $E(t)$ from s_{SSB+C}.
Observe that

$$\begin{aligned} E(t) &= [A^2 + 2m(t)A + m^2(t) + m_h^2(t)]^{1/2} \qquad (22) \\ &= A\left[1 + \frac{2m(t)}{A} + \frac{m^2(t)}{A^2} + \frac{m_h^2(t)}{A^2}\right]^{1/2} \end{aligned}$$

If $A >> |m(t)|$ or $|m_h(t)|$, $E(t)$ can be approximated as

$$E(t) \approx A\left[1 + \frac{2m(t)}{A}\right]^{1/2}$$

Using a series expansion and discarding higher order terms due to $m/A << 1$, we have

$$E(t) \approx A\left[1 + \frac{m(t)}{A} + \cdots\right] = A + m(t)$$

It is evident that for a large carrier, the SSB+C can be demodulated by an envelope detector.

In AM, we need $A > m_p = |m(t)|$, while SSB+C, we need $A >> |m(t)|$. Therefore, SSB+C is very inefficient.

Vestigial Sideband Modulation (VSB)

Some observations

- SSB modulation is well suited for transmission of voice signals (or for all signals which exhibit a lower component at $f \approx 0$).
- DSB generation is much simpler, but requires twice the signal bandwidth.
- VSB modulation represents a compromise between SSB and DSB modulation systems.
- Simply stated VSB: one sideband is passed almost completely whereas just a trace (or vestige) of the other sideband is retained.

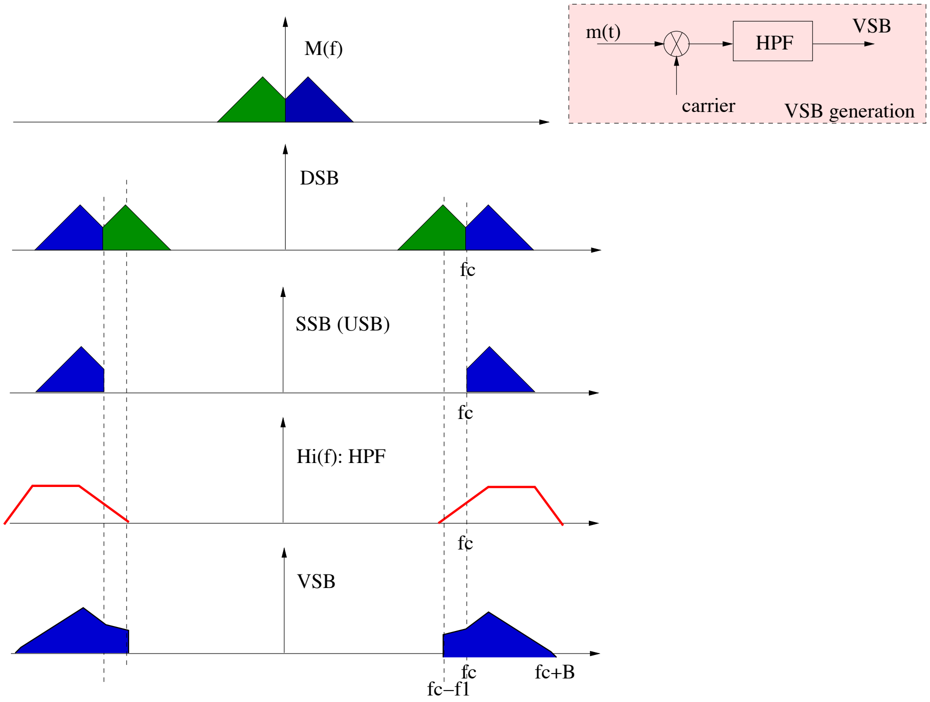
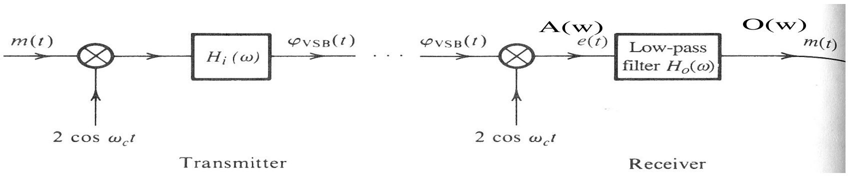


Figure 8: Illustration – VSB.

where $H_i(f)$ is vestigial shaping filter that produces VSB from DSB. It allows the transmission of one sideband, but suppresses the other sideband, not completely, but gradually. How do we design $H_i(f)$ to generate VSB signal?



$$\Phi_{VSB}(f) = \Phi_{DSB-SC} \cdot H_i(f) = [M(f+f_c) + M(f-f_c)] \cdot H_i(f) \tag{23}$$

It can be demodulated by multiplying the carrier:

$$\begin{aligned}
A(f) &= [\Phi_{VSB}(f+f_c) + \Phi_{VSB}(f-f_c)] \\
&\approx [M(f-2f_c) + M(f)]H_i(f-f_c) + [M(f+2f_c) + M(f)] \cdot H_i(f+f_c) \\
&= [H_i(f-f_c) + H_i(f+f_c)]M(f) + \text{other terms} \\
O(f) &\approx H_0(f)[H_i(f-f_c) + H_i(f+f_c)]M(f) = M(f)
\end{aligned}$$

Thus, we require

$$H_0(f) = \frac{1}{H_i(f+f_c) + H_i(f-f_c)} \quad |f| \le B \tag{24}$$

Furthermore, if we choose

$$H_i(f-f_c) + H_i(f+f_c) = 1 \quad |f| \le B$$

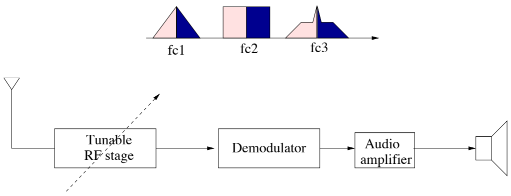
the output filter is just a simple low-pass filter.

Further observations

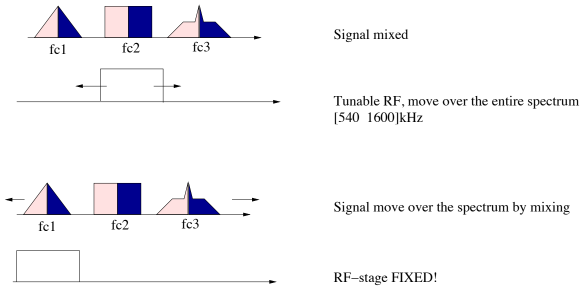
- $\lim_{f_v \to 0}[VSB] = SSB$, $\lim_{f_v \to B}[VSB] = DSB$
- VSB is demodulated using coherent detector.
- As an alternative, we can use VSB+C (envelope detector)
- B_T(VSB)$\approx 1.25 B_T$(SSB)
- $1/3 \ge \eta(AM) > \eta(VSB+C) > \eta(SSB+C)$

Superheterodyne Receiver

- Consider the FDM signal, to receive this signal and to tune in to a particular “channel”, we may require a receiver with the following structure



- The above system will function as required. However, the design and implementation of a tunable front-end, the RF stage, with sharp cut-off frequencies and high gain over a wide-range of frequencies, is a difficult task.
- Consider the following scenarios



- We know how to “mix” the input signal, *i.e.*, how to move the spectrum up and down (multiplying the input signal with the output of a local oscillator).
- We can also design a fixed frequency RF stage which has all the desired filtering and amplication properties.
- **Heterodyning**: translating or shifting in frequency. The concept which we described in very general terms above is called heterodyning. This technique consists of either down-converting or up-converting the input signal to some convenient frequency.
- We use a fixed **Intermediate Frequency (IF)** band. IF is fixed and is independent of the f_c (the carrier frequency) of the signal we receive.

- Commercial AM broadcast $f_{IF} = 455$ kHz. Carrier frequency assignment $f_c \in [540, 1600]$ kHz.
- Let us consider an AM radio station broadcasting at the carrier frequency of $f_c = 1000$ kHz, then

$$f_{LO} = f_c + f_{IF} = 1000 + 455 = 1455 \ kHz$$

- Image station: $2f_{IF}$ above f_c,

$$f_{image} = f_c + 2f_{IF} = 1000 + 2 \cdot 455 = 1910 \ kHz$$

  would also appear simultaneously at the IF output if it were not filtered out by the RF filter.

- The RF filter is hard to provide selectivity against adjacent stations separated by 10 kHz, but it can provide reasonable selectivity against a station separated by 910 kHz.

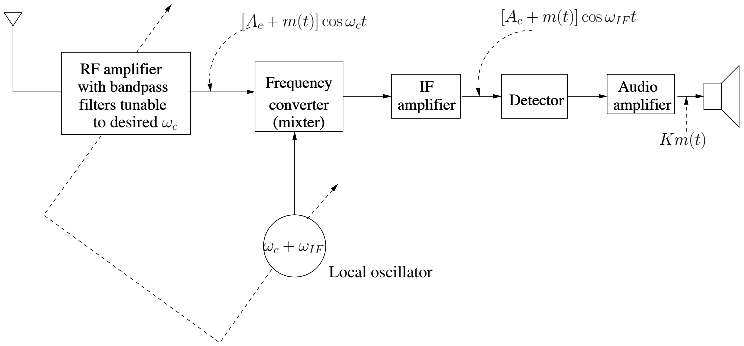


Figure 9: FM stereo transmitter.