Space charge effect

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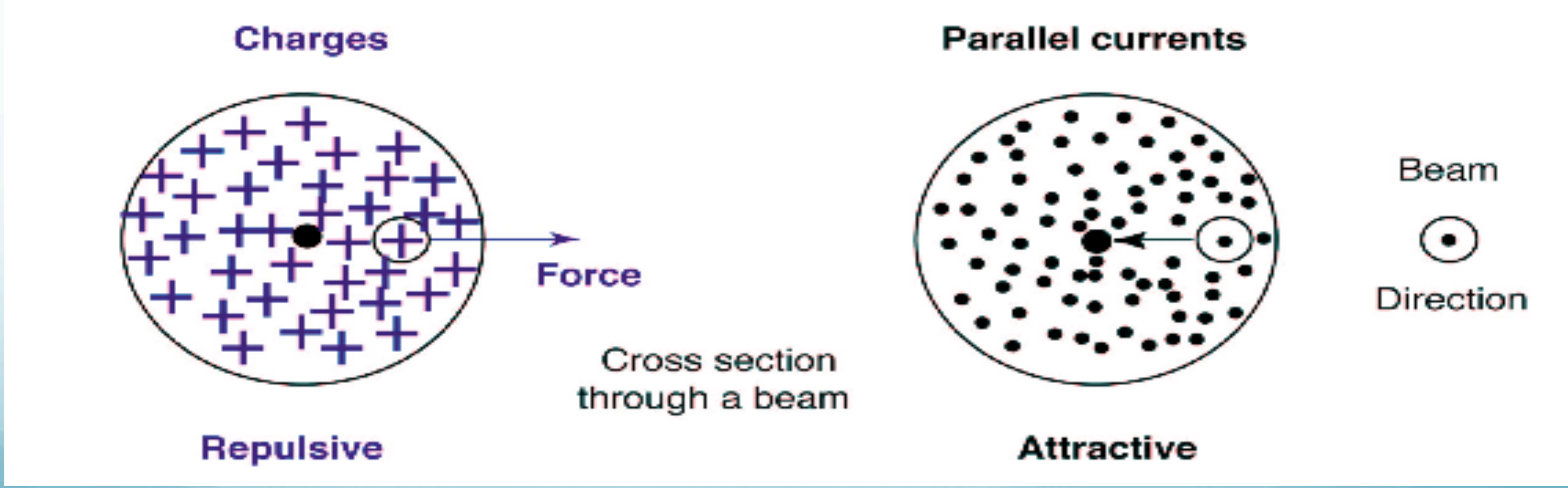
Outline

- Space charge overview (Direct space charge)
- Space charge in KV envelope equation
- Transverse tune shift
- Beam Break up effect (Indirect space charge)
- Longitudinal tune shift
- Compensation technique for S.C

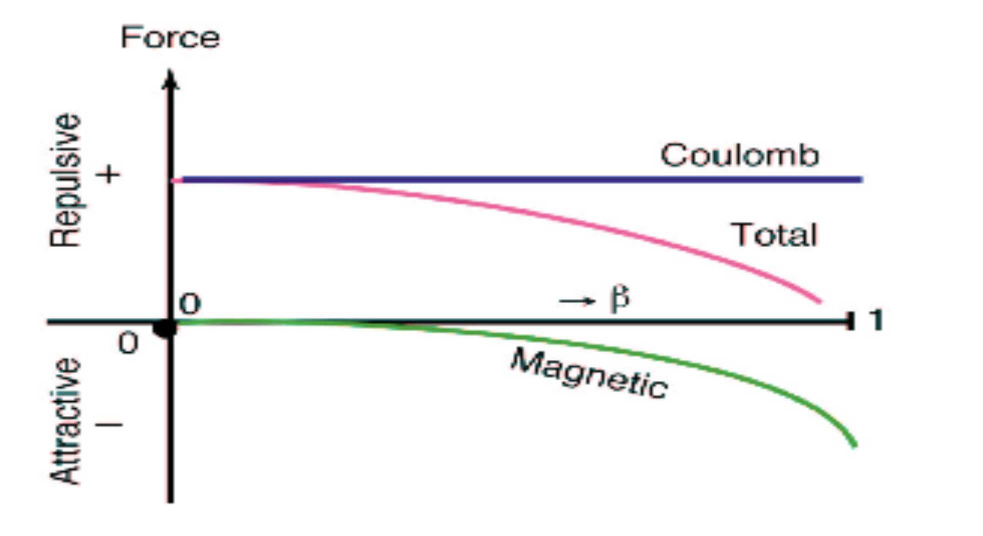
Space charge

- "Collective effect" space charge is the coulomb force the particle experiences by its surrounding
- Proportional to its intensity
- Adversely affect the beam stability



Cancelation at ultrarelativity

- Repulsive force by radial E field is canceled by azimuthal B field at relativistic limit



$$E_r = \frac{I}{2\pi\varepsilon_0\beta c}\frac{r}{a^2}\,.$$

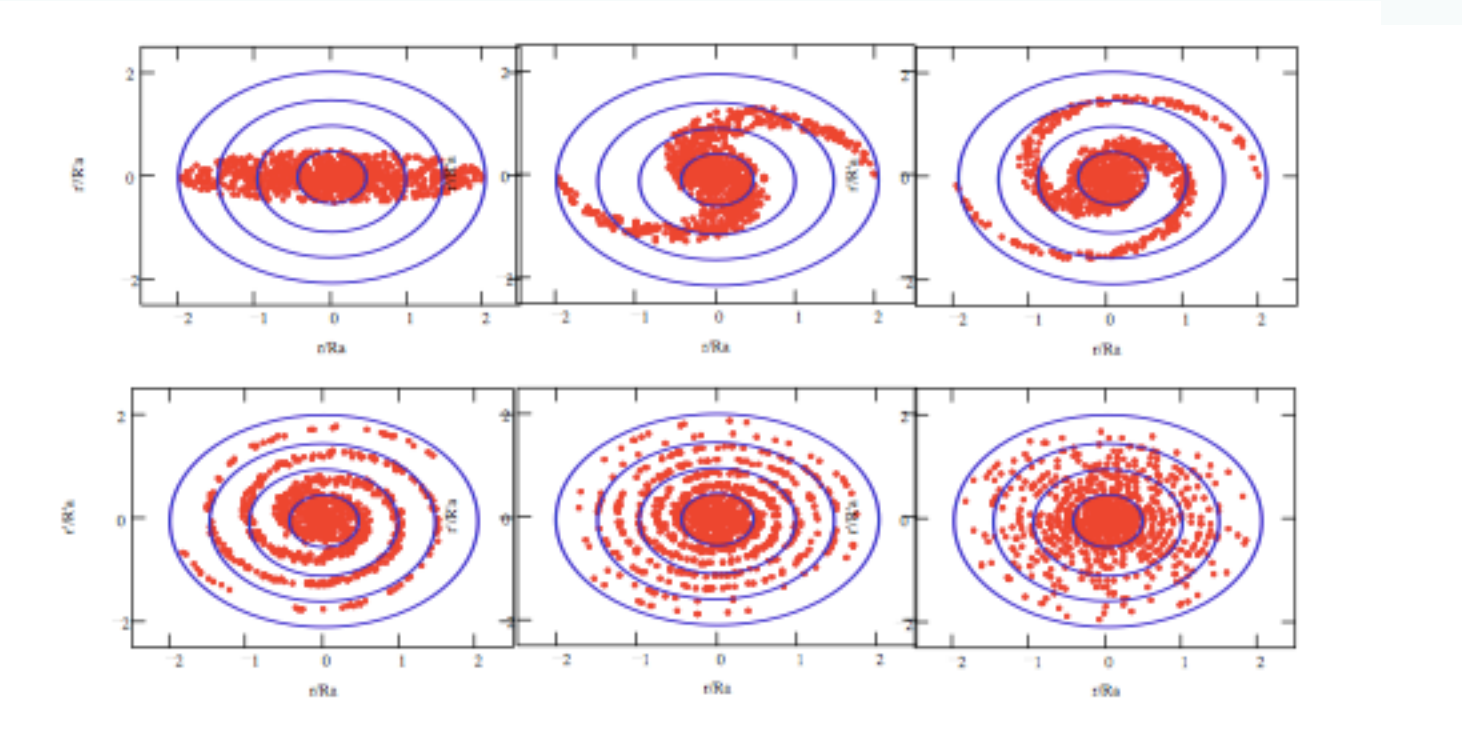
$$B_\phi = \frac{I}{2\pi\varepsilon_0 c^2}\frac{r}{a^2}\,.$$

$$F_r = e(E_r - v_s B_\phi)$$

$$F_r = \frac{eI}{2\pi\varepsilon_0\beta c}(1-\beta^2)\frac{r}{a^2} = \frac{eI}{2\pi\varepsilon_0\beta c}\frac{1}{\gamma^2}\frac{r}{a^2}$$

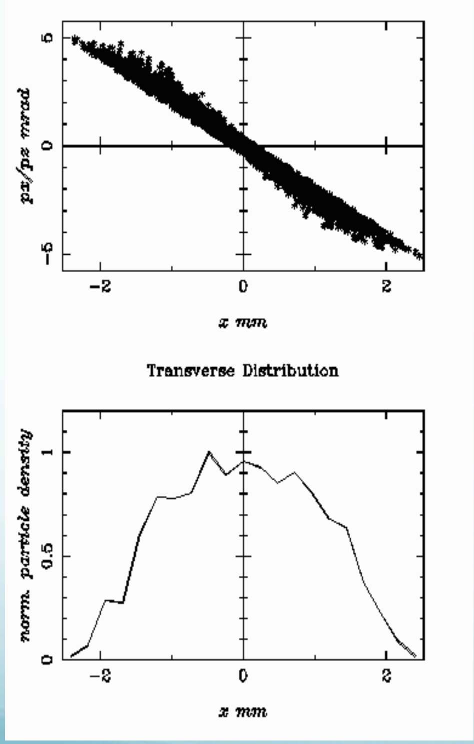
Nonlinearity of S.C

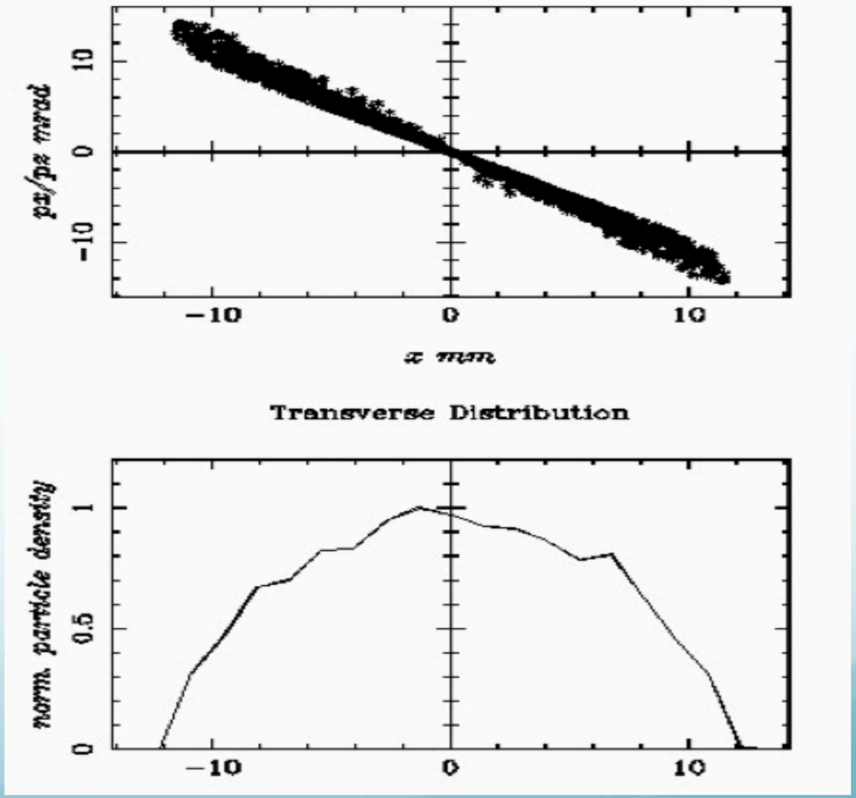
- Although S.C has linear form for uniform distribution, it gives rise to non-linear form for more realistic dist.



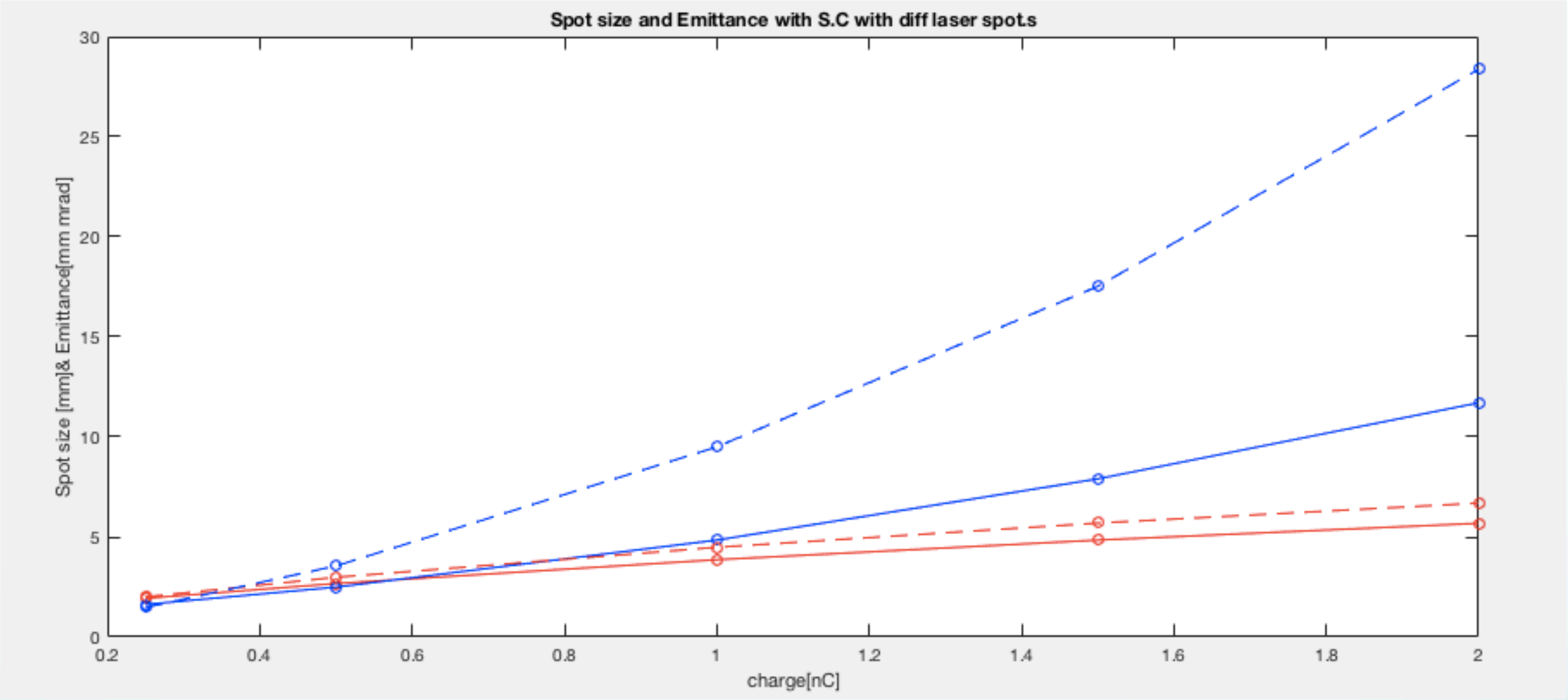
Phase space SRF112Mhz Gun

- 2000 particles with 2nC and 1.5MeV with and without S.C





Charge as variance



- Fig. S.C effect for 112 SRF Gun
- Spotsize and emittance with charge 0.25 ~ 2 nC
- Solid line has 1.5mm and dashed line has 0.75mm laser spot size

S.C in envelope equation

- Envelope equation
- Simplest consistent model incorporating applied focusing, space-charge defocusing, and thermal defocusing forces
- Starting point of almost all practical machine design!

Hill's equation

- Transverse particle dynamics
- Deviation from reference particle in Frenet-Serret coordinate

$$x''(s) + \kappa_x(s)x(s) = 0$$

$$\begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = \mathbf{M}_x(s \mid s_i) \cdot \begin{pmatrix} x(s_i) \\ x'(s_i) \end{pmatrix}$$

$$x(s) = A_i w(s) \cos \psi(s)$$

$$w(s + L_p) = w(s) \qquad w(s) > 0$$

Hill's eqn with Floquent theorem, periodic lattice condition gives x with periodic amplitude

Courant Snyder Invariant

- Hill's eqn (linear beam dynamics) gives rise to invariant.

$$x(s) = A_i w(s) \cos\psi(s)$$

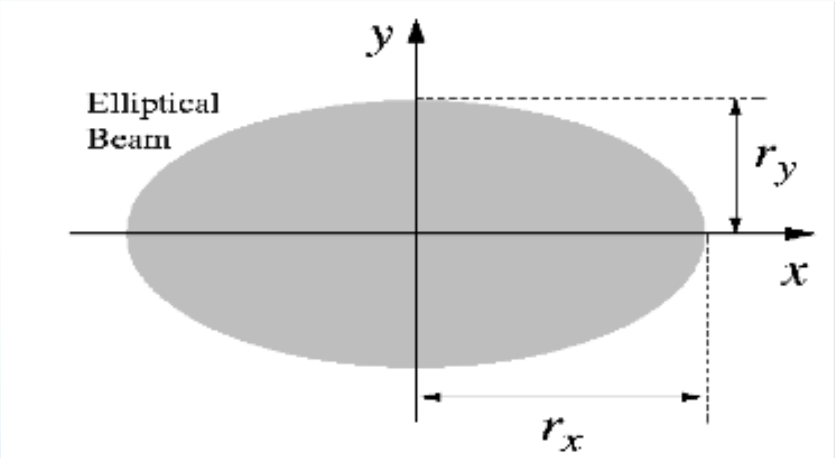
$$x'(s) = A_i w'(s) \cos\psi(s) - \frac{A_i}{w(s)} \sin\psi(s)$$

$$\left(\frac{x}{w}\right)^2 + (wx' - w'x)^2 = A_i^2 = \text{const}$$

This Ai is called Courant Snyder Invariant
Energy of the beam, amplitude of oscillation

KV distribution

- 4D KV. D gives elliptical uniform dist in any 2d projection
- Not realistic but highly investigated and used for accelerator designing.



- So the Poisson eqn would be calculated as

Space charge in KV.D

- Poison eqn is written by

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\phi=\begin{cases}-\frac{\lambda}{\pi\epsilon_0 r_x r_y}, & \text{if } \frac{x^2}{r_x^2}+\frac{y^2}{r_y^2}<1\\ 0, & \text{if } \frac{x^2}{r_x^2}+\frac{y^2}{r_y^2}>1\end{cases}$$

- Analytical solution of space charge potential inside of the beam is linear form

$$\phi=-\frac{\lambda}{2\pi\epsilon_0}\left\{\frac{x^2}{r_x(r_x+r_y)}+\frac{y^2}{r_y(r_x+r_y)}\right\}+\text{const}$$

Envelope equation

- Define emittance as maximum C.S invariant

$$\varepsilon_x \equiv \mathrm{Max}(A_{xi}^2)$$

- Means the edge of the distribution

$$r_x(s) = \sqrt{\varepsilon_x} w_x(s)$$

- Hill's eqn can be written by envelope rx as follows

$$r_x''(s) + \kappa_x(s) r_x(s) - \frac{2Q}{r_x(s) + r_y(s)} - \frac{\varepsilon_x^2}{r_x^3(s)} = 0$$

Envelope equation

- Projection of 4D invariant KV distribution
- If not KV dist, E.E is given equivalently by moments(statistical information)

$$r_x'' + \kappa_x r_x - \frac{2Q}{r_x + r_y} - \frac{\varepsilon_x^2}{r_x^3} = 0$$

$$r_y'' + \kappa_y r_y - \frac{2Q}{r_x + r_y} - \frac{\varepsilon_y^2}{r_y^3} = 0$$

Terms:	Applied Focusing	Space-Charge Defocusing	Thermal Defocusing
	Lattice	Perveance	Emittance

Matched Solution:

$$r_x(s + L_p) = r_x(s)$$

$$r_y(s + L_p) = r_y(s)$$

$$\kappa_x(s + L_p) = \kappa_x(s)$$

$$\kappa_y(s + L_p) = \kappa_y(s)$$

Transverse phase shift

- Space charge acts as defocusing

$$\kappa_x^{\text{eff}}(s) = \kappa_x(s) - \frac{2Q}{[r_x(s) + r_y(s)]r_x(s)}$$

- Generate tune shift from designated P.A is called depressed P.A

$$\Delta Q_{\mathrm{y}} \cong -\frac{r_0 N}{2\pi E_{\mathrm{y}} \beta^2 \gamma^3} \frac{2}{(1+\sqrt{\bar{\beta}_{\mathrm{x}} E_{\mathrm{x}} / \bar{\beta}_{\mathrm{y}} E_{\mathrm{y}}})} \,.$$

$$0 \quad \leq \quad \frac{\sigma}{\sigma_0} \quad \leq \quad 1$$

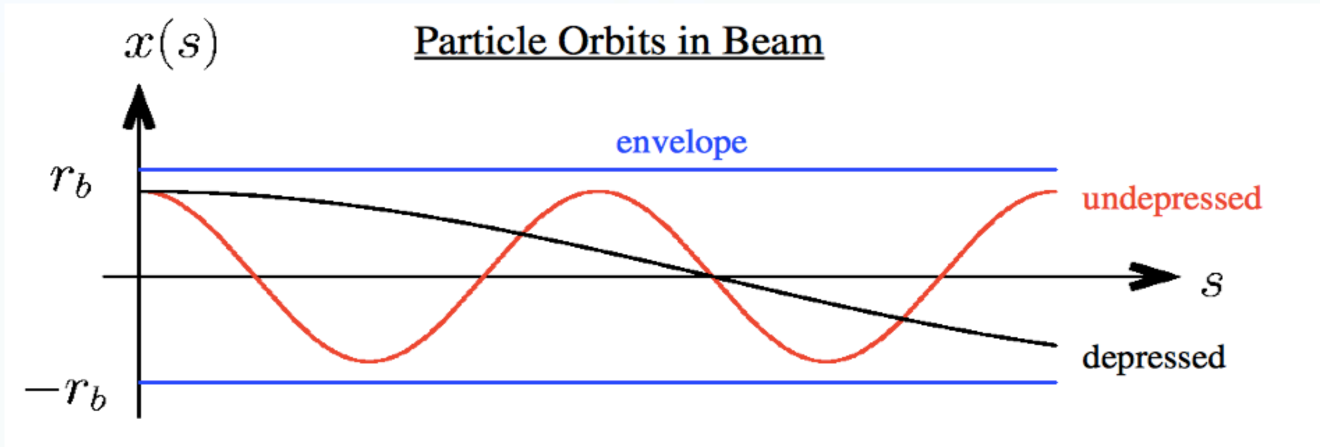
$$\varepsilon \rightarrow 0 \qquad\qquad Q \rightarrow 0$$

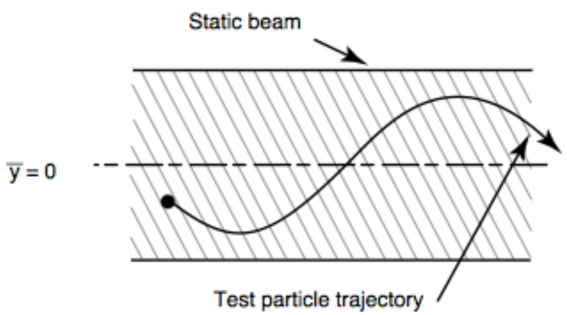
- Each limit is called S.C or Emittance dominated beam

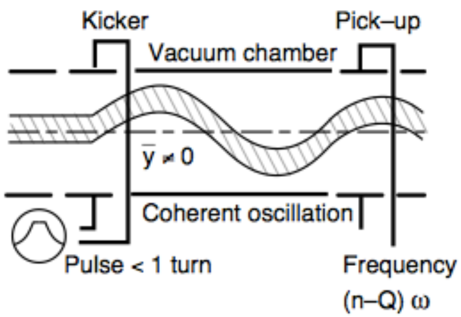
Incoherent tune shift by S.C

- With constant focusing strength

$$\frac{\sigma}{\sigma_0} = 0.2$$

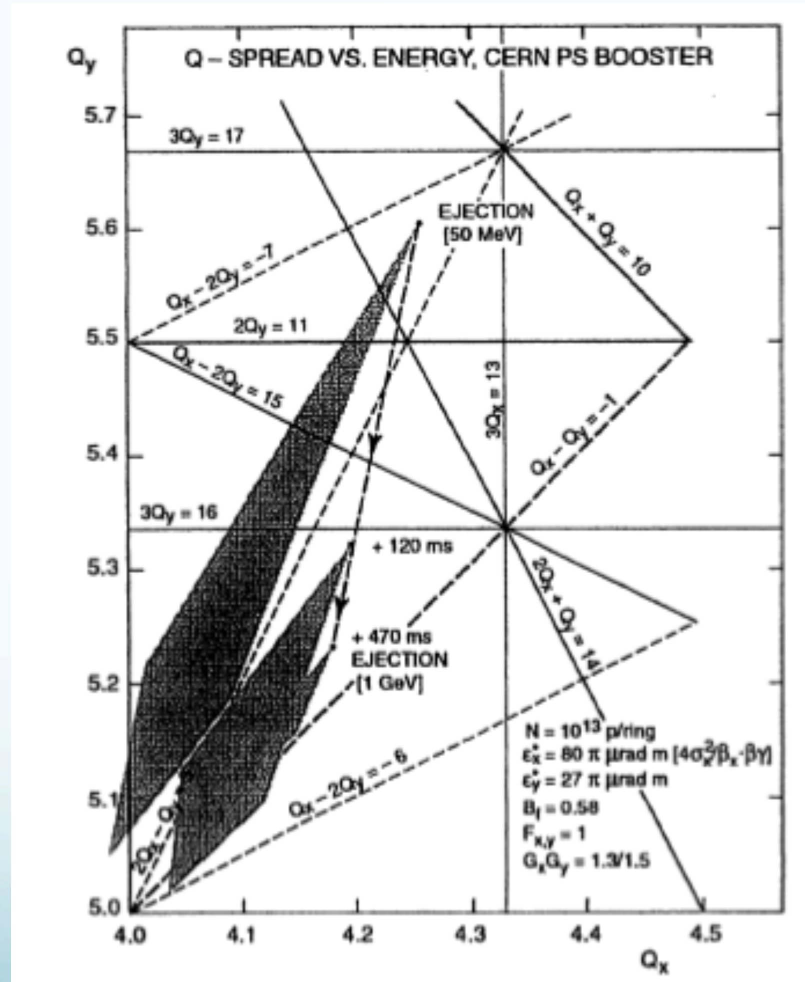






Incoherent T.S

- It is cast into tune spread => more likely to cross the resonance tune => undesirable beam loss
- CERN Proton Synchrotron Booster



Longitudinal tune shift

- Longitudinal eqn of motion follows

$$z'' + \left(\frac{\nu_{s0}}{R}\right)^2 z = -\frac{3Nr_0\eta}{\beta^2\gamma^3\hat{z}^3}\left(\ln\frac{b}{a} + \frac{1}{2}\right) z$$

$$\Delta\nu_s = \frac{3Nr_0\eta R^2}{2\beta^2\gamma^3\hat{z}^3\nu_{s0}}\left(\ln\frac{b}{a} + \frac{1}{2}\right)$$

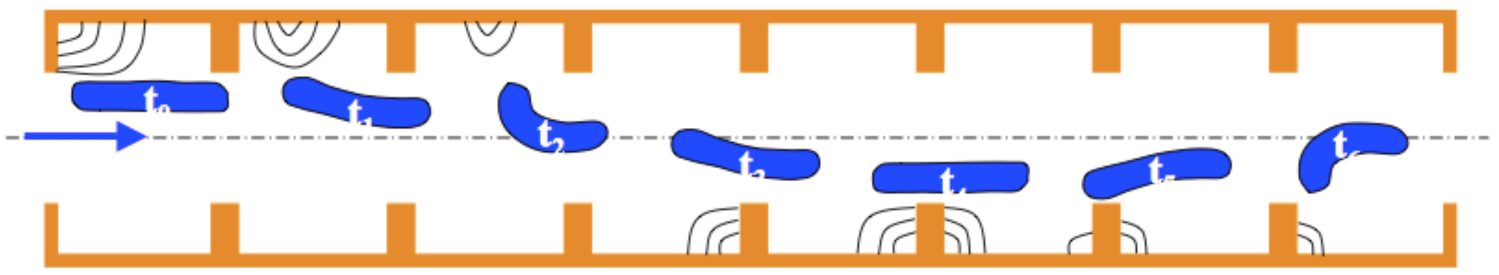
Unlikely the transverse one, longitudinaly tune shift could be focusing or defocusing depending on whether it is below or above transition

Indirect space charge effect

- Beam interacts with the surrounding (pipe)
- beam induces surface charges or currents into this environment that act back on the beam, possibly resulting in an ‘indirect’ space-charge
- Called Wake field
- Example) Beam breakup instability
- Bunch entering with offset from the center of the pipe in linac creates transverse wake field and it will acts back on the tail

Indirect space charge effect

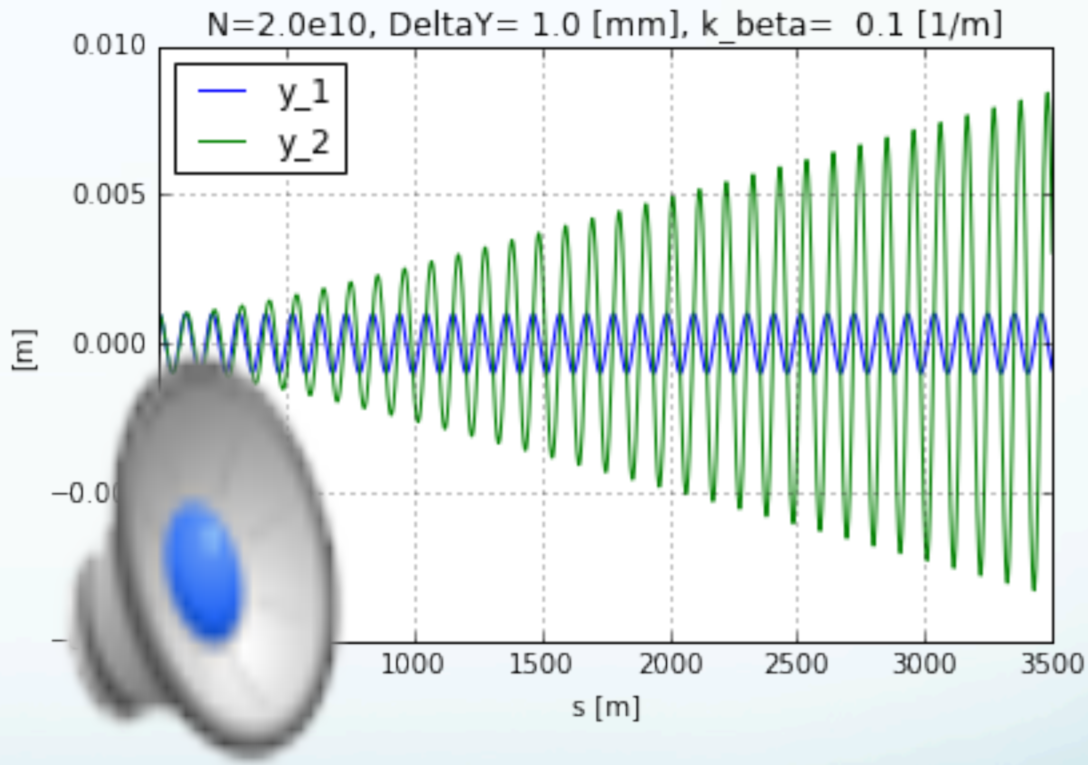
- BBU instability
- In long linac with high current beam, this effects amplifying the distortion of the beam shape into a “banana” like shape.



Snapshots of a single bunch traversing a SLAC structure

Indirect space charge effect

- BBU instability

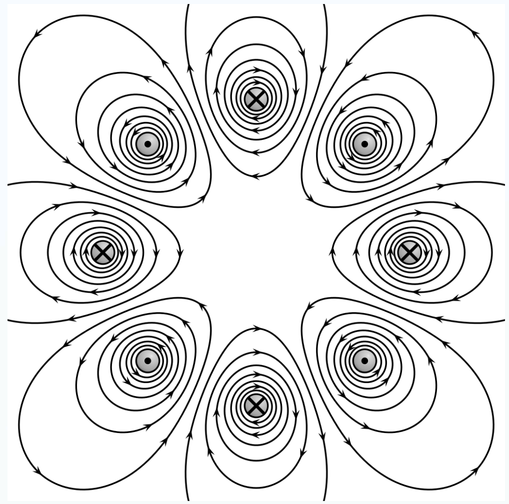


So far..

- Non linearity of S.C causes instability of the beam such as emittance growth, incoherent tune shift => tune spread!!
- How can we compensate those effect?
- A lot of work is going on for this subject.

Compensation of S.C

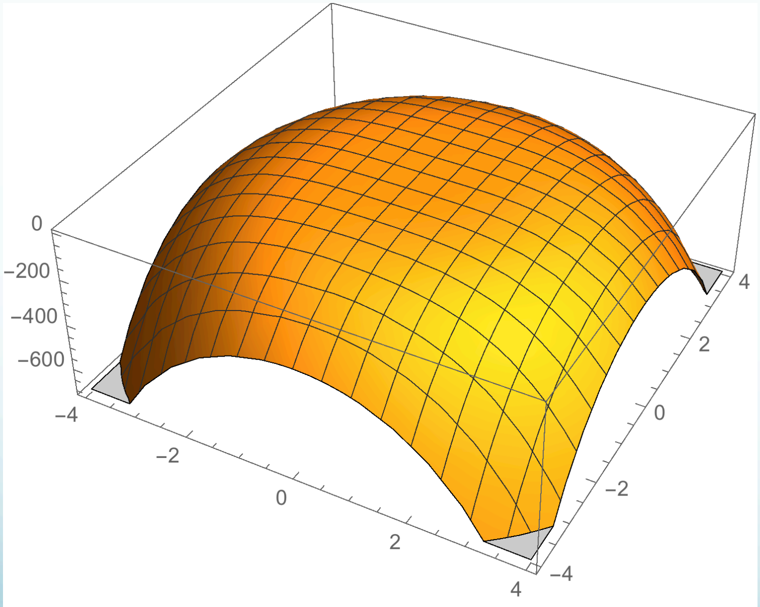
- Octupole



Octupole fields could cancel the next-to-leading term in the s.-c. force. For a round beam, the 4th order of term of the direct s.-c. potential varies as $(x^4 + 2x^2y^2 + y^4)$, while the potential of an octupole is proportional to $(x^4 - 6x^2y^2 + y^4)$. Therefore, at least two families of octupoles are needed to reduce the s.-c. tune spread, which are placed

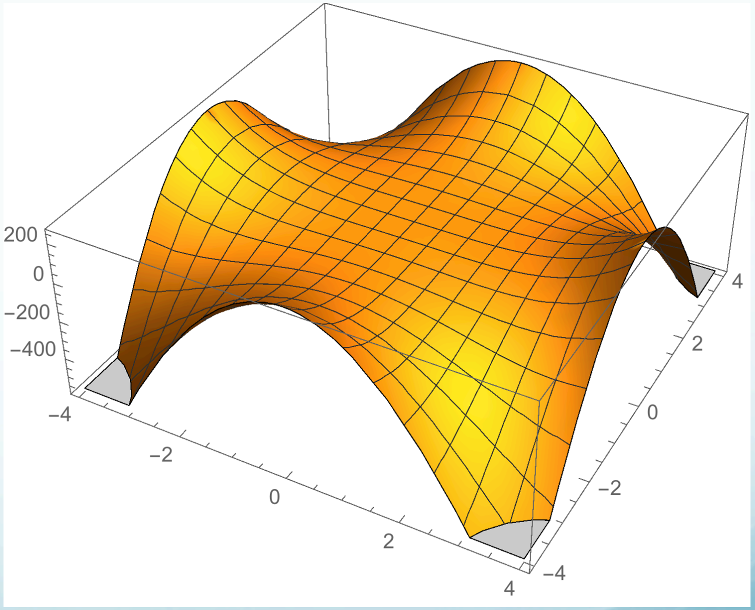
Compensation of S.C

- Potential of family of Octupole can suppress the 2nd leading order of S.C



Space charge

+



Octupole

Compensation of S.C

- Compensate the S.C force by static force with electrons
- **Electron lenses**, in which a negatively charged electron beam collides with the proton beam inside a strong solenoid field, could also compensate the s.c.

Assuming: free space, coasting beam, ρ(r)

- Proton to Proton

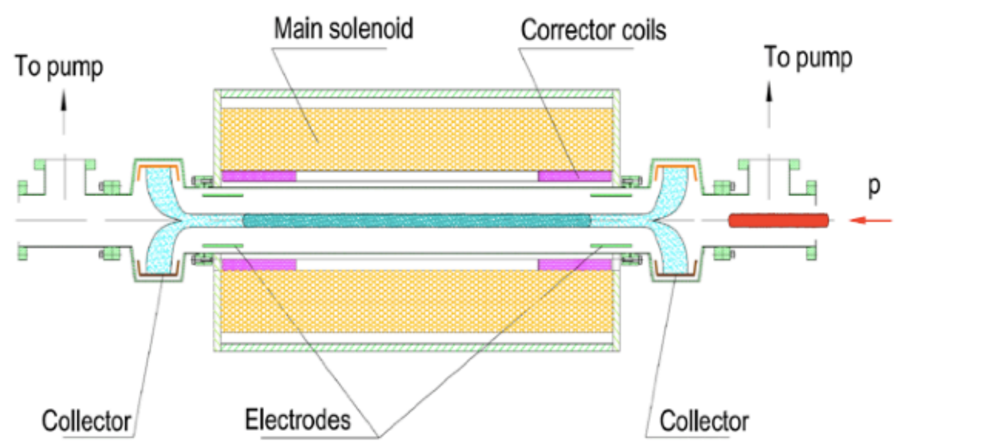
$$F_p(r) = e\left(E_{rp}(r) - v_p B_{\theta p}(r)\right) = \frac{e^2(1-\beta_p^{\ 2})}{\varepsilon_0 r}\int^r r'\rho_p(r')dr'$$

- Electron to Proton

$$F_e(r) = -\frac{e^2(1-\beta_e\beta_p)}{\varepsilon_0 r}\int_0^r r'\rho_e(r')dr'$$

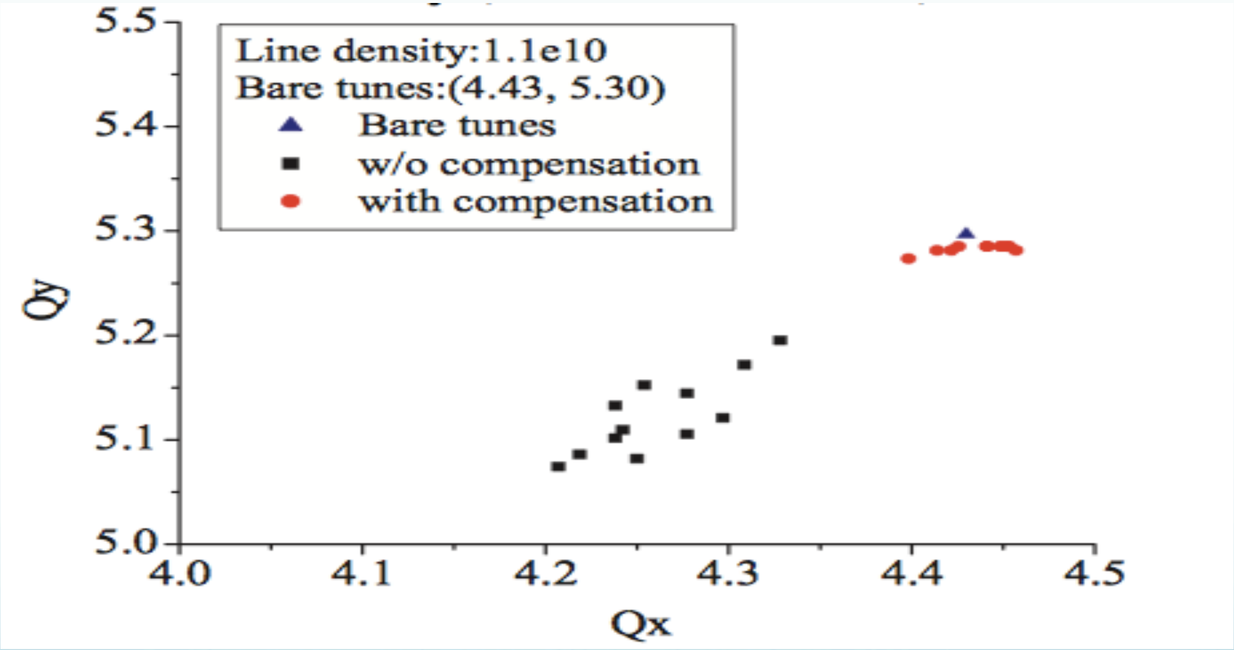
- Compensation

$$F_p(r) + F_e(r) = 0$$



Compensation of S.C

- Simulation for PS booster at CERN
- 4 electron lens with 10 keV, 1.2A



- Incoherent tune spread is suppressed by seeing 12 particles located at sigma to 2 sigma

Summary

- Nonlinearity of S.C causes instability such as emittance growth
- Introduced the S.C effect in EV envelope eqn
- Transverse and longitudinal tune shift and its incoherence could cause undesirable beam loss
- S.C compensation technique is going on and challenging topic

References

- https://acceleratorinstitute.web.cern.ch/acceleratorinstitute/ACINST89/Schindl_Space_Charge.pdf
- SPACE-CHARGE COMPENSATION OPTIONS FOR THE LHC INJECTOR COMPLEX∗ M. Aiba, M. Chanel, U. Dorda, R. Garoby
- https://acceleratorinstitute.web.cern.ch/acceleratorinstitute/ACINST89/Schindl_Space_Charge.pdf
- http://uspas.fnal.gov/materials/09UNM/Unit_4_Lecture_13_Beam_loading_&_wakefileds.pdf
- Transverse Equilibrium Distributions* Prof. Steven M. Lund
- USPAS2016 Collective Instability by Alex Chao

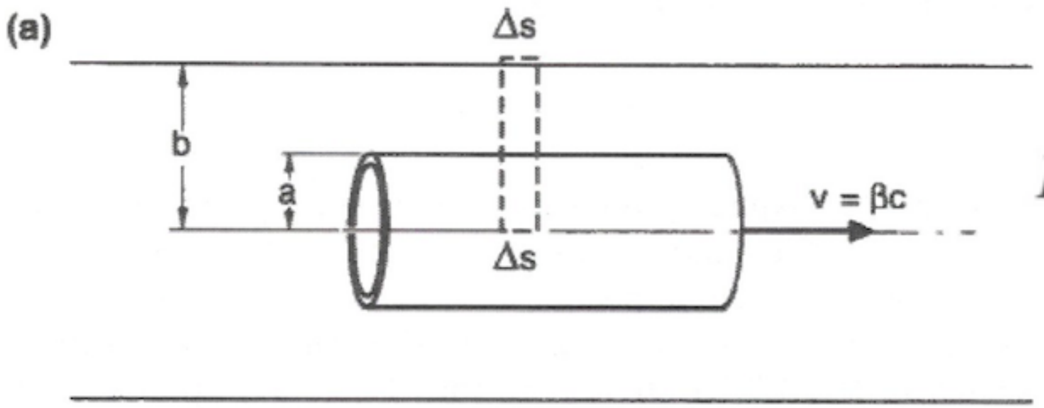
Thank you for your attention!!

Longitudinal S.C

- Unbunched uniform longitudinal dist gives only transverse field by symmetry
- Non uniform dist gives rise to S.C in s direction.
- Elamda(s-betact) with lamda being line charge density
- S.C proportional to lamda prime and 1/gamma can be gessed.

Longitudinal S.C

- Faraday's law gives Es



$$E_s = -\frac{2e}{\gamma^2}\lambda'(s-\beta ct)\ln\frac{b}{a}$$

$$E_s\,\Delta s \;+\; 2e\left[\lambda(s+\Delta s-\beta ct)-\lambda(s-\beta ct)\right]\int_a^b \frac{dr}{r}$$

$$= -\frac{2e\beta}{c}\frac{\partial\lambda(s-\beta ct)}{\partial t}\,\Delta s\int_a^b\frac{dr}{r}$$

Longitudinal S.C

- Opening angle is small
- So Er and Bseta would be

$$E_r = \frac{B_\theta}{\beta} = 2e\lambda(s - \beta ct) \begin{cases} 0 & \text{if } r < a \\ \frac{1}{r} & \text{if } a < r < b \end{cases}$$

- Line integral for Faraday's law gives Es as follows

$$\oint d\vec{\ell} \cdot \vec{E} = -\frac{1}{c}\frac{\partial}{\partial t}\int d\vec{A} \cdot \vec{B}$$