Vestigial Sideband Modulation

KEEE343 Communication Theory

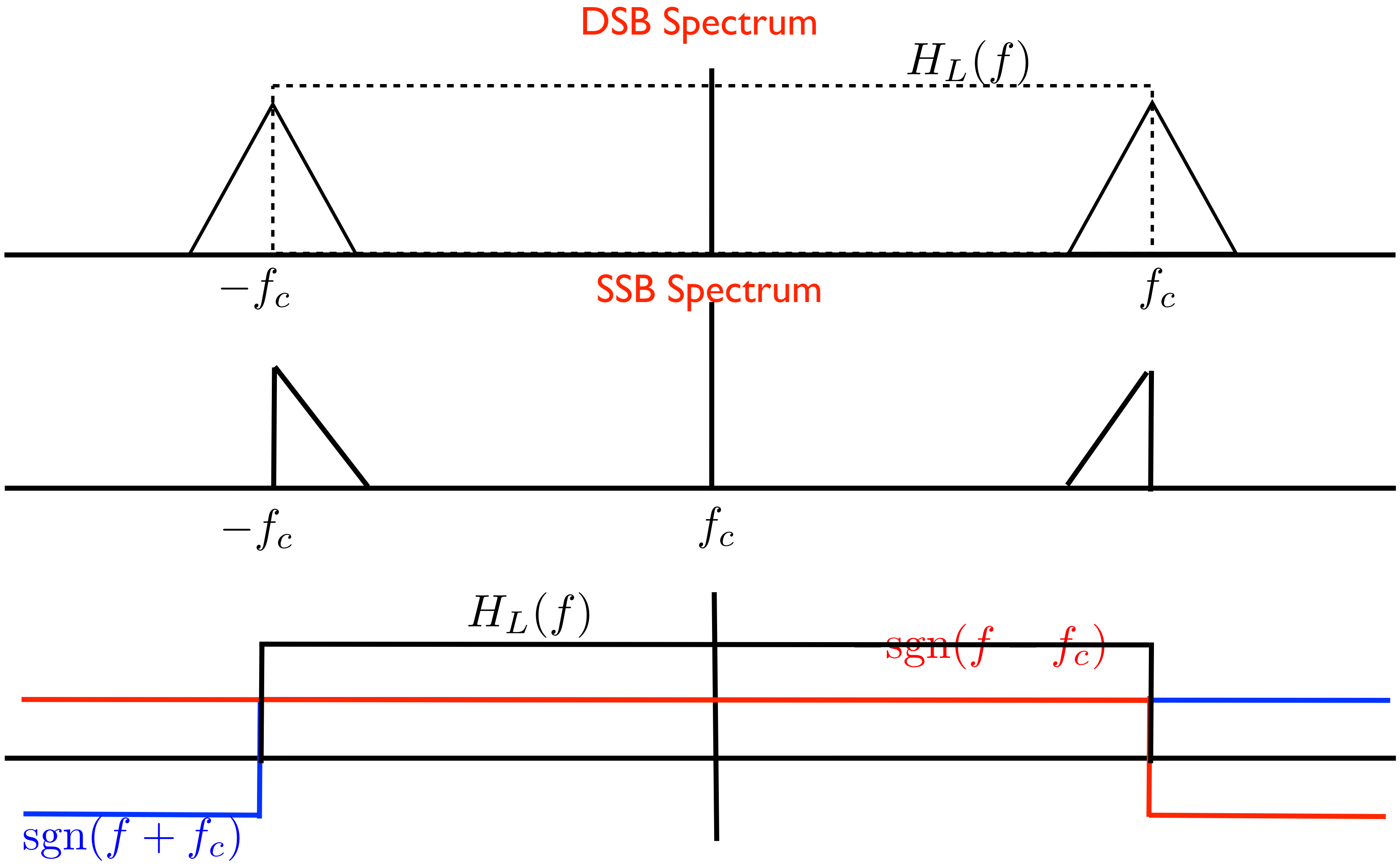
Lecture #11, April 7, 2011

Prof. Young-Chai Ko
koyc@korea.ac.kr

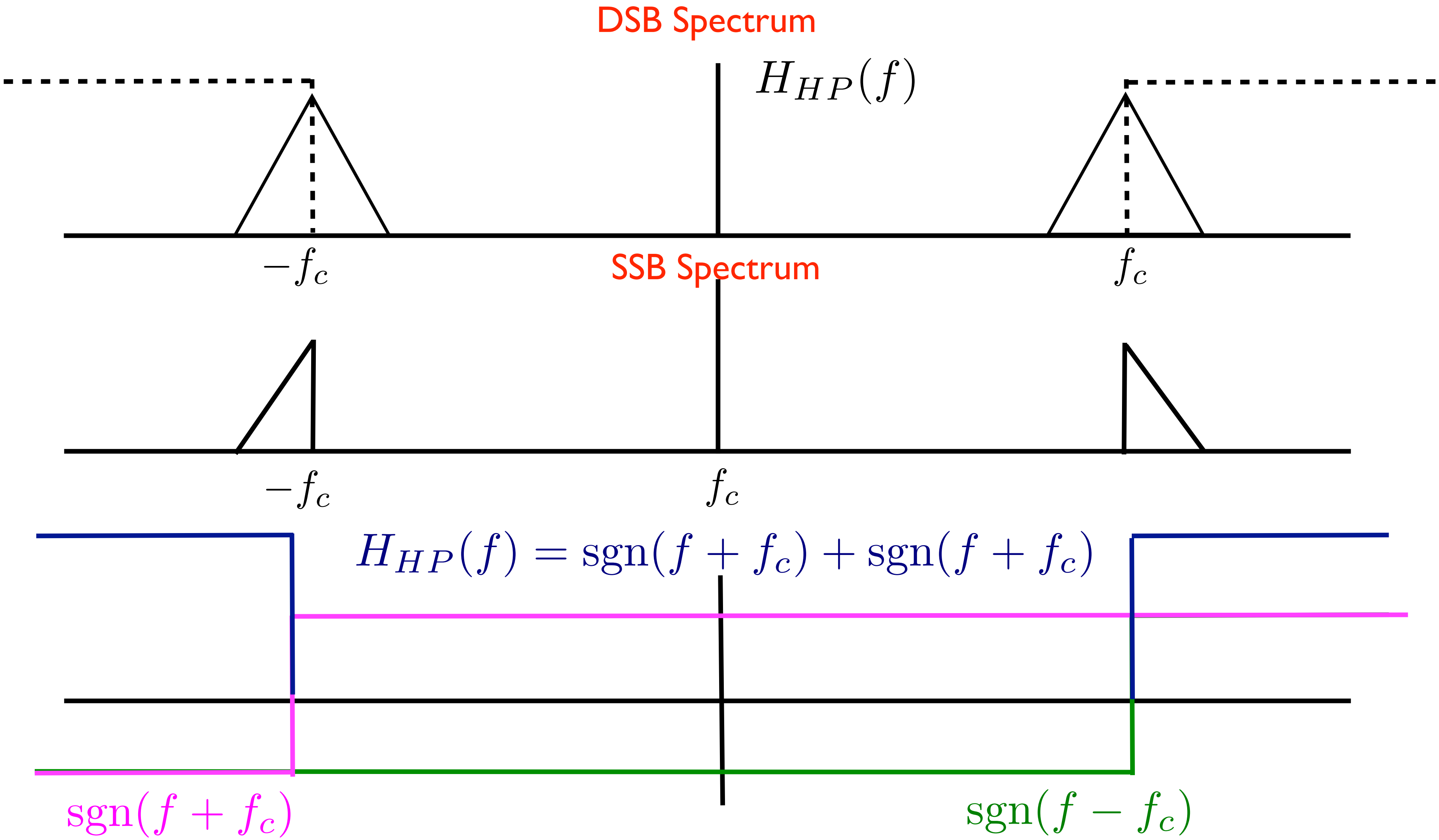
Summary

- Vestigial sideband modulation
- Baseband representation of modulated wave
- Baseband representation of pass-band filter
- Frequency division multiplexing
- Introduction to Angle Modulation

Generation of LSB SSB Using Wideband Low-Pass Filter



Generation of USB SSB Using High-Pass (or Passband) Filter



SSB Modulated Wave

- Lower Sideband SSB

$$s_{LSB}(t) = \frac{1}{2}m(t)\cos(2\pi f_c t) + \frac{1}{2}\hat{m}(t)\sin(2\pi f_c t)$$

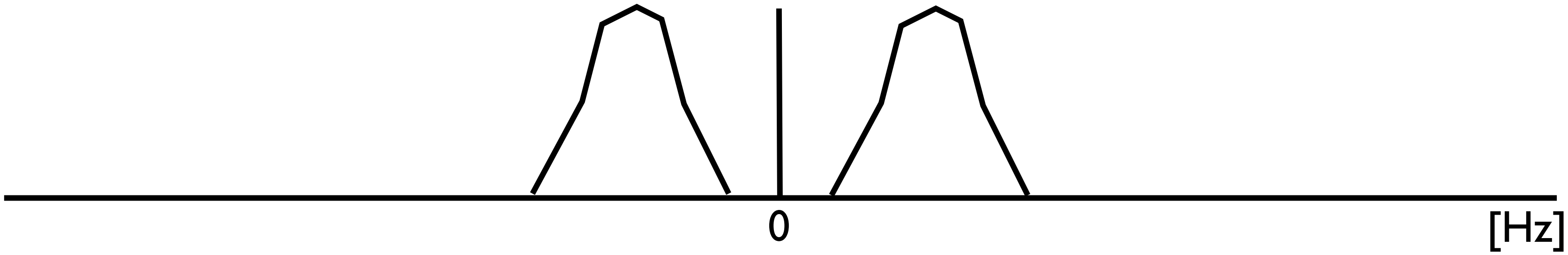
- Upper sideband SSB

$$s_{USB}(t) = \frac{1}{2}m(t)\cos(2\pi f_c t) - \frac{1}{2}\hat{m}(t)\sin(2\pi f_c t)$$

Applications of SSB and Difficulties in Implementing SSB

- Two difficulties of SSB modulations
  - Designing and implementing the sharp Low-pass (or High-pass/pass-band) filter is not easy for circuit designer.
  - Hence, the message signal which does not contain the significant energy in the DC area is often modulated by the SSB such as the speech signal.

Spectrum of Speech Signal (Example)



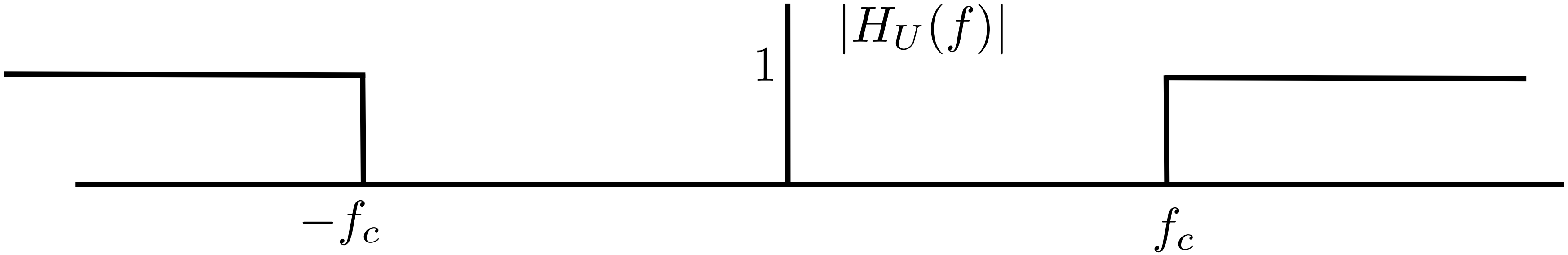
- However, the SSB cannot be applicable for the message signal which contains the significant energy around zero frequency such as video signal, computer data, and etc.

Vestigial Sideband (VSB) Modulation

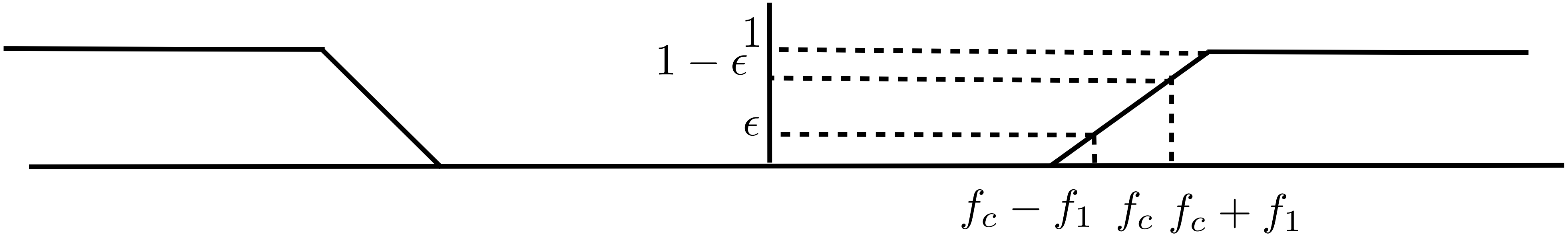
- VSB Modulation
  - Modulation to overcome the two difficulties of the SSB modulations.
  - Allow a small amount, or vestige, of the unwanted sideband to appear at the output of an SSB modulator
    - The design of the sideband filter is simplified since the need for sharp cutoff at the carrier frequency is eliminated.
    - In addition, a VSB system has improved low-frequency response and can even have dc response.

Idea of VSB Modulator

- Pass-band (or High-pass) filter for USB-SSB modulation



- The filter below is much easier to design and implement



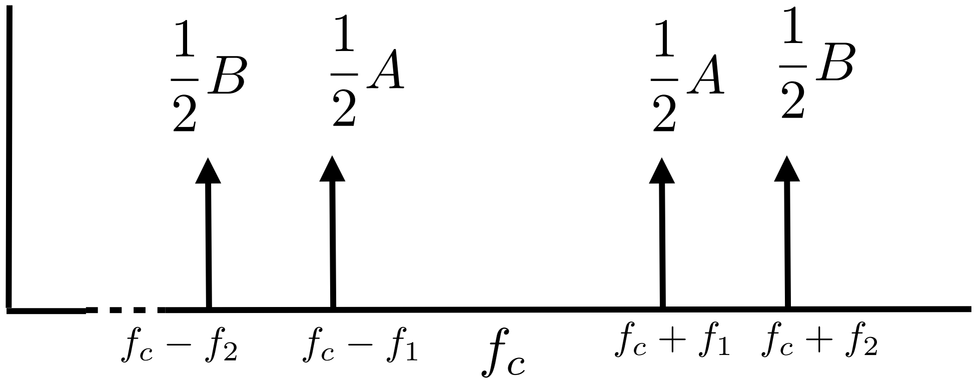
- Consider the two-tone message signal given as

$$m(t) = A\cos(2\pi f_1 t) + B\cos(2\pi f_2 t)$$

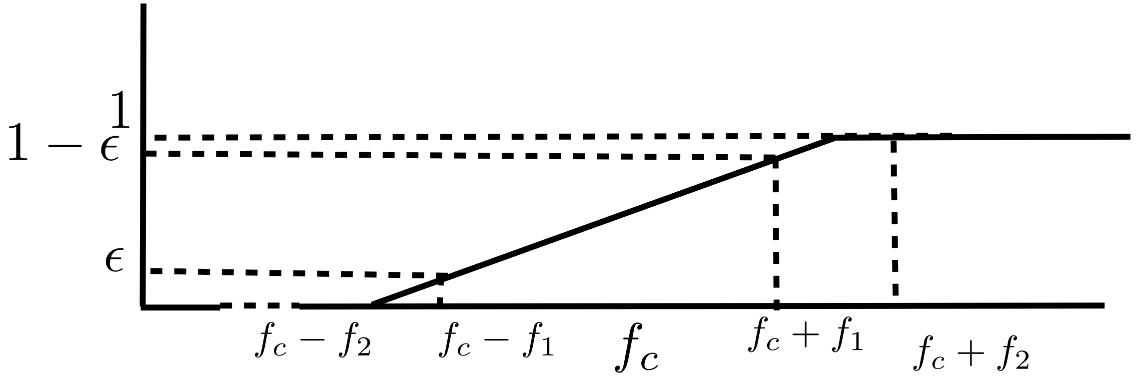
- Message signal multiplied by the carrier wave, that is, DSB signal

$$\begin{aligned} e_{DSB}(t) &= (A\cos(2\pi f_1 t) + B\cos(2\pi f_2 t)) \cdot \cos(2\pi f_c t) \\ &= \frac{1}{2}A\cos(2\pi(f_c + f_1)t) + \frac{1}{2}A\cos(2\pi(f_c - f_1)t) \\ &\quad + \frac{1}{2}B\sin(2\pi(f_c + f_1)t) + \frac{1}{2}B\sin(2\pi(f_c - f_1)t) \end{aligned}$$

- Spectrum of DSB signal

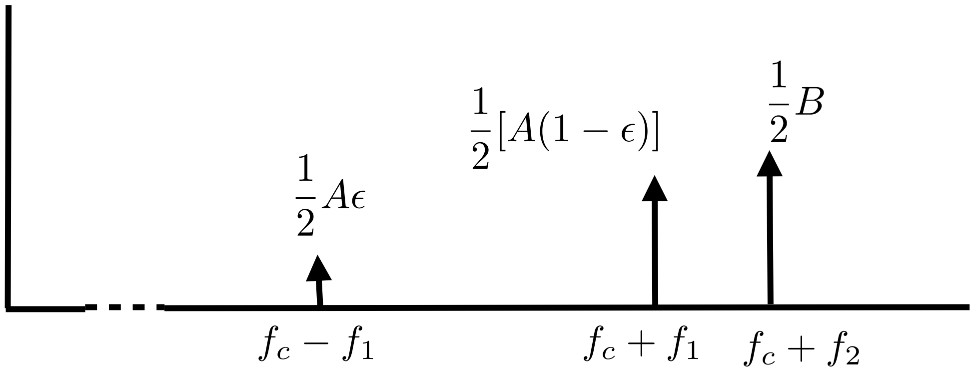


- Frequency response of the VSB filter



- Output response

$$
\begin{aligned}
s(t) &= \frac{1}{2}A\epsilon\cos(2\pi(f_c - f_1)t) \\
&+\frac{1}{2}A(1-\epsilon)\cos(2\pi(f_c + f_1)t) \\
&+\frac{1}{2}B\cos(2\pi(f_c + f_2)t)
\end{aligned}
$$



Demodulation of VSB Signal (Coherent method)

- Downconvert (by Multiplying $4\cos(2\pi f_c t)$) and low pass filtering
    - Downconvert

$$\begin{aligned} d(t) &= s(t)\cdot 4\cos(2\pi f_c t) \\ &= \frac{1}{2}A\epsilon\cos(2\pi(f_c - f_1)t)\cdot 4\cos(2\pi f_c t) \\ &\quad + \frac{1}{2}A(1-\epsilon)\cos(2\pi(f_c + f_1)t)\cdot 4\cos(2\pi f_c t) \\ &\quad + \frac{1}{2}B\cos(2\pi(f_c + f_2)t)\cdot 4\cos(2\pi f_c t) \end{aligned}$$

$$\boxed{\cos(2\pi(f_c + f_1)t)\cdot\cos(2\pi f_c t) = \frac{1}{2}\left[\cos(2\pi(2f_c + f_1)t) + \cos(2\pi f_1 t)\right]}$$

➡ Low-Pass Filtering ➡ $\frac{1}{2}\cos(2\pi f_1 t)$

- Signal after Low-pass filtering

$$
\begin{aligned}
\nu(t) &= A\epsilon\cos(2\pi f_1 t) + A(1-\epsilon)\cos(2\pi f_1 t) + B\cos(2\pi f_2 t) \\
&= A\cos(2\pi f_1 t) + B\cos(2\pi f_2 t)
\end{aligned}
$$

Television Signals

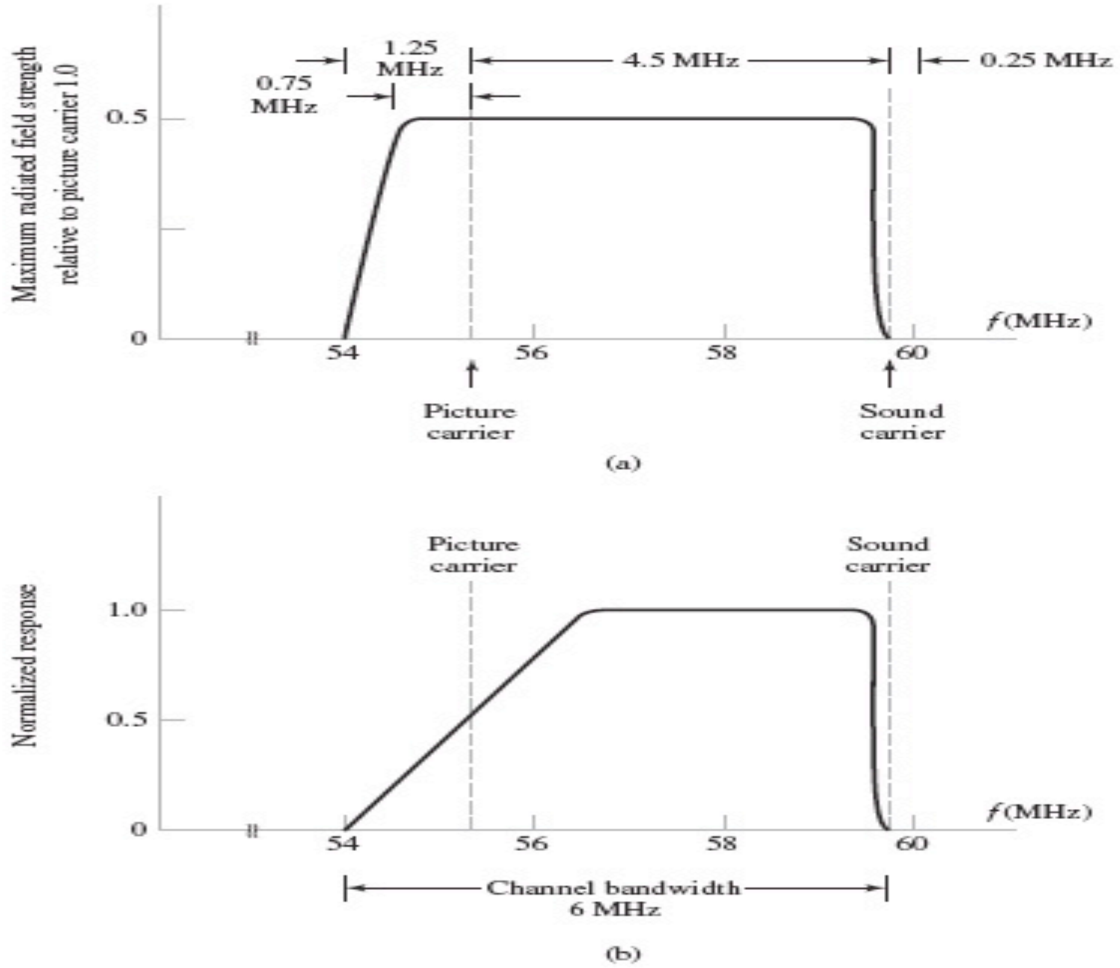


FIGURE 3.28 (*a*) Idealized amplitude spectrum of a transmitted TV signal. (*b*) Amplitude response of a VSB shaping filter in the receiver.

[Ref: Haykin & Moher, Textbook]

Baseband Representation of Modulated Waves

- DSB modulated wave signal

$$s_{DSB}(t) = Am(t)\cos(2\pi f_c t)$$

- SSB modulated wave signal

$$s_{SSB}(t) = \frac{1}{2}Am(t)\cos(2\pi f_c t) \pm \frac{1}{2}A\hat{m}(t)\sin(2\pi f_c t)$$

- In general, we can write the "linear modulated wave" as

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

Carrier wave with frequency f_c

$$c(t) = \cos(2\pi f_c t)$$

quadrature-phase version of the carrier

$$\hat{c}(t) = \sin(2\pi f_c t)$$

Orthogonal each other

- We can rewrite the modulated wave as

$$s(t) = \underbrace{s_I(t)}_{\text{in-phase component of s(t)}} c(t) - \underbrace{s_Q(t)}_{\text{quadrature(-phase) component of s(t)}} \hat{c}(t)$$

- Introduce the complex envelop of the modulated wave s(t)

$$\tilde{s}(t) = s_I(t) + j s_Q(t)$$

- Define the complex carrier wave

$$\tilde{c}(t) = c_I(t) - j c_Q(t)$$

- Consider the following

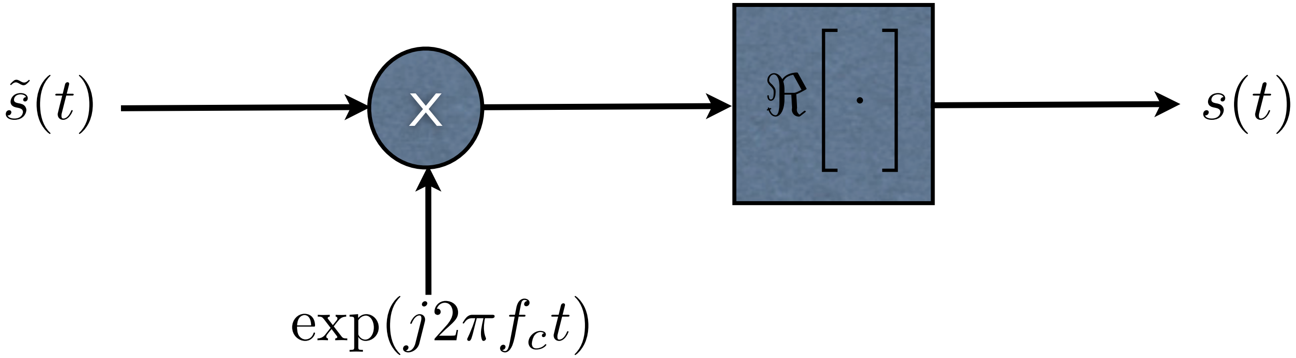
$$\tilde{s}(t) \cdot \exp(j2\pi f_c t) \quad = \quad \Big[s_I(t) + j s_Q(t)\Big] \cdot \Big[\cos(2\pi f_c t) + j\sin(2\pi f_c t)\Big]$$

- Real term

$$\Re\Big[\tilde{s}(t) \cdot \exp(j2\pi f_c t)\Big] \quad = \quad s_I(t) \cdot \cos(2\pi f_c t) - s_Q(t) \cdot \sin(2\pi f_c t)$$

- Imaginary term

$$\Im\Big[\tilde{s}(t) \cdot \exp(j2\pi f_c t)\Big] \quad = \quad s_I(t)\sin(2\pi f_c t) + s_Q(t)\cos(2\pi f_c t)$$



- Now consider

$$\tilde{s}(t) = s_I(t) + js_Q(t) = a(t)e^{j\theta(t)}$$

where

$$a(t) = \sqrt{s_I^2(t) + js_Q^2(t)}, \qquad \theta(t) = \tan^{-1}\frac{s_Q(t)}{s_I(t)}$$

- Then we can represent the modulated wave as

$$\begin{aligned} s(t) &= \Re\left[\tilde{s}(t)e^{j2\pi f_c t}\right] = \Re\left[a(t)e^{j\theta(t)}e^{j2\pi f_c t}\right] \\ &= \Re\left[a(t)e^{j[2\pi f_c t + \theta(t)]}\right. \\ &= a(t)\cos[2\pi f_c t + \theta(t)] \end{aligned}$$

- Three different representation of modulated wave using its equivalent baseband signal

$$\begin{aligned} s(t) &= s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) \\ &= \Re\left[\tilde{s}(t)e^{j2\pi f_c t}\right] \\ &= a(t)\cos[2\pi f_c t + \theta(t)] \end{aligned}$$

Superheterodyne Receiver

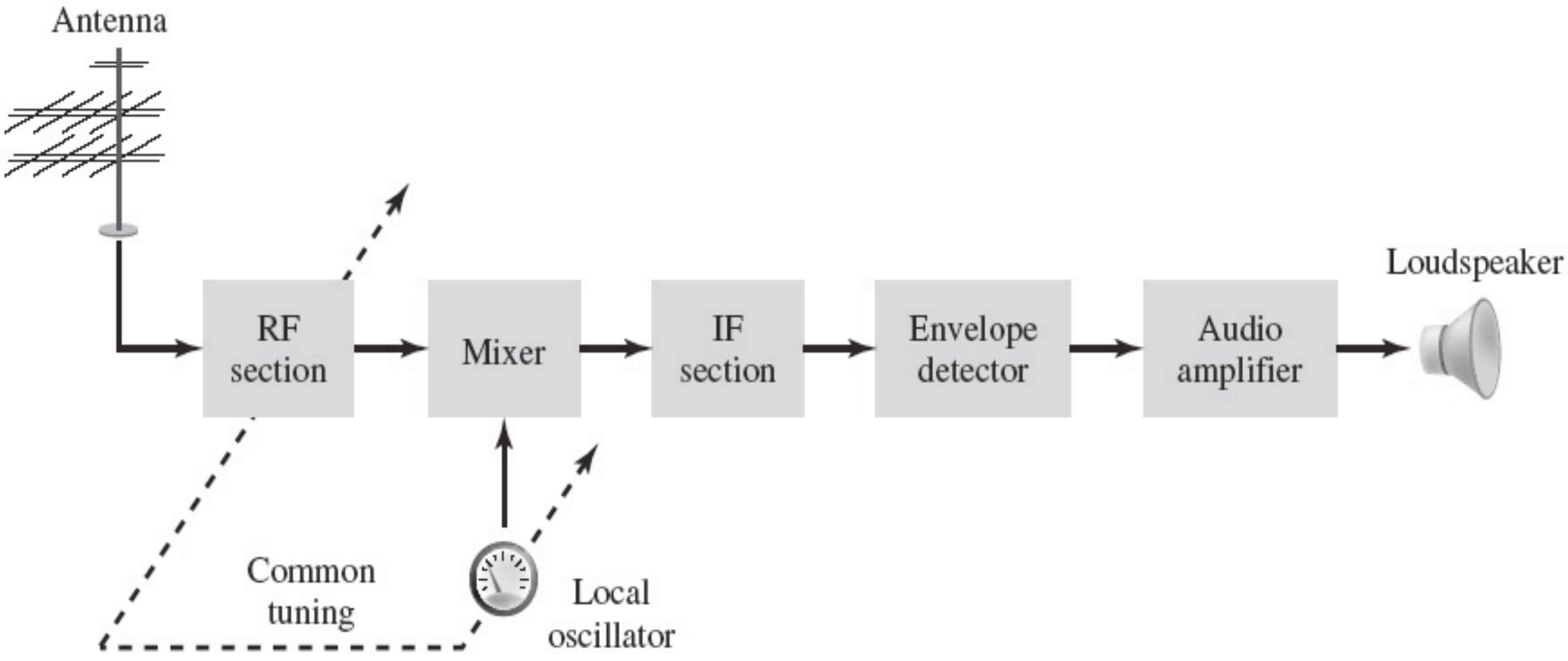


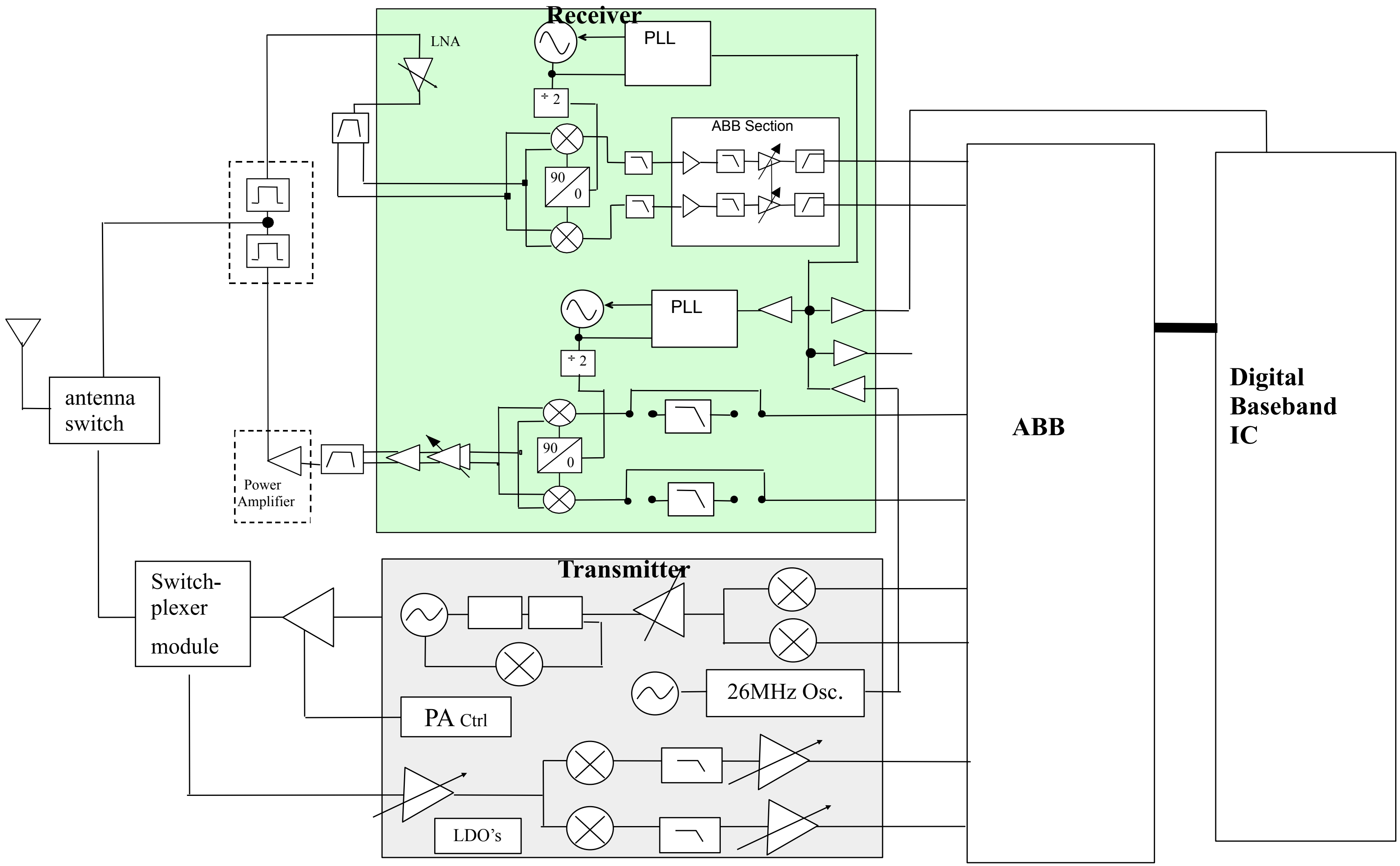
FIGURE 3.27 Basic elements of an AM radio receiver of the superheterodyne type.

TABLE 3.2 *Typical Frequency Parameters of AM and FM Radio Receivers*

	AM Radio	*FM Radio*
RF carrier range	0.535–1.605 MHz	88–108 MHz
Mid-band frequency of IF section	0.455 MHz	10.7 MHz
IF bandwidth	10 kHz	200 kHz

[Ref: Haykin & Moher, Textbook]

Communication Chipset Architecture



Frequency-Division Multiplexing

- To transmit a number of communication signals over the same channel, the signals must be kept apart so that they do not interfere with each other, and thus they can be separated at the receiving end.
- FDM (Frequency division multiplexing)
- TDM (Time division multiplexing)
- SDM (Space division multiplexing)
- CDM (Code division multiplexing)

Block Diagram of FDM

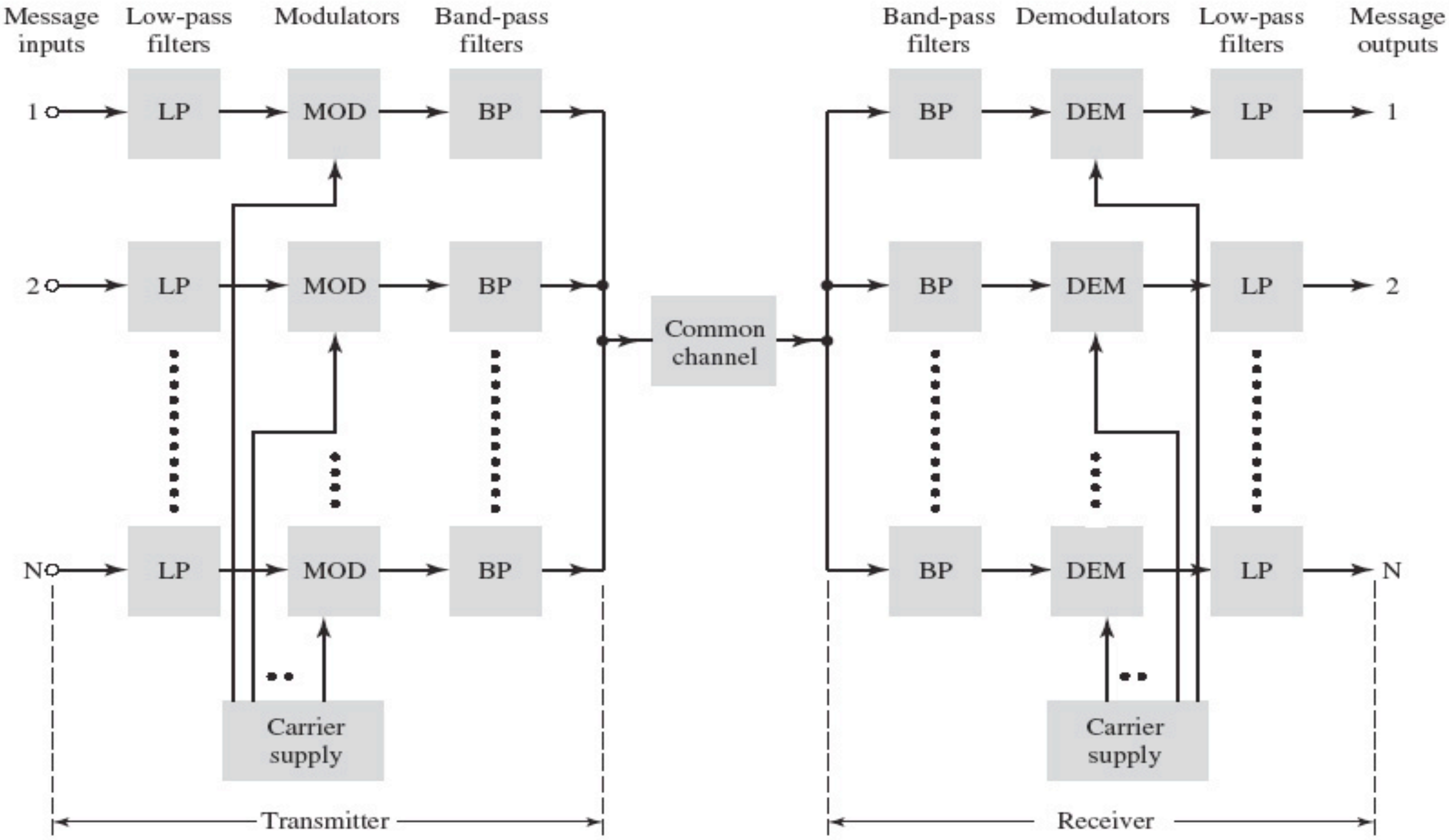


FIGURE 3.29 Block diagram of frequency-division multiplexing (FDM) system.

Angle Modulation

- Basic Definition of Angle Modulation

$$s(t) = A_c \cos[\theta_i(t)] = A_c \cos[2\pi f_c t + \phi_c]$$

  - Phase modulation (PM) if

$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

  - Frequency modulation (FM) if

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau$$