Polya's Problem Solving Techniques

In 1945 George Polya published the book How To Solve It which quickly became his most prized publication. It sold over one million copies and has been translated into 17 languages. In this book he identifies four basic principles of problem solving.

Polya's First Principle: Understand the problem

This seems so obvious that it is often not even mentioned, yet students are often stymied in their efforts to solve problems simply because they don't understand it fully, or even in part. Polya taught teachers to ask students questions such as:

- Do you understand all the words used in stating the problem?
- What are you asked to find or show?
- Can you restate the problem in your own words?
- Can you think of a picture or diagram that might help you understand the problem?
- Is there enough information to enable you to find a solution?

Polya's Second Principle: Devise a plan

Polya mentions that there are many reasonable ways to solve problems. The skill at choosing an appropriate strategy is best learned by solving many problems. You will find choosing a strategy increasingly easy. A partial list of strategies is included:

- Guess and check • Look for a pattern
- Make an orderly list • Draw a picture
- Eliminate possibilities • Solve a simpler problem
- Use symmetry • Use a model
- Consider special cases • Work backwards
- Use direct reasoning • Use a formula
- Solve an equation • Be ingenious

Polya's Third Principle: Carry out the plan

This step is usually easier than devising the plan. In general, all you need is care and patience, given that you have the necessary skills. Persist with the plan that you have chosen. If it continues not to work discard it and choose another. Don't be misled, this is how mathematics is done, even by professionals.

Polya's Fourth Principle: Look back

Polya mentions that much can be gained by taking the time to reflect and look back at what you have done, what worked, and what didn't. Doing this will enable you to predict what strategy to use to solve future problems.

So starting on the next page, here is a summary, in the master's own words, on strategies for attacking problems in mathematics class. This is taken from the book, How To Solve It, by George Polya, 2nd ed., Princeton University Press, 1957, ISBN 0-691-08097-6.

Step One
Understand the Problem

Can you state the problem in your own words?
What are you trying to find or do?
What are the unknowns?
What information do you obtain from the problem?
What information, if any, is missing or not needed?

Step Two
Make a Plan

Look for a pattern.
Remember related problems.
Break the problem down into different parts.
Make a table.
Make a diagram.
Write an equation.
Use a guess and check.
Work backward.
Identify a subgoal.

George Polya's Steps to Problem Solving

Step Three
Do the Plan

Implement the strategy in Step 2 and perform the necessary math computations.
Check each step of the plan as you do it.
Keep an accurate record of your work.
Organize your work into easy to understand visuals.
Double check your math work.

Step Three
Look Back

Check the results in the original problem.
Interpret the solution in terms of the original problem. Does your answer make sense? Is it reasonable?
Determine whether there is another method of finding the solution.
If possible, determine other related or more general problems for which the techniques will work