Design and analysis of T and inverted L beams-

Theory and Examples

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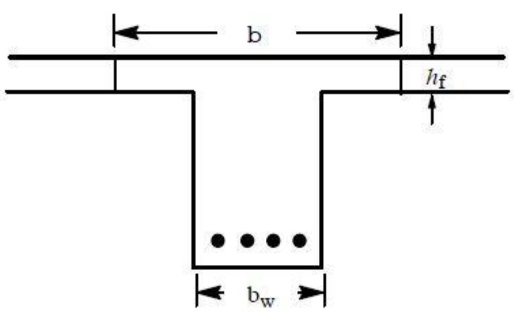
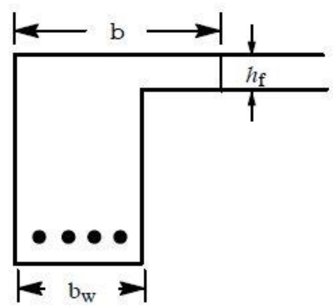
Reference Book: Design of Reinforced Concrete by Jack C. McCormac and Russell H. Brown, Clemson University, 9th Edition, 2014.

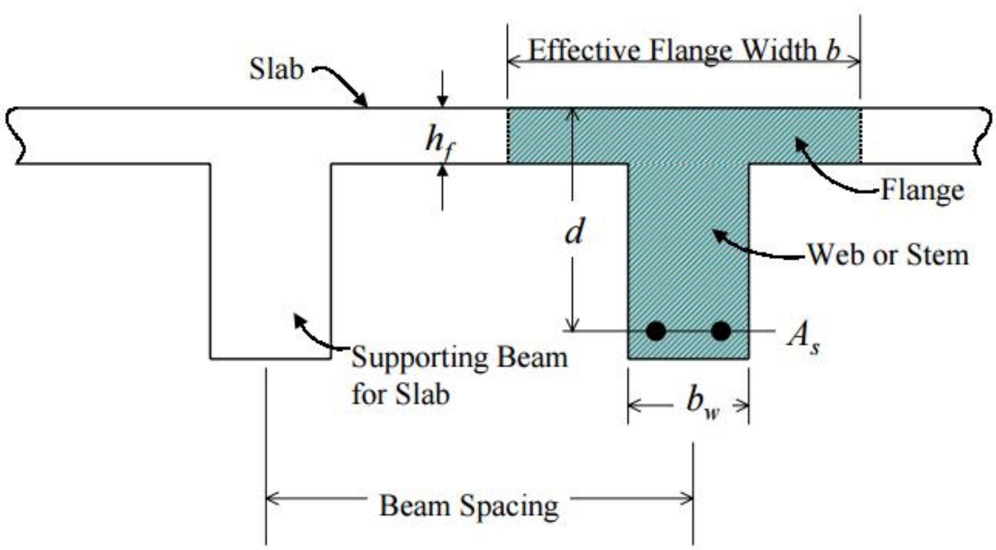
Reinforced concrete floor systems normally consist of slabs and beams that are placed monolithically. As a result, the two parts act together to resist loads. In effect, the beams have extra widths at their tops, called *flanges*, and the resulting T-shaped beams are called *T beams*. The part of a T beam below the slab is referred to as the *web* or *stem*. The beams may be inverted L shaped if it is edge or spandrel beam.
The analysis of T beams is quite similar to the analysis of rectangular beams in that the specifications relating to the strains in the reinforcing are identical.



T beam



L or inverted L beam



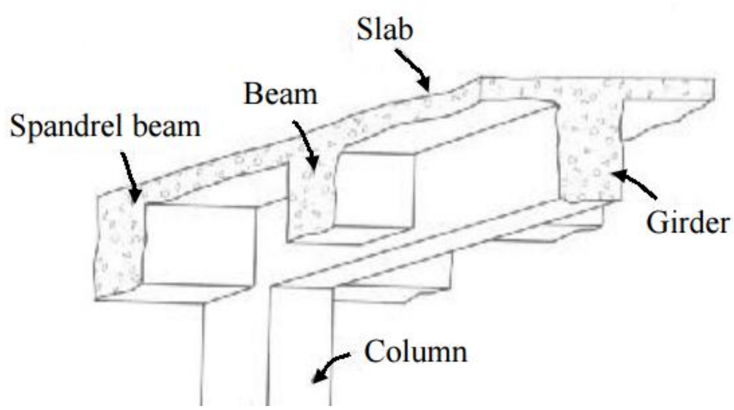
For T beams with flanges on both sides of the web, the code states that the effective flange width may not exceed one-fourth of the beam span, and the overhanging width on each side may not exceed eight times the slab thickness or one-half the clear distance to the next web.

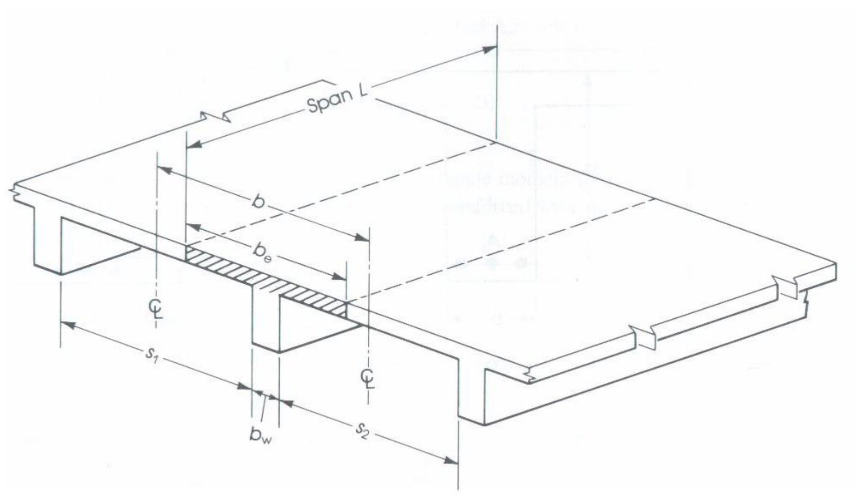
Inverted L-beam: If there is a flange on only one side of the web, the width of the overhanging flange cannot exceed one-twelfth the span, 6h_f, or half the clear distance to the next web (ACI 8.12.3).
As slab and beams are casted monolithically it is permitted to include the contribution of the slab in beam. Effective width of the flange can be calculated as per ACI 318 section 8.10.2 which is given in the following table.

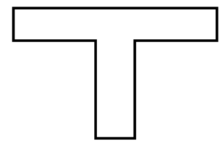
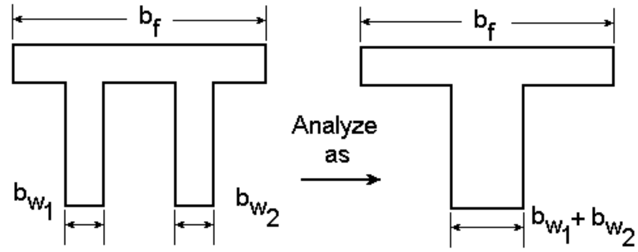
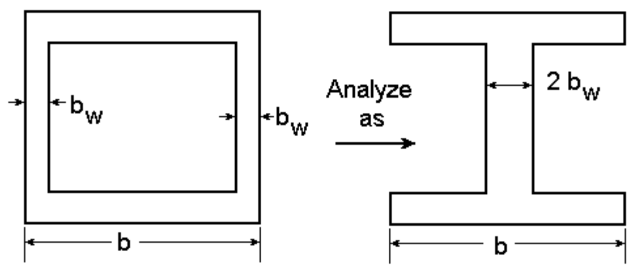
Table: Effective flange width of beam according to ACI

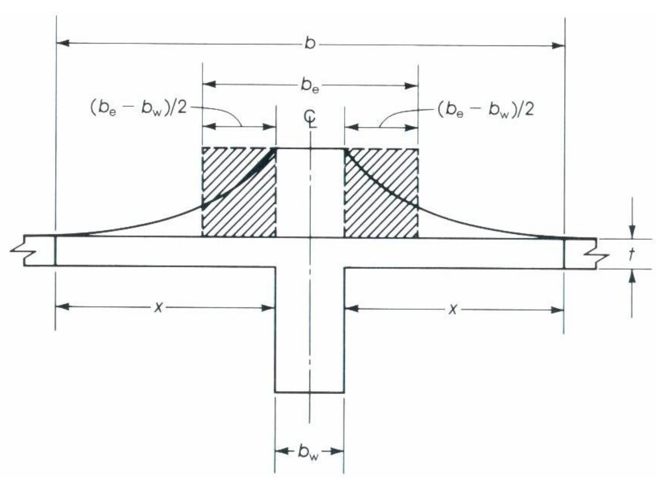
T-Beam	L-Beam
1. $b \leq \frac{\text{Span}}{4}$ 2. $b \leq b_w + 16h_f$ 3. $b \leq$ average clear distance to adjacent webs + b_w **The smallest of three values control**	1. $b \leq b_w + \frac{\text{Span}}{12}$ 2. $b \leq b_w + 6h_f$ 3. $b \leq b_w + \frac{\text{C/C beam distance}}{2}$ **The smallest of three values control**

■ Beam and Girder Floor System





Various Possible Geometries of T-Beams	
Single Tee	
Twin Tee	
Box	

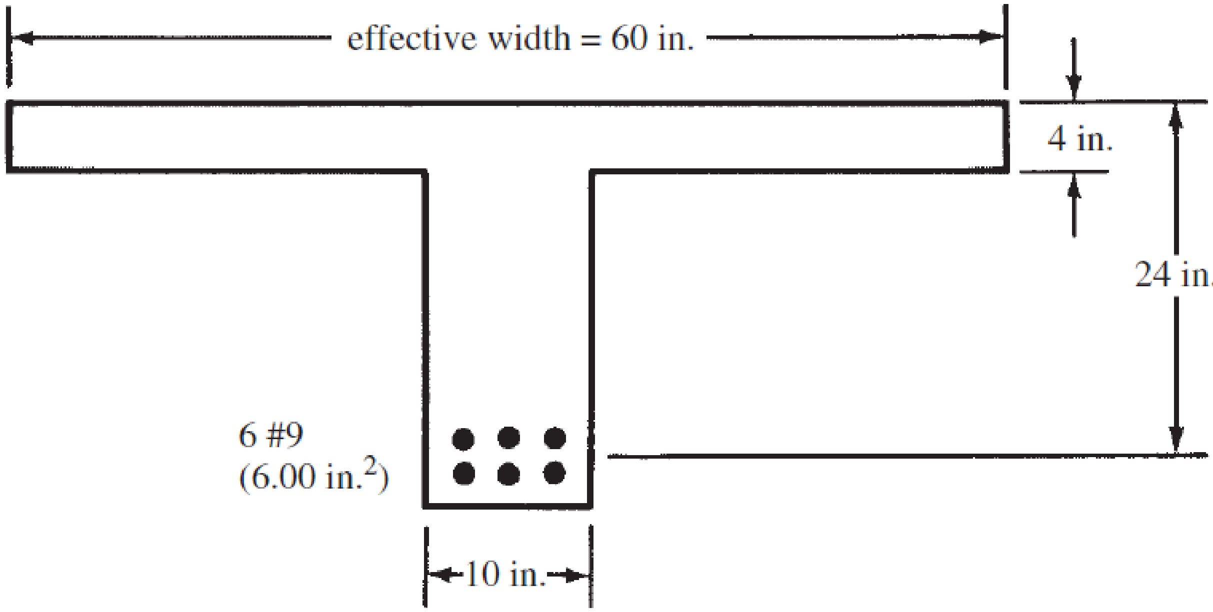


If the neutral axis falls within the slab depth analyze the beam as a rectangular beam, otherwise as a T-beam.

Analysis of T-Beam

Example 1

Determine the design strength of the T beam shown in the Figure below, with $f'c$ = 4000 psi and fy = 60,000 psi. The beam has a 30-ft span and is cast integrally with a floor slab that is 4 in. thick. The clear distance between webs is 50 inches.



SOLUTION

Check Effective Flange Width

$$b \le 16h_f + b_w = 16(4\text{ in.}) + 10\text{ in.} = 74\text{ in.}$$

$$b \le \text{ average clear distance to adjacent webs } + b_w = 50\text{ in.} + 10\text{ in.} = 60\text{ in.} \leftarrow$$

$$b \le \frac{\text{span}}{4} = \frac{30\text{ ft}}{4} = 7.5\text{ ft} = 90\text{ in.}$$

Checking $A_{s\,min}$

$$A_{s\ min} = \frac{3\sqrt{f'_c}}{f_y} b_w d = \frac{(3\sqrt{4000}\text{ psi})}{60{,}000\text{ psi}}(10\text{ in.})(24\text{ in.}) = 0.76\text{ in.}^2$$

$$\text{nor less than } \frac{200 b_w d}{f_y} = \frac{(200)(10\text{ in.})(24\text{ in.})}{60{,}000\text{ psi}} = 0.80\text{ in.}^2 \leftarrow$$

$$< A_s = 6.00\text{ in.}^2 \quad \underline{\underline{\text{OK}}}$$

Computing *T*

$$T = A_s f_y = (6.00\text{ in.}^2)(60\text{ ksi}) = 360\text{ k}$$

Determining A_c

$$A_c = \frac{T}{0.85f'_c} = \frac{360 \text{ k}}{(0.85)(4 \text{ ksi})} = 105.88 \text{ in.}^2$$

$< \text{flange area} = (60 \text{ in.})(4 \text{ in.}) = 240 \text{ in.}^2$ $\quad \therefore$ Compression stress block, a, is in flange

Calculating a, c, and ϵ_t

$$a = \frac{105.88 \text{ in.}^2}{60 \text{ in.}} = 1.76 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{1.76 \text{ in.}}{0.85} = 2.07 \text{ in.}$$

$$\epsilon_t = \left(\frac{d-c}{c}\right)(0.003) = \left(\frac{24 \text{ in.} - 2.07 \text{ in.}}{2.07 \text{ in.}}\right)(0.003)$$

$$= 0.0318 > 0.005$$ $\quad \therefore$ Section is ductile and $\phi = 0.90$

Calculating ϕM_n

Obviously, the stress block is entirely within the flange, and the rectangular formulas apply. However, using the couple method as follows:

$$\text{Lever arm} = z = d - \frac{a}{2} = 24 \text{ in.} - \frac{1.76 \text{ in.}}{2} = 23.12 \text{ in.}$$

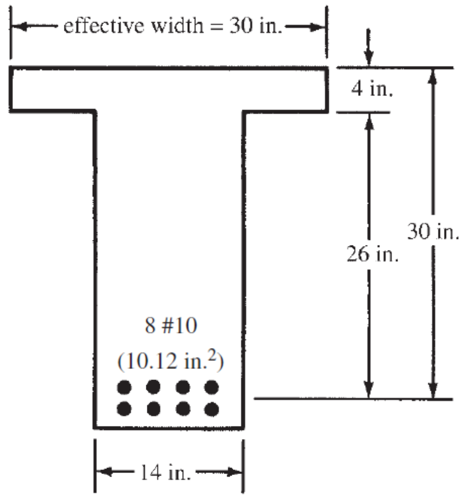
$$\phi M_n = \phi T z = (0.90)(360 \text{ k})(23.12 \text{ in.})$$

$$= 7490.9 \text{ in-k} = \underline{\underline{624.2 \text{ ft-k}}}$$

Analysis of T-Beam

Example 2- Moment Capacity of T beam

Compute the design strength for the T beam shown in the Figure below, in which $f'c$ = 4000 psi and fy = 60,000 psi.



SOLUTION

Checking $A_{s\ min}$

$$A_{s\ min} = \frac{3\sqrt{4000}\ \text{psi}}{60{,}000\ \text{psi}}(14\ \text{in.})(30\ \text{in.}) = 1.33\ \text{in.}^2$$

$$\text{nor less than } \frac{(200)(14\ \text{in.})(30\ \text{in.})}{60{,}000\ \text{psi}} = 1.40\ \text{in.}^2 \leftarrow$$

$$< A_s = 10.12\ \text{in.}^2 \quad \underline{\underline{\text{OK}}}$$

Computing T

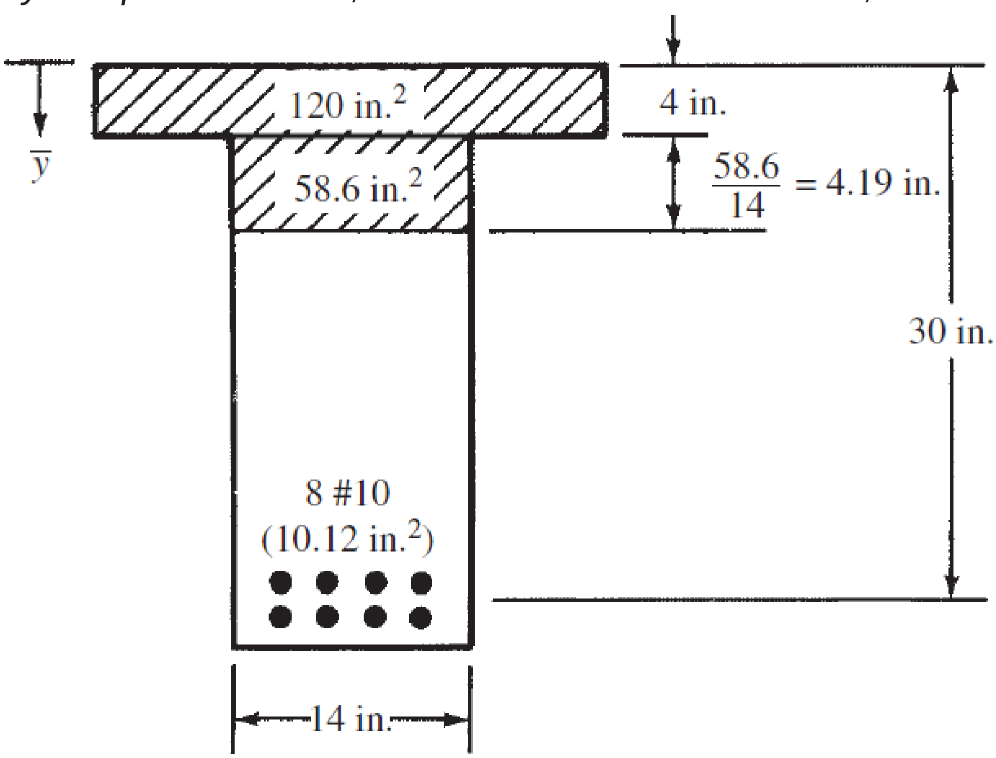
$$T = A_s f_y = (10.12\ \text{in.}^2)(60\ \text{ksi}) = 607.2\ \text{k}$$

Determining A_c and Its Center of Gravity

$$A_c = \frac{T}{0.85f'_c} = \frac{607.2\ \text{k}}{(0.85)(4\ \text{ksi})} = 178.59\ \text{in.}^2$$

$$> \text{flange area} = (30\ \text{in.})(4\ \text{in.}) = 120\ \text{in.}^2$$

Obviously, the stress block must extend below the flange to provide the necessary compression area, 178.6 in^2 − 120 in^2 = 58.6 in^2,



Computing the Distance $\bar{y}$ from the Top of the Flange to the Center of Gravity of A_c

$$\bar{y} = \frac{(120 \text{ in.}^2)(2 \text{ in.}) + (58.6 \text{ in.}^2)\left(4 \text{ in.} + \frac{4.19 \text{ in.}}{2}\right)}{178.6 \text{ in.}^2} = 3.34 \text{ in.}$$

The Lever Arm Distance from T to C = 30.00 in. − 3.34 in. = 26.66 in. = z

Calculating a, c, and ϵ_t

$$a = 4 \text{ in.} + 4.19 \text{ in.} = 8.19 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{8.19 \text{ in.}}{0.85} = 9.64 \text{ in.}$$

$$\epsilon_t = \frac{d-c}{c}(0.003) = \left(\frac{30 \text{ in.} - 9.64 \text{ in.}}{9.64 \text{ in.}}\right)(0.003) = 0.00634$$

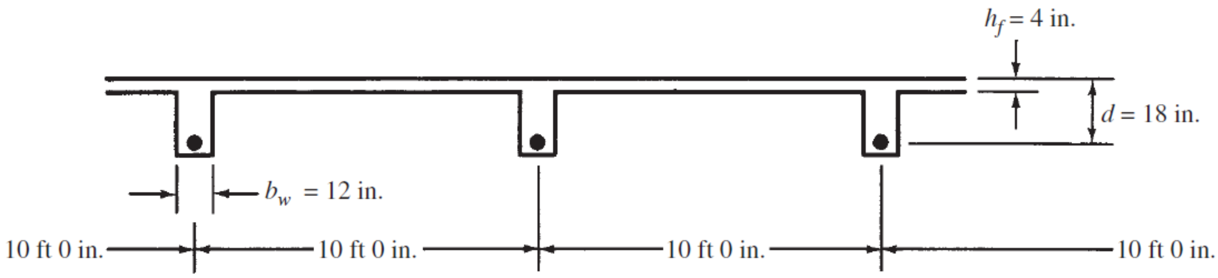
$$> 0.005 \qquad \underline{\therefore \text{ Section is ductile and } \phi = 0.90}$$

Calculating ϕM_n

$$\phi M_n = \phi T z = (0.90)(607.2 \text{ k})(26.66 \text{ in.}) = 14{,}569 \text{ in-k}$$
$$= \underline{\underline{1214 \text{ ft-k}}}$$

Design of T beam

Example 3: Design a T beam for the floor system shown in the figure below for which *bw* and *d* are given. M_D = 80 ft-k, M_L = 100 ft-k, *f'c* = 4000 psi, *fy* = 60,000 psi, and simple span = 20 ft.



SOLUTION

Effective Flange Width

(a) $\frac{1}{4}$ ft × 20 ft = 5 ft = 60 in.
(b) 12 in. + (2)(8)(4 in.) = 76 in.
(c) 10 ft = 120 in.

Computing Moments Assuming $\phi = 0.90$

$$M_u = (1.2)(80 \text{ ft-k}) + (1.6)(100 \text{ ft-k}) = 256 \text{ ft-k}$$

$$M_n = \frac{M_u}{\phi} = \frac{256 \text{ ft-k}}{0.90} = 284.4 \text{ ft-k}$$

Assuming a Lever Arm *z* Equal to the Larger of 0.9*d* or *d* − (h_f/2)

$$z = (0.9)(18 \text{ in.}) = \underline{\underline{16.20 \text{ in.}}}$$

$$z = 18 \text{ in.} - \frac{4 \text{ in.}}{2} = 16.00 \text{ in.}$$

Trial Steel Area

$$A_s f_y z = M_n$$

$$A_s = \frac{(12 \text{ in/ft})(284.4 \text{ ft-k})}{(60 \text{ ksi})(16.20 \text{ in.})} = 3.51 \text{ in.}^2$$

Computing Values of *a* and *z*

$$0.85 f'_c A_c = A_s f_y$$

$$(0.85)(4 \text{ ksi})(A_c \text{ in.}^2) = (3.51 \text{ in.}^2)(60 \text{ ksi})$$

$$A_c = 61.9 \text{ in.}^2 < (4 \text{ in.})(60 \text{ in.}) = 240 \text{ in.}^2 \qquad \underline{\underline{\therefore \text{N.A. in flange}}}$$

$$a = \frac{61.9 \text{ in.}^2}{60 \text{ in.}} = 1.03 \text{ in.}$$

$$z = d - \frac{a}{2} = 18 \text{ in.} - \frac{1.03 \text{ in.}}{2} = 17.48 \text{ in.}$$

Calculating A_s with This Revised *z*

$$A_s = \frac{(12 \text{ in/ft})(284.4 \text{ ft-k})}{(60 \text{ ksi})(17.48 \text{ in.})} = 3.25 \text{ in.}^2$$

Computing Values of *a* and *z*

$$A_c = \frac{(3.25 \text{ in.}^2)(60 \text{ ksi})}{(0.85)(4 \text{ ksi})} = 57.4 \text{ in.}^2$$

$$a = \frac{57.4 \text{ in.}^2}{60 \text{ in.}} = 0.96 \text{ in.}$$

$$z = 18 \text{ in.} - \frac{0.96 \text{ in.}}{2} = 17.52 \text{ in.}$$

Calculating A_s with This Revised z

$$A_s = \frac{(12 \text{ in/ft})(284.4 \text{ ft-k})}{(60 \text{ ksi})(17.52 \text{ in.})} = 3.25 \text{ in.}^2 \qquad \underline{\underline{\text{OK, close enough to previous value}}}$$

Checking Minimum Reinforcing

$$A_{s\ \min} = \frac{3\sqrt{f'_c}}{f_y} b_w d = \frac{3\sqrt{4000} \text{ psi}}{60{,}000 \text{ psi}}(12 \text{ in.})(18 \text{ in.}) = 0.68 \text{ in.}^2$$

but not less than

$$A_{s\ \min} = \frac{200 b_w d}{f_y} = \frac{(200)(12 \text{ in.})(18 \text{ in.})}{60{,}000 \text{ psi}} = 0.72 \text{ in.}^2 < 3.25 \text{ in.}^2 \qquad \underline{\underline{\text{OK}}}$$

Computing c, ϵ_t, and ϕ

$$c = \frac{a}{\beta_1} = \frac{0.96 \text{ in.}}{0.85} = 1.13 \text{ in.}$$

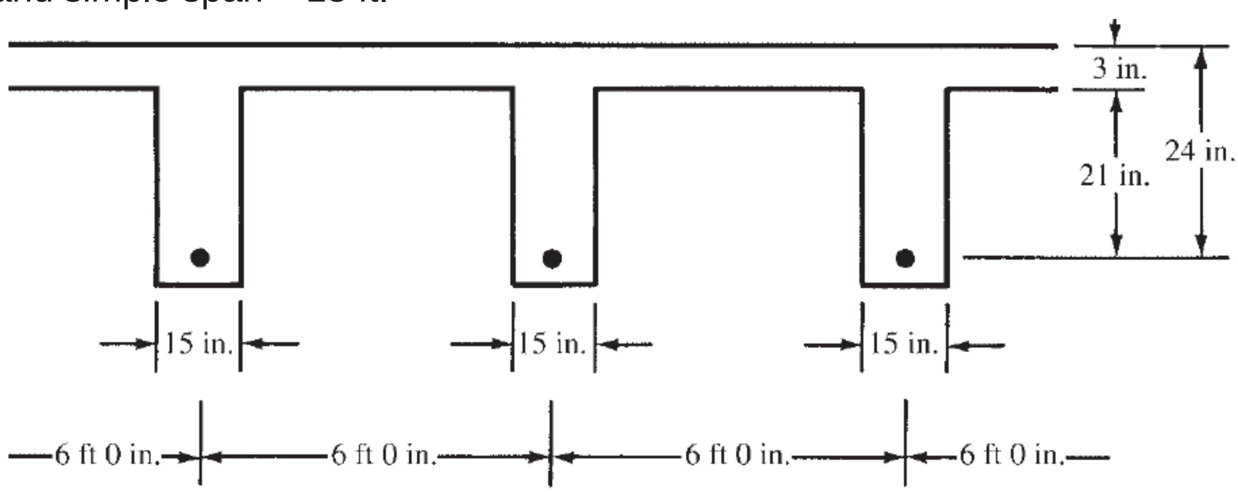
$$\epsilon_t = \left(\frac{d-c}{c}\right)(0.003) = \left(\frac{18 \text{ in.} - 1.13 \text{ in.}}{1.13 \text{ in.}}\right)(0.003)$$

$$= 0.045 > 0.005 \qquad \underline{\therefore \phi = 0.90 \text{ as assumed}}$$

$$\underline{\underline{A_{s\ \text{reqd}} = 3.25 \text{ in.}^2}}$$

Example 4:

Design a T beam for the floor system shown in Figure below, for which *bw* and *d* are given. M_D = 200 ft-k, M_L = 425 ft-k, *f'c* = 3000 psi, *fy* = 60,000 psi, and simple span = 18 ft.



Effective Flange Width

(a) $\frac{1}{4}$ ft × 18 ft = 4 ft 6 in. = $\underline{\underline{54\text{ in.}}}$

(b) 15 in. + (2) (8) (3 in.) = 63 in.

(c) 6 ft = 72 in.

Moments Assuming $\phi = 0.90$

$$M_u = (1.2)(200\text{ ft-k}) + (1.6)(425\text{ ft-k}) = 920\text{ ft-k}$$

$$M_n = \frac{M_u}{0.90} = \frac{920\text{ ft-k}}{0.90} = 1022\text{ ft-k}$$

Assuming a Lever Arm *z*

(Note that the compression area in the slab is very wide, and thus its required depth is very small.)

$$z = (0.90)(24\text{ in.}) = 21.6\text{ in.}$$

$$z = 24\text{ in.} - \frac{3\text{ in.}}{2} = \underline{22.5\text{ in.}}$$

Trial Steel Area

$$A_s = \frac{(12\text{ in/ft})(1022\text{ ft-k})}{(60\text{ ksi})(22.5\text{ in.})} = 9.08\text{ in.}^2$$

Checking Values of *a* and *z*

$$A_c = \frac{(60\text{ ksi})(9.08\text{ in.}^2)}{(0.85)(3\text{ ksi})} = 213.6\text{ in.}^2$$

The stress block extends down into the web, as shown in Figure 5.11.

Computing the Distance $\bar{y}$ from the Top of the Flange to the Center of Gravity of A_c

$$\bar{y} = \frac{(162\text{ in.}^2)(1.5\text{ in.}) + (51.6\text{ in.}^2)\left(3\text{ in.} + \frac{3.44\text{ in.}}{2}\right)}{213.6\text{ in.}^2} = 2.28\text{ in.}$$

$$z = 24\text{ in.} - 2.28\text{ in.} = 21.72\text{ in.}$$

$$A_s = \frac{(12\text{ in/ft})(1022\text{ ft-k})}{(60\text{ ksi})(21.72\text{ in.})} = 9.41\text{ in.}^2$$

The steel area required (9.41 in.^{2}) could be refined a little by repeating the design, but space is not used to do this. (If this is done, $A_s = 9.51$ in.^{2}.)

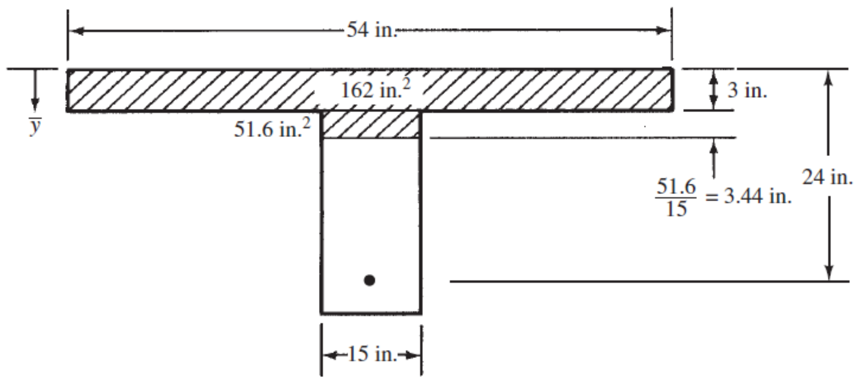
Checking Values of ϵ_t and ϕ

$$a = 3\text{ in.} + 3.44\text{ in.} = 6.44\text{ in.}$$

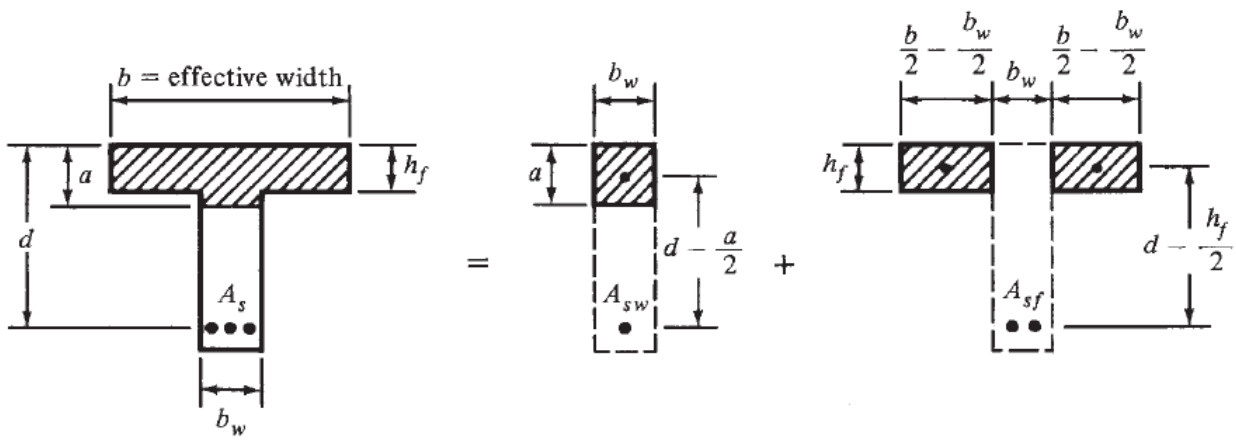
$$c = \frac{a}{\beta_1} = \frac{6.44\text{ in.}}{0.85} = 7.58\text{ in.}$$

$$\epsilon_t = \left(\frac{d-c}{c}\right)(0.003) = \left(\frac{24\text{ in.} - 7.58\text{ in.}}{7.58\text{ in.}}\right)(0.003)$$

$$= 0.00650 > 0.005 \qquad \therefore \phi = 0.90\text{ as assumed}$$



Alternate Solution



The compression force provided by the overhanging flange rectangles must be balanced by the tensile force in part of the tensile steel, A_{sf}, while the compression force in the web is balanced by the tensile force in the remaining tensile steel, A_{sw}.

For the overhanging flange, we have

$$0.85f'_c(b - b_w)(h_f) = A_{sf}f_y$$

from which the required area of steel, A_{sf}, equals

$$A_{sf} = \frac{0.85f'_c(b - b_w)h_f}{f_y}$$

The design strength of these overhanging flanges is

$$M_{uf} = \phi A_{sf}f_y\left(d - \frac{h_f}{2}\right)$$

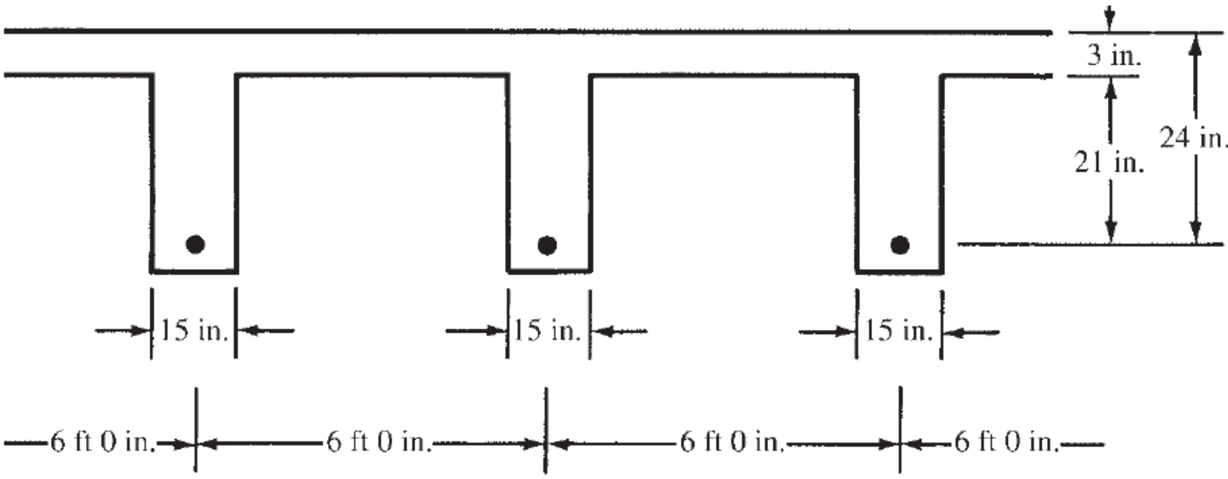
The remaining moment to be resisted by the web of the T beam and the steel required to balance that value are determined next.

$$M_{uw} = M_u - M_{uf}$$

The steel required to balance the moment in the rectangular web is obtained by the usual rectangular beam expression.

Example 5:

Design a T beam for the floor system shown in Figure below, for which *bw* and *d* are given. M_D = 200 ft-k, M_L = 425 ft-k, *f'c* = 3000 psi, *fy* = 60,000 psi, and simple span = 18 ft.



SOLUTION

First assume $a \le h_f$ (which is very often the case). Then the design would proceed like that of a rectangular beam with a width equal to the effective width of the T-beam flange.

$$\frac{M_u}{\phi b d^2} = \frac{920 \text{ ft-k } (12{,}000 \text{ in-lb/ft-k})}{(0.9)(54 \text{ in.})(24 \text{ in.})^2} = 394.4 \text{ psi}$$

$$\rho = 0.0072 \text{ (from Appendix A, Table A.12)}$$

$$a = \frac{\rho f_y d}{0.85 f'_c} = \frac{0.0072(60 \text{ ksi})(24 \text{ in.})}{(0.85)(3 \text{ ksi})} = 4.06 \text{ in.} > h_f = 3 \text{ in.}$$

The beam acts like a T beam, not a rectangular beam, and the values for ρ and a above are not correct. If the value of a had been $\le h_f$, the value of A_s would have been simply ρbd = 0.0072(54 in.) (24 in.) = 9.33 in.2. Now break the beam up into two parts and design it as a T beam.

Assuming $\phi = 0.90$

$$A_{sf} = \frac{(0.85)(3\text{ ksi})(54\text{ in.} - 15\text{ in.})(3\text{ in.})}{60\text{ ksi}} = 4.97\text{ in.}^2$$

$$M_{uf} = (0.9)(4.97\text{ in.}^2)(60\text{ ksi})\left(24\text{ in.} - \frac{3}{2}\text{ in.}\right) = 6039\text{ in-k} = 503\text{ ft-k}$$

$$M_{uw} = 920\text{ ft-k} - 503\text{ ft-k} = 417\text{ ft-k}$$

Designing a Rectangular Beam with $b_w = 15$ in. and $d = 24$ in. to Resist 417 ft-k

$$\frac{M_{uw}}{\phi b_w d^2} = \frac{(12\text{ in/ft})(417\text{ ft-k})(1000\text{ lb/k})}{(0.9)(15\text{ in.})(24\text{ in.})^2} = 643.5\text{ psi}$$

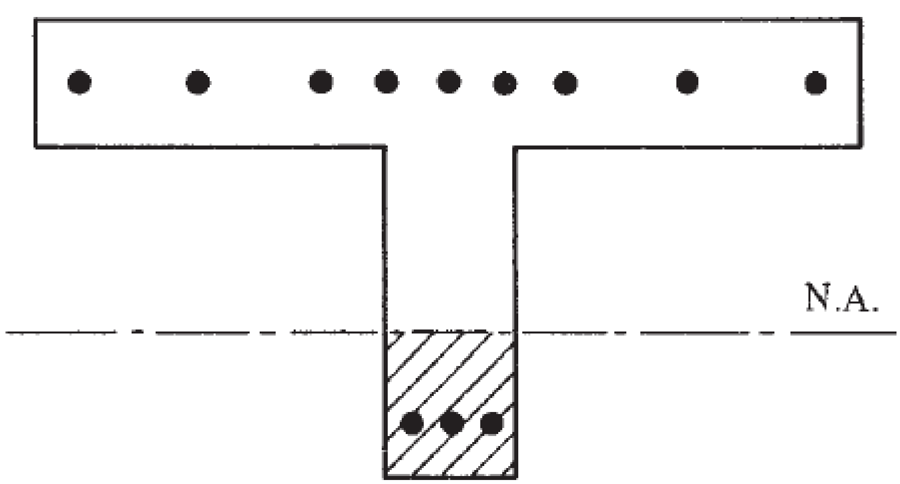
$$\rho_w = 0.0126 \text{ (from Appendix A, Table A.12)}$$

$$A_{sw} = (0.0126)(15\text{ in.})(24\text{ in.}) = 4.54\text{ in.}^2$$

$$A_s = 4.97\text{ in.}^2 + 4.54\text{ in.}^2 = \underline{\underline{9.51\text{ in.}^2}}$$

Design of T Beams for Negative Moments

When T beams are resisting negative moments, their flanges will be in tension and the bottom of their stems will be in compression, as shown in Figure below. Obviously, for such situations, the rectangular beam design formulas will be used. Section 10.6.6 of the ACI Code requires that part of the flexural steel in the top of the beam in the negative-moment region be distributed over the effective width of the flange or over a width equal to one-tenth of the beam span, whichever is smaller. Should the effective width be greater than one-tenth of the span length, the code requires that some additional longitudinal steel be placed in the outer portions of the flange. The intention of this part of the code is to minimize the sizes of the flexural cracks that will occur in the top surface of the flange perpendicular to the stem of a T beam subject to negative moments.



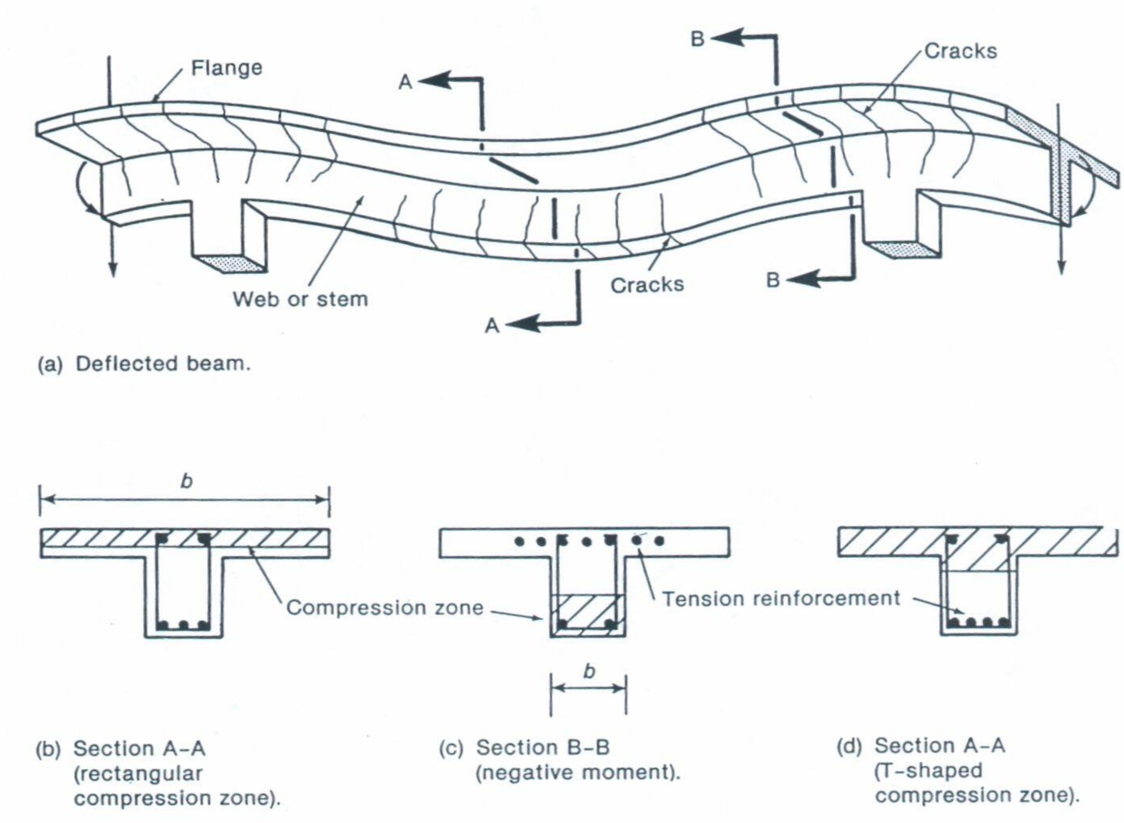
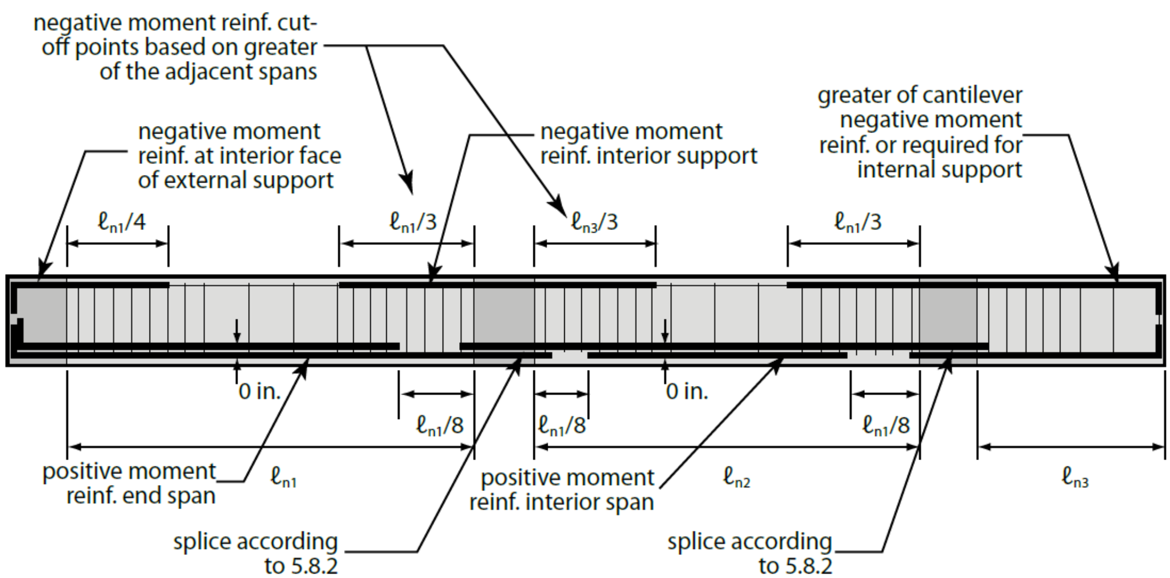


Figure: Positive and Negative Moment Regions in a T-beam



Reinforcement for beams and joists supported by beams or girders.