

Cambridge, Massachusetts

May 25, 1953

Chairman, Departmental Committee on Graduate Students
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:

In partial fulfillment of the requirements for the degree of
Master of Science in Mechanical Engineering, we hereby submit our
thesis entitled "Excitation of Gyroscopic Precession in Overhung
Rotors at Critical Speeds".

Very truly yours,

Chaovana Na Sylvanta

Daniel B. DeBra

EXCITATION OF GYROSCOPIC PRECESSION IN
OVERHUNG ROTORS AT CRITICAL SPEEDS

by

Chaovana Na Sylvanta & Daniel B. DeBra

Submitted to the Department of Mechanical Engineering on
May 18, 1953 in partial fulfillment of the requirements for the
degree of Master of Science in Mechanical Engineering.

The investigation of gyroscopic precession of overhung rotors has been complicated by the fact that it is difficult to observe the vibrations which occur at various speeds. These vibrations are not always of a simple nature so a means of clearly defining the path and frequency is necessary in order to understand what happens at resonant speeds. With the use of a microscope and stroboscopic light for observing the end of the shaft, very accurate results can be obtained and are described in this thesis.

Observations were made carefully throughout the speed range of the equipment to note all vibrations which occur in that range.

The excitation of the vibrations which have been observed in overhung rotors has been explained for some cases. A large portion of the work done for this thesis was the investigation of the effects of shaft flatness and eccentricity of the center of mass of the disk on the excitation of retrograde precession of the same frequency as the solid body rotation. Work was done investigating each effect separately and various combinations of the two effects.

Thesis Supervisor: J. P. Den Hartog
Title: Professor of Mechanical Engineering

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INTRODUCTION

The investigation of gyroscopic precession of overhung rotors has been complicated by the fact that it is difficult to observe the vibration which occur at various speeds. These vibrations are not always of a simple nature so a means of clearly defining the path and frequency is necessary in order to understand what happens at resonant speeds. With the use of stroboscopic light for observing the end of the shaft, very accurate results can be obtained and are described in this paper.

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The excitation of the vibrations which have been observed in overhung rotors has been explained for some cases. A large portion of the work done for this thesis was the investigation of the effects of shaft flatness and eccentricity of the center of gravity of the disk to the center line of rotation on the excitation of retrograde precession of the same frequency as the solid body rotation. Work was done investigating each effect separately and various combinations of the two effects.

APPARATUS

The apparatus used in the experiment consisted of a speed control system, disks, shafts and bearing housing, strobotac and a cathetometer which is a traveling vernier microscope.

The motor and generator used were big in order to control and maintain the speed in the critical range since at a particular resonant speed a large amount of energy is absorbed by the vibration as the amplitude builds up. A 3.75 KW 250 V. D.C. motor and generator set was used to supply power at constant voltage to the driving motor which was separately mounted. The driving motor was a variable speed 3.5 - 5.0 H.P. D.C. motor. The shaft was coupled to the driving motor by means of rubber hose. This coupling was chosen to prevent or to minimize vibration transmitted from the motor to the shaft. This system provided a speed control range from 550 to 1800 rpm. which could be regulated and maintained to ± 3 rpm.

The bearing housing was constructed so that it could be filled with oil. It had two precision ball bearings placed about 6 inches apart in a stiff bracket which was mounted in turn on a four inch wooden block and a one half ton bed plate. The driving motor was also mounted on the bed plate.

The shaft and disk were made of steel and the complete dimensions are shown in Fig. 1, 2, & 3. Two disks were used in the experiments, disk No.2 was made smaller in order to reduce the gyroscopic effect. Disk No. 1. had six equally spaced holes for balancing

3.

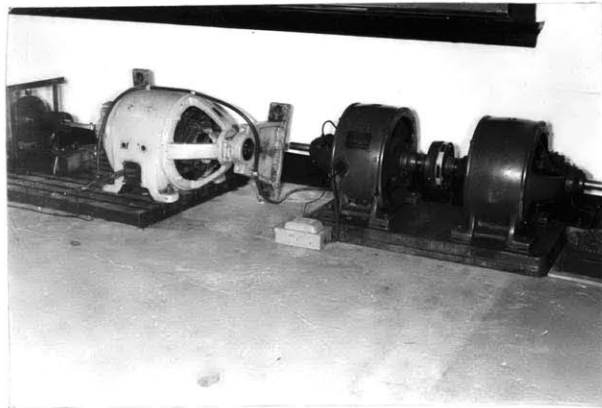
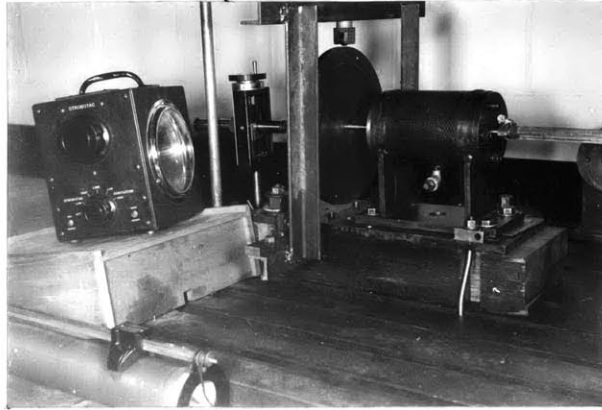
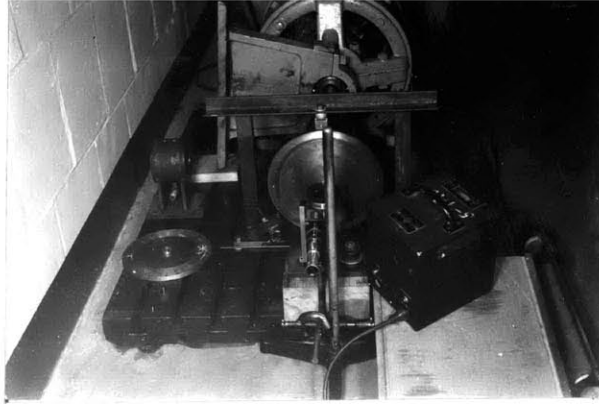
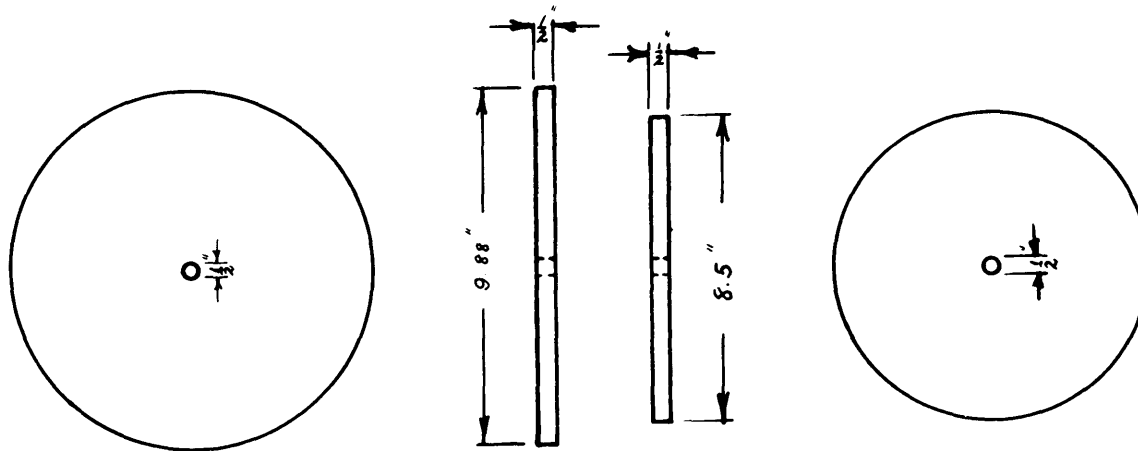


Fig. 1.



DISK NO 1

DISK NO 2

FIG. 2

FIG. 3

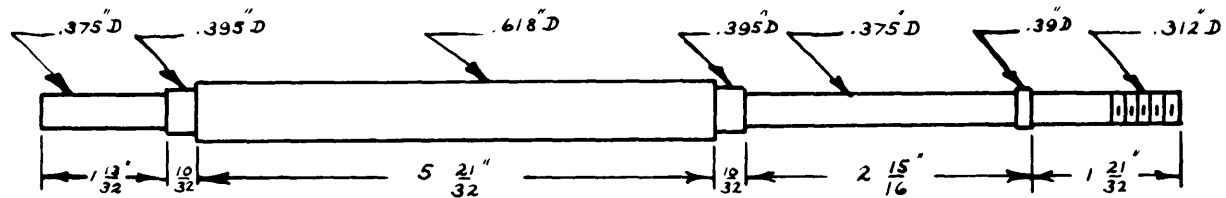


FIG. 4.

TYPICAL SHAFT

4.

and disk No. 2. had twenty-four.

Shaft No. 1. was milled on two sides to accentuate the flat shaft effect. Shaft No. 2. was later modified and called shaft No. 3. by filing the stiff side of the shaft in order to make bending stiffness equal in all directions.

The stiffness of the shaft was carefully determined in each case by loading and measuring the stiffness deflection. This was done in various directions since the stiffness was not equal in all directions. The value used on the calculations and plottings was the average value of the maximum and minimum stiffness. It was found for shaft No. 2. that the difference between the maximum and minimum stiffness was about 8 %, which had a considerable effect.

The amplitude and the path of the whirl was observed by a cathetometer at the end of the shaft. The observation was made possible by making a very small round dot at the center of the end of the shaft which protruded beyond the disk. This dot was made by putting a drop of solder on the end of the shaft to provide a good smooth surface. A small round piece of colored scotch tape with a pin prick 0.09 mm wide was put on the solder. While looking through the cathetometer or microscope the dot was centered by sliding the tape back and forth as the disk was slowly rotated. When the dot was at the center of rotation a quick drying cement was applied and allowed to set. By rotating the disk at such a^a slow speed, dynamic deflection effects were avoided and the dot could be considered at the true center of solid body rotation. In order to be seen more distinctly, the pin prick was filled with a fine talcum powder. When such a point is properly

located, it exactly describes the path of whirl of the shaft.

The vibrations or whirls were observed through the microscope with the aid of stroboscopic light. Vibrations can be made to stand still visually by adjusting the frequency of light flashes to be the same as that of the shaft vibration. They will also appear to be frozen if the frequency of the vibration is some integral multiple of the flashing frequency, but for any other frequency of the strobotac that stops the motion, the vibration will appear to have more than one image. The frequencies of vibration were checked in this way by finding the highest frequency which there appears a single dot. If a strobotac is operated at a frequency slightly lower than the frequency of the vibration, the dot will drift in its regular path at the frequency equal to the difference in frequency of the strobotac and vibration. If the vibration is a multiple of the stroboscopic light, the drift speed will be the difference between that multiple of flashing speed and the speed of vibration. The drift is therefore proportional to and in the direction of the true whirl. This fact is extremely useful in observing several types of vibrations at once. To take a simple case let us look at a negative whirl at twice the speed of the solid body rotation. If we observe the center point at the end of the shaft, we will see it describes a circular path. If we count the number of times it drifts around the circle we will find that it is twice as many as the drift of solid body rotation about the center point and at the same time we will also see that it moves in opposite direction. When more than one type of whirl is present the situation will become

more complicated. If, for example, in the case above the center point was moved so it was a little eccentric, then there would be a minus two whirl and a plus one whirl as well as the solid body drift. These two paths when superimposed describe a three-leaf rose once for every rotation of the solid body drift if the amplitude of whirl is the same size. The locus of several types of whirl which might occur in this work have been plotted and are shown in Fig. 5, 6, & 7 page 15-17.

DISCUSSION

When a rotor is mounted in pedestal type bearings the stiffness of the system is usually greater in the vertical direction than in the horizontal direction. This effects the resonant speeds of the various modes of vibration, and in the case of minus one whirl is, it is believed, indirectly the cause of excitation. Even when the shaft and the rotor are carefully balanced there is still a slight eccentricity of the center of gravity to the center line of rotation. This eccentricity causes a centrifugal force. When there is no other dynamic deflection in the shaft except the deflection due to this centrifugal force, the force and deflection can be divided into horizontal and vertical components which will be sine and cosine functions of time and speed of rotation. The difference in the bearing stiffness of the system causes the natural frequency of minus one whirl to occur over a larger range (from 1395 to 1425 rpm.) , and indeed gives a natural vibration which has a larger magnitude in the horizontal direction than the vertical direction at the lower end of critical range. When the critical speed is approached from below, the horizontal component of the centrifugal force excites the natural vibration which occurs in the horizontal direction at that speed. This centrifugal force whose horizontal component excites the horizontal vibration has a vertical component which is 180 degrees out of phase with a vertical force which would excite the vertical component of minus one in a similar manner. This force cannot take as much work out of the system

as the horizontal force puts in. As the path becomes elliptical, the displacement in the vertical direction is smaller than in the horizontal direction and the vertical force cannot act through as a great distance. The net horizontal force on the other hand acts through an increasingly longer distance and so adds more energy to the system. The excitation occurs in a similar manner when approaching the range from above. As the speed changes and the center of critical range is approached the precessional moment tending to flatten the plus one ellipse effects the transition to a pure minus one whirl.

This agrees very well with what is observed in practice. Many runs were made with different combinations of eccentricity and flat shaft effect and in each case the pattern was precisely the same. As the amplitude of vibration built up at either end of resonant speed range, the path of the whirl of the shaft was very clearly an elongated path in a horizontal direction at the lower end and in the vertical direction at the higher end of the speed range. In going from a horizontal to a vertical position the orientation shifted gradually and the path became closer to a minus one in the middle of the range. As the amount of unbalance was increased, the amplitude of the minus one increased. There was not a direct proportionality between the unbalance and the amplitude of the minus one and this may be due to a change of damping and because as the path is changed there is a change in the centrifugal force.

The orientation of the center of the mass of the disk with respect to the shaft had a pronounced effect on the amplitude of the minus one which it is felt was due to a variation in the unbalance

which results from turning the disk on the shaft. In any shaft there is a slight curvature or bend which may be caused by machining. The center of any rotation therefore is not at the geometric center of the end of the shaft. When the center of mass of the disk is placed on the shaft so it is in the same direction from the center of rotation as the deflection of the bend in the shaft, the effect of eccentricity is magnified. This effect is additive at the worst position and subtractive when the center of mass is turned 180 degrees from there. The effect of the bent shaft is therefore doubled in the extreme positions.

The effect of varying the amount of shaft flatness was not obvious at first. It appeared that when the shaft was made more nearly "round" (the difference in stiffness in the maximum and minimum directions was reduced to less than 3%) the amplitude of the minus one decreased. The amplitude which is given in the data is the size of the path that was seen. The path, however, as explained before was often made up of more than one pure motion superimposed. The path of maximum amplitude of minus one was made up of a pure minus one, a plus one due to the eccentricity of the dot itself and a vertical displacement of twice the frequency due to the flat shaft effect. It may be noted that since the resonant speed of minus one for disk No. 2. was several hundred rpm. from the resonant speed of plus two whirl, the excitation of flat shaft caused only a vertical displacement instead of the circular path which occurs at resonance and explains the shape of the elongated triangular paths which were observed. As the flat shaft effect was reduced the amplitude of the forced vertical displacement was

removed and the path not only became more nearly a circular minus one but also showed a smaller amplitude. The reduction in size, however, was only due to the reduction of the vertical displacement.

When the dot at the end of the shaft is placed a little bit off center there is an apparent plus one whirl viewed through the microscope. The whirl can be visually removed by making the center of whirl coincide with that point. An unbalance placed 180 degrees from the point with respect to the center of rotation can remove the plus one path of the point at a particular speed. It was found that by running the disk at 1500 rpm. with the correct amount of unbalance at the proper location found by trial and error, the dot could be made to appear at the center of rotation. Then running the system in the range of minus one the purest minus one whirl observed was obtained. Even with these conditions a slight effect of plus two was noticeable and the circular path was slightly flattened on three sides.

When the speed of the shaft is one-half the natural speed of either plus two or minus two whirl, these modes are excited by the flat shaft effect. "Flat shaft" refers to a shaft which does not have equal bending stiffness in all directions, and hence has a maximum and minimum stiffness at right angles to each other. There is an alternating vertical force set up while a shaft rotates due to unequal stiffness when it is overhung in a gravitational field. This force acts twice in each direction for each revolution of the shaft. When this force acts at the same frequency as the natural vibration it can add energy to the system. It is interesting to note that since the exci-

tation is purely in the direction of the gravitational force, it is independent of the direction of whirl it excites. The force acts only on the vertical component of the displacement to do work and this component of the displacement can lag the force for plus or minus two so that net work can be added to the system for either vibration at their resonant speeds.

In the first runs that were made, disk No. 1 and shaft No. 1 were used. A vibration occurred with this combination at 624 rpm. which appeared to be a plus three whirl. It was not observed through the microscope, but the disk was scribed with chalk and marks appeared at three distinct points on the disk indicating plus or minus three whirl. At 624 rpm. the only natural whirl which is close to a multiple of three times the shaft rotation is the first positive mode so the motion is assumed to have been plus three. When operating with shaft No. 2 and disk No. 2 a combined plus one and plus three path was clearly observed. No experimental work was done to observe the effects on the plus three whirl of changing unbalance and shaft flatness so there is no indication that these effects are the cause of excitation. It is felt that friction is not responsible for this roughness because the frequencies remained an exact multiple of each other during all observations.

At any speed of rotation there are four resonant whirling speeds which may occur. To have a vibration a force is needed which can excite any one of these whirls. So far only vibrations which are some integral multiple of the solid body speed have been

discussed. Frictional forces, however, are not in general directly related to the solid body speed in a simple manner. Several cases of roughness were observed which occurred at whirling speeds which were not simple multiples of the solid body rotation, and it was felt that they must have been excited by friction in the bearing. At times some roughness occurred which was only transient. It was observed to build up and die out in ten or fifteen seconds, and would not appear again. In one observation at minus one, there was a frictional vibration superimposed which was not observed at any other time. Since the disk had not been run for a while it was felt that the bearings were dry. The housing was filled with oil and the superimposed roughness disappeared. The oil, however, had no effect on two frictional cases that were seen regularly.

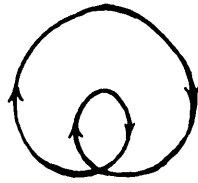
LOCUS OF SUPERIMPOSED CIRCULAR PATHS OF VARIOUS FREQUENCIES

In some cases the natural frequencies of whirl are close together which causes the path to be the combination of several types of circular whirl. Some exciting forces, by their nature, are present at all speeds and cause small amplitude vibrations at speeds outside their critical ranges. When an eccentricity of the center of gravity is present, the amount can be large enough to have an effect of a plus one whirl everywhere even though the speed of operation is far from the primary critical speed. This plus one whirl will superimpose on and distort the shape of the whirl whose natural frequency occurs at that speed and may be misleading to the interpretation of results. The locus of various combinations of whirl are shown here to help in the interpretation of what was seen.

In each case the locus has been drawn for the cases where the amplitude of the two component paths are equal and differ by a factor of one and one half. The symbol $[+1]$ refers to the amplitude of a point which is travelling in a circular path at an angular speed of one positive unit. The positive direction of the angular speed is shown clockwise. $[-3]$ would therefore refer to the amplitude of a point traveling in the opposite direction at three times the speed of $[+1]$.



$$[+1] = [+2]$$



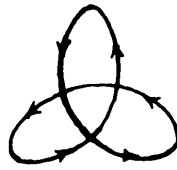
$$[+1] < [+2]$$



$$[+1] > [+2]$$



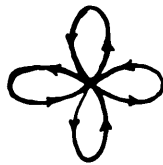
$$[+1] = [-2]$$



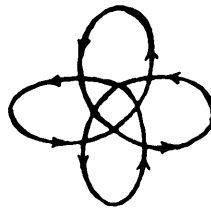
$$[+1] < [-2]$$



$$[+1] > [-2]$$



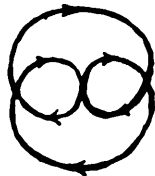
$$[+1] = [-3]$$



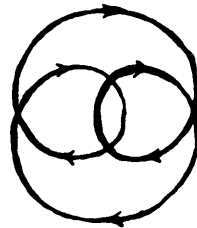
$$[+1] < [-3]$$



$$[+1] > [-3]$$



$$[+1] = [+3]$$



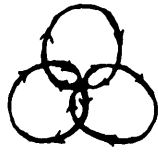
$$[+1] < [+3]$$



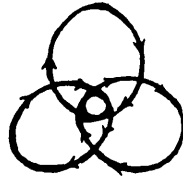
$$[+1] > [+3]$$

LOCUS OF SUPERIMPOSED CIRCULAR PATHS OF VARIOUS FREQUENCIES

Fig. 5



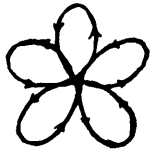
$$[+1] = [+4]$$



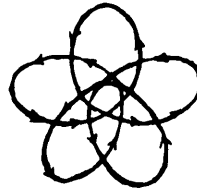
$$[+1] < [+4]$$



$$[+1] > [+4]$$



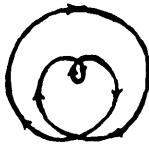
$$[+1] = [-4]$$



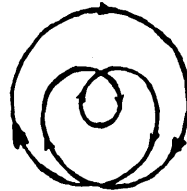
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$$[+1] > [-4]$$



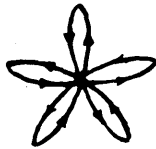
$$[+2] = [+3]$$



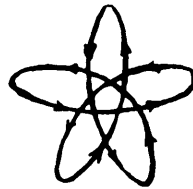
$$[+2] < [+3]$$



$$[+2] > [+3]$$



$$[+2] = [-3]$$



$$[+2] < [-3]$$



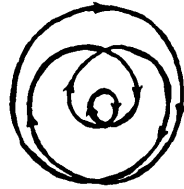
$$[+2] > [-3]$$

LOCUS OF SUPERIMPOSED CIRCULAR PATHS OF VARIOUS FREQUENCIES

Fig. 6



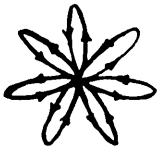
$$[+3] - [+4]$$



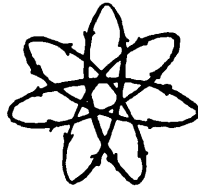
$$[+3] < [+4]$$



$$[+3] > [+4]$$



$$[+3] - [-4]$$



$$[+3] < [-4]$$



$$[+3] > [-4]$$



$$[+1] - [-1]$$



$$[+1] < [-1]$$



$$[+1] > [-1]$$

LOCUS OF SUPERIMPOSED CIRCULAR PATHS OF VARIOUS FREQUENCIES

Fig. 7

CALCULATION OF THE NATURAL FREQUENCIES

For the calculation of the critical speeds of overhung shaft, we make use of the results of Green's thesis. The equation governing the system is

$$\left[1 - \omega_\omega^2 \frac{m l^3}{3 E_s I_s} \right] \left[1 + \omega_\omega (2 \omega_\lambda - \omega_\omega) \frac{I_d l}{E_s I_s} \right] + \omega_\omega^3 (2 \omega_\lambda - \omega_\omega) I_d \frac{m l^4}{4 E_s^2 I_s^2} = 0 \quad [1]$$

where

ω_λ	angular velocity of the shaft rotation	rad/sec.
ω_ω	angular velocity of whirl	rad/sec.
m	mass of the disk	lb-sec ² /in.
l	length of the shaft	in.
E_s	modulus of elasticity of the shaft	lb/in ²
I_s	moment of inertia of cross section of the shaft	in ⁴
I_d	moment of inertia of the disk about its diameter	in ⁴

This equation may be put in the dimensionless form by introducing

$$K = \omega_\omega \sqrt{\frac{m l^3}{3 E_s I_s}}, \quad D = I_d / m l^2 \quad \text{and} \quad h = \omega_\lambda / \omega_\omega$$

the equation becomes

$$3 K^4 D (2h - 1) + K^2 [4 - 12 D (2h - 1)] - 4 = 0 \quad [2]$$

For disk No.1. the values were found by direct measurement
as follows:

deflection due to unit load ($l^3/3E_sI_s$)	7.8×10^{-4} in/lb.
mass of the disk	2.525×10^{-2} in-lb/sec ²
diameter of the disk	9.88 in.

By substituting these values in equation [2] we get,

$$\omega_\omega^4 - 2 \omega_\lambda \omega_\omega^3 - 364,000 \omega_\omega^2 + 407,000 \omega_\lambda \omega_\omega + 81.5 \times 10^8 = 0$$

This equation will give four roots of ω_ω for each value of ω_λ
and is plotted in Fig. 8 page 20.

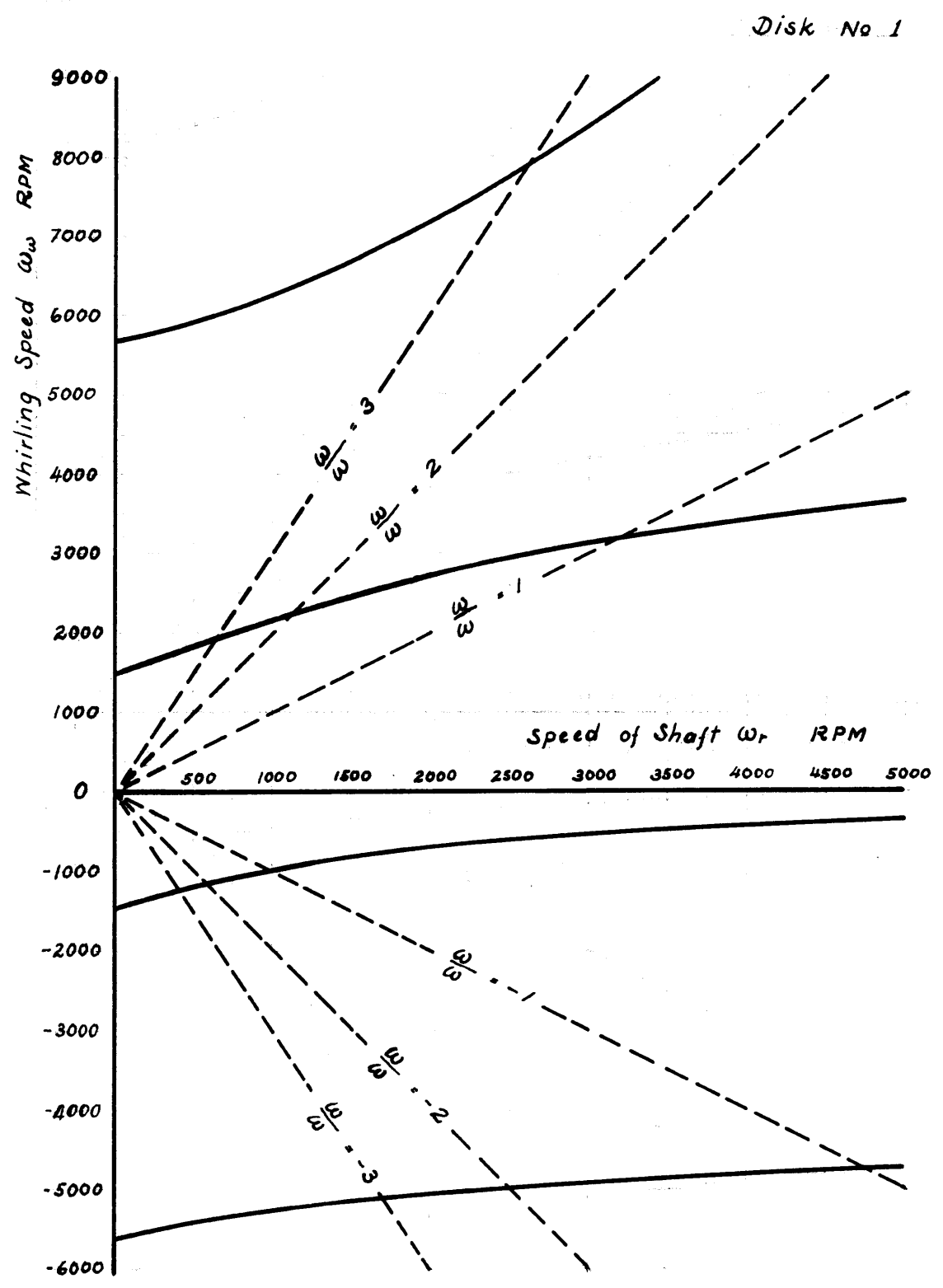
Similarly for disk No. 2. the following values were determined

deflection due to unit load	7.6×10^{-4} in/lb
mass of the disk	1.91×10^{-2} in-lb/sec ²
diameter of the disk	8.5 in.

For which equation [2] takes the form

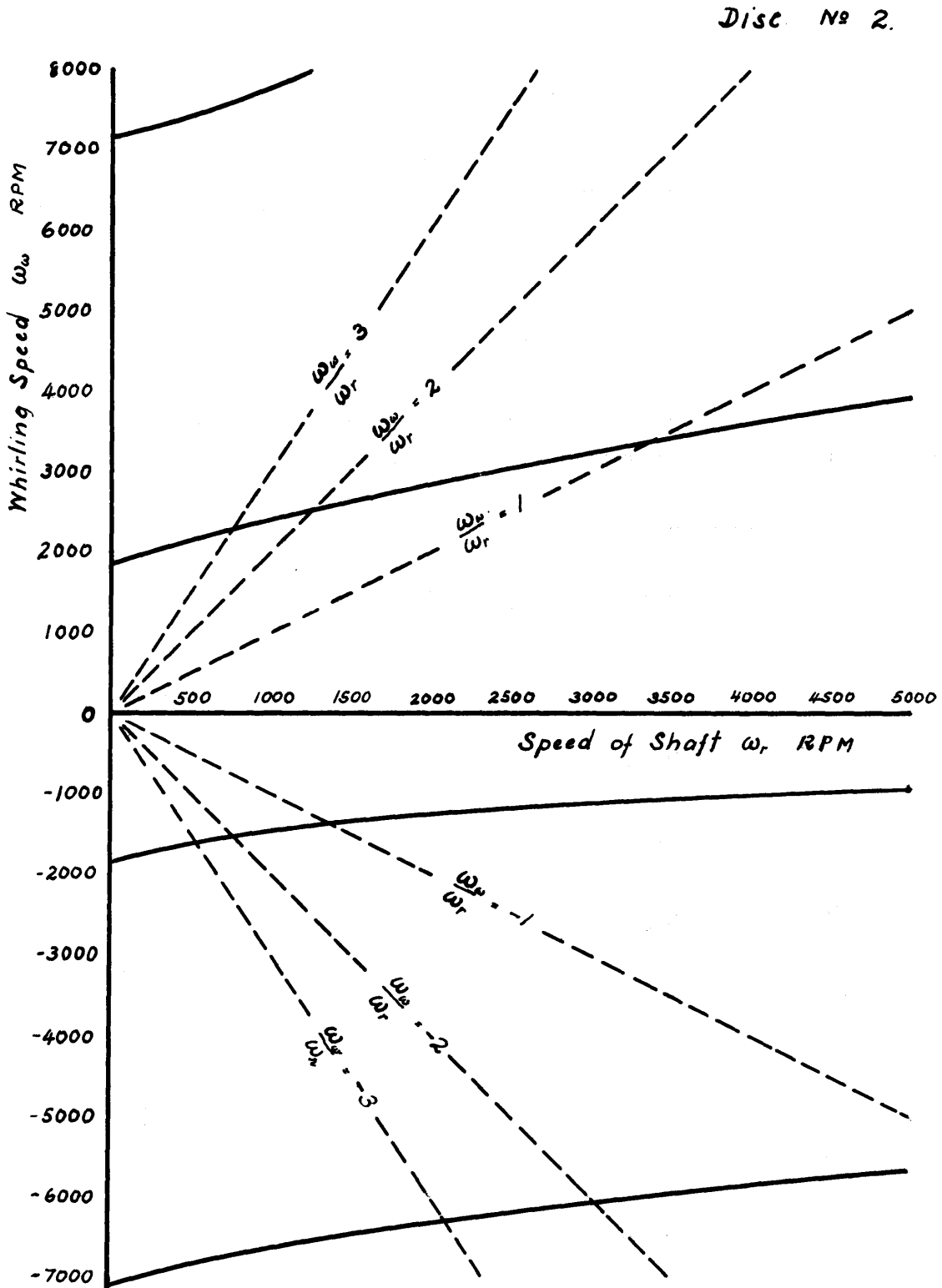
$$\omega_\omega^4 - 2 \omega_\lambda \omega_\omega^3 - 138,000 \omega_\omega^2 + 276,000 \omega_\lambda \omega_\omega + 112 \times 10^8 = 0$$

and is plotted in Fig. 9 page 21.



Critical Whirling Speeds of a Rotating Disk on an Overhung Rotor

FIG 8.



Critical Whirling Speeds of a Rotating Disk on an Overhung Shaft.

FIG. 9

D A T A.

CRITICAL SPEED OBSERVATION

With disk No. 1.

Type of Whirl	Calculated Critical speed rpm.	Observed Speed	Observed path	Amplitude mm.
4	480 & 1740	----	--	--
3	650 & 2630	----	--	--
2	1100	1020	circular	.24
1	3200	----	--	--
- 1	1000 & 4750	1050	Y	.30*
- 2	580 & 2510	610	circular	.24
- 3	420 & 1710	----	--	--
- 4	310 & 1300	----	--	--

Note: shaft No. 2. with average stiffness of 7.8×10 inch/lb.

* by height

With disk No. 2.

Type of Whirl	Calculated Critical speed rpm.	Observed Speed	Observed path	Amplitude mm.
4	540 & 2243	----	--	--
3	750 & 3484	770	⊗	.27
2	1253	1250	circular	.25
1	3400	----	--	--
- 1	1400 & 5600	1412	see data page 24-29.	
- 2	792 & 3060	790	circular	.18
- 3	533 & 2100	----	--	--
- 4	420 & 1612	----	--	--









Note: shaft No. 2. with average stiffness of 7.8×10 inch/lb.














THE EFFECT OF LOCATION OF ECCENTRICITY OF MASS CENTER WITH
RESPECT TO THE SHAFT FOR MINUS ONE WHIRL














θ = angle measured from a certain reference point marked
on the shaft.














(the region of minus one whirl is between 1395 - 1425 rpm.)














θ = 0 degree

Speed of Shaft rpm.	Path of Whirl	Angle from vertical line degrees	Maximum amplitude mm.	Ratio max/min (width)
1395		70	.20	∞
1400		45	.25	3
1405		35	.30	3
1410		25	.30	4
1415		15	.30	5
1420-5		0	.18	∞
• 15 degrees				
1395		70	.25	∞
1400		65	.25	3

Speed of shaft rpm.	Path of Whirl	Angle from vertical line degrees	Maximum amplitude mm.	Ratio max/min (width)
1405		35	.28	3
1410		20	.35	3
1415		5	.30	6
1420-5		0	.30	∞
$\theta = 30$ degrees				
1395		70	.35	∞
1400		60	.35	6
1405		45	.40	3
1410		30	.42	2.5
1415		15	.40	3
1420-5		5	.30	∞
$\theta = 45$ degrees				
1395		80	.30	∞
1400		70	.45	6
1405		45	.50	4

Speed of Shaft rpm.	Path of Whirl	Angle from vertical line degrees	Maximum amplitude mm.	Ratio max/min (width)
1410		20	.45	5
1415		10	.40	5
1420-5		0	.35	∞
$\theta = 60$ degrees				
1395		70	.30	∞
1400		60	.30	6
1405		30	.40	3
1410		20	.45	4
1415		10	.46	5
1420-5		5	.35	∞
$\theta = 75$ degrees				
1395		80	.30	∞
1400		60	.35	6
1405		30	.40	3
1410		20	.40	3

Speed of shaft rpm.	Path of Whirl	Angle from vertical line degrees	Maximum amplitude mm.	Ratio max/min (width)
1415		10	.35	3
1420-5		0	.31	∞
$\theta = 90$ degrees				
1395		80	.30	∞
1400		60	.35	6
1405		30	.40	3
1410		25	.43	3
1415		20	.44	4
1420-5		10	.35	∞
$\theta = 105$ degrees				
1395		80	.35	∞
1400		70	.38	5
1405		60	.40	4
1410		45	.45	4
1415		30	.47	3

Speed of Shaft RPM.	Path of Whirl	Angle from vertical line degrees	Maximum amplitude mm.	Ratio max/min (width)
1420-5		10	.39	∞
$\theta = 120$ degrees				
1395		75	.35	∞
1400		60	.40	5
1405		50	.40	4
1410		30	.45	4
1415		15	.48	5
1420-5		5	.39	∞
$\theta = 135$ degrees				
1395		75	.35	∞
1400		50	.40	5
1405		30	.43	4
1410		20	.45	3.5
1415		10	.42	4
1420-5		0	.36	∞

AMPLITUDES AT MAXIMUM AND MINIMUM LOCATIONS ABOUT THE SHAFT

	Angle from vertical line degrees	Maximum amplitude mm.	Ratio of axes of ellipse	Speed of shaft rpm.
45	45	.50	4	1405
135	20	.45	3.5	1410
225	30	.28	3	1410
315	35	.30	3	1410

RESONANT SPEEDS FOR FRICTIONAL EXCITATION

Speed of shaft rpm.	Disk No.	Maximum amplitude mm.	Speed of whirl rpm.
700	1	.05	--
910	1	.07	-1060
1130	1	.07	-960
1400	2	.10	--

EQUIPMENT

Motor - generator set	Westinghouse Electric Mfg. Co. direct current 3.75 kw 250 v. 15 amp. 1675 rpm.
Driving motor	General Electric Co. direct current variable speed motor 5 h.p. at 1800 rpm. 3.5 h.p. at 550 rpm. with 2000 ohm speed control rheostat.
Cathetometer (sliding vernier microscope)	Gaertner Scientific Corp. .001 mm accuracy or .0004 inch accuracy
Strobotac	General Radio Co. range 500 to 11,000 rpm.

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