Review of Antenna theory

Dr. Sandra Cruz-Pol
Electrical and Computer Engineering
University of Puerto Rico at Mayaguez

What is an antenna?

- An antenna is a passive structure that serves as transition between a transmission line and air used to transmit and/or receive electromagnetic waves.

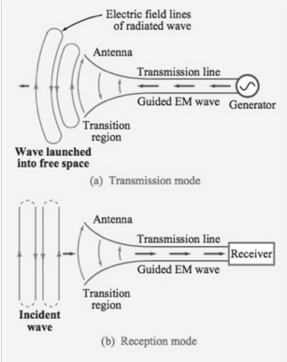


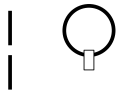
Figure 3-1: Antenna as a transducer between a guided electromagnetic wave and a free-space wave, for both transmission and reception.

Antenna

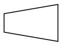
Ulaby, 1999

Types of antennas

- Can be divided into two groups
  - **Wire** antennas:
    - dipoles, loops, Yagi-Uda…



  - **Aperture** antennas:
    - parabolic, horns, microstrip antennas…



http://www.kyes.com/antenna/antennatypes/antennatypes.html
http://en.wikipedia.org/wiki/Antenna_(electronics)#Overview

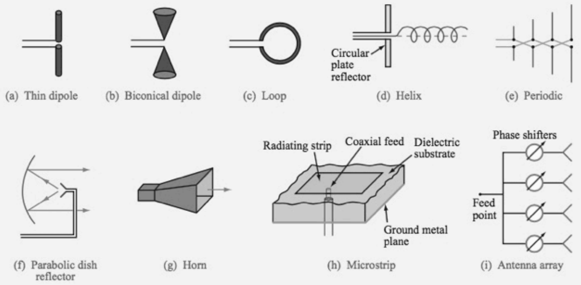
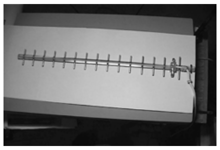


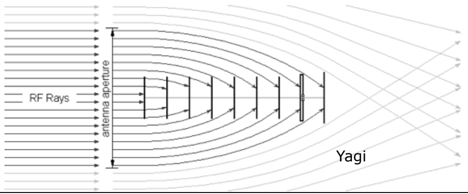
Figure 3-2: Various types of antennas.

Wire antennas

Log periodic

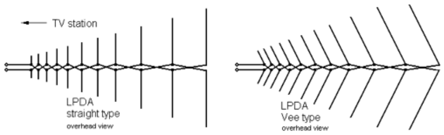


Yagi



Yagi

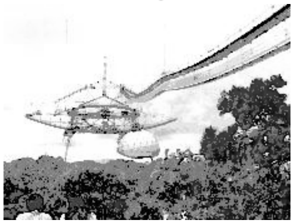
Wire antennas



Log periodic

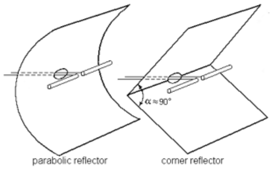
Yagi-Uda with reflector

Aperture antennas

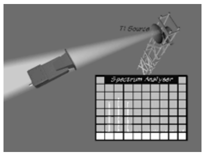


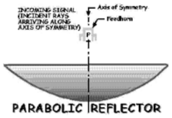
Spherical (main reflector) with Gregorian feed

Dipole with parabolic and corner reflector



Reflector and Pyramidal horn antennas





Far Field

Far field

- The distance at which the fields transmitted by an antenna (spherical) can be approximated to plane waves.
- It's defined as

$$\boxed{r_{ff} = 2D^2/\lambda}$$

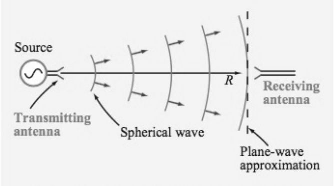


Figure 3-3: Far-field plane-wave approximation.

D = is the largest physical dimension of the antenna
λ = wavelength of operation
r_{ff} = distance from the antenna to the observation point

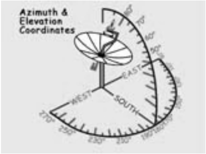
Related parameters

- Solid angle, Ω_A and Radiation intensity, U
- Radiation pattern, P_n, sidelobes, HPBW
- Far field zone, r_{ff}
- Directivity, D or Gain, G
- Antenna radiation impedance, R_{rad}
- Effective Area, A_e

All of these parameters are expressed in terms of a **transmission** antenna, but are identically applicable to a **receiving** antenna. We'll also study:

- Friis Transmission Equation
- Radar Equation

Spherical coordinates



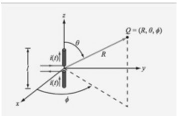
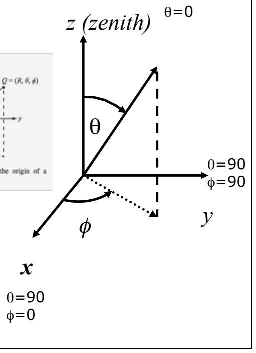


Figure 3-4: Short dipole placed at the origin of a spherical coordinate system.



ϕ= azimuth

θ= elevation

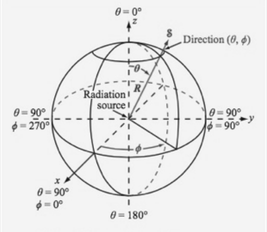


Figure 3-5: Spherical coordinate system.

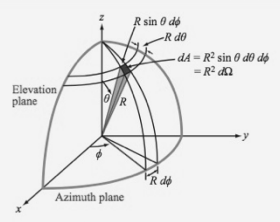
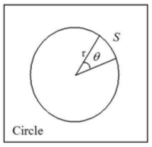
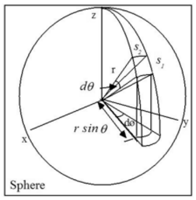


Figure 3-7: Definition of solid angle $d\Omega = \sin\theta \, d\theta \, d\phi$.

Solid Angle





$s = \theta r$ = arco

$s_1 = r\,d\theta \quad s_2 = r \sin\theta\, d\phi$

$dA = s_1 s_2$

$dA = r^2 \quad \theta\, d\phi\, d\theta$

$r^2\, d\Omega$

θ = ángulo plano

$d\Omega$

- *El arco total en un círculo:* $= 2\pi r$
- *Ángulo total:* $= 2\pi$ [radianes]
- *El área total en una esfera:* $= 4\pi r^2$
- *Ángulo sólido total:* $=4\pi$ [rad^2] $=4\pi$ [sr]

1 steradian (sr) = (1 radian)2

Radiation Intensity

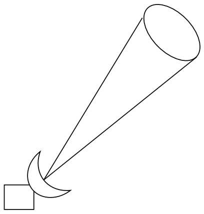
- **Is the power density per solid angle:**

$$U = r^2 S_r \qquad [\mathrm{W/sr}]$$

where

$$S_r = \frac{1}{2}\mathrm{Re}\{E \times H^*\}\hat{r} \quad [\mathrm{W/m^2}]$$

is the power density also known as Poynting vector.



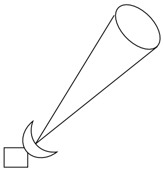
Total radiated power by antenna

- **Can be calculated as;**

$$P_{rad} = \int U \cdot d\Omega \ [\mathrm{W}]$$

or

$$P_{rad} = \int S_r \cdot dA \quad [\mathrm{W}]$$



Radiation Pattern

- **Radiation pattern is the 3D plot of the gain, but usually the two dimensional horizontal and vertical cross sections of the radiation pattern are considered.**
- **Refers to the variation of the relative amplitude of the radiation as a function of direction.**

Field pattern:

$$E_n(\theta,\phi) = \frac{E(\theta,\phi)}{E_{\max}(\theta,\phi)}$$

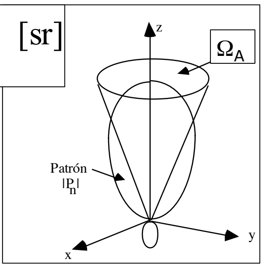
Power pattern:

$$F_n(\theta,\phi) = \frac{S(\theta,\phi)}{S_{\max}(\theta,\phi)} = \frac{U(\theta,\phi)}{U_{\max}(\theta,\phi)}$$

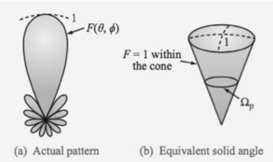
Where U is the radiation intensity to be defined later.

Total Solid Angle of an antenna

$$\Omega_A = \iint_{4\pi} F_n(\theta,\phi)\, d\Omega \quad [\mathrm{sr}]$$



Is as if you changed the **radiation pattern beam** of an antenna into a **pencil beam shape** and find out what's the equivalent solid angle occupied by this pattern.



(a) Actual pattern (b) Equivalent solid angle

Figure 3-10: The pattern solid angle Ω_p defines an equivalent cone over which all the radiation of the actual antenna is concentrated with uniform intensity equal to the maximum of the actual pattern.

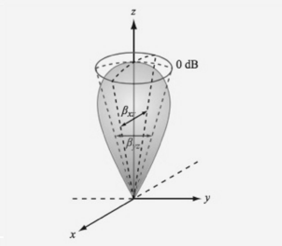


Figure 3-11: The solid angle of a unidirectional radiation pattern is approximately equal to the product of the half-power beamwidths in the two principal planes; that is, $\Omega_p \approx \beta_{xz}\beta_{yz}$.

Isotropic antenna

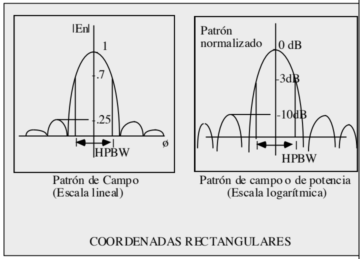
- It's an hypothetic antenna, i.e., it does not exist in real life, yet it's used as a measuring bar for real antenna characteristics.
- It's a point source that occupies a negligible space. Has no directional preference.
- Its pattern is simply a sphere so it has $\Omega_A = \Omega_{isotropic} = 4\pi$ [steradians].

$$\Omega_{\mathrm{isotropic}} = \iint_{4\pi} (1)\, d\Omega$$

$$\int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} (1) \sin\theta \, d\theta \, d\phi = 4\pi$$

Radiation Pattern

- Whenever we speak of radiation patterns, we normally mean we are at a distance far enough from the antenna known as the **far field**.



Note that when plotted in decibels, the power and field patterns look exactly the same.

Radiation Pattern of

Short dipole antenna

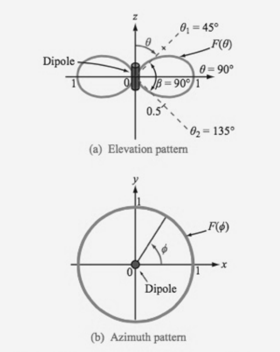
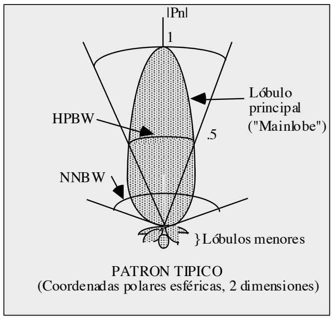
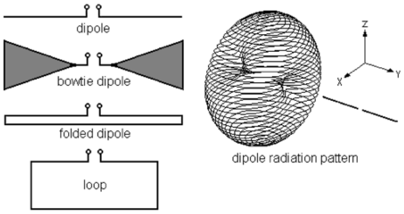


Figure 3-6: Radiation patterns of a short dipole.

Pattern – polar plot



Dipole antenna pattern



Note the radiation pattern is
donut shaped.

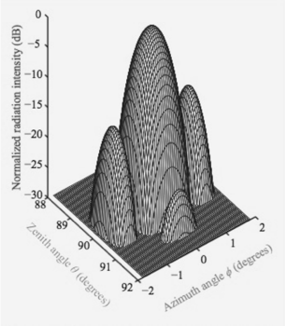
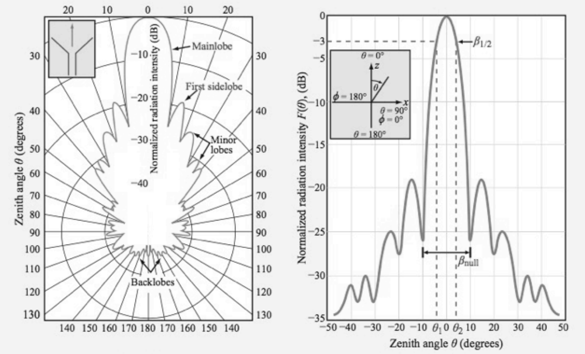


Figure 3-8: Three-dimensional pattern of a narrow-beam antenna.



(a) Polar diagram

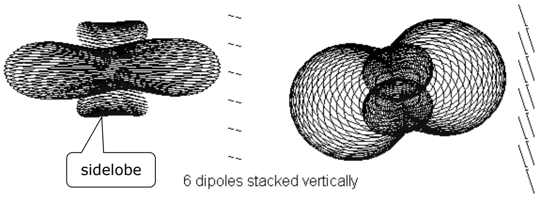
(b) Rectangular plot

Figure 3-9: Representative plots of the normalized radiation pattern of a microwave antenna in (a) polar form and (b) rectangular form.

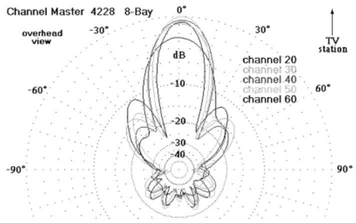
Sidelobes

- Antennas sometimes show **side lobes** in the radiation pattern.
- Side lobes are peaks in gain other than the main lobe (the "beam").
- Side lobes have bad impact to the antenna quality whenever the system is being used to determine the direction of a signal, for example in RADAR systems.

Sidelobes of dipole arrays



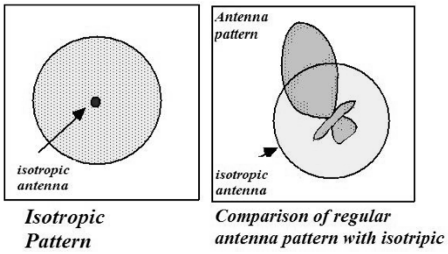
Antenna Pattern with sidelobes



Many applications require sidelobe levels
(SLL) to be below -20dB.

OTG @ Cornelia

Gain or Directivity



An isotropic antenna and a practical antenna fed with the same power. Their patters would compare as in the figure on the right.

Directivity and Gain

- All practical antennas radiate more than the isotropic antenna in some directions and less in others.
- Gain is inherently directional; the gain of an antenna is **usually measured in the direction which it radiates best**.

$$D = D_{\max}(\theta,\varphi) = \mathsf{S}_{\max} / \mathsf{S}_{ave} = U_{\max} / U_{ave}$$

If lossless antenna, $G=D$

Gain or Directivity

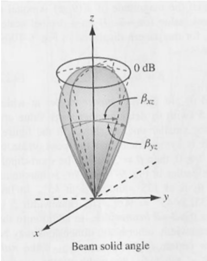
- Gain is measured by comparing an antenna to a model antenna, typically the <u>isotropic antenna</u> which radiates equally in all directions.

$$D(\theta,\phi) = \frac{4\pi r^2}{\iint_{4\pi} F_n(\theta,\phi)\, d\Omega} = \frac{4\pi}{\Omega_p}$$

$$D_o = \Omega_{\text{isotropic}}/\Omega_{\text{A}}$$

Directivity

- For an antenna with a single main lobe pointing in the z-direction , Ω_A can be approximated to the product of the HPBW



$$\Omega_A \cong \beta_{xz}\beta_{yz}$$

then

The Directivity:

$$D = 4\pi/\Omega_{\text{A}} \cong \frac{4\pi}{\beta_{xz}\beta_{yz}} = \frac{4\pi}{\lambda^2} A_e$$

Beamwidth, HPBW

- Is the “distance” in radians o degrees between the direction of the radiation pattern where the radiated power is half of the maximum.

$$10 \;\log\; 0.5 \;=\; \text{-}3\; dB$$

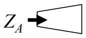
$$20 \;\log\; 0.707 \;=\; \text{-}3\; dB$$

for "pencil beam" shape;

$$HPBM \approx 70^o \frac{\lambda}{D}$$

Antenna Impedance

- An antenna is “seen" by the generator as a load with impedance Z_A, connected to the line.



$$Z_A = (R_{rad} + R_L) + jX_A$$

- **The real part is the radiation resistance plus the ohmic resistance**.
  - Minimizing impedance differences at each interface will reduce SWR and maximize power transfer through each part of the antenna system.
  - Complex impedance, Z_A, of an antenna is related to the electrical length of the antenna at the wavelength in use.
    - The impedance of an antenna can be matched to the feed line and radio by adjusting the impedance of the feed line, using the feed line as an impedance transformer.
    - More commonly, the impedance is adjusted at the load (see below) with an antenna tuner, a balun, a matching transformer, matching networks composed of inductors and capacitors, or matching sections such as the gamma match.

Antenna efficiency, η

- **Efficiency** is the ratio of power put into the antenna terminals to the power actually radiated
- Radiation in an antenna is caused by radiation resistance which can only be measured as part of total resistance including loss resistance.

$$P_{rad} = \eta P_{in}$$

$$G = \eta D$$

Radiation Resistance

- The antenna is connected to a T.L., and it “sees” it as an impedance.
- The power radiated is $P_{rad} = \frac{1}{2} I_o^2 R_{rad}$
- The loss power is $P_{loss} = \frac{1}{2} I_o^2 R_L$

$$\eta = \frac{P_{rad}}{P_{rad} + P_{loss}} = \frac{R_{rad}}{R_{rad} + R_{loss}}$$

Antenna polarization

- The **polarization** of an antenna is the polarization of the signals it emits.
  - The ionosphere changes the polarization of signals unpredictably, so for signals which will be reflected by the ionosphere, polarization is not crucial.
  - However, for line-of-sight communications, it can make a tremendous difference in signal quality to have the transmitter and receiver using the same polarization.
  - Polarizations commonly considered are *vertical*, *horizontal*, and *circular*.

Antenna Bandwidth

- The **bandwidth** of an antenna is the range of frequencies over which it is effective, usually centered around the operating or resonant frequency.
  - The bandwidth of an antenna may be increased by several techniques, including using thicker wires, replacing wires with *cages* to simulate a thicker wire, tapering antenna components (like in a feed horn), and combining multiple antennas into a single assembly and allowing the natural impedance to select the correct antenna.

Effective Area

- How a Rx antenna extracts energy from incident wave and delivers it to a load?

$$A_e = \frac{P_{rec}}{S_{inc}} = \frac{\lambda^2 D}{4\pi}$$

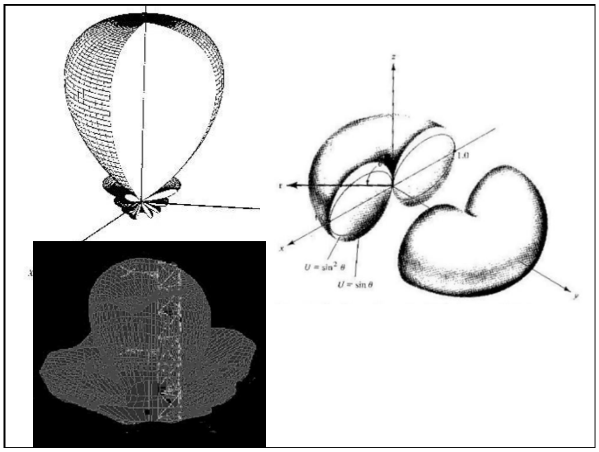
Above is valid for any antenna under matched-load conditions

Example

- **Determine the direction of maximum radiation , pattern solid angle, directivity and HPBW in the *y-z* plane for an antenna with normalized radiation intensity given by**

$$F(\theta,\phi)=\begin{cases}\cos^2\theta & \text{for } 0\le\theta\le\dfrac{\pi}{2} \quad \text{and } 0\le\phi\le 2\pi\\ 0 & \text{elsewhere}\end{cases}$$

$$\boxed{Answers\,:(0,0),\frac{2\pi}{3},6,90^o}$$



Communication set up

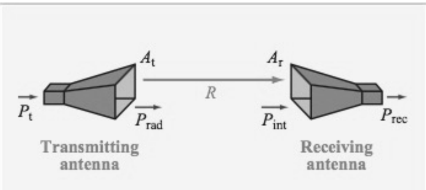
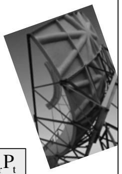


Figure 3-13: Transmitter–receiver configuration.

Friis Transmission Eq.

- **In any communication link, there is a transmitting antenna and a receiver with a receiver antenna.**



TX

$$S_{isotr}=\frac{P_t}{4\pi R^2}$$

$$P_t=G_tS_{isotr}=\frac{G_tP_t}{4\pi R^2}=\frac{A_tP_t}{\lambda^2R^2}$$

$$P_{rec}=A_rS_t=\frac{A_tA_rP_t}{\lambda^2R^2}$$

RX

$$P_{rec}=\frac{G_tG_rP_t\lambda^2}{(4\pi R)^2}$$

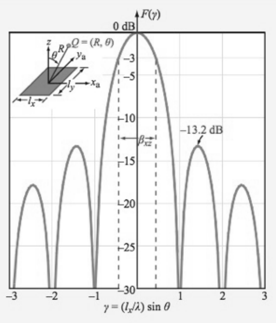
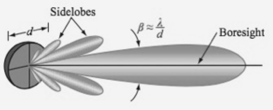
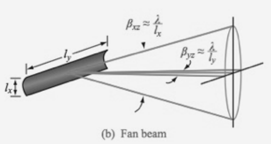


Figure 3-17: Normalized radiation pattern of a uniformly illuminated rectangular aperture in the *x–z* plane (*φ* = 0).



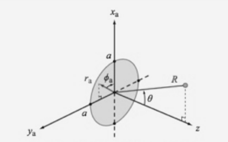
(a) Pencil beam



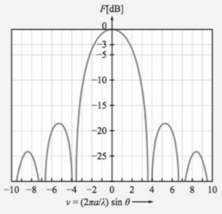
(b) Fan beam

Figure 3-18: Radiation patterns of (a) a circular reflector and (b) a cylindrical reflector (sidelobes not shown).

Circular aperture



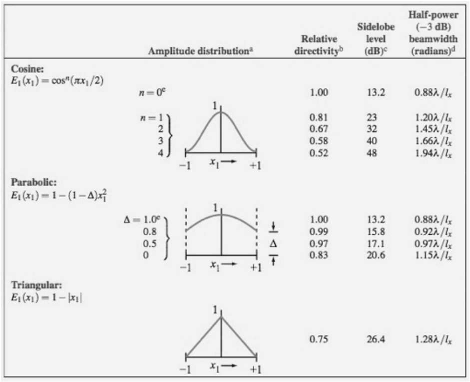
(a) Circular aperture in the x_a-y_a plane



(b) Radiation pattern

Figure 3-19: Circular aperture and its radiation pattern.

Radiation Pattern for several aperture distributions



	Amplitude distribution[a]	Relative directivity[b]	Sidelobe level (dB)[c]	Half-power (−3 dB) beamwidth (radians)[d]
Cosine: $E_1(x_1) = \cos^n(\pi x_1/2)$				
	$n = 0^e$	1.00	13.2	$0.88\lambda/l_x$
	$n = 1$	0.81	23	$1.20\lambda/l_x$
	2	0.67	32	$1.45\lambda/l_x$
	3	0.58	40	$1.66\lambda/l_x$
	4	0.52	48	$1.94\lambda/l_x$
Parabolic: $E_1(x_1) = 1-(1-\Delta)x_1^2$				
	$\Delta = 1.0^e$	1.00	13.2	$0.88\lambda/l_x$
	0.8	0.99	15.8	$0.92\lambda/l_x$
	0.5	0.97	17.1	$0.97\lambda/l_x$
	0	0.83	20.6	$1.15\lambda/l_x$
Triangular: $E_1(x_1) = 1-\|x_1\|$		0.75	26.4	$1.28\lambda/l_x$

Table 3-2: Radiation patterns produced by the amplitude distribution function $E(r_1) = (1-r_1^2)^n$ over a circular aperture of diameter d.

n	Relative directivity[a]	Sidelobe level (dB)[b]	Half-power (−3 dB) beamwidth (radians)
0 (uniform)	1.00	17.6	$1.02\lambda/d$
1	0.75	24.6	$1.27\lambda/d$
2	0.55	30.6	$1.47\lambda/d$
3	0.45	36.1	$1.65\lambda/d$

The variable $r_1 = (2/d)r_a$ and $0 \le r_1 \le 1$. [a]Relative to a uniform aperture distribution. [b]Below maximum intensity.

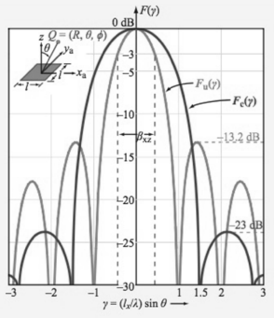


Figure 3-20: Comparison of the antenna pattern $F_u(\gamma)$ of a uniformly illuminated aperture with $F_c(\gamma)$ for a cosine-tapered illuminated aperture.

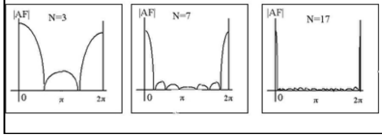
Antenna Arrays

- Uses many antennas synchronized with each other to increase
- Pattern multiplication, AF

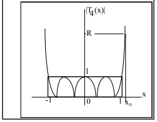
$$E(r) = \begin{bmatrix}\text{Patrón}\\ \text{antena}\\ \text{individual}\end{bmatrix}\begin{bmatrix}\text{Factor}\\ \text{del}\\ \text{Arreglo}\end{bmatrix}$$

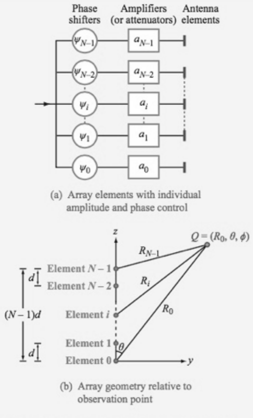
$$AF_N = \left[\frac{\sin\frac{N\psi}{2}}{N\sin\frac{\psi}{2}}\right]$$

Uniform illumination



Tschebyscheff Illumination





(a) Array elements with individual amplitude and phase control

(b) Array geometry relative to observation point

Figure 3-21: Linear-array configuration and geometry.

Linear Arrays

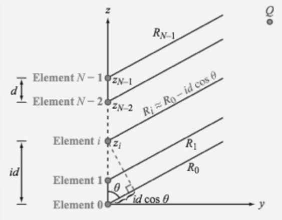
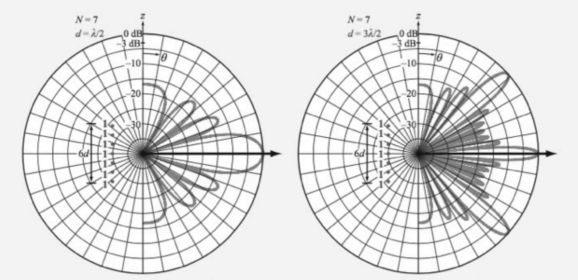


Figure 3-22: The rays between the elements and a far-away observation point are approximately parallel lines. Hence, the distance $R_i \approx R_0 - id\cos\theta$.

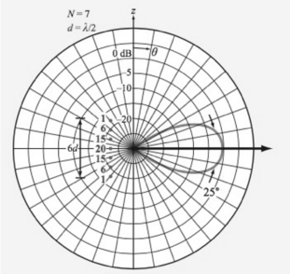
Uniform Distribution



(a) Uniform distribution, $d = \lambda/2$

(b) Uniform distribution, $d = 3\lambda/2$

Binomial Distribution



(c) Binomial distribution

Phase (Electronic) Scanning

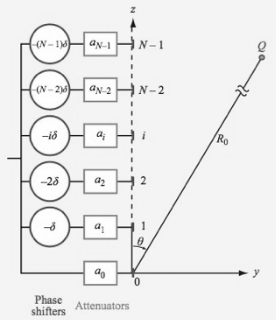


Figure 3-24: The application of linear phase.

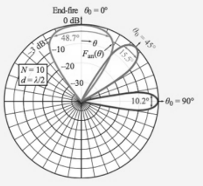
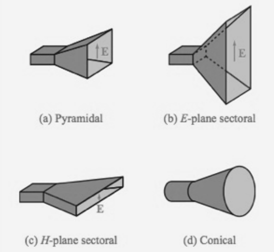


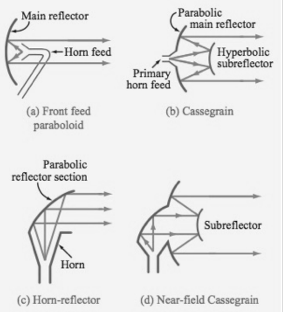
Figure 3-25: Normalized array pattern of a 10-element array with $\lambda/2$ spacing between adjacent elements. All elements are excited with equal amplitude. Through the application of linear phase across the array, the main beam can be steered from the broadside direction ($\theta_0 = 90°$) to any scan angle θ_0. Equiphase excitation corresponds to $\theta_0 = 90°$.

Horn Antennas



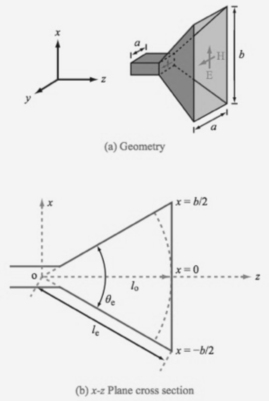
(a) Pyramidal
(b) *E*-plane sectoral
(c) *H*-plane sectoral
(d) Conical

Figure 3-26: Commonly used types of horn antennas.



(a) Front feed paraboloid
(b) Cassegrain
(c) Horn-reflector
(d) Near-field Cassegrain

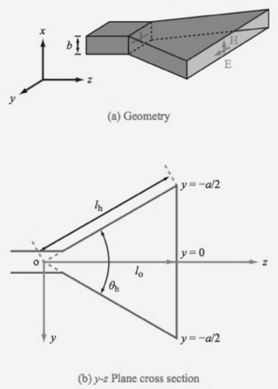
Figure 3-27: Horn-reflector antennas.



(a) Geometry

(b) *x*-*z* Plane cross section

Figure 3-28: *E*-plane sectoral horn geometry and coordinates.



(a) Geometry

(b) *y*-*z* Plane cross section

Figure 3-29: *H*-plane sectoral horn geometry and coordinates.

Horn Antennas

E-sectorial, H-sectorial

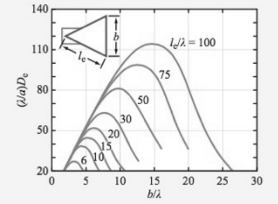


Figure 3-30: Directivity of *E*-plane sectoral horns with aperture height *b* and width *a* (based on Fig. 16.4 of Schelkunoff and Friis, 1952).

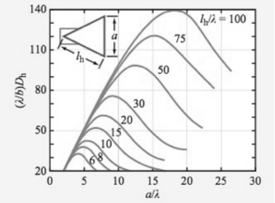


Figure 3-31: Directivity of *H*-plane horns of width *a* and height *b* (based on Fig. 16.3 of Schelkunoff and Friis, 1952).

Optimum Conical Horn

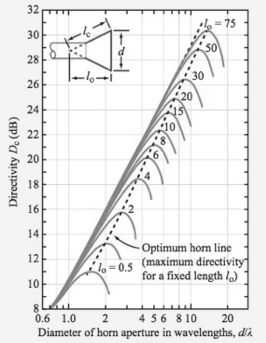
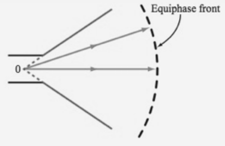


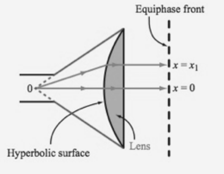
Figure 3-32: The directivity of a conical horn as a function of axial length and aperture diameter.

Horn Antenna

with hyperbolic lens to transform to uniform phase front



(a) Empty sectoral horn



(b) Lens-corrected sectoral horn

Figure 3-33: A hyperbolic lens of the appropriate index of refraction can transform the phase front to a uniform distribution across the aperture.

Radiation thru slots

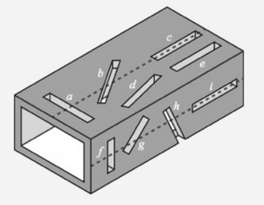


Figure 3-34: Various types of slots cut in the walls of a rectangular waveguide. Slots c and f do not radiate, because they do not interrupt the flow of surface current (see Fig. 3-35).

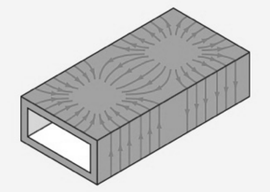
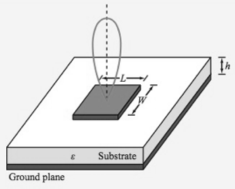
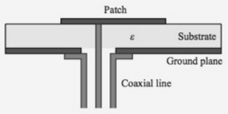


Figure 3-35: The flow of surface currents in the walls of a rectangular waveguide excited with a TE_{10} mode.

Antennas



(a) Microstrip antenna



(b) Side view with coaxial feed line

Figure 3-39: (a) Top view of a rectangular patch antenna and (b) side view showing how it connects to a coaxial transmission line.

Patch Antennas

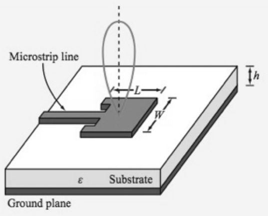


Figure 3-40: Microstrip patch antenna fed by a microstrip transmission line.

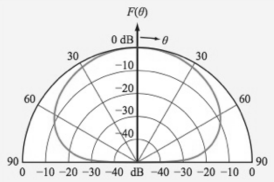


Figure 3-41: Far-field patterns of a rectangular patch on an infinite ground plane (Jackson, 2007).

Patch input Impedance

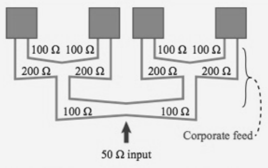


Figure 3-42: Tapered lines to match 100-Ω patches to a 50-Ω line (R. E. Munson, 1974).

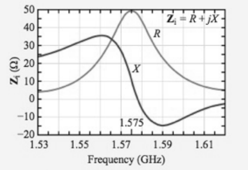
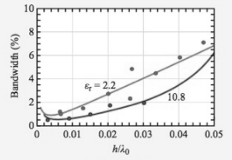


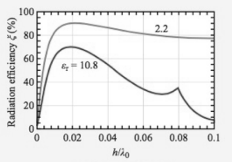
Figure 3-43: Input impedance $\mathbf{Z}_i$ of a microstrip antenna with $L = 6.255$ cm, $W = 9.383$ cm, $h = 0.1524$ cm, and $\varepsilon_r = 2.2$ (Jackson, 2007).

Patch antennas

Bandwidth and Radiation Efficiency



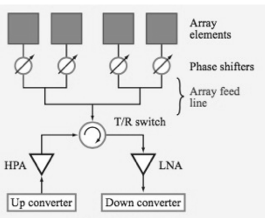
(a) Bandwidth



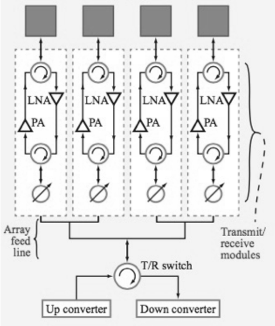
(b) Radiation efficiency

Figure 3-44: (a) Bandwidth and radiation efficiency versus the normalized substrate thickness for a moderate-permittivity substrate and a high-permittivity substrate (Jackson, 2007).

Conventional & Active Array



(a) Conventional array



(b) Active array

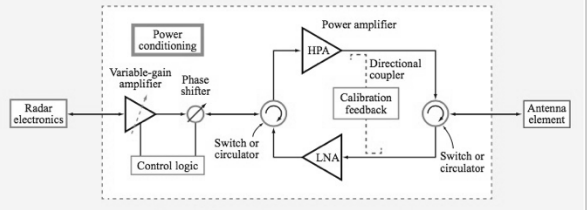
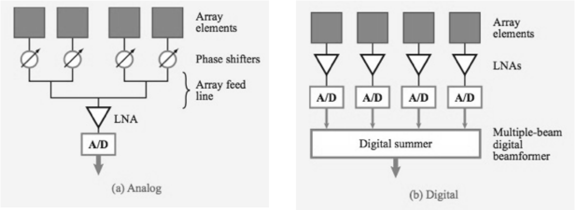
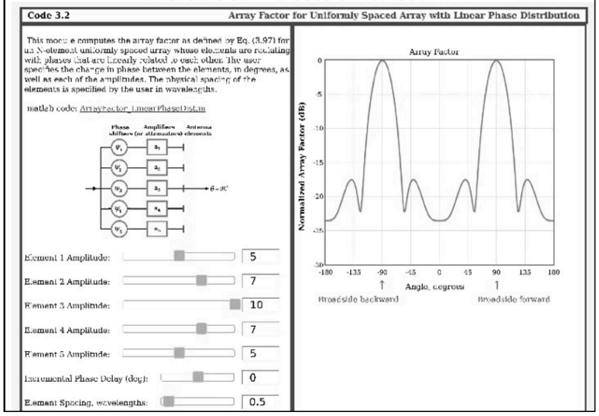


Figure 3-46: Design of an active transmit/receive module.

Analog & Digital Beamforming



Available at mrs.eecs.umich.edu



Code 3.2 **Array Factor for Uniformly Spaced Array with Linear Phase Distribution**

This module computes the array factor as defined by Eq. (3.97) for an N-element uniformly spaced array whose elements are radiating with phases that are linearly related to each other. The user specifies the change in phase between the elements, in degrees, as well as each of the amplitudes. The physical spacing of the elements is specified by the user in wavelengths.

matlab code: ArrayFactor_LinearPhaseDist.m

Element 1 Amplitude: 5

Element 2 Amplitude: 7

Element 3 Amplitude: 10

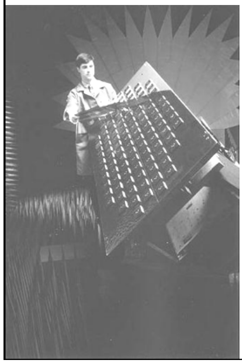
Element 4 Amplitude: 7

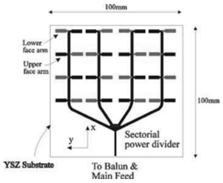
Element 5 Amplitude: 5

Incremental Phase Delay (deg): 0

Element Spacing, wavelengths: 0.5

Planar Arrays





$$AF_n(\theta,\phi) = \left[\frac{1}{M}\frac{\sin\left(\frac{M}{2}\psi_x\right)}{\sin\left(\frac{\psi_x}{2}\right)}\right]\left[\frac{1}{N}\frac{\sin\left(\frac{N}{2}\psi_y\right)}{\sin\left(\frac{\psi_y}{2}\right)}\right]$$

Assigned Problems
ch. 3 Antennas

- 1-4, 7-9, 11-13, 15-22, 28-30, 32-34, 36-42