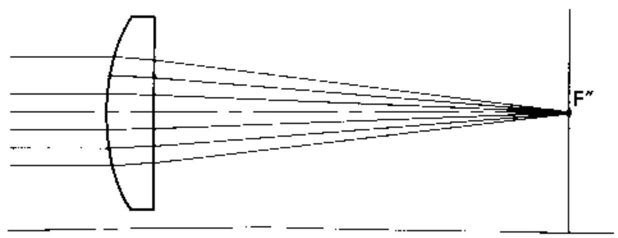
OPTICAL ENGINEERING NOTE #9
APPLICATIONS OF CONJUGATE RATIOS

Solving the Simple Lens Formula for a variety of object distances yields the corresponding image distances. The combination of object and image distances are called **conjugate pairs**, as the two distances are work together.

The **conjugate ratio**, image distance divided by object distance, is also known as the **magnification**. Unit magnification occurs when the image distance = object distance, (at a symmetric twice focal length distance as in **Figure 2**), and larger than life when the object is between 2f and 1f, **Figure 3**. But the typical picture-taking situation, **Figure 1**, shows that the image distance < object distance, resulting in a **fractional magnification** or **minification**.

OBJECT AT INFINITY CONJUGATE RATIO, Figure 0*: This graphic also illustrates the concept of focal length. Rays from a point on an object, so far away that there is essentially no angle between the top and bottom rays entering the lens, will be reunited after travelling all that way at a distance one focal length away from the lens.



Plugging in infinity as the object distance in the magnification equation means we end up dividing by it, and the mathematical consequences means we have zero magnification**. Yet projecting an image of the sun with a magnifying glass shows that there is some dimension, with longer focal length lenses providing larger images.

To find the magnification at this particular conjugate ratio the angular magnification must be computed, as shown in the reprint of the OEReports article in the following Handout, **MAGNIFICATION**.

The most obvious optical device that uses this infinite conjugate ratio is the telescope objective. Angular magnificaton of these rays is accomplished by the telescope's eyepiece, which is placed its focal length away from the image delivered by the objective,

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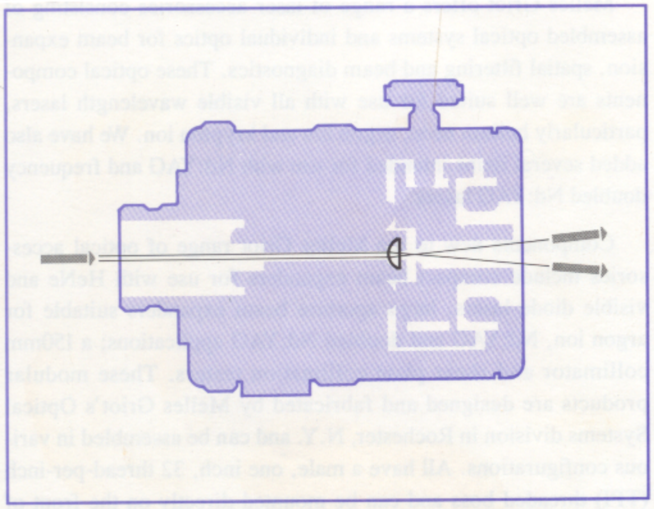
*. The source book for **Figures 1 -5**, Vitalized Physics, Robert H. Carleton, College Entrance Book Company, 1960 were already numbered, so the prelude was assigned 0.

**. "Any number divided by infinity = 0."

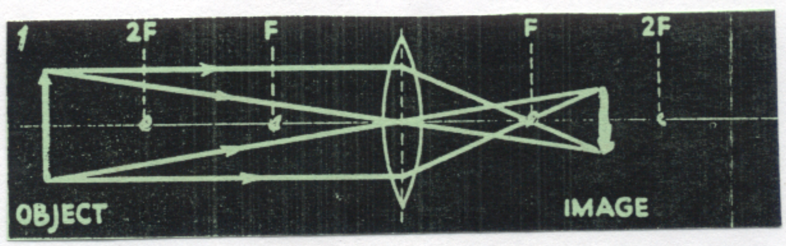
an example of a **Figure 4** application.

Laser beams are usually considered to be composed of parallel rays, or collimated, which makes them look like they are coming from infinity. A lens to focus the beam to a small spot for drilling, welding, soldering, etc., is used in this conjugation configuration.



SPATIAL FILTERING produces a beam with smoother irradiance distribution.

If there is no target for the laser beam to converge onto at the focus, the light will continue on its way, but diverging from that point, expanding so that the beam will illuminate a larger area. Sometimes a pinhole is placed at the focal point in a device called a <u>spatial filter</u> which "cleans up" diverging laser beams by blocking stray light diffracting from the surface of the optics in the path of the laser beam.



FINITE CONJUGATE RATIOS, Figure 1: This is the typical picture taking zone. The object is greater than 2f away and the image is formed between 1f and 2f behind the lens. Object distance > image distance, so the magnification < 1, or minified.

At the nearest focus of the typical 50mm normal lens on a 35mm camera, usually 60cm,

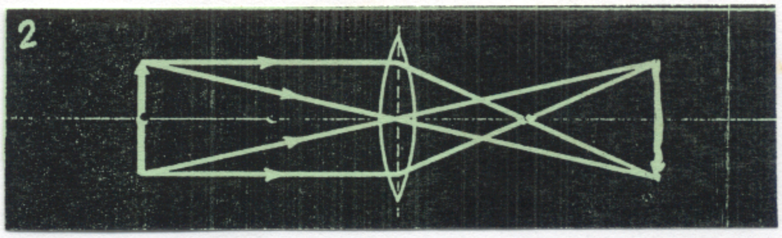
$$m = \frac{f}{Obj - f}$$

or 50cm/600cm - 50cm = 1/11. Objects at this particular conjugate distance will be rendered 1/11th their normal size in the image. This demagnification will allow a person's head and shoulders to fully fill the film format.

Imaging optics for photo-, cinemato-, and video- graphic purposes are designed to minimize aberrations for conjugate ratios from infinity on down to about 1/10X.

A beautiful demonstration of the **Simple Lens Formula** can be performed with the diverging cone of light generated by passing a laser beam through a lens or spatial filter as mentioned above, and moving the point source nearer and further from the lens and watching the image spot react.

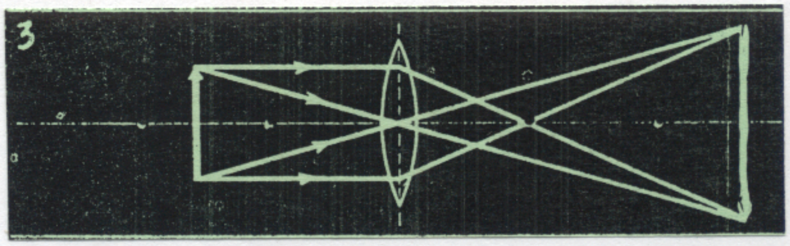
UNIT MAGNIFICATION, Figure 2: Object and Image positions are symmetric about the lens; the most common application is in Xerox machines, to make a copy that is the same size as the orignal. The object and image are both two focal lengths away from the principal plane of the lens.



This fact provides a simple trick for finding the focal length of a compound lens. Image a ruler onto another ruler, so that the size of the image of the first ruler is coincident with the markings on the second in the image plane. When this is done, measure the distance from one ruler to the other, divide by 4, and the quotient is the focal length. Half of the ruler-to-ruler distance locates the nodal point of the lens.

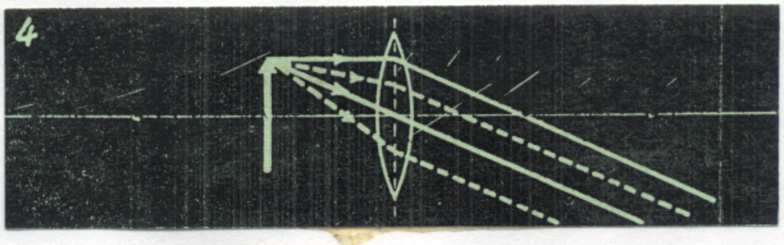
The definition of f/# is focal length divided by the size of the aperture. But this is true only for the infinite conjugate ratio. The light-gathering power of a lens is greatly diminished at this 1:1 ratio or other ratios near it, as the true f/# is really the focal distance divided by the diameter of the aperture. At unit magnification the focal distance is twice that of the infinite one, so the f/# is doubled, meaning that the intensity at the film plane is one-fourth of the light expected at the marked f/#. (For example, a marked f/2 becomes f/4 at this magnification.)

MAGNIFICATION, Figure 3: The object is placed between 1f and 2f, and the image is thrown far away frm the lens and is larger than life. Notice the symmety between this case and **Figure 1**; the object and image distances are interchanged.



Photographic enlargers usually magnify in the range of 2X to about 16X; slide projectors about 10X to 40X; movie prrojectors up to even a hundred times! Microscope objectives range from 1X up to 100X.

IMAGE AT INFINITY CONJUGATE DISTANCE, Figure 4: Since the infinitely far parallel rays focus 1f away from the lens, the image rays of an object placed 1f from a lens will exit parallel. The image will be formed at infinity.

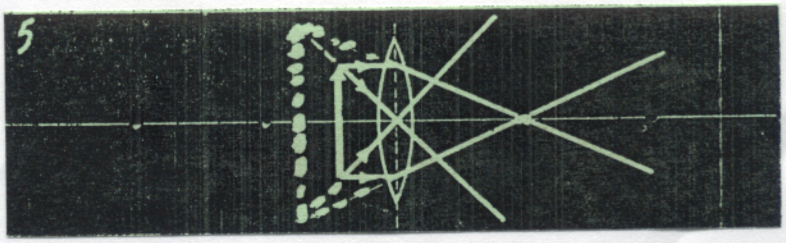


Collimators, which make parallel beams of light, work at this conjugate ratio, either using white light issuing from a pinhole or a spatially filtered laser beam. Telescope eyepieces are placed one focal length away from the intermediate image supplied by the objective in the tube, so that the output bundle is parallel.

Close-up supplementary lenses for cameras use this same principle. The object to be photographed is placed 1f from the supplementary lens; no image is formed, just parallel bundles of rays, which enter the prime camera lens focussed at infinity, willing to accept parallel rays.

Some magnifiers use this trick, like an Agfa Loupe. The stand positions the object 1f away; the viewer's eye is focussed at infinity while observing something much closer than the normal near point. Near-sighted students can check this by removing their glasses while observing through the Loupe; the image exiting its eyepiece will apppear fuzzy like other distant objects.

NEGATIVE CONJUGATE DISTANCE, Figure 5: If an object is positioned less than 1f away from the lens, a virtual image is formed that is further behind the lens than object. Unlike the negative lens whose virtual image is always smaller than the object, this is a magnified virtual image. The most obvious use is in magnifying glasses, a` la Sherlock Holmes, to see more detail. This is also the configuration used in drop-in slide viewers or movie editors to enlarge the transparency.



But the placing of the image further back than the object finds an important use in camera viewfinders, Viewmasters and other stereoscopes, and virtual reality or other helmet mounted displays. The groundglass or monitor seen in a camera viewfinder is only a few centimeters away from the eye, much nearer than the average near point. With the groundglass positioned much less than 1f away, the image can be found several feet away, in a zone that most people can focus on.