Insurance Equilibrium with Monoline and Multiline Insurers*

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Abstract

We study a competitive insurance industry in which insurers have limited liability, face frictional costs in holding capital, and offer coverage over a range of risk classes. We distinguish monoline and multiline industry structures, and provide what we believe are the first propositions indicating the conditions under which each structure is optimal. We relate the equilibrium results to the core concept used for coalition games. Markets for which the risks are limited in number, asymmetric or correlated may be best served by monoline insurers. Markets characterized by a large number of essentially independent risks, on the other hand, will be best served by many multiline firms, each with a different level of capital. Our results are consistent with the observed structures in the insurance industry, and have implications more broadly for financial services industries, including banking.

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1 Introduction

An important distinction in insurance markets is whether insurance companies have monoline or multiline structures. A monoline structure requires that the insurer dedicates its capital to pay claims on its single line of business, thus eliminating the diversification benefit in which a multiline firm can apply its capital to pay claims on any and all of its insurance lines.[1] The choice between a monoline and multiline structure therefore affects the states of the world in which insurer default occurs. For example, the market for mortgage default insurance, an industry with monoline structures, is currently at significant risk to default on its obligations as a result of the subprime mortgage crisis. On the one hand the monoline structures limit the amount of capital available to cover losses and may therefore increase the risk of default; on the other hand the structures may protect policyholders on other lines from facing insurer default (see Jaffee (2006)). Indeed, United States' insurance regulations require that certain high-risk insurance lines be provided on a monoline basis. Given the importance of the topic, it is surprising that no framework exists that can be used to systematically evaluate the costs and benefits of monoline insurance structures, which structures will prevail in an insurance industry, and the resulting default risks. In this paper, we develop such a framework.

In a friction-free market, the optimal outcome in an economy with risk-neutral insurers and risk-averse policy holders is, of course, well-known. In this case, the insurance companies should insure all risk, and the industry structure is irrelevant since all policy holders are fully insured and without counterparty risk regardless of the structure. In practice, two factors together make such an outcome infeasible. First, virtually all insurers are now limited liability corporations, which eliminates the unlimited recourse to partners' external (private) assets that was once common.[2] To avoid counterparty risk, a large amount of capital therefore needs to be held within the firm. Second, the excess costs of holding capital, such as corporate taxes, asymmetric information, and agency costs, create deadweight costs to holding internal (on balance sheet) capital, providing a strong incentive for the insurers to limit the amount of internal capital they hold.[3] In practice, policyholders therefore face counterparty risk, and it is then *a priori* unclear what is the optimal structure. One may conjecture that the benefits of diversification are

[1]Monoline structures do not preclude an insurance holding company from owning an amalgam of both monoline and multiline subsidiaries. Within a holding company, the role of a monoline structure is to restrict the capital of each monoline division to paying claims for that division alone.

[2]The insurer Lloyds of London once provided a credible guarantee to pay all claims, based on the private wealth of its "names" partners. In the aftermath of large asbestos claims, however, Lloyds now operates primarily as a standard "reserve" insurer with balance sheet capital and limited liability.

[3]This second factor was first emphasized with respect to general corporate finance by Froot, Scharsfstein, and Stein (1993), and Froot and Stein (1998) later extended that analysis to financial intermediaries. More specifically for insurance firms, Cummins (1993), Merton and Perold (1993), Jaffee and Russell (1997), Myers and Read (2001), and Froot (2007) all emphasize the importance of various accounting, agency, informational, regulatory, and tax factors in raising the cost of internally held capital.

more important in this situation, since diversification allows an insurer to pay claims with very high probability even with a limited amount of internal capital. Along this line of reasoning, one may think that the fully diversified industry structure, in which one large multiline insurer covers all risks, is the optimal outcome, since it leads to a maximal level of diversification.[4]

In this paper, we introduce a parsimonious model of an industry of insurance firms with limited liability and costly internal capital. Each insurer has the option to offer coverage against one or more of the existing insurance lines to risk-averse policy holders. We consider a competitive market, in which insurance companies (insurers) compete to attract risk averse agents (insurees) who wish to insure risks. This competition severely restricts the monoline and multiline structures that may exist in equilibrium. We also relate the equilibrium results to the core concept used in coalition games. We know of no other paper that provides an analytic framework for determining the industry structure that will prevail for an insurance industry that may contain both monoline and multiline firms.

Our model has several important industry structure implications: The fully diversified outcome—with one large insurer in the market—is typically *not* the outcome that will prevail even though this is the outcome that maximizes the diversification benefits. Instead, the industry will be served by several multiline companies, each holding a different amount of capital reflecting a different degree of safety, and it may even be optimal for some companies to choose a monoline structure. The reason is that different levels of internal capital may be optimal for different types of risks. By choosing a multiline structure an insurer is forced to choose a single "compromise" level of internal capital. In contrast, with a monoline structure each insurance line can be served by an insurer with an amount of internal capital tailored for that specific line, which may outweigh the benefits of diversification. If there are enough insurance lines in an economy with well-behaved risk distributions, the vast majority of policy holders will be served by multiline insurers, but there will be many such insurers so the outcome is still far away from the fully diversified case.

Moreover, some special cases aside, there may always be a role for some monoline insurers. With few insurance lines in the economy, or risk distributions that are not well-behaved—in that they are heavy-tailed or heavily dependent—the diversification benefits of a multiline structure may break down completely and the industry may instead be dominated by monoline structures. To support our intuition, we provide an example of an economy with two insurance lines, in which we can completely characterize when monoline and multiline insurance structures are optimal.

Our results fit broadly with what is observed in practice. For one thing, insurance companies operate with different ratings—AAA, AA, A, etc.—reflecting different amounts

[4] Or, equivalently, the optimal outcome could be to have one large reinsurance company that takes on all risk.

of capital and safety. For another thing, catastrophe lines of insurance illustrate one type of risks in which the benefits of diversification may be low enough to make a monoline structure preferable. The catastrophe lines create a potential bankruptcy risk if a "big one" should create claims that exceed the insurer's capital resources. Thus, an insurer that offers coverage against both traditional diversifiable risks and catastrophe risks may impose a counterparty risk on the policyholders of its traditional lines, a counterparty risk that would not exist if the insurer did not offer coverage on the catastrophe line. This negative externality for the policyholders on the diversifiable lines can be avoided if catastrophe insurers operate on a monoline basis, each one offering coverage against just one risk class and holding just the amount of capital that its policyholders deem optimal.

Our paper is related to Zanjani (2002), who also studies the effect of costly internal capital for multiline insurers. The focus of Zanjani's analysis, however, is to understand the price effects of variations in capital-to-premium ratios, whereas our focus is to understand the optimal industry structure. Also, Zanjani makes several simplifying assumptions, for example, that risks are normally distributed and that the demand function for insurance is exogenously given, whereas we allow for general one-sided loss distributions, and endogenize the demand. Relaxing the first assumption may be especially important for catastrophe insurance lines, which are known to have heavy-tailed distributions, see Ibragimov, Jaffee, and Walden (2009). To endogenize policy holders' demand, it is necessary to model the payouts of a multiline insurer to different claimholders in different states of the world. Here we apply an ex post payout rule, whereby the available funds are paid to claimants on a prorated basis in case of insurer default. Our paper is also related to Ibragimov, Jaffee, and Walden (2011), who analyze risk-sharing among intermediaries in a reduced form framework, focusing on the the distributional properties of the risks.

Our model is explicitly developed in the context of an insurance market, but the framework is also applicable to monoline structures within the financial services industry, as illustrated by, the Glass Steagall Act that forced U.S. commercial banks to divest their investment bank divisions, a clear monoline restriction. Winton (1995) develops a banking industry equilibrium in which larger banks have the advantage of greater diversification, but the drawback of a lower capital ratio. Our model studies this tradeoff in a substantially more general context by endogenizing the amount of capital held by each firm, by considering multiple categories of risks, and by evaluating both thin-tailed and heavy-tailed risk distributions. Leland (2007) also develops a model in which single-activity operating corporations can choose the optimal debt to equity ratio, whereas multiline conglomerates obtain a diversification benefit but can only choose an average debt to equity ratio for the overall firm. Thus, here too there is a tension between the diversification benefit associated with a multiline structure and the benefit of separating risks allowed by a monoline structure.[5]

[5] Diamond (1984) also notes the comparable role that diversification may play for banks, insurers, and operating

The paper is organized as follows: In Section 2, we provide the framework for our analysis. In Section 3, we show in an example with two insurance lines that little can be said about the industry structure in the general case. In Section 4, we then focus on markets with many independent risks, and show that in such markets the multiline outcome will be predominant, but that there may still be room for some monoline companies even as the number of insurance lines tends to infinity. Finally, Section 5 concludes.

2 The model

We first study the case of only one insured risk class to introduce the basic concepts and notation, and then proceed to the main focus of our study, with multiple risk classes. Our set-up follows Ibragimov, Jaffee, and Walden (2010) and Jaffee and Walden (2011) closely.

2.1 One risk class

Consider the following one-period model of a competitive insurance market. At $t = 0$, an *insurer* (i.e., an insurance company) in a competitive insurance market sells insurance against an idiosyncratic risk (risk class), $\tilde{L} \geq 0$ (throughout the paper we use the convention that losses take on positive values) to an *insuree*. The expected loss of the risk is $\mu_L = E[\tilde{L}]$, $\mu_L < \infty$. It is natural to think of the risk as an insurance line. This interpretation is straightforward if risks are perfectly correlated within an insurance line, in which case a representative insuree exists. We would also expect similar results to hold when an insurance line consists of many i.i.d. risks with identical insurees, although the analysis would become more complex in this case. The key is that each insuree within a risk class faces an identical choice, so that a "representative insuree" can be defined in each risk class.

The insuree is risk averse, with expected utility function u, where u is a strictly concave, increasing, twice continuously differentiable function defined on the whole of $\mathbb{R}_{-}$, and we further assume that $u'(0) \geq C_1 > 0$, and $u''(x) \leq C_2 < 0$, for constants C_1, C_2, for all $x \leq 0$. For some of the results we need to impose stronger conditions on u. We also require that the risk cannot be divided between multiple insurers. We note that sharing risks is uncommon in practice, reflecting the fixed costs of evaluating risks and selling policies, as well as the agency problems between insurers when handling split insurance claims. Finally, we assume that expected utility, U, is finite,

$$U = Eu(-\tilde{L}) > -\infty. \tag{1}$$

conglomerates. However, his analysis does not consider the possible advantage of the monoline structure when the diversification benefits are limited.

Later, in the general case with multiple risk classes, we will assume that all insurees have the same utility function, and that (1) holds for each risk class.

For many types of individual and natural disaster risks, such as auto and earthquake insurance, etc., it seems reasonable to assume that risks are idiosyncratic, i.e., that they do not carry a premium above the risk-free rate in a competitive market for risk although, of course, there will be some mega-disasters and corporate risks for which it is not true. In the analysis here, therefore, we assume that the risks are idiosyncratic in this sense.

At $t = 0$, the insurer takes on risks, receives premium payments, and contributes its own equity capital. The premiums and contributed capital are invested in risk-free assets, so that the assets A are available at $t = 1$, at which point losses are realized. Without loss of generality, we normalize the risk-free discount rate to zero.

The insurer has limited liability, and satisfies all claims by paying $\tilde{L}$ to the insuree, as long as $\tilde{L} \leq A$. But, if $\tilde{L} > A$, the insurer pays A and defaults on the additional amount that is due.[6] Thus, the payment is

$$Payment = \min(\tilde{L}, A) = \tilde{L} - \max(\tilde{L} - A, 0) = \tilde{L} - \tilde{Q}(A),$$

where $\tilde{Q}(A) = \max(\tilde{L} - A, 0)$, i.e., $\tilde{Q}(A)$ is the payoff to the option the insurer has to default. As shown in Cummins and Mahul (2004), see also Jaffee and Walden (2011), the optimal contract with limited liability and capital has this form, but includes a deductible. In the analysis here, we assume that the deductible is zero, since this property of the insurance contract is of second order importance to our analysis, and since this assumption simplifies the analysis substantially. When obvious, we suppress the A dependence, e.g., writing $\tilde{Q}$ instead of $\tilde{Q}(A)$. The premium for the insurance is P.

With unlimited liability and no friction costs, the price for $\tilde{L}$ risk in the competitive market is

$$P_L = \mu_L,$$

because the risk is idiosyncratic. More generally, given that there is a market for risk that admits no arbitrage, there is a risk-neutral expectations operator that decides the premium in a competitive insurance market. Since the risk is idiosyncratic, the risk-neutral expectation coincides with the true expectation for the risks that we consider. Similarly, the value of the option to default is

$$P_Q = E[\tilde{Q}(A)] = \mu_Q.$$

We assume, however, that there are deadweight frictional costs that apply when an

[6]For some lines of consumer insurance (e.g. auto and homeowner), there exist state guaranty funds through which the insurees of a defaulting insurer are supposed to be paid by the surviving firms for that line. In practice, delays and uncertainty in payments by state guaranty funds leave insurees still facing a significant cost when an insurer defaults; see Cummins (1988). More generally, our analysis applies to all the commercial insurance lines and catastrophe lines for which no state guaranty funds exist.

insurer holds internal capital; we refer to these as the excess costs of internal capital. The most obvious source is the taxation of corporate income, although asymmetric information, agency issues and bankruptcy costs may create similar costs. We specify the excess cost of internal capital as δ per unit of capital, i.e., δ provides a reduced form summary of the total excess cost per unit risk.[7] This assumption is comparable to the standard corporate finance assumption of a tax shield provided by corporate debt. The difference is that insurers hold net positive positions in financial instruments as capital assets, while most operating corporations are net debt issuers. Our model shows that, even with the deadweight cost of internal capital, insurers maintain net positive positions in financial assets precisely because it reduces the counterparty risk faced by their policyholders. The result is that to ensure that a capital amount A is available at $t = 1$, $(1 + \delta)A$ needs to be reserved at $t = 0$. Since the market is competitive and the cost of internal capital is δA, the premium charged for the insurance is

$$P = P_L - P_Q + \delta A = \mu_L - \mu_Q + \delta A. \tag{2}$$

The premium setting and capital allocations build on the no-arbitrage, option-based, technique, introduced to insurance models by Doherty and Garven (1986), then extended to multiline insurers by Phillips, Cummins, and Allen (PCA, 1998) and Myers and Read (MR, 2001), and further developed in Ibragimov, Jaffee, and Walden (2010).

Since we assume, in line with practice, that premiums are paid upfront, to ensure that A is available at $t = 1$, the additional amount of $A + \delta A - P = A - P_L + P_Q$ needs to be contributed by the insurer. Through the remainder of the paper, we shall refer to A as the insurer's assets or capital, depending on the context, it being understood that the amount $P_L - P_Q + \delta A$ is paid by the insurees as the premium, and the amount $A - P_L + P_Q$ is contributed by the insurer's shareholders. The total market structure is summarized in Figure 1.

It is natural to ask why insurees, recognizing that insurers impose the costs of holding internal capital, would not instead purchase their coverage directly in the market for risk. The answer is that here, as in any model of financial intermediation, there must be other costs, arising from transactions, contracting, or asymmetric information, which cause agents to prefer to deal with the intermediary. In this paper, we simply make the assumption that insurees do not have direct access to the market for risk and that they can obtain coverage only through the insurers.

There is also the question whether the primary insurance firms can eliminate their counterparty risk by transferring their risks to reinsurance firms. The answer is no.

[7] This is precisely the assumption used in a series of papers by Froot, Scharfstein, and Stein (1993), Froot and Stein (1998), and Froot (2007). It also implies that an additional dollar of equity capital raises the firm's market value by less than a dollar. Since we assume a competitive insurance industry, this excess cost is recovered through the higher premiums charged policyholders. It is also for this reason that the amount of capital is chosen to maximize policyholder utility.

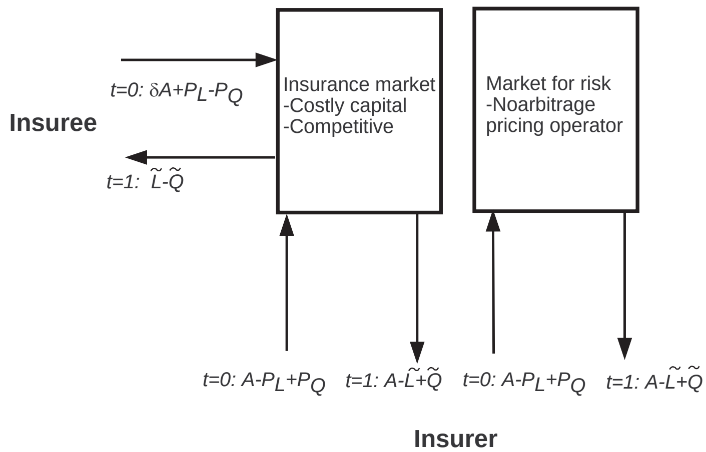

Figure 1: *Structure of model. Insurers can invest in market for risk and in a competitive insurance market. There is costly capital, so to ensure that A is available at $t = 0$, $(1 + \delta)A$ needs to be reserved at $t = 1$. The premium, $\delta A + P_L - P_Q$, is contributed by the insuree and $A - P_L + P_Q$ by the insurer. The discount rate is normalized to zero. Competitive market conditions imply that the premium for insurance is $P = P_L - P_Q + \delta A$.*

The basic reason is that the reinsurers would then create the same counterparty risk vis-à-vis the primary insurer. Of course, if the reinsurer can create a more diversified portfolio of insurance risks, the amount of counterparty risk may be reduced. This is fully incorporated in our model, since we allow, as one possible equilibrium structure, that the industry consist of massively multiline firms that hold all the insurance risks and obtain all possible benefits of diversification. In this sense, our model fully incorporates reinsurance, although we refer to all the firms simply as insurers.

2.2 Multiple risk classes

The generalization to the case when there are multiple risk classes requires an additional assumption regarding the timing at which claims are made. Here we follow PCA by assuming that claims on all the insured lines are realized at the same time, $t = 1$.[8] The result is that at $t = 1$ the insurer either pays all claims in full (when assets exceed total claims) or defaults (when total claims exceed the assets).

If coverage against N risks is provided by one multiline insurer, the total payment made to all policyholders with claims, taking into account that the insurer may default,

[8]This assumption is invaluable in that it allows tractability in computing the risk sharing and risk transfer attributes of the equilibrium outcomes of the model. It does, however, also mean that we are unable to study a variety of explicitly dynamic questions. Although the study of such dynamic factors would certainly provide additional insight, we believe that they would not change the basic results that are emphasized in this paper.

is

$$Total\ Payment = \tilde{L} - \max(\tilde{L} - A, 0) = \tilde{L} - \tilde{Q}(A),$$

where $\tilde{L} = \sum_i \tilde{L}_i$ and $\tilde{Q}(A) = \max(\tilde{L} - A, 0)$. The shortfall in total assets for a defaulting insurer is allocated across insurance lines in proportion to the actual claims by line. This is the so-called *ex post* sharing rule, see the extensive discussion in Ibragimov, Jaffee, and Walden (2010). With this rule, the payments made to insuree i is then

$$Payment_i = \frac{\tilde{L}_i}{\tilde{L}} A = \tilde{L}_i - \frac{\tilde{L}_i}{\tilde{L}} \tilde{Q}(A). \tag{3}$$

Theoretically, this may of course not be the optimal sharing rule in a friction-free market. However, the rule is overwhelmingly used in practice, which indicates that there are frictions — e.g., in the form contract complexity and non-contractability of payments in some states of the world — that make other potentially superior contracts unimplementable. We therefore take the ex post sharing rule as given.

Following Ibragimov, Jaffee, and Walden (2010), we define the binary default option

$$\tilde{V}(A) = \begin{cases} 0 & \tilde{L} \leq A, \\ 1 & \tilde{L} > A, \end{cases} \tag{4}$$

and the price for such an option in the competitive friction free market, $P_V = E[\tilde{V}]$. The total price for the risks is, $P \stackrel{\text{def}}{=} \sum_i P_i$, where P_i is the premium for insurance against risk i. It follows that

$$P_i = P_{L_i} - r_i P_Q + v_i \delta A, \tag{5}$$

where

$$r_i = E\left[\frac{\tilde{L}_i}{\tilde{L}} \times \frac{\tilde{Q}}{P_Q}\right], \qquad v_i = E\left[\frac{\tilde{L}_i}{\tilde{L}} \times \frac{\tilde{V}}{P_V}\right]. \tag{6}$$

Thus, (3-6) completely characterize payments and prices for all policyholders in the general case with multiple risk classes.

3 Equilibrium market structure with two risk classes

Consider now a market in which there are two risk classes, each of which is to be insured by a representative insuree. For simplicity, we assume that the insurees have expected utility functions defined by $u(x) = -(-x+t)^\beta$, $\beta > 1$, $t \geq 0$, $x < 0$, and that the risks, $\tilde{L}_1$ and $\tilde{L}_2$, have (scaled) Bernoulli distributions: $\mathbb{P}(\tilde{L}_1 = 1) = p$, $\mathbb{P}(\tilde{L}_1 = 0) = 1 - p$, $\mathbb{P}(\tilde{L}_2 = 2) = q$, $\mathbb{P}(\tilde{L}_2 = 0) = 1 - q$, $\text{corr}(\tilde{L}_1, \tilde{L}_2) = \rho$. Depending on $0 < p < 1$ and $0 < q < 1$, there are restrictions on the correlation, ρ. For example, ρ can only be equal to 1 if $p = q$.

There are two main alternative market structures in this case. The two risk classes may be insured by two separate monoline insurers, or alternatively by one multiline insurer. In addition, with the interpretation that each risk class contains a large number of perfectly correlated identical risks, a multiline insurer may choose to offer insurance against fractions of risk classes, e.g., selling insurance against half of all the risks in one class, and against all the risks in the other class. We first focus on the case where no such fractional offerings occur, and then note that such a "fractional" approach is not optimal in this example.

To analyze the market structure given a fixed level of capital and premiums, although quite straightforward, may give misleading results because the capital held and the structure chosen are jointly determined. For example, an insurance company choosing to be massively multiline may choose to hold a lower level of capital than the total capital of a set of monoline firms insuring the same risks. We therefore need to allow the level of capital to vary. Specifically, we will compare a multiline structure where one insurer sells insurance against both risk classes when reserving capital A, with a monoline structure where two monoline companies insure the two risk classes, reserving capital A_1 and A_2, respectively.

Given our assumptions about competitive markets, we would expect a multiline insurer to dominate if it can choose a level of capital that makes insurees in both lines better off than what they can get from a monoline insurer. On the other hand, if there is a way for a monoline insurer in the first risk class to choose a level of capital that improves the situation for the first insuree, then we would expect this insuree to go with the monoline insurer, and the monoline outcome will then dominate. A similar argument can be made if a monoline offering dominates the multiline outcome for the second risk class.

To formalize this intuition, we let $U_1^{MONO}(A_1)$ and $U_2^{MONO}(A_2)$ denote the expected utilities of the first and second insuree with monoline insurance, when capital A_1 and A_2 is reserved, respectively. Similarly, $U_1^{MULTI}(A)$ and $U_2^{MULTI}(A)$ denotes the expected utilities of the first and second insuree when insured by a multiline insurer with capital A. The multiline structure is said to dominate if there is a level of internal capital, A, such that for $U_1^{MULTI}(A) > U_1^{MONO}(A_1)$ for all A_1, and $U_2^{MULTI}(A) > U_2^{MONO}(A_1)$, for all A_2. Otherwise, the monoline structure is said to dominate.

In case of a monoline market structure, we would expect the competitiveness between insurers to lead to an outcome where the level of capital is chosen to maximize the insuree's expected utility. From the analysis in Jaffee and Walden (2011) we know that under general conditions there is a unique level of capital, $A^* \geq 0$ that maximizes the insuree's expected utility. Thus, the multiline outcome dominates if there is a level of capital, A such that $U_1^{MULTI}(A) > U_1^{MONO}(A_1^*)$ and $U_2^{MULTI}(A) > U_2^{MONO}(A_2^*)$.

We study the case when $\beta = 7$, $t = 1$, $\delta = 0.2$, $p = 0.25$, and $q = 0.65$. When the industry is structured as two monoline firms, the optimal capital levels are $A_1^* = 0.78$ and $A_2^* = 1.53$ respectively, i.e., these are the level of capitals that maximize the respective expected utilities of the two insurees, respectively. The expected utility for insuree 1 and 2, respectively, for these choices of monoline capital are $U_1^{MONO}(A_1^*) = -12.06$, and $U_2^{MONO}(A_2^*) = -933.7$, respectively. These levels are represented by the solid straight horizontal and vertical lines in Figure 2.

The curves in Figure 2 show the multiline outcome as a function of capital, A, for correlations $\rho \in \{0.1, 0.2, 0.3, 0.4\}$. When correlations are low, the outcome can be improved for both insuree classes by moving to a multiline solution, reaching an outcome somewhere on the efficient frontier of the multiline utility possibility curve. For $\rho = 0.4$, however, insuree class 2 will not participate in the multiline solution, regardless of capital, since the utility is always lower than what he achieves in a monoline offering. The monoline outcome will therefore prevail.

So far, we have compared the situations where the multiline insurer insures both risk classes completely. If the risk classes are divisible, e.g., because they represent a large number of small risks, a multiline insurer could also choose to insure some risks, e.g., insuring all risks in one risk class and one half of the risks in the other. It is straightforward to show that when $\rho = 0.4$ in the previous case, *any* multiline insurance structure against a fraction α of the first risk class and $1 - \alpha$ against the second will make either insurees in the first risk class or in the second risk class worse off. Thus, optimality of the single line structure holds in this more general setting too.

In Figure 3, we plot the regions in which the monoline and multiline solutions will occur respectively, as a function of q and ρ. We use the parameter values $p = 0.1$, $\beta = 1.2$, $\delta = 0.01$ and $t = 1$. The figure shows that, all else equal, increasing the correlation decreases the prospects for a multiline solution. Also, increasing the asymmetry $(q - p)$ between risks decreases the prospects for a multiline outcome. The two regions labeled "not feasible" arise because certain combinations of q and p are not possible for the assume parameter values.

The intuition behind these results is quite straightforward, once we realize the multiple forces at play. First, diversification — a major rationale for having insurance and reinsurance in the first place — benefits a multiline structure. It allows the insurer to decrease the risk of default, given a constant level of capital per unit of risk insured. Alternatively, it allows the insurer to decrease the amount of capital reserved for a constant level of risk, decreasing the total cost of internal capital. This effect underlies our prior that multiline structures should be more efficient than monoline structures. But a multiline structure also forces insurees in the two lines to agree on or to compromise on the level of capital. Further, it introduces additional uncertainty for insurees who now need to worry about the losses in other lines as well as in their own line, leading to higher counterparty risk. These two effects benefit a monoline structure. Of course, if full contracting freedom was

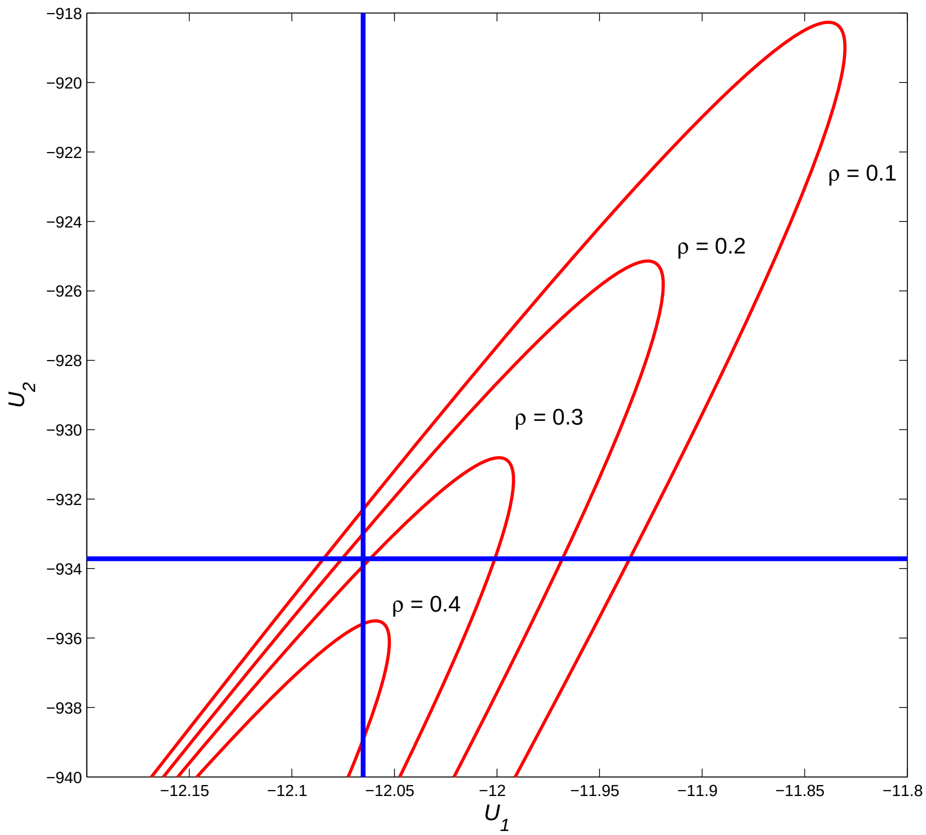

Figure 2: *The solid vertical and horizontal lines show optimal expected utility for insuree 1 and 2 respectively when the industry is structured as two monoline firms (with optimal capital levels* $A_1 = -12.06$ *and* $A_2 = -933.7$*). The curved lines show the utility combinations for a multiline insurer, based on 4 different correlations between risks 1 and 2, and for all possible capital levels, A. The monoline outcome dominates when* $\rho = 0.4$*, because the multiline structure is suboptimal for insuree 2. For* $\rho = 0.3$*,* $\rho = 0.2$ *and* $\rho = 0.1$*, the multiline structure dominates since it is possible to improve expected utility for insuree 2, as well as for insuree 1. Parameters:* $p = 0.25$*,* $q = 0.65$*,* $\delta = 0.2$*,* $\beta = 7$*.*

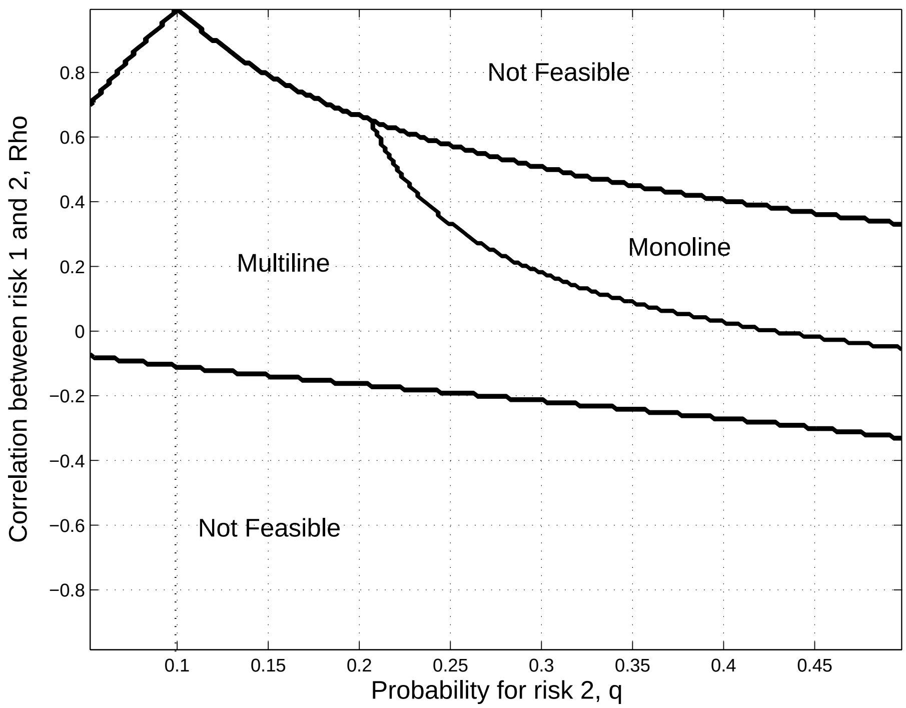

Figure 3: *Regions of q and ρ, in which monoline and multiline structure is optimal. All else equal: Increasing ρ (correlation), given q makes monoline structure more likely. Increasing asymmetry of risks $(q-p)$ also makes monoline structure more likely. Correlations can not be arbitrary for the two (Bernoulli) risks, so there are combinations of q and ρ that are not feasible. Parameters: $p = 0.1$, $\delta = 0.01$, $\beta = 1.2$.*

allowed, a very complex payoff contract could be written by the multiline insurance that mitigated these negative effects of the multiline structure, but given that the ex post sharing rule is the one predominantly seen in practice, we reiterate our view that frictions *de facto* make such complex contracts unfeasible.

We note that the results in this two-line example confirm with our intuition in that when correlation between lines increase, a monoline structure becomes optimal. This is of course the situation when diversification is of least value. Also, when asymmetry between the risk classes increases, a monoline structure becomes optimal, in line with the intuition that such asymmetries may make one insuree relatively worse off. For example, insurees in low-risk property and casualty insurance lines may not be willing to group together with a insurees in high risk, heavy-tailed, catastrophe insurance line, since their capital requirements may be quite different.

Given these results, it is natural to ask whether it is possible to draw *any* general conclusions about when monoline or multiline structures will prevail. The problem is made particularly difficult due to the complex tradeoff between the advantages and disadvantages of each possible monoline or multiline structure. In the next section, however, we show that in the case where there are many insurance classes, with risks that have limited asymmetry and dependence, the typical outcome is one with many multiline structures, although there may still be a role for a few monoline insurers even in this case.

4 Equilibrium market structure with many risk classes

We first extend the concepts from the previous section to include markets with multiple risks classes, using a more formal approach for the arguments to be precise.

4.1 Definition of market structure

Consider an insurance market, in which M insurers sell insurance against $N \geq M$ risk classes, each risk class held by a representative insuree. The set of risk classes (and equivalently, the set of insurees) is $X = \{1, \ldots, N\}$, which is partitioned into $\mathcal{X} = \{X_1, X_2, \ldots, X_M\}$, where $\cup_i X_i = X$, $X_i \cap X_j = \emptyset, i \neq j$, $X_i \neq \emptyset$. The partition represents how the risks are insured by M monoline or multiline insurers. The total industry structure is characterized by the duple, $\mathcal{S} = (\mathcal{X}, \mathbf{A})$, where $\mathbf{A} \in \mathbb{R}_+^M$ is a vector with i:th element representing the capital available in the firm that insures the risks for agents in X_i. Here, we use the notation $\mathbb{R}_+ = \{x \in \mathbb{R} : x \geq 0\}$ and $\mathbb{R}_- = \{x \in \mathbb{R} : x \leq 0\}$. We call $\mathbf{A}$ the capital allocation and $\mathcal{X}$ the industry partition. If we want to stress the set of insurees included in a market, we write $\mathcal{S}_X$. The number of sets in the industry partition is denoted by $M(\mathcal{X})$. Two polar cases are the fully multiline industry partition, $\mathcal{X}^{MULTI} = \{\{0, 1, \ldots, N\}\}$ and the monoline industry partition,

$\mathcal{X}^{MONO} = \{\{0\}, \{1\}, \ldots, \{N\}\}$. Of course, for a fully multiline industry structure, $M = 1$ and $\mathbf{A} = A$. Given an industry structure, $\mathcal{S}$, the premium in each line is uniquely defined through (5), and we write $P_i = P_i(\mathcal{S})$.

For N risks, $\tilde{L}_1, \ldots, \tilde{L}_N$, and a general industry structure, $\mathcal{S} = (\mathcal{X}, \mathbf{A})$, when the ex post sharing rule is used, the residual risk for an insuree (i.e., the net risk after claims are received), $i \in X_j$, is then

$$\tilde{K}_i(\mathcal{S}) = \frac{\tilde{L}_i}{\sum_{i' \in X_j} \tilde{L}_{i'}} \min\left(A_i - \sum_{i' \in X_j} \tilde{L}_{i'}, 0\right). \tag{7}$$

His expected utility is therefore $U_i(\mathcal{S}) = Eu_i(-P_i(\mathcal{S}) + \tilde{K}_i(\mathcal{S}))$.

Given the rich complexity we have already seen in the case with two risk classes, we have been unable to provide a complete characterization for the general case with any fixed number of multiple risk classes. However when there are numerous independent risk classes available, asymptotic analysis becomes feasible. Our first objective is to understand how powerful diversification is in providing value to the insurees in this case, which we analyze in the next section. We then analyze what the implications are for equilibrium outcomes, in the following two sections. Going forward, we therefore assume that the risk classes are independent, and that they are numerous — in a sense that we will make precise.

4.2 Feasible outcomes

We use the certainty equivalent as a measure of the size of a risk. For a specific utility function, u, the certainty equivalent of risk $\tilde{L}$, $CE_u(-\tilde{L}) \in \mathbb{R}_-$ is defined such that $u(CE_u(-\tilde{L})) = E[u(-\tilde{L})] - u(0)$.

When capital is costly, $\delta > 0$, it is not possible to obtain the friction-free outcome, in which full insurance is offered at the price of expected losses, μ_L. To ensure that the friction cost is not so high as to rule out all insurance purchases, we assume that the cost of holding capital is sufficiently small compared with expected losses, such that

Condition 1 $CE_u(-P(A) - \tilde{Q}(A)) < -\mu_L(1 + \delta)$ *for all* $A \in [0, \mu_L]$.

This condition implies that the risk is potentially insurable in that if an insurer could guarantee default-free insurance against a risk by holding capital just equal to the expected loss and by setting a premium equal to the expected loss plus the cost of holding the internal capital, the insuree would purchase such insurance, rather than bearing the risk himself.

Given costly capital, under Condition 1 the best possible risk-free outcome is for the insuree to reach a certainty equivalent of $-\mu_L(1 + \delta)$. We therefore call an outcome in which an insuree obtains $CE_u = -\mu_L(1+\delta)$ the *ideal risk-free outcome with costly internal capital.* In practice, an insurer holding capital equal to the expected loss would still create a counterparty risk, so this is indeed an ideal outcome.

What can we say about industry structure when there are many risks available? Intuitively, when capital is costly and there are many risks available, we would expect an insurer to be able to diversify by pooling many risks and, through the law of large numbers, choose an efficient A^* per unit of risk. Therefore, the multiline structure should be more efficient in mitigating risk than the monoline structure.[9] The argument is very general, as long as there are enough risks to pool, and these risks are independent. For example, under quite general (technical) conditions the multiline business can reach an outcome arbitrarily close to the ideal risk-free outcome with costly internal capital. We have:

Theorem 1 *Consider a sequence of insurees, $i = 1, 2, \ldots$, with expected utility functions, $u_i \equiv u$, holding independent risks $\tilde{L}_i$. Suppose that u'' is bounded by a polynomial of degree q, $E\tilde{L}_i^p \le C$ for $p = 2 + q + \epsilon$ and some $C, \epsilon > 0$, and $E(\tilde{L}_i) \ge C'$, for some $C' > 0$. Then, regardless of the cost of internal capital, $0 < \delta < 1$, as the number of risks in the economy, N, grows, a fully multiline industry, $\mathcal{X}^{MULTI} = \{\{1, \ldots, N\}\}$ with capital $A = \sum_{i=1}^{N} \mu_{L_i}$, reaches an outcome that converges to the ideal risk-free outcome with costly internal capital, i.e.,*

$$\min_{1 \le i \le N} CE_u(-P_i((\mathcal{X}^{MULTI}, A)) + \tilde{K}_i((\mathcal{X}^{MULTI}, A))) = -\mu_{Li}(1 + \delta) + o(1).$$

This Theorem extends the Diamond (1984) and Winton (1995) results concerning risk-free intermediary debt, to cover multiple loan classes and an explicit cost of internal capital. Theorem 1 can be further generalized in several directions, e.g., to allow for dependence. For example, as follows from the proof of the Theorem, it also holds for all (possibly dependent) risks $\tilde{L}_i$ with $E|\tilde{L}_i|^p < C$ that satisfy the Rosenthal inequality.[10]

[9] This type of diversification argument is, for example, underlying the analysis and results in both Jaffee (2006) and Lakdawalla and Zanjani (2006).

[10] The Rosenthal inequality (see Rosenthal (1970)) and its analogues are satisfied for many classes of dependent random variables, including martingale-difference sequences (see Burkholder (1973) and de la Peña, Ibragimov, and Sharakhmetov (2003) and references therein), many weakly dependent models, including mixing processes (see the review in Nze and Doukhan (2004)), and negatively associated random variables (see Shao (2000) and Nze and Doukhan (2004)). Furthermore, using the Phillips-Solo device (see Phillips and Solo (1992)) in a similar fashion of the proof of Lemma 12.12 in Ibragimov and Phillips (2004), one can show that Theorem 1 also holds for correlated linear processes $\tilde{L}_i = \sum_{j=0}^{\infty} c_j \epsilon_{i-j}$, where (ϵ_t) is a sequence of i.i.d. random variables with zero mean and finite variance and c_j is a sequence of coefficients that satisfy general summability assumptions. Several works have focused on the analysis of limit theorems for sums of random variables that satisfy dependence assumptions that imply Rosenthal-type inequalities or similar bounds (see Serfling (1970), Móricz, Serfling, and Stout (1982) and references therein). Using general Burkholder-Rosenthal-type inequalities for nonlinear functions of sums of (possibly dependent) random variables (see de la Peña, Ibragimov, and Sharakhmetov (2003) and

Theorem 1 shows that, with enough risks, a solution can be obtained arbitrarily close to the ideal risk-free outcome with costly internal capital. Theorem 2 below shows the opposite, that with too few risks it is not possible to get arbitrarily close to the ideal risk-free outcome with costly internal capital:

Theorem 2 *Consider a sequence of insurees, $i = 1, 2, \ldots$. If, in additions to the assumptions of Theorem 1, the risks are uniformly bounded: $\tilde{L}_i \leq C_0 < \infty$ (a.s.) for all i, and $Var(\tilde{L}_i) \geq C_1$, for some $C_1 > 0$, for all i, then for every $\epsilon > 0$, there is an n such that $\lim_{\epsilon \searrow 0} n(\epsilon) = \infty$ and such that, as N grows, any partition with $\mathbf{A}_j = \sum_{i \in X_j} \mu_{L_i}$ for all j and*

$$\min_{1 \leq i \leq N} CE_u(-P_i((\mathcal{X}, \mathbf{A})) + \tilde{K}_i((\mathcal{X}, \mathbf{A}))) \geq \mu_{L_i}(1 + \delta_i) - \epsilon, \tag{8}$$

must have $|X_i| \geq n$ for all $X_i \in \mathcal{X}$, i.e., any $X_i \in \mathcal{X}$ must contain at least n risks.

Similarly to Theorem 1, Theorem 2 can be generalized. For example, the condition of uniformly bounded risks can be relaxed. Specifically, if the utility function, u, has deceasing absolute risk aversion, then the Theorem holds if the *expectations* of the risks are uniformly bounded ($E[\tilde{L}_i] < C$) for all i.

These two results illuminate the power of diversification to eliminate counterparty risk as long as there are a sufficient number of risk classes and the loss distributions are sufficiently well behaved. They also address the issue of risk of default driven by other risk classes that we saw in the previous section, since no such risk is present after full diversification. The results do not, however, address the optimal industry structure when capital levels are endogenous. In fact, it is straightforward to show that the ideal risk-free outcome with costly internal capital has a capital level that is "too high" in the sense that insurees would always prefer a lower level of capital than the risk-free level.

The intuition behind this result is simple. The marginal utility benefit of decreasing capital, A, slightly below the ideal risk-free outcome, in terms of reduced cost of capital, will always outweigh the marginal cost of introducing some risk, since the former factor is a first order effect, whereas the latter factor is of second order close to the ideal risk-free outcome. In general, insurees will have different opinions about how large the capital reductions should be, and may therefore prefer to be served by different insurers, to avoid having to compromise on capital levels. The results in this section are therefore of limited use in determining which outcome will be observed in equilibrium. This is our focus in the next two sections.

references therein), one can also obtain extensions of Theorem 1 to the case of losses that satisfy nonlinear moment assumptions.

4.3 A strategic game

In the multiline case, there are many possible industry structures, $\mathcal{S}$. We wish to extend the concept of dominated industry structures that we used in the two risk-class example to the general multiline setting. We note that the arguments made in the Section 3 are quite similar to those made in coalition games without transferable payoffs (see, e.g., Osborne and Rubinstein (1984)). Specifically, an insuree-centered interpretation of the comparison between monoline and multiline outcomes in Section 3 would be to view it as a decision between the two insurees to form a "coalition" or not. Such a coalition dominates if it is possible to make both insurees better off than they would be "alone" — i.e., in the monoline outcome. In other words, if there are choices of capital for which the multiline outcome Pareto dominates the monoline outcome, this outcome will prevail. Equivalently, the *core* of this coalition game contains the multiline outcome in that case.

We extend this parallel to markets with several risk classes. For N agents with utility functions, u_i, $1 \leq i \leq N$, where each agent wishes to insure risk $\tilde{L}_i$, an industry structure, $\mathcal{S}'$, Pareto dominates another industry structure, $\mathcal{S}$, if $E[u_i(-P_i(\mathcal{S}) + \tilde{K}_i(\mathcal{S}))] \leq E[u_i(-P_i(\mathcal{S}') + \tilde{K}_i(\mathcal{S}'))]$ for all i and $E[u_i(-P_i(\mathcal{S}) + \tilde{K}_i(\mathcal{S}))] < E[u_i(-P_i(\mathcal{S}) + \tilde{K}_i(\mathcal{S}'))]$ for at least one i. We also say that $\mathcal{S}'$ is a Pareto improvement of $\mathcal{S}$. An industry structure, $\mathcal{S}$, for which there is no Pareto improvement is said to be Pareto efficient. An industry structure, $(\mathcal{X}, \mathbf{A})$, is said to be constrained Pareto efficient (given $\mathcal{X}$), if there is no $\mathbf{A}'$ such that $(\mathcal{X}, \mathbf{A}')$ is a Pareto improvement of $(\mathcal{X}, \mathbf{A})$. An industry partition $\mathcal{X}$ is said to be Pareto efficient if there is a capital allocation, $\mathbf{A}$, such that $(\mathcal{X}, \mathbf{A})$ is Pareto efficient. Given the finiteness of expected utility for all insurees (1), the continuity of expected utility as a function of capital for any given multiline structure that follows from (7), and the linear cost of capital, it follows that there is a constrained Pareto efficient outcome for any given industry partition, $\mathcal{X}$.

Following this intuition of coalition games and the arguments from Section 3, we formalize the concept that an industry structure is unstable if it is possible to achieve higher expected utility for any specific insuree by introducing a monoline offering:

Definition 1 *An industry structure, $\mathcal{S}$, is said to be* robust to monoline blocking, *if there is no insuree, $i \in \{1, \ldots, N\}$ such that $U_i(\mathcal{S}) < U_i^{MONO}(A)$ for some $A \geq 0$, where $U_i^{MONO}(A)$ is the expected utility insuree i achieves under a monoline offering with capital A.*

This definition captures exactly the intuition from the example with two risk classes. However, with multiple lines, there may of course be other ways to improve the situation for a subset of insurees. One such improvement, which may be especially easy to implement is by "merging" insurers. If a more efficient offering can be made to all insurees in one or more insurance companies, we would expect competition to lead to such mergers.

We define:

Definition 2 *An industry structure, $\mathcal{S}_X$, is said to be* robust to aggregation*, if there is no set of insurers in $\mathcal{X}$, $X_{k_1} \in \mathcal{X}, \ldots, X_{k_n} \in \mathcal{X}$, $k_1 < k_2, \ldots < k_n$, $n \geq 1$, such that an insurer can make a fully multiline offering to all insurees in $Y = \cup_{i=1}^{n} X_{k_i}$ by choosing some capital level A, so that all insurees are at least as well of as they were before, and at least one insuree is strictly better off, i.e., $U_i(\mathcal{S}_Y) \geq U_i(\mathcal{S}_X)$ for all $i \in Y$ and the inequality is strict for at least one $i \in Y$. Here, $\mathcal{S}_Y = (\{Y\}, A)$.*

In other words, robustness to aggregation means that there is no aggregate structure that Pareto dominates for all the insurees of one or more insurers. A market that is not robust to aggregation would potentially be unstable, since mergers and acquisitions would take place in such a market. We note that robustness to aggregation implies constrained Pareto efficiency, since an insurer covering risk classes X_i, who chose a Pareto inefficient level of capital could otherwise be improved upon by choosing $Y = X_i$ with a superior capital level.

We now introduce an equilibrium concept that captures robustness to both monoline blocking and aggregation:

Definition 3 *The* equilibrium set*, $\mathcal{O}$, is defined as the set of industry structures, $\mathcal{S}$, that are robust to both monoline blocking and aggregation.*

The restrictions on the equilibrium set are quite weak, which implies that the set can be quite large, and specifically that it is never empty:[11]

Proposition 1 *The equilibrium set is nonempty, $\mathcal{O} \neq \emptyset$.*

The weak assumptions needed are a major strength of our results. One may wish to impose additional assumptions to further restrict the possible industry outcomes. Our main results on industry structure will hold for all elements in $\mathcal{O}$, so they will of course then also hold for any subset of $\mathcal{O}$. For example, a stronger restriction on the outcome would be imposed by using the core concept:

Definition 4 *An industry structure $\mathcal{S}_X$ is said to be* robust to all blocking *if there is no set of insurees, $Y \subset X$, such that an insurer can make a fully multiline offering to all*

[11] This result is easily seen. Given a constrained Pareto efficient industry structure, $\mathcal{S}^0$, which is robust to monoline blocking, it is either the case that $\mathcal{S} \in \mathcal{O}$, or that a Pareto improvement can be achieved by aggregation, leading to a new constrained Pareto efficient industry structure, $\mathcal{S}^1$. This new structure is obviously also robust to monoline blocking. The argument can be repeated and since the number of risk classes is finite, it must terminate for some $\mathcal{S}^n \in \mathcal{O}$. $\mathcal{S}_0$ can now be chosen to be a constrained Pareto efficient *monoline* structure, which per definition is robust to monoline blocking, and the result follows.

insurees in Y by choosing some capital level A, so that all insurees are at least as well of as they were before, and at least one insuree is strictly better off, i.e., $U_i(\mathcal{S}_Y) \geq U_i(\mathcal{S}_X)$ for all $i \in Y$ and the inequality is strict for at least one $i \in Y$. Here, $\mathcal{S}_Y = (\{Y\}, A)$.

Robustness to all blocking is a stronger condition than joint robustness to monoline blocking and aggregation, since it implies that there is no structure that Pareto dominates for *some* (not all) the insurees of one or more insurers. Such blocking would allow for cherry picking of risk classes among several multiline insurers, and one may conjecture that such competitive moves are more difficult to carry out in practice than the ones defining the equilibrium set, which is why we do not include them in our definition of the equilibrium set.

Definition 5 *The* core, *$\mathcal{C}$, is defined as the set of industry structures, $\mathcal{S}$, that are robust to all blocking.*

It follows immediately that $\mathcal{C} \subset \mathcal{O}$, so all results that hold for all industry structures in the equilibrium set will also hold for industry structures in the core. There is no guarantee that the core is nonempty though. In fact, we may easily imagine a situation with three risk classes, in which it is optimal for two insurees to be insured by a multiline company. This situation is similar to a majority game, since any structure with two insurees may be blocked by one in which one of the two insurees is replaced by the third. There core is therefore empty in this case. The analysis of the conditions under which the core is nonempty in our setting provides an interesting avenue for future research.

We stress that although we use terminology from coalition games in our analysis, our market mechanism is *not* based on coalition. It is the competitiveness of insurers that allows insurees to ensure outcomes that can not be improved by other industry structures.

4.4 Equilibrium outcome

We show that under some additional technical assumptions about the risks, when there are many risk classes, massively multiline industry structures dominate monoline structures, and that equilibrium outcomes will contain very few monoline structures.

We introduce the following definitions: An industry partition is said to be massively multiline if, as the number of lines in the industry, N, grows, the average number of lines per insurer grows without bounds, i.e., $\lim_{N\to\infty} N/M(\mathcal{X}) = \infty$.[12] Here, $M(\mathcal{X})$, defined in Section 2.2, is the number of insurers in the industry. We let $\mathcal{X}^{MASS}$ (or $\mathcal{X}_N^{MASS}$, if we want to stress the number of risks) represent a massively multiline industry partition.

For some of the results, we need conditions on the behavior of the risks. Specifically, the following condition ensures that the risks are not too "asymmetric":

[12] Strictly speaking, we are analyzing a sequence of industries, with a growing number of risks.

Condition 2 *There are positive, functions, $g : \mathbb{R}_+ \to \mathbb{R}_+$, and $h : \mathbb{R}_+ \to \mathbb{R}_+$ such that, g is strictly decreasing, h is nonincreasing, $\lim_{x\to\infty} h(x) = 1$, and for all i,*

$$F_i(x) \in [g(h(x)x), g(x)].$$

Here, $F_i(x)$ is the complementary cumulative distribution function of $\tilde{L}_i$, $F_i(x) \stackrel{\text{def}}{=} \mathbb{P}(\tilde{L}_i > x)$.

We also need the risks to be insurable in the sense that the cost of capital is not too high and the insurees are not too close to risk-neutral to wish to insure the risks. One natural assumption is that the outcome in which no insurance is offered is Pareto dominated. It turns out that we need a stronger condition:

Condition 3 *There is an $\epsilon > 0$, such that, for each risk, i, the optimal capital for a monoline insurer is greater than ϵ, i.e., $A_i^* > \epsilon$, for all i.*

Both conditions ensure, in different ways, that the risks do not become degenerate for large i. Obviously, if the risks are i.i.d., $g = F_1$ and $h(x) \equiv 1$ can be chosen in condition 2.

We now have

Theorem 3 *Under the conditions of Theorem 1, if the risks, $\tilde{L}_i$, have absolutely continuous distributions with strictly positive probability density functions on $\mathbb{R}_+$. Then, for large N,*

1. *If conditions 2 and 3 are satisfied, there is a constant, $C < \infty$, such that any industry structure in the equilibrium set has at most C monoline insurers, regardless of N, i.e., any $\mathcal{X} \in \mathcal{O}$ has at most C monoline insurers.*

2. *If conditions 2 and 3 are satisfied, then there is a massively multiline industry partition in the equilibrium set, $\mathcal{X}^{MASS} \in \mathcal{O}$.*

3. *If the risks are i.i.d. and if condition 3 is satisfied, then the fully multiline industry partition is in the equilibrium set, $\mathcal{X}^{MULTI} \in \mathcal{O}$.*

4. *If condition 1 is satisfied, then the fully multiline industry partition, $\mathcal{X}^{MULTI}$, with capital $A = \sum_i \mu_{L_i}$ Pareto dominates the fully monoline insurance structure, $\mathcal{X}^{MONO}$.*

All these results are for economies in which many lines are present, with independent risks that are not too asymmetric. The first result shows that monoline structures may still exist, but in an economy with many lines there will be relatively few risk classes insured by monoline insurers. We note that since this result holds for all elements in the equilibrium set, it will also hold for all elements in a refinement of this set, e.g., in the core. The second result shows that there are massively multiline industry structures in the equilibrium set. The third result shows that if the risks are identically distributed, there is a market with one fully multiline insurer in the equilibrium set. The fourth result shows conditions under which a fully multiline firm with almost no risk Pareto dominates the monoline structure.

The difference between the second and third result in Theorem 3 is important. If the risks are identically distributed, then the agents, having the same utility functions, will all agree upon the optimal level of internal capital, A^*. They may therefore agree to insure in one fully multiline company. If, on the other hand, the risks have different distributions (or equivalently, if the utility functions are different), then the insurees will typically disagree about what is the optimal level of internal capital. Recall that increasing capital has two offsetting effects. It decreases the risk of insurer default and thereby increases the expected utility of the insurees, but it also increases the total cost of internal capital, and thereby decreases the expected utility. With many different types of risks, it may therefore be optimal to have several massively multiline insurance firms that all choose different levels of internal capital, instead of one fully multiline company.

To see that the fully multiline outcome may indeed not be in the equilibrium set, even when very many risks are present, consider the following example in which the second but not the third condition of Theorem 3 is satisfied: There are N independent Bernoulli distributed risk classes, $\mathbb{P}(\tilde{L}_i = 1) = p$, $\mathbb{P}(\tilde{L}_i = 0) = 1 - p$, $i = 1, \ldots, N$, and additionally N independent risk-classes with scaled Bernoulli distributions, $\mathbb{P}(\tilde{L}_i = 2) = q$, $\mathbb{P}(\tilde{L}_i = 0) = 1 - q$, $i = N + 1, \ldots, 2N$. All insurees have quadratic utility functions $u(x) = x - \rho x^2$, $x \leq 0$, $\rho > 0$. Here, N is a very large number — so large so that perfect diversification is basically achieved if all N risk classes are insured by the same insurer.[13] The parameters are $p = 19/25$, $q = 1/2$, $\delta = 1/8$, and $\rho = 2$.

In this case, a massively multiline structure in which there are two insurers, one covering all risk classes of the first type, and one covering all risk classes of the second, belongs to the equilibrium set, whereas the fully multiline outcome, in which one insurer covers all risks, does not belong to the equilibrium set. This can be seen in Figure 4. With the massively multiline outcome, capital of $A_1 = N \times 0.060$ is reserved by the insurer who insures the first N (Bernoulli) risks, and $A_2 = N \times 0.74$, is chosen by the insurer who insures the latter N (scaled Bernoulli) risks, achieving expected utilities of $U_1^{MASS} = -1.139$ and $U_2^{MASS} = -1.72$, for agents insuring the first and second risk classes, respectively. These utilities are shown by the two dash-dotted lines in the figure.

Both these outcomes are robust to monoline blocking, since the optimal monoline

[13] We show the asymptotic results, as N tends to infinity. Since all variables almost surely converge to this asymptotic result as N grows, for large enough finite N the asymptotic results hold in large but finite industries too.

offerings achieve lower expected utilities of $U_1^{MONO} = -1.14$ and $U_2^{MONO} = -1.88$, respectively. These utilities of the monoline outcomes, which are shown as solid lines in Figure 4, occur at capital levels of $A_1^{MONO} = 0$ and $A_2^{MONO} = 0.94$, respectively.

The massively multiline outcome is also robust to aggregation. To see this, note that the solid curve in the figure, which represents possible expected utility of the two insuree types, as a function of capital level, is below the massively multiline outcome for the second type of insurees at all capital levels, except for $A = 0.49$, at which point it is lower for the first type of insuree (about -1.18, compared with -1.139 for the massively multiline outcome). Thus, the massively multiline outcome with two insurees does indeed belong to the equilibrium set, $\mathcal{O}$.

To see that the fully multiline outcome does *not* belong to the equilibrium set, note that for all choices of capital, it is either the case that the expected utility of an insuree of type 1 is lower than that obtained in a monoline offering, or the expected utility of an insuree of type 2 is lower, or both. Thus, monoline blocking is always possible, regardless of A, and the fully multiline outcome therefore does not belong to $\mathcal{O}$.

The results in Theorem 3 together suggest that when there is a large number of essentially independent risks that are thin-tailed, a monoline insurance structure is never optimal and that massively multiline industry structures may instead occur. For standard risks — like auto and life insurance — it can be argued that these conditions are reasonable. On the contrary, a multiline structure is more likely to be suboptimal if risks are not numerous, of they are dependent, and/or asymmetric across risk classes. The intuition of why is clear. First, if there are few insurance lines, the diversification benefits may be limited. Second, if risks are dependent and/or asymmetric, the negative externalities faced by policy holders in one insurance line, in case of insurer default are more severe. In both cases, this works against the multiline outcome.

Catastrophe risks, in particular, appear to satisfy all these conditions under which a multiline structure may be suboptimal. Consider, for example, residential insurance against earthquake risk in California.[14] The outcome for different households within this area will obviously be heavily dependent when an earthquake occurs, making the pool of risks essentially behave as one large risk, without diversification benefits. Moreover, many other catastrophic risks are known to have heavy tails. This further reduces the diversification benefits. Thus, even though an earthquake in California and a hurricane in Florida, and a flood on the Mississippi river may be considered independent events, the gains from diversification of such risks may be limited due to their heavy-tailedness. Interestingly, most California earthquake insurance is provided through the quasi-public California Earthquake Authority, most hurricane risk in Florida is reinsured through the state's Florida Hurricane Catastrophe Fund, and most U.S. flood insurance is provided by the National Flood Insurance Program. All three of these entities are monoline in the

[14] See, e.g., Ibragimov, Jaffee, and Walden (2009).

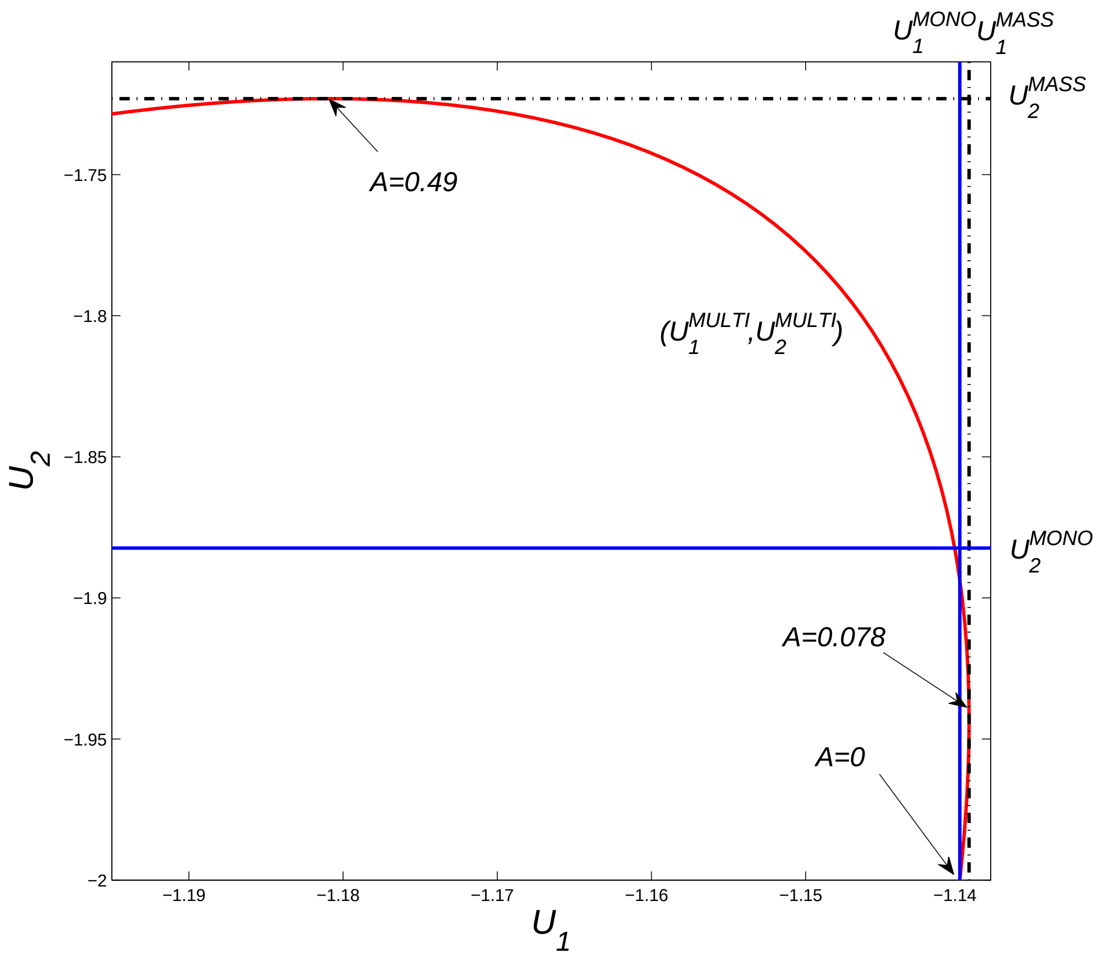

Figure 4: *Example where a massively multiline industry structure belongs to the equilibrium set, but not the fully multiline structure. The massively multiline structures (dash-dotted lines) are robust to monoline blocking (solid lines) since their utility is higher. Further, they are robust to aggregation, since the utility of one type of insurees is always dominated in the fully multiline outcome by at least one monoline offering, regardless of amount of capital (solid curve is always below one of solid lines, regardless of A). Parameters:* $p = 19/25$, $q = 1/2$, $\delta = 1/8$, $\rho = 2$.

sense that they provide insurance or reinsurance only for the designated catastrophe.

5 Concluding remarks

This paper develops a model of the insurance market under the assumptions of limited liability, costly capital and competition. Our unique contribution is to develop a framework to determine the industry structure in terms of which insurance lines are provided by monoline versus multiline insurers. We employ an equilibrium concept in which dominated firm structures are eliminated by new entrants that offer preferred structures. A resulting equilibrium is robust to the entry of any new monoline provider and to aggregation of any two insurance providers. Capital levels and premiums are set optimally for the given equilibrium industry structure.

We derive three important properties for any such equilibrium. First, we show that the multiline structure dominates when the benefits of diversification are achieved because the underlying lines are numerous and independent. Second, even with such numerous independent lines, the industry may be served by several multiline companies, each holding a different amount of capital, as opposed to one multiline company serving all lines. There is thus a limit to the benefit of diversification even in this case with numerous risk factors. Third, the monoline structure may be the efficient form when the risks are difficult to diversify because they are limited in number and/or heavy-tailed, as is characteristic of various catastrophe lines.

Our results are consistent with the observed structure of the insurance industry by lines. Consumer lines such as homeowners and auto insurance are dominated by multiline insurers, while the catastrophe lines of bond and mortgage default insurance are available only on a monoline basis. Furthermore, it is a feature of our model that the default probability for a monoline bond or mortgage default insurer would likely be higher than it is for a multiline firm, offering coverage only on highly diversifiable lines.

Appendix

Proof of Theorem 1: To simplify the notation, in this proof we write L_i instead of $\tilde{L}_i$, L instead of $\tilde{L}$, and μ_i instead of μ_{L_i}. Moreover, C represents an arbitrary positive, finite constant, i.e., that does not depend on N. The condition that u is twice continuously differentiable with u'' bounded by a polynomial of degree q implies that

$$|u''(z)| \le C(1 + |z|^q), \qquad z \le 0, \tag{9}$$

for some constant, $C > 0$.

Moreover, condition $E|L_i|^p \le C$, together with Jensen's inequality implies that $E|L_i - \mu_i|^p \le C$, $\sigma_i^2 \le (E|L_i - \mu_i|^p)^{2/p} \le C$. Using the Rosenthal inequality for sums of independent mean-zero random variables, we obtain that, for some constant $C > 0$,

$$E|\sum_{i=1}^{N}(L_i - \mu_i)|^p \le C \max\Big(\sum_{i=1}^{N} E|L_i - \mu_i|^p, \Big(\sum_{i=1}^{N}\sigma_i^2\Big)^{p/2}\Big), \tag{10}$$

and, thus,

$$\begin{aligned} N^{-p}E|\sum_{i=1}^{N}(L_i - \mu_i)|^p &\le CN^{-p}\max\Big(\sum_{i=1}^{N} E|L_i - \mu_i|^p, \Big(\sum_{i=1}^{N}\sigma_i^2\Big)^{p/2}\Big) \\ &\le CN^{-p/2}\max\Big(N^{-p/2} \times CN, N^{-p/2} \times (CN)^{p/2}\Big) \\ &\le CN^{-p/2} \\ &\to 0, \end{aligned} \tag{11}$$

as $N \to \infty$, where the second to last inequality follows since $p \ge 2$.

Take $A = \sum_{i=1}^{N} \mu_i$. This represents a fully multiline insurer, who chooses internal capital to be equal to total expected losses. Denote

$$y_i = -\mathcal{K}_i(\mathcal{S}) = L_i \max\Big(1 - \frac{A}{\sum_{i=1}^{N} L_i}, 0\Big) = L_i \max\Big(1 - \frac{\sum_{i=1}^{N}\mu_i}{\sum_{i=1}^{N} L_i}, 0\Big).$$

From (5) and (7), the expected utility of insuree i is then $Eu(-\mu_i(1+\delta) + Ey_i - y_i + \delta\mu_i(b_i - 1))$. Here,

$$b_i \stackrel{\text{def}}{=} E\left[\frac{L_i}{\mu_i} \times \frac{\sum_{i=1}^{N}\mu_i}{\sum_{i=1}^{N} L_i} \times \frac{\tilde{V}}{E[\tilde{V}]}\right], \tag{12}$$

and $\tilde{V} = \Big(\sum_{i=1}^{N} L_i - \sum_{i=1}^{N}\mu_{L_i}\Big)_+$, in line with the definition of the binary default option in Section 2.2. Clearly, for large N, $b_i \to 1$ uniformly over the i's, i.e., $\lim_{N\to\infty}\max_{1\le i\le N}|1 - b_i| = 0$, so $Eu(-\mu_i(1 + \delta) + Ey_i - y_i + \delta\mu_i(b_i - 1)) \to Eu(-\mu_i(1 + \delta) + Ey_i - y_i)$ uniformly.

Using a Taylor expansion of order one around $-\mu_i(1+\delta)$, and the polynomial bound, (9), for u'', we get

$$u(-\mu_i(1+\delta) + Ey_i - y_i) = u(-\mu_i(1+\delta)) + u'(-\mu_i(1+\delta))(Ey_i - y_i) + \frac{u''(\xi(y_i))}{2}(y_i - Ey_i)^2,$$
$$\xi(y_i) \in [-\mu_i(1+\delta), -\mu_i(1+\delta) + Ey_i - y_i], \forall y_i.$$

Therefore,

$$Eu(-\mu_i(1+\delta) + Ey_i - y_i) = u(-\mu_i(1+\delta)) + \frac{1}{2}E\left(u''(\xi(y_i))(y_i - Ey_i)^2\right),$$

so

$$|Eu(-\mu_i(1+\delta) + Ey_i - y_i) - u(-\mu_i(1+\delta))| \le C \times E\left((1 + |y_i|^q)(y_i - Ey_i)^2\right) \le C' \times (E|y_i| + E|y_i|^{2+q}).$$

If the right hand side is small, then the expected utility is close to $u(-\mu_i(1+\delta))$. To complete the proof, it thus suffices to show that

$$E|y_i|^{2+q} \to 0 \tag{13}$$

as $N \to \infty$, where the speed of convergence does not depend on i, i.e., the convergence is uniform over i.

By Jensen's inequality, evidently, $E|L_i|^p \le C$ for $p = 2+q+\epsilon$ with $0 < \epsilon \le 2+q$. For such p and ϵ, using the obvious bound $\max\Big(1 - \frac{\sum_{i=1}^N \mu_i}{\sum_{i=1}^N L_i}, 0\Big) \le 1$ and Hölder's inequality, we get, under the conditions of the Theorem,

$$\begin{aligned} E|y_i|^{2+q} = E\Big|L_i \max\Big(1 - \frac{\sum_{i=1}^N \mu_i}{\sum_{i=1}^N L_i}, 0\Big)\Big|^{2+q} &\le E|L_i|^{2+q}\Big(\max\Big(\frac{\sum_{i=1}^N (L_i - \mu_i)}{\sum_{i=1}^N L_i}, 0\Big)\Big)^{\epsilon} \le \\ E|L_i|^{2+q}\Big|\frac{\sum_{i=1}^N (L_i - \mu_i)}{\sum_{i=1}^N L_i}\Big|^{\epsilon} &\le \Big(E|L_i|^p\Big)^{\frac{2+q}{p}} \Big(E\Big|\frac{\sum_{i=1}^N (L_i - \mu_i)}{\sum_{i=1}^N L_i}\Big|^p\Big)^{\epsilon/p} \le \\ C\Big(E\Big|\frac{\sum_{i=1}^N (L_i - \mu_i)}{\sum_{i=1}^N \mu_i}\Big|^p\Big)^{\epsilon/p} &\le C\Big(N^{-p}E\Big|\sum_{i=1}^N (L_i - \mu_i)\Big|^p\Big)^{\epsilon/p}. \end{aligned} \tag{14}$$

From (14) and (11) it follows that (13) indeed holds, and, since the constants do no depend on i, the convergence is uniform over i.

We now go from expected utility to certainty equivalents. Expected utility is $u(-\mu_i(1+\delta)) - \epsilon = u(-\mu_i(1+\delta) - c) = u(-\mu_i(1+\delta)) - cu'(\xi)$, where $\xi \in (-\mu_i(1+\delta) - c, -\mu_i(1+\delta))$, so $c = \epsilon/u'(\xi) \le \epsilon/u'(-\mu_i(1+\delta)) \le \epsilon/u'(0)$, and $\epsilon \to 0$ therefore implies that $c \to 0$. The proof is complete. ∎

Proof of Theorem 2:

By Taylor expansion, for all x, y, $u(x+y) = u(x) + u'(x)y + u''(\zeta)\frac{y^2}{2}$, where ζ is a number between x and $x+y$. Since u'' is bounded away from zero: $-u'' \ge C > 0$, we, therefore, get

$$u(x+y) \le u(x) + u'(x)y - C\frac{y^2}{2} \tag{15}$$

for all x, y. Using inequality (15), in the notations of the proof of Theorem 1, we obtain

$$\begin{aligned} &Eu(-\mu_i(1+\delta) + Ey_i - y_i) \le \\ E\Big[u(-\mu_i(1+\delta)) + u'(-\mu_i(1+\delta))(Ey_i - y_i) - C&\frac{(y_i - Ey_i)^2}{2}\Big] = \\ &u(-\mu_i(1+\delta)) - CVar(y_i). \end{aligned} \tag{16}$$

Consequently, if $u(-\mu_i(1+\delta)) - Eu(-\mu_i(1+\delta) + Ey_i - y_i) < \epsilon$, then $Var(y_i) < \epsilon' = \epsilon/C$. We therefore wish to show that, for a fixed number of risks, N, $Var(y_i) \ge \epsilon_N > 0$.

We need the following two lemmas:

Lemma 1 *For $x_2 \ge x_1$, if X is a random variable, such that $\mathbb{P}(X \le x_1) = a$ and $\mathbb{P}(X \ge x_2) = b$, then $Var(X) \ge \frac{\min(a,b)(x_2 - x_1)^2}{4}$.*

Proof: Define $e = \frac{x_2 - x_1}{2} \ge 0$. Define $M = E(X)$and assume that $M \ge x_1 + e$. Moreover, let ϕ denote X's p.d.f. Then $Var(X) = \int (x-M)^2\phi(x)dx \ge \int_{x \le x_1}(x-M)^2\phi(x)dx \ge \int_{x \le x_1}(x - x_1 - e)^2\phi(x)dx \ge e^2 a = \frac{a(x_2-x_1)^2}{4}$. A similar argument shows that if $M < x_1 + e$, then $Var(X) \ge \frac{b(x_2-x_1)^2}{4}$, and since either $M \ge x_1 + e$ or $M < x_1 + e$ the lemma follows.

Lemma 2 *If X is a random variable, such that $0 \le X \le C < \infty$, $E(X) = \mu$ and $Var(X) = \sigma^2 > 0$, then there are constants $d > 0$ and $\epsilon > 0$, that only depend on C, μ and σ^2, such that $\mathbb{P}(X \le \mu - \epsilon) \ge d$ and $\mathbb{P}(X \ge \mu + \epsilon) \ge d$.*

Proof Let ϕ be X's p.d.f. For a small $\epsilon > 0$, we have

$$\begin{aligned} \sigma^2 &= \int_0^C (x-\mu)^2\phi(x)dx \\ &= \int_{|x-\mu|<\epsilon}(x-\mu)^2\phi(x)dx + \int_{|x-\mu|\ge\epsilon}(x-\mu)^2\phi(x)dx \\ &\le \epsilon^2 + \mu\int_{x\le\mu-\epsilon}|x-\mu|\phi(x)dx + (C-\mu)\int_{x\ge\mu+\epsilon}|x-\mu|\phi(x)dx. \end{aligned} \tag{17}$$

Moreover, $\int_{|x-\mu|<\epsilon}(x-\mu)\phi(x)dx + \int_{|x-\mu|\ge\epsilon}(x-\mu)\phi(x)dx = 0$, and $\left|\int_{|x-\mu|<\epsilon}(x-\mu)\phi(x)dx\right| \le \epsilon$, so

$$\int_{x\ge\mu+\epsilon}|x-\mu|\phi(x)dx + \epsilon \ge \int_{x\le\mu-\epsilon}|x-\mu|\phi(x)dx.$$

Plugging this into (17) yields

$$C\int_{x\ge\mu+\epsilon}|x-\mu|\phi(x)dx \ge \sigma^2 - \epsilon^2 - \mu\epsilon.$$

However, since $\int_{x\ge\mu+\epsilon}|x-\mu|\phi(x)dx \le (C-\mu)\int_{x\ge\mu+\epsilon}\phi(x)dx = (C-\mu)\mathbb{P}(X \ge \mu+\epsilon)$, we arrive at

$$\mathbb{P}(X \ge \mu+\epsilon) \ge \frac{\sigma^2-\epsilon^2-\epsilon\mu}{C(C-\mu)}.$$

A similar argument implies that

$$\int_{x\le\mu-\epsilon}|x-\mu|\phi(x)dx + \epsilon \ge \int_{x\ge\mu+\epsilon}|x-\mu|\phi(x)dx,$$

which, when plugged into (17) yields

$$C\int_{x\le\mu-\epsilon}|x-\mu|\phi(x)dx \ge \sigma^2 - \epsilon^2 - \epsilon(C-\mu),$$

and since $\int_{x\le\mu-\epsilon}|x-\mu|\phi(x)dx \le \mu\int_{x\ge\mu+\epsilon}\phi(x)dx = \mu\mathbb{P}(X \le \mu-\epsilon)$, we arrive at

$$\mathbb{P}(X \le \mu-\epsilon) \ge \frac{\sigma^2-\epsilon^2-\epsilon(C-\mu)}{C\mu}.$$

Thus, by defining

$$d \stackrel{\text{def}}{=} \min\left(\frac{\sigma^2-\epsilon^2-\epsilon\mu}{C(C-\mu)}, \frac{\sigma^2-\epsilon^2-\epsilon(C-\mu)}{C\mu}\right), \tag{18}$$

the lemma follows.

We note that the condition in Theorem 1, that $EL_i \ge C'$ for some $C' > 0$, actually is implied by the conditions that $Var(L_i) > C_1$ and that $L_i \le C_0$, by the following lemma

Lemma 3 *If X is a random variable, such that $0 \leq X \leq C_0 < \infty$, then $E(X) \geq \frac{Var(X)}{C_0}$.*

Proof: Define $e = E(X)$ and let ϕ denote X's p.d.f. We have $Var(X) = \int_0^C x^2\phi(x)dx - e^2 \leq \int_0^C x^2\phi(x)dx \leq C\int_0^C x\phi(x)dx = C \times E[X]$.

Now, from the conditions of Theorem 2, it follows that the L_i's satisfy the conditions of lemma 2, and that ϵ and d can be chosen not to depend on i. Moreover, $y_i = L_i \max\left(1 - \frac{\sum_i \mu_i}{\sum_i L_i}, 0\right)$, so $\mathbb{P}(y_i \leq 0) \geq \mathbb{P}(\sum_i L_i \leq \sum_i \mu_i) \geq \prod_{i=1}^N \mathbb{P}(L_i \geq \mu_i) \geq \prod_{i=1}^N \mathbb{P}(L_i \geq \mu_i + \epsilon) \geq d^N$, where d is defined in (18).

Similarly, if $L_i \geq \mu_i + \epsilon$ for all i, then $y_i = L_i\left(1 - \frac{\sum_i \mu_i}{\sum_i L_i}\right) \geq (\mu_i + \epsilon)\left(1 - \frac{\sum_i \mu_i}{\sum_i \mu_i + N\epsilon}\right) = (\mu_i + \epsilon) \times \left(1 - \frac{1}{1 + \frac{N\epsilon}{\sum_i \mu_i}}\right) \geq \mu_i \times \left(1 - \left(1 - \frac{N\epsilon}{NC}\right)\right) = \mu_i \times \frac{\epsilon}{C}$, so $\mathbb{P}(y_i \geq \frac{\mu_i \epsilon}{C}) \geq \prod_{i=1}^N \mathbb{P}\left(L_i \geq \mu_i + \epsilon\right) \geq d^N$.

Now, from lemma 3, $\mu_i \geq \frac{\sigma^2}{C}$ for all i, so we have $\mathbb{P}(y_i \geq \frac{\sigma^2\epsilon}{C^2}) \geq d^N$. Therefore, y_i satisfies all the conditions of lemma 1, with $x_1 = 0$, $x_2 = \frac{\sigma^2\epsilon}{C^2}$, $a = b = d^N$, and therefore

$$Var(y_i) \geq C_N, \qquad \text{where } C_N = \frac{\sigma^4\epsilon^2}{4C^4}\left(\min\left(\frac{\sigma^2 - \epsilon^2 - \epsilon\mu}{C(C-\mu)}, \frac{\sigma^2 - \epsilon^2 - \epsilon(C-\mu)}{C\mu}\right)\right)^N > 0.$$

The constant, C_N, depends on N, σ^2 and C, but not on the specific distributions of the L_i's, so for a fixed N,

$$Eu(-\mu_i(1+\delta) + Ey_i - y_i) \leq u(-\mu_i(1+\delta)) - C \times C_N. \tag{19}$$

The argument, so far, has been for $Eu(\mu_i(1+\delta) + Ey_i - y_i)$, whereas the utility is $Eu(\mu_i(1+\delta) + Ey_i - y_i + \delta\mu_i(b_i - 1))$, where b_i is defined in (12). However, since by definition $\sum_{i=1}^N b_i = 1$ for all N, it is clear that for all N, there must be an i, such that $b_i - 1 \leq 0$. For such an i, $Eu(\mu_i(1+\delta) + Ey_i - y_i + \delta\mu_i(b_i - 1)) < Eu(\mu_i(1+\delta) + Ey_i - y_i)$, so $Eu(\mu_i(1+\delta)) - Eu(\mu_i(1+\delta) + Ey_i - y_i) > \epsilon \Rightarrow Eu(\mu_i(1+\delta)) - Eu(\mu_i(1+\delta) + Ey_i - y_i + \delta\mu_i(b_i - 1)) > \epsilon$. Thus, the argument also goes through for the utility of agent.

A similar argument as in the proof of Theorem 1, takes us from utilities to certainty equivalents: $u(-\mu_i(1+\delta) - c) = u(-\mu_i(1+\delta)) - \epsilon \Rightarrow c = \epsilon/u'(-\xi)$, $\xi \leq C_0$, so $c \geq \frac{\epsilon}{u'(-C_0)}$, so if ϵ is bounded away from 0, so is c. We are done.

∎

Proof of Theorem 3: We use the same notation as in the proof of Theorem 1. We prove the third result first, since it will help us in proving the other results.

3. For the fully multiline industry structure, with capital $A = \beta\sum_i \mu_i$, the utility of agent i is $Eu(-\mu_i(1+\delta\beta) + Ey_i - y_i)$, where

$$y_i = -\mathcal{K}_i(\mathcal{S}) = L_i \max\left(1 - \frac{A}{\sum_{i=1}^N L_i}, 0\right) = L_i \max\left(1 - \beta\frac{\sum_{i=1}^N \mu_i}{\sum_{i=1}^N L_i}, 0\right). \tag{20}$$

Here, since we we will use the results for non i.i.d. risks when proving (1.) and (2.), we use the i-subscript, even though it is not needed in the i.i.d. case, in which $\mu_i \equiv \mu$.

Assume that $(\mathcal{X}^{MONO}, (A_1, A_2, A_3, \ldots,))$ is a constrained Pareto efficient industry structure. Then, $\mathcal{S}^{MONO} = (\mathcal{X}^{MONO}, (A_1, A_1, A_1, \ldots,))$ is obviously Pareto equivalent, i.e., it provides the same expected utility for all insurees, i, as $(\mathcal{X}^{MONO}, (A_1, A_2, A_3, \ldots,))$ does. Such an industry structure is characterized by $\beta \stackrel{\text{def}}{=} A/\mu$. If we show that, regardless of β, for large enough N, and some β^{MULTI}, the industry partition, $\mathcal{X}^{MULTI}$, with capital, A, where $A = \beta^{MULTI} \times \sum_i \mu_i$, Pareto dominates S^{MONO}, then we are done, since $\mathcal{X}^{MULTI}$ is already per definition robust to aggregation.

For $\beta^{MULTI} = 1$, the argument of Theorem 1 implies that $Eu(-\mu_i(1+\delta\beta^{MULTI}) + Ey_i - y_i) \to u(-\mu_i(1+\delta))$, i.e., the expected utility converges to the ideal risk-free outcome with costly capital, as

N grows, and that the convergence is uniform in i. The utility from insuring with a monoline insurer with $\beta^{MONO} = 1$, on the other hand, is $Eu(-\mu_i(1+\delta) + Ey_i - y_i)$, where (20) implies that $y_i = \max(L_i - \beta^{MONO}\mu_i, 0)$. Now, $Ey_i - y_i$ is obviously second order stochastically dominated (SOSD) by 0, so for $\beta^{MONO} = 1$, the fully multiline partition with $\beta^{MULTI} = 1$, will obviously dominate the monoline offering. Similarly, the multiline partition with $\beta^{MULTI} = 1$ dominates any monoline offering with $\beta^{MONO} > 1$, since there is still residual risk for such a monoline offering *and* the total cost of internal capital is higher — effects that both make the insuree worse off.

It is also clear that for large N, β^{MULTI} for any constrained Pareto efficient outcome must lie in $[0, 1+o(1)]$, since internal capital is costly, which imposes a linear cost of increasing β, and, by the law of large numbers, all risk eventually vanishes for $\beta^{MULTI} = 1$. Thus, there is always a constrained Pareto efficient solution in $[0, 1+o(1)]$, given the fully multiline industry partition.

If we show that, for a given $0 < \beta < 1$, the fully multiline outcome will dominate monoline offerings *with the same* β, then we are done, since, regardless of a candidate β^{MONO}, choosing $\beta^{MULTI} = \beta^{MONO}$ will lead to a Pareto improvement ($\beta^{MONO} = 0$ is strictly dominated by some $\beta^{MONO} > \epsilon/\mu$ from condition 3, so we do not need to consider $\beta^{MONO} = 0$).

For the fully multiline outcome, it is clear from (20), using an identical argument as in the proof of Theorem 1 that y_i converges uniformly in i to $(1-\beta)L_i$, and the expected utility of agent i converges uniformly to

$$Eu(-\mu_i(1+\delta\beta) + (1-\beta)(\mu_i - L_i)). \tag{21}$$

On the other hand, for the monoline offering, the expected utility is

$$Eu(-\mu_i(1+\delta\beta) + Ey_i - y_i), \tag{22}$$

where

$$y_i = \max(L - \beta\mu_i, 0) = (1-\beta)\max\left(\mu_i - \frac{\mu_i - L}{1-\beta}, 0\right). \tag{23}$$

We define $z_i \stackrel{\text{def}}{=} \mu_i - L_i \in (-\infty, \mu_i)$ and $\alpha \stackrel{\text{def}}{=} \frac{1}{1-\beta} \in (1, \infty)$. Equations (21-23) imply that if

$$(1-\beta)z_i \succeq (1-\beta)\left(E\left[\max\left(\mu_i - \alpha z_i, 0\right)\right] - \max\left(\mu_i - \alpha z_i, 0\right)\right) \tag{24}$$

for all $\alpha \in (1, \infty)$, where $\succeq$ denotes second order stochastic dominance, then the third part of Theorem 3 is proved. However, (24) is equivalent to

$$z_i \succeq E\left[\max\left(\mu_i - \alpha z_i, 0\right)\right] - \max\left(\mu_i - \alpha z_i, 0\right),$$

and by defining $x_i \stackrel{\text{def}}{=} \max\left(\mu_i - \alpha z_i, 0\right)$, and $w_i \stackrel{\text{def}}{=} Ex_i - x_i$, to

$$z_i \succeq w_i.$$

We define $q_i(\alpha) \stackrel{\text{def}}{=} Ex_i$. From the definitions in Section 2: $\tilde{Q}(A) = \max(L - A, 0)$ and $P_Q(A) = E[\tilde{Q}(A)]$, it follows that $q_i(\alpha) = \alpha P_Q\left(\left(1 - \frac{1}{\alpha}\right)\mu_i\right)$. Therefore,

$$w_i = q_i(\alpha) - \max\left(\mu_i - \alpha z_i, 0\right). \tag{25}$$

To show that z_i second order stochastically dominates w_i, we use the following lemma, which follows immediately from the theory in Rothschild and Stiglitz (1970):

Lemma 4 *if z and w are random variables with absolutely continuous distributions, and distribution functions $F_z(\cdot)$ and $F_w(\cdot)$ respectively, $z \in (\underline{z}, \overline{z})$, $-\infty \leq \underline{z} < \overline{z} \leq \infty$ (a.s.) and the following conditions are satisfied:*

1. *$Ez = Ew$,*

2. *$F_z(x) < F_w(x)$ for all $\underline{z} < x < x_0$, for some $x_0 > \underline{z}$,*

3. $F_z(x) = F_w(x)$ *at exactly one point,* $x^* \in (\underline{z}, \overline{z})$.

Then $z \succ w$*, i.e.,* z *strictly SOSD* w*.*

Clearly, $Ez_i = Ew_i = 0$, so the first condition of lemma 4 is satisfied. For the second condition, it follows from (25), that for $x < q_i(\alpha)$,

$$\begin{aligned} F_{w_i}(x) &= \mathbb{P}(w_i \le x) = \mathbb{P}(q_i(\alpha) - \mu_i + \alpha z_i \le x) \\ &= \mathbb{P}(z_i \le (x + \mu_i - q_i(\alpha))/\alpha) \\ &= F_{z_i}((x + \mu_i - q_i(\alpha))/\alpha). \end{aligned} \tag{26}$$

Since $\alpha > 1$, it is clear that for small enough x, $(x + \mu_i - q_i(\alpha))/\alpha > x$, and therefore $F_{z_i}(x) < F_{w_i}(x)$, so the second condition is satisfied.

To show that the third condition is satisfied, we need the following lemma

Lemma 5 $q_i(\alpha) \ge \mu_i$ *for all* $\alpha > 1$.

Proof: Denote by ϕ, the probability distribution function (or measure) of L_i. Thus, $\mu_i = \int_0^\infty L\phi(L)dL$. We have, for $r > 0$:

$$\int_0^r L\phi(L)dL \le r\int_0^r \phi(L)dL,$$

which leads to

$$\begin{aligned} \mu_i - \int_0^r L\phi(L)dL \ge \mu_i - r\int_0^r \phi(L)dL \quad &\Leftrightarrow \quad \int_r^\infty L\phi(L)dL \ge \mu_i - r\int_0^r \phi(L)dL \\ &\Leftrightarrow \quad \int_r^\infty L\phi(L)dL + \int_0^r \phi(L)dL - r \ge \mu_i - r \\ &\Leftrightarrow \quad \int_r^\infty L\phi(L)dL - r\left(1 - \int_0^r \phi(L)dL\right) \ge \mu_i - r \\ &\Leftrightarrow \quad \int_r^\infty L\phi(L)dL - r\int_r^\infty \phi(L)dL \ge \mu_i - r \\ &\Leftrightarrow \quad \int_r^\infty (L-r)\phi(L)dL \ge \mu_i - r \\ &\Leftrightarrow \quad \frac{\mu_i}{\mu_i - r}\int_r^\infty (L-r)\phi(L)dL \ge \mu_i \\ &\Leftrightarrow \quad \frac{\mu_i}{\mu_i - r}\int_0^\infty \max(L-r,0)\phi(L)dL \ge \mu_i \\ &\Leftrightarrow \quad \frac{\mu_i}{\mu_i - r}E\left[\max(L-r,0)\right] \ge \mu_i. \\ &\Leftrightarrow \quad \frac{\mu_i}{\mu_i - r}P_Q(r) \ge \mu_i. \end{aligned} \tag{27}$$

Now, for $r < \mu_i$, define $\alpha = \frac{\mu_i}{\mu_i - r} \in (1, \infty)$, implying that $r = \left(1 - \frac{1}{\alpha}\right)\mu_i$. Then, the last line of (27) can be rewritten

$$q_i(\alpha) = \alpha P_Q\left(\left(1 - \frac{1}{\alpha}\right)\mu_i\right) \ge \mu_i.$$

This completes the proof of lemma 5.

Since $z \in (\underline{z}, \overline{z}) = (-\infty, \mu_i)$, and the p.d.f. is strictly positive, $F_{z_i}(\mu_i) = 1$ and, $F_{z_i}(x) < 1$ for $x < \mu_i$. We also note that for $x < q_i(\alpha)$, $F_{w_i}(x) < 1$ is given by (26), and for $x \ge q_i(\alpha)$, $F_{w_i}(x) = 1$.

Since $\overline{z} \le q_i(\alpha)$ (by lemma 5) the only points at which $F_{z_i}(x) = F_{w_i}(x)$ are therefore points at which $F_{z_i}(x) = F_{z_i}((x + \mu_i - q_i(\alpha))/\alpha)$, i.e., for points at which

$$x = \frac{x + \mu_i - q_i(\alpha)}{\alpha} \Rightarrow x = \frac{\mu_i - q_i(\alpha)}{\alpha - 1}. \tag{28}$$

Now, since there is a unique solution to (28), the third condition is indeed satisfied. Thus, z_i SOSD w_i, so a monoline offering is inferior for any insuree with strictly concave utility function, and therefore the third part (3.) of Theorem 3 holds.

2. It was shown in the proof of the third part of the Theorem (3.) above that when the risks are i.i.d., a massively multiline industry partition Pareto dominates the monoline one. There are two complications when extending the proof to nonidentical distributions. The first, main, complication is that insurees may no longer agree on the optimal level of capital, i.e., they have different β_i's. Some insurees may face "small" risks and thereby opt for limited capital, to save on costs of internal capital (a low β), whereas others may wish to have a close to full insurance (a high β). A fully multiline industry structure has only one β, and it may therefore be optimal to have several insurance companies, each with a different β. As the number of lines tends to infinity, however, within each such company the number of lines grows, making the industry structure massively — but not fully — multiline.

The second complication is technical: Since the risks are no longer i.i.d., and each insuree may therefore have a different β, most insurees will not be at their optimal β when insuring with a multiline firm, although they will be close. There is therefore a trade off between the utility loss of being at a suboptimal β, versus the utility gains of diversification, in the multiline partition. We need quantitative bounds, as opposed to the qualitative SOSD bounds in the third part of the Theorem above, that show that when N increases, for most insurees the second effect dominates the first. This will be ensured by condition 2.

If condition 2 fails, even though each insuree may eventually wish to be insured by a multiline industry, for any fixed N, it could still be that most insurees prefer a monoline solution, e.g., if insurees $i = 1, \ldots, N/2$ prefer a massively multiline insurer, whereas insurees $i = N/2 + 1, \ldots, N$ prefer to insure with a fully multiline insurance company. In this case, $M(S) = N/2 + 1$, so the average number of lines converges to 2 and the industry structure is thus not massively multiline.

The expected utility of insuree $i \in X_j$, is $Eu(-\mu_i(1 + \delta\beta) + Ey_i - y_i + \beta\delta\mu_i(b_i - 1))$, where

$$y_i = -\mathcal{K}_i(\mathcal{S}) = L_i \max\Big(1 - \frac{A}{\sum_{i\in X_j} L_i}, 0\Big) = L_i \max\Big(1 - \beta\frac{\sum_{i\in X_j}\mu_i}{\sum_{i\in X_j} L_i}, 0\Big), \tag{29}$$

and

$$b_i \stackrel{\text{def}}{=} E\left[\frac{L_i}{\mu_i} \times \frac{\sum_{i\in X_j}\mu_i}{\sum_{i\in X_j} L_i} \times \frac{\tilde{V}}{E[\tilde{V}]}\right]. \tag{30}$$

A similar argument as in Theorem 1 implies that $b_i \to 1$ uniformly, so for large $|X_j|$, we can study $Eu(-\mu_i(1 + \delta\beta) + Ey_i - y_i) \to Eu(-\mu_i(1 + \delta\beta) + (1 - \beta)(\mu - L_i))$.

The following two lemmas are needed to show the result.

Lemma 6 *Under the conditions of Theorem 3.1, there is a $C > 0$, that does not depend on i, such that for all $0 \le \beta \le 1$,*

$$\left|\frac{\partial Eu(-\mu_i(1 + \delta\beta) + (1 - \beta)(\mu_i - L_i))}{\partial\beta}\right| \le C. \tag{31}$$

Proof Define $Z(\beta) \stackrel{\text{def}}{=} Eu(-\mu_i(1 + \delta\beta) + (1 - \beta)(\mu_i - L_i))$. It immediately follows that Z is concave, so $\max_{0\le\beta\le1} |Z'(\beta)|$ is realized at either $\beta = 0$ or $\beta = 1$. The derivative of Z is

$$Z'(\beta) = E[(-(1 + \delta) + L_i)u'(-\mu_i(1 + \delta\beta) + (1 - \beta)(\mu_i - L_i))],$$

so $|Z'(0)| = |E[((-1 - \delta) + L_i)u'(-L_i)]| \le C(|E[u'(0) + L_i u''(-L_i)| + |E[L_i u'(0)| + L_i^2 u''(-L_i)]| \le C(|u'(0)(\mu_i + 1) + E[(L_i + L_i^2)u''(-L_i)]|) \le C$. Here, the last inequality follows from the assumptions in Theorem 1: Since $|u''(-x)| \le c_1 + c_2 x^q$ and $|E[L_i^2 u''(-L_i)]| \le E[L_i^2(c_1 + c_2 L_i^q)] \le c_1 \times E[L_i^2] + c_2 \times E[L_i^{2+q}] \le C'$.

Thus, $|Z'(0)|$ is bounded by a constant, and moreover, since a Taylor expansion yields that $|Z'(1)| \leq |Z'(0)| + \max_{0\leq\beta\leq 1}|Z''(\beta)|$, as long as $|Z''(\beta)|$ is bounded in $0 \leq \beta \leq 1$ (independently of i), $|Z'(1)|$ will also be bounded by a constant. We have

$$\begin{aligned} |Z''(\beta)| &= |E[(-(1+\delta)+L_i)^2 u''(-\mu_i(1+\delta\beta)+(1-\beta)(\mu_i - L_i))]| \\ &\leq |E[(-(1+\delta)+L_i)^2 u''(-\mu_i(1+\delta)-L_i))| \\ &\leq E[(c_1 + c_2 L_i + c_3 L_i^2)(c_4 + c_5 L_i^q)] \leq C. \end{aligned}$$

The lemma is proved.

Lemma 6 immediately implies that for β close to the optimal β^*, the utility will be close to the utility at the optimum, since $|Z(\beta+\epsilon) - Z(\beta)| \leq C\epsilon$. The second condition, that the utility provided from the asymptotic fully multiline company is bounded away from the utility provided by the monoline insurer, for a fixed β, follows from the following lemma.

Lemma 7 *If condition 2 is satisfied, then for any $\epsilon > 0$, there is a constant, $C > 0$, such that, for all $\beta \in [\epsilon, 1)$,*

$$Eu(-\mu_i(1+\delta\beta)+(1-\beta)(\mu_i - L_i)) - Eu(-\mu_i(1+\delta\beta)+Ey_i - y_i) \geq C. \quad (32)$$

Here, $y_i = \max(L_i - \beta\mu_i, 0)$, is the payoff of the option to default for the monoline insurer, with capital $\beta\mu_i$.

Proof: In the notation of the proof of the third result (3.), equation (32) can be rewritten as $Eu(-c + z_i(1-\beta)) \geq Eu(-c + w_i(1-\beta)) + C$, where $c \stackrel{\text{def}}{=} -\mu_i(1+\delta\beta)$, $z_i \stackrel{\text{def}}{=} (\mu_i - L_i)$ and $w_i \stackrel{\text{def}}{=} \frac{1}{1-\beta}(Ey_i - y_i)$. We know from the proof that z_i SOSD w_i, so $Eu(-c + z_i(1-\beta)) > Eu(-c + w_i(1-\beta))$, but we need a bound that is independent of i.

From (26), we know that

$$F_{w_i} - F_{z_i} = F_{z_i}((x + \mu_i - q_i(\alpha))/\alpha) - F_{z_i}(x) \quad (33)$$

Here, $\alpha = \frac{1}{1-\beta} \in [\frac{1}{\epsilon}, \infty)$. We know from the definition of q_i that $q_i(\alpha) \leq \alpha \times \mu_i$. The conditions of Theorem 1, immediately imply that $\mu_i \leq C$ for some C independent of i, so $q_i(\alpha)$ is bounded for each α, independently of i, by $\alpha \times C_0$. If we denote by F_i, the c.d.f of $-L_i$, then, since $z_i = \mu_i - L_i$, it follows that $F_{z_i}(x) = F_i(x - \mu_i)$. The risk faced by the insuree is $\frac{1}{\alpha}z_i$ in the asymptotic multiline case, and $\frac{1}{\alpha}w_i$ in the monoline case. Therefore, (33) implies that

$$e(x,\alpha) \stackrel{\text{def}}{=} F_i(x + \mu_i - q_i(\alpha)) - F_i(\alpha x)$$

is the difference between the c.d.f.'s of the monoline and fully insured offerings for largely negative x. Since $q_i(\alpha) \leq C_0\alpha$, $e(x,\alpha) \geq F_i(x - C_0\alpha) - F_i(\alpha x)$. we can choose x negative enough so that $h(-x) < \frac{\alpha x - 1}{x - C_0\alpha}$. Then, for $x' < x$, from condition 2, it follows that

$$e(x',\alpha) \geq g(-h(-x')(x' - C_0\alpha)) - g(-\alpha x') \geq g(-\alpha x' - 1) - g(-\alpha x').$$

This bound is independent of i, and since g is continuous and strictly decreasing, for each α it is the case that on $[-x-1, -x]$,

$$F_i(x + \mu_i - q_i(\alpha)) - F_i(\alpha x) > \epsilon_\alpha > 0. \quad (34)$$

We have, for a general risk, R, with c.d.f. F_R, and support on $(-\infty, r)$, where R is thin-tailed enough so expected utility is defined,

$$Eu[R] = \int_{-\infty}^{r} u(x)dF_R = u(r) - \int_{-\infty}^{r} u'(x)F_R(x)dx. \quad (35)$$

Now, since u is strictly concave, u' is decreasing, but positive, so if, $E[R_1] = E[R_2]$, $F_{R_1} > F_{R_2} + \epsilon$, on $[a,b]$, $\epsilon > 0$, and if F_{R_1} and F_{R_2} only cross at one point, then $Eu[R_1] - Eu[R_2] = \int_{-\infty}^{r} u'(x)(F_{R_2}(x) -$

$F_{R_1}(x))dx \geq (\inf_{x\leq r}|u''(x)|) \times \epsilon \times \frac{b-a}{2}$.

Therefore, (34), implies that $Eu(-c+z_i(1-\beta)) \geq Eu(-c+w_i(1-\beta))+C_\alpha$, and since ϵ_α is continuous in α, we can take the minimum over $\alpha \in [1, 1/\epsilon]$, to get a bound that does not depend on α. The lemma is proved.

We have thus shown that, away from $\beta = 0$, the asymptotic fully multiline solution is uniformly better (in i) than the monoline solution, for each β (lemma 7), and that, as long as a β close to the optimal β is offered, the decrease in utility for the insuree is small (lemma 6). This, together with the uniform convergence toward the asymptotic risk as the number of lines covered by an insurer grows, implies that there is an $\epsilon^* > 0$ and a $C^* < \infty$, such that any firm with a β within ϵ^* distance from the optimal β for the monoline offering, and with at least C^* insurees, will make all insurees strictly better off than the monoline insurer.

Now, since $\beta \in [\epsilon, 1]$ is in a compact interval, as the number of lines grows, most insurees will be close to many insurees, making massively multiline offerings feasible for the bulk of the insurees. In fact, defining $T_N(i, \epsilon)$ to be the number of insurees, who have β's within ϵ distance from insuree i, i.e., for which $|\beta_i - \beta_j| \leq \epsilon$, in the economy with insurees $1 \leq i \leq N$, it is the case that $\forall \epsilon > 0$, and $\forall C < \infty$, all but a bounded number of agents belong to $\{i : T_N(i, \epsilon) \geq C\}$, due to a standard compactness argument. If this were not the case, there would be a sequence of insurees, i_j, $j = 1, \ldots$, with β's not close to other insurees, and since such a sequence must have a limit point, there must be an agent with more than C neighbors within ϵ distance, contradicting the original assumption.

It is clear from the previous argument that for agents in $\{i : T_N(i, \epsilon) \geq C\}$, as N grows, multiline solutions will be optimal. Specifically, for $\epsilon = \epsilon^*$, a sequence of $C \to \infty$ can be chosen, and an N large enough, such that $\{i : T_N(i, \epsilon) \geq C\}/N > 1 - \delta$, for arbitrary $\delta > 0$. Then it follows immediately that all insurees can be covered by insurers with at least $C/2$ insurees, where insurer n chooses $\beta_n = n\epsilon^*/2$. This leads to a massively multiline structure, since the average number of insurees is greater than or equal to $(1 - \delta)C/2$, and, as we have shown, it Pareto dominates the monoline industry partition. We are done.

2. The Theorem follows immediately from the compactness argument made toward the end of the proof of the second result, (2.). If there would be a sequence of insurees, i_j, $j = 1, \ldots$, for which it is Pareto optimal to insure with a monoline insurer, there must be a limit point, i.e., a $\beta^* \in [\epsilon, 1]$ for which, for each $\epsilon > 0$, there is an infinite number of insurees — each insuree having optimal capital β_{i_j} — such that $|\beta_{i_j} - \beta^*| < \epsilon$. However, the argument in (2.) then immediately implies that a multiline firm with $\beta^{MULTI} = \beta^*$ can make a Pareto improvement, contradicting the original assumption that the industry partition robust to aggregation.

4. Since, from Theorem 2, with $\beta = 1$ (i.e. with $A = \sum_i \mu_i$), the fully multiline outcome converges to the ideal risk-free outcome with costly internal capital, and this outcome strictly dominates any offering that can be provided by a monoline insurer, the fourth part (4.) follows immediately.

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