Global Lateral Buckling of I-Shaped Girder Systems

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Abstract: A closed form solution for elastic global buckling of twin girder systems interconnected with cross frames is derived. Current design specifications for such systems only consider individual girder buckling between cross frames. The solution, which is suitable for design specifications, was developed for a uniform moment loading condition. Finite-element analyses (FEAs) were used to verify the closed form solution and extend it to more practical loading conditions. FEA showed that the load height condition had only a minor effect for twin girders compared to the published effects on single girders. Both singly and doubly symmetric sections were studied and showed that the girder spacing and the in-plane moment of inertia of the girders are the principal variables controlling global buckling of twin girders. The number and size of the intermediate cross frames had little effect. A method for improving the global buckling capacity through the use of a partial top flange lateral bracing system is presented along with a design example.

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Introduction

Lateral-torsional buckling (LTB) of a girder is a failure mode that involves lateral movement and twist of the girder cross section. The classic solution for LTB of a simply supported girder (Timoshenko and Gere 1961) bent about the strong axis by a uniform moment, M_o, is

$$M_o = \frac{\pi}{L_b}\sqrt{EI_yGJ + \frac{\pi^2E^2I_yC_w}{L_b^2}} \tag{1}$$

where, L_b=distance between points along the length where twist is prevented; E=modulus of elasticity; G=shear modulus (usually taken as $E/2.6$ for metals); I_y=weak axis (lateral) moment of inertia; J=torsional constant; and C_w=warping constant. For I-shaped girders $J=\Sigma(wt^3/3)$, where w and t are the respective width and thickness of each of the plate elements that make up the girder cross section. For a doubly symmetric I-shaped section $C_w=I_y(h_o/2)^2$, where h_o is the distance between flange centroids. It should be noted that Eq. (1) was derived using only the no twist condition at each end of the unbraced length. Pure lateral displacement of the cross section at the brace points does not affect the critical buckling moment.

The design equations for LTB of doubly symmetric beams in bridge and building design specifications are based on Eq. (1). In many applications, braces are used to reduce L_b thus increasing M_o so that yielding, not buckling, controls the strength of the girder. Although a lateral truss system can be used to stabilize the compression flange of girders, diaphragms or cross frames interconnecting two girders spaced at a distance S as shown in Fig. 1 are frequently used. Properly designed cross frames as shown in Fig. 1(c) act as torsional braces (restrain twist) that force compression flange lateral movement of individual girders to occur between braces as shown by the dashed line in Fig. 1(a). Design methods for torsional braces are given in AISC (2005) and Yura (2001).

Current design specifications only consider the lateral buckling of individual girders. Although the common design practice is to evaluate the lateral buckling capacity of the girder between braces, a lower global system buckling mode over the span L_g represented by the dotted lines in Fig. 1(b) may occur depending on the geometry of the girders. The global buckling of the girder system has traditionally not been a design consideration because it appears that system bending is about the weaker x-x axis. When bending occurs about the smaller of the two principal axes of a doubly symmetric system (both cambered and uncambered), lateral buckling will not be a practical design consideration. A history and critique of this concept which initiated more than 100 years ago with Michell (1899), is given by Yura and Widianto (2005). If there is a properly designed lateral diagonal truss system connecting the two girders, then the system moment of inertia about the y-axis, I_{ys}, will be greater than the system x-axis moment of inertia, $I_{xs}=2I_x$. However, if only cross frames interconnect the two girders, $I_{ys}\approx 2I_y$, so $I_{xs}>I_{ys}$ and global buckling is a possibility.

Some recent bridge failures during construction have been attributed to global lateral buckling. The Marcy Pedestrian Bridge was a U-shaped girder with internal K-frames connected to the two webs that failed during casting of the concrete deck (Weidlinger 2003). The authors are also aware of a bridge widening project in Texas where a twin girder with several cross frames

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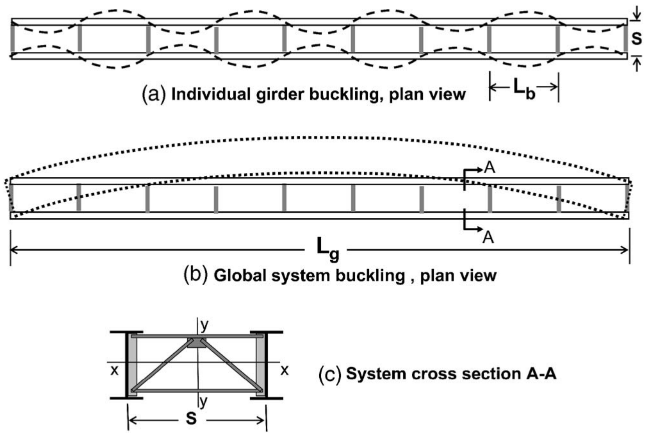

Fig. 1. Twin I-girder system

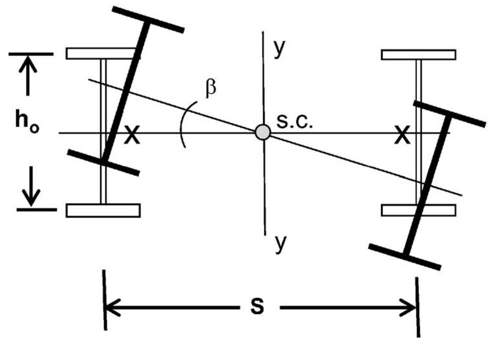

Fig. 2. System twist

twisted significantly during the deck pours. Multigirder systems interconnected by cross frames or diaphragms with large span-to-width ratios are susceptible to global lateral buckling. Although twin girder systems are the most susceptible to the global mode, systems with more girders can also fail globally if the girders are relatively closely spaced.

Komatsu et al. (1983) presented a global solution for twin girders interconnected by diaphragms. The out-of-plane rigidity of the diaphragm was the controlling variable (Vierendeel action). If the diaphragm connection detail provides zero out-of-plane restraint, the Komatsu global solution resolves to the sum of the buckling capacity of the two individual girders. The in-plane stiffness of the diaphragms is ignored.

The purpose of this paper is to develop a method for determining the global lateral buckling capacity of multigirder systems interconnected by cross frames suitable for use in design specifications. In the next section the global lateral buckling solution for a doubly symmetric twin girder system subjected to uniform moment is derived. The girders are interconnected by cross frames or diaphragms that are assumed to prevent relative twist between the girders. In the later sections of the paper, finite-element analyses (FEA), are used to study the effects of distributed load, load height, in-plane cross frame stiffness, and girder monosymmetry. Adjustments to the basic solution are developed to account for these effects. Although twin girder systems are particularly susceptible to the system mode, systems with large span/width ratios having more than two girders were also considered in the investigation. An example problem in the Appendix illustrates the application of the design recommendations. Finally a technique for improving the global lateral buckling capacity through the selective application of a few top flange diagonal braces is discussed.

Derivation of Basic Solution

The differential equations of lateral bending and torsion used by Timoshenko to derive Eq. (1) are equally applicable for the doubly symmetric twin girder system shown in Fig. 1(c). The only variation necessary is the evaluation of the lateral bending rigidity and torsional rigidity of the system. It is assumed that the cross frames are sufficiently stiff in their plane to maintain the same angle of twist for both girders as shown in Fig. 2 (cross frame omitted). It is also assumed that the cross frame-girder connection details are pinned so there is no cross-frame Vierendeel effect. With these assumptions, the cross frames will not affect the lateral bending system rigidity, which is $2EI_y$.

Referring to Fig. 2, the torsional resistance of the system has three components: shear stresses in the individual plates (St. Venant), lateral flange bending (warping), and vertical I_x bending (system warping). The St. Venant rigidity is $2GJ$. The total warping rigidity (two girders plus system) is

$$2E\frac{I_y h_o^2}{4} + 2E\frac{I_x S^2}{4} \tag{2}$$

Substituting $2EI_y$, $2GJ$, and Eq. (2) for the EI_y, GJ, and EC_w, respectively, in Eq. (1) gives the total elastic global lateral buckling moment of the twin girder system, M_g

$$M_g = 2\frac{\pi}{L_g}\sqrt{EI_y GJ + \frac{\pi^2 E^2 I_y}{4L_g^2}(I_y h_o^2 + I_x S^2)} \tag{3}$$

All the section properties in Eq. (3) are those of the single girder. Eq. (3) is valid for girders that have twist restrained at the ends of the girders and warping deformation unrestrained. The global buckling moment given by Eq. (3) is similar to Eq. (1) except for the addition of the $I_x S^2$ term. For many twin girder geometric arrangements, the $I_x S^2$ term dominates. Retaining only the term containing $I_x S^2$ under the radical gives the following conservative estimate of the global buckling moment

$$M_{gs} = \frac{\pi^2 SE}{L_g^2}\sqrt{I_y I_x} \tag{4}$$

The simplified global buckling moment, M_{gs}, provides a reasonable estimate of the buckling capacity and is recommended for determining if global buckling is a concern. For the twin girder system, the global critical moment per girder is one half of the value given by Eqs. (3) or (4).

In Table 1 the results from Eqs. (3) and (4) are compared with independent buckling results from the ANSYS (2003) finite-element program for a twin girder system subjected to uniform moment. The girders were simply supported with a span of

Table 1. Global Lateral Buckling Solutions

	Buckling stress (MPa)		
	Girder spacing S		
Analysis type	2.03 m	2.77 m	3.81 m
FEA	147	197	267
Eq. (3)	148	196	270
Eq. (4)	143	195	268

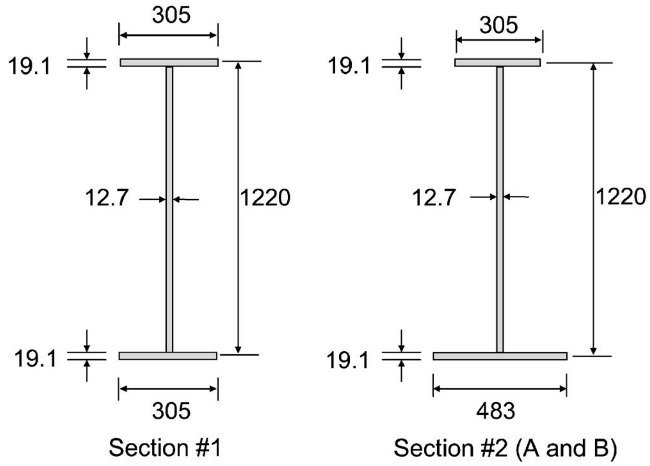

Fig. 3. Cross sections studied (mm)

51.8 m. The cross section consisted of 810 mm × 18.7 mm flange plates and a 1,925 mm × 25.4 mm web plate. The span, depth, I_x, and cross-frame spacing are similar to the Marcy girder. The critical buckling moments for the three girder spacings shown in Table 1 were determined and converted to buckling stresses by dividing by the section modulus. The results from ANSYS and Eq. (3) are nearly identical. On average, the simplified Eq. (4) is only 1.2% conservative. All the buckling stresses are below the yield stress of typical bridge materials indicating that global buckling is a serious issue for the example shown. Eq. (4) indicates that the global lateral buckling strength is directly proportional to the girder spacing. When the girder spacing is less than the girder depth, Eq. (3) should be used because M_{gs} will become too conservative. Eqs. (3) and (4) were derived for the case of uniform moment and doubly symmetric sections. In the following sections, adjustments are developed through FEA analyses to account for more practical loading conditions and singly symmetric sections. The interaction between cross-frame stiffness and global lateral buckling will also be demonstrated.

Finite-Element Model

The finite-element program ANSYS (2003) was used to study the buckling behavior of twin girder systems with cross frames for bracing. Linear elastic materials were utilized in all analyses. The cross sections of the two girders that were studied are shown in Fig. 3. Section #1 is a doubly symmetric section, while Section #2 is a singly symmetric section. Analyses were conducted with two orientations for the singly symmetric section: (A) the small flange subjected to compression; and (B) the large flange subjected to compression. The cross sections of the girders were modeled using eight-node shell elements: two shell elements for each flange and four elements through the web depth. The number of elements along the girder length was typically selected so that the element aspect ratios were as close to unity as possible. Shell elements were also used to model transverse web stiffeners at supports and at the locations of concentrated loads.

Twin girder systems with simple supports were analyzed with loading that caused compression in the top flange. Although twist was restrained by cross frames at the ends of the girders, the cross sections were free to warp. In addition to uniform moment loading, two other loading cases were considered in the investigation: a single concentrated load applied at midspan, and a uniformly distributed load applied along the member length. Analyses were conducted with points of load application located at the top flange, midheight, and bottom flange. Several different values of the cross frame spacing along the girder length were studied.

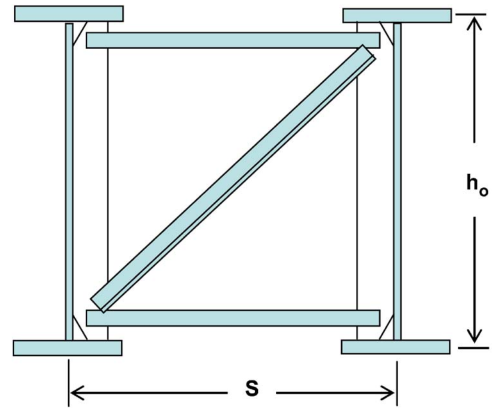

Fig. 4. Tension-only cross frame

The single diagonal cross frame shown in Fig. 4 was modeled using truss elements, thus eliminating Vierendeel action. Although most cross frames have two diagonals, the members that are employed often consist of angles, which have relatively low buckling strengths. The "tension-only" cross-frame system conservatively neglects the compression diagonal and its relatively low buckling strength. The cross frames were full depth members that framed into the girders at the top and bottom of the web. The three members comprising the cross frame had the same area. Although span-to-depth ratios of 15–25 were considered, the results presented will focus on systems with span-to-depth ratios of 20. Similar trends were observed with the other span-to-depth ratios that were analyzed.

Twin Girder FEA Results

Buckling analyses were conducted on different twin girder systems with varying cross-frame stiffness. The stiffness of a single diagonal cross frame, β_b, is (Yura 2001)

$$\beta_b = \frac{A_b E S^2 h_o^2}{2L_d^3 + S^3} \tag{5}$$

where A_b = area of each cross-frame horizontal and diagonal member; and $L_d = \sqrt{h_o^2 + S^2}$ = length of the diagonal. For a given geometry, the cross-frame stiffness is directly proportional to A_b. The critical buckling moments of the twin girder system, M_{cr}, are nondimensionalized by the buckling moments of the two individual girders $2M_o$ given by Eq. (1) with L_b taken as the span length. In all cases considered in this section, the span length of the simply supported girders was 24.4 m. At the end supports, twist and lateral movement are prevented but warping is permitted.

Doubly Symmetric Section

Effect of Diaphragm Stiffness

Fig. 5 shows FEA results that demonstrate the effect of β_b on the LTB of twin girder systems with girders spaced at 1.52 and 3.05 m. In both cases, there were three evenly spaced intermediate cross frames. The twin girders were subjected to uniform mo-

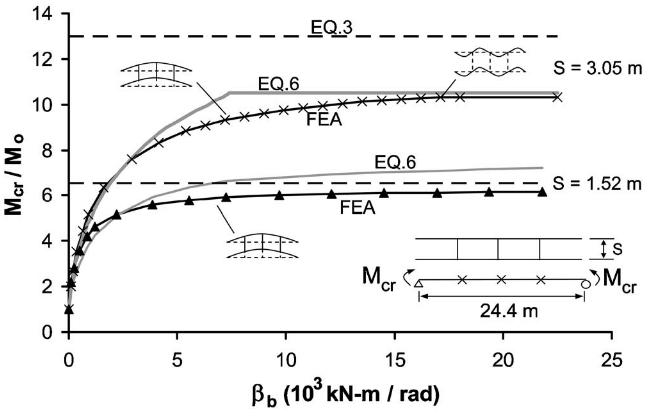

Fig. 5. Effect of cross-frame stiffness

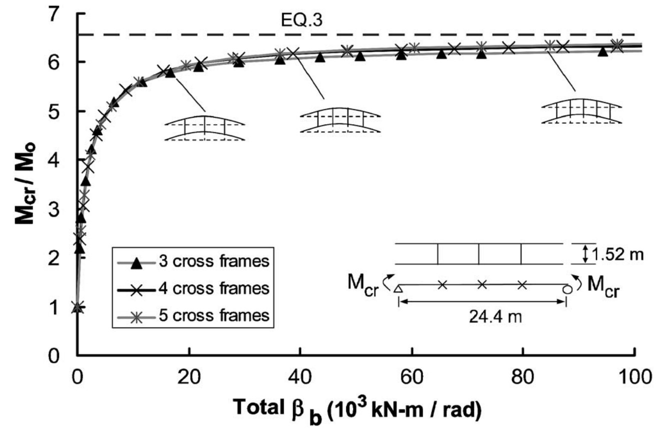

Fig. 6. Effect of number of cross frames

ment. Sketches depicting the plan view of the FEA buckled shape are shown on the graph. For $S=1.52$ m, the FEA model always buckled in a global half-sine buckled shape. For very small values of β_b, the system behaves as two independent girders, governed by Eq. (1). As the cross-frame stiffness increases the critical moment approaches the dashed line given by Eq. (3). In the derivation of Eq. (3), the diaphragms were assumed very rigid all along the length (no distortion as depicted in Fig. 2). At $\beta_b=20{,}000$ kN m/rad, the FEA results are 6% smaller than predicted by Eq. (3). Cross frames constructed with relatively small $L51\times51\times6.4$ (United States $L2\times2\times0.25$) angle sections provide this level of stiffness. Cross frames with five times this stiffness (100,000 kN m/rad) only increase the critical moment by 0.6%.

When the girder spacing increases from 1.52 to 3.05 m, the global buckling moment given by Eq. (3) is almost doubled. However, the twin girder system never gets to this load level. The FEA results show that when $\beta_b\geq17{,}000$ kN m/rad, buckling of the compression flange occurs between the diaphragms instead of global buckling. M_{cr} for buckling between cross frames is also predicated by Eq. (1) with $L_b=24.4\text{ m}/4=6.1$ m.

The global mode of buckling can also be predicted from the LTB equation for a beam with continuous torsional bracing along the length in Yura (2001)

$$M_g=\sqrt{(M_o^2)+EI_y\bar{\beta}_T}\leq M_1 \tag{6}$$

where $\bar{\beta}_T$=effective torsional brace stiffness per unit length of girder. For the number (n) of intermediate cross frames, $\bar{\beta}_T=n\beta_T/L_g$. M_g is limited to M_1, the moment corresponding to buckling between cross frames. Eq. (6) is based on the Taylor and Ojalvo (1966) torsional bracing solution with the brace stiffness adjusted to account for in plane flexibility of the girders (Helwig et al. 1993). The relationship between β_b and β_T is given by

$$\frac{1}{\beta_T}=\frac{1}{\beta_g}+\frac{1}{\beta_b} \tag{7}$$

where $\beta_g=12S^2EI_x/L^3$. In Fig. 5 the results from Eq. (6) follow the trend of the FEA results and provide reasonable estimates of the buckling capacity. For the system mode of buckling that occurred with $S=1.52$ m, Eq. (6) overestimates the capacity relative to Eq. (3) by approximately 10%.

Increasing the number of cross frames for the case of $S=1.52$ m does not significantly affect the global buckling capacity as illustrated in Fig. 6. The horizontal axis shows the total cross frame stiffness applied to the span ($\beta_b\times n$). The curves for three, four, and five cross frames are almost identical, indicating that for global buckling the sum of the cross frame stiffness is the dominate variable and not the actual number or spacing of the cross frames.

Loading Conditions

Eqs. (1) and (3) were derived for beams subjected to uniform moment. Distributed and concentrated loads produce moment gradients along the span. For single girders, Eq. (1) is typically modified by a C_b factor to account for moment gradient. For uniform vertical load applied at the centroid $C_b=1.12$; for a concentrated vertical load at midspan, $C_b=1.35$ (SSRC 1998).

FEA were conducted on the $S=1.52$ m, three cross-frame system shown in Fig. 5 with uniformly distributed vertical load and with a single concentrated vertical load at midspan. The cross-frame member area was varied and the relationship between M_{cr} and β_b followed the trend shown in Fig. 5. The ratio of M_{cr} for the distributed load to M_{cr} for uniform moment was 1.12, which is exactly the same as the C_b value for single girders. Similarly, the M_{cr} ratio for the concentrated load case was 1.35. Therefore, Eqs. (3) and (4) can be adjusted directly by the published C_b values for single girders.

The position of the load on the cross section for single girders can have a significant effect on the LTB capacity. Top flange loading reduces M_{cr} for centroid loading by an approximate factor of $1/1.4=0.7$ (Helwig et al. 1997; SSRC 1998), whereas bottom flange loading improves the buckling strength by a 1.4 factor. To study the effects of top and bottom flange loading conditions, finite-element analyses for the $S=1.52$ m twin girder system were performed. The ratio of top flange M_{cr} to centroid M_{cr} was 0.956 for the distributed load and 0.952 for the concentrated load at midspan. The top flange loading effect was less than 5%. Similar results were obtained for bottom flange loading. Thus for doubly symmetric twin girder systems, the load height effect can be ignored.

Singly Symmetric Sections

The LTB formula for a doubly symmetric single girder given by Eq. (1), cannot be used for a singly symmetric section when the smaller flange is in compression as depicted in Fig. 7. The approach given in AASHTO (2002) in which Eq. (1) is used but with $2I_{yc}$ substituted for I_y where I_{yc} is the moment of the compression flange about the y-y axis is fairly accurate compared to

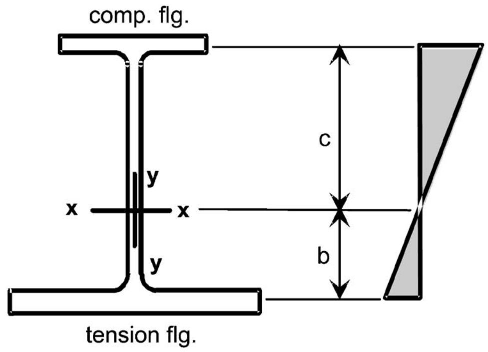

Fig. 7. Singly symmetric I-shape

the exact solution by Galambos (1968). For torsionally braced singly symmetric girders, Yura (2001) showed that an effective moment of inertia, $I_{\text{eff}}=I_{yc}+(b/c)I_{yt}$, should be substituted for I_y in the bracing term of Eq. (6) where I_{yt}=moment of inertia of the tension flange and b and c=distances from the centroidal axis to the tension and compression flanges, respectively (see Fig. 7). Substituting $2I_{yc}$ and I_{eff} into Eq. (3) and including the C_b factor gives the following design formula for global LTB for both singly and doubly symmetric twin girders:

$$M_{\text{gl}} = 2C_b \frac{\pi E}{L_g} \sqrt{\frac{I_{yc}J}{1.3} + \frac{\pi^2 I_{yc}^2 h_o^2}{L_g^2} + \frac{\pi^2 I_{\text{eff}} I_x S^2}{4L_g^2}} \qquad (8)$$

or conservatively

$$M_{\text{gls}} = C_b \frac{\pi^2 SE}{L_g^2} \sqrt{I_{\text{eff}} I_x} \qquad (9)$$

FEA solutions and Eq. (8) are compared in Fig. 8 for twin girder Sections #2A and #2B subjected to uniform moment. The girders are spaced at 1.52 m as in the previous cases. Four intermediate cross frames were required to avoid buckling between the cross frames. At β_b=50,000 kN m/rad, which represents a practical brace stiffness that is easily attainable with relatively small structural shapes, Eq. (8) is within 1% of the FEA solutions. Top flange uniform loading for Sections 2A and 2B give ratios of Eq. (8)/FEA of 1.08 and 1.06, respectively. Midspan concentrated loads applied at the top flange for Sections 2A and 2B gave ratios of Eq. (8)/FEA of 1.15 and 1.08, respectively. The top flange loading effect for the concentrated vertical load at midspan is more significant (15% when the compression flange is the smaller flange) than the top flange loading effect noted earlier for the doubly symmetric section (5%). Therefore, for singly symmetric sections loaded at the top flange, reasonable accuracy of the critical global buckling can be obtained by using $0.9M_{\text{gl}}$ from Eq. (8).

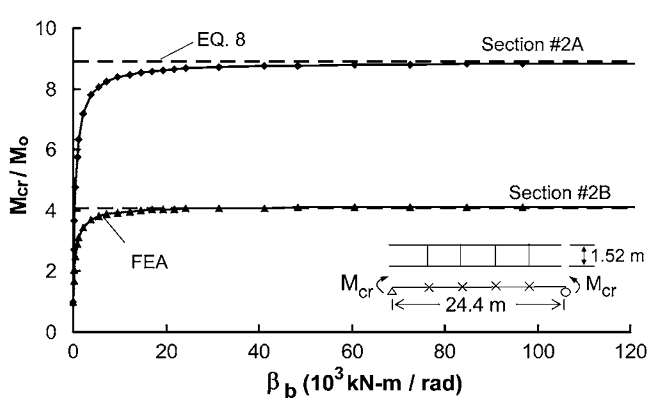

Fig. 8. Singly symmetric global buckling

The failure of the Marcy Pedestrian Bridge due to system buckling was mentioned in the "Introduction" of the paper. The Marcy Bridge consisted of a trapezoidal box girder section that did not have a top flange lateral truss. Eq. (8) could be directly applied to an open box girder system such as the Marcy Bridge with reasonable accuracy provided only half of the bottom flange is used in the calculation of the I_{eff}. However, box girder systems (straight and curved) should be designed with top flange lateral trusses to improve the torsional stiffness of each girder. The stiffness of the quasi-closed box girder system will be extremely high and girder stability from the system mode of buckling will generally not be an issue.

Systems with More Than Two Girders

Although twin girder systems are the most susceptible to the system global mode, finite-element analyses showed that global LTB can also occur on systems with three or more closely spaced I-girders. It was found that Eq. (8), which was derived for twin girder systems, could be adjusted for three or more girders. For a three girder system, replace I_{yc} and J in Eq. (8) with $3/2I_{yc}$ and $3/2J$, and define S as $2S$, which is the distance between the two exterior girders. For four girders, replace the I_{yc}, J, and S terms in Eq. (8) with $2I_{yc}$, $2J$, and $3S$, respectively. In the simplified formulations, Eqs. (4) and (9), substitute the distance between exterior girders for S.

Improving Global LTB Strength

When global LTB controls the strength of I-girder systems interconnected with cross frames, options for increasing the strength are somewhat limited. Based on Eq. (9), the global LTB strength can be improved by increasing either the girder spacing S or the cross-section moment of inertia. Geometry may limit the increase in S and an increase in cross section for a condition that exists only during the construction stage may not be economical. An alternative is to add a top-flange diagonal system for a few panels at the supports. For U-shaped steel girders (trapezoidal steel girders) such as those used in the Marcy Bridge (Weidlinger 2003), top flange diagonal bracing applied over a distance of $0.2L_g$ at each end provided warping end restraint corresponding to "fixed ends" (Yura and Widianto 2005). With such a partial top flange diagonal system, Eqs. (8) or (9) can be used to predict the global LTB strength, M_{glw}, by substituting an effective length to account for the end restraint. A true fixed end condition would correspond to an effective length of $0.5L_g$ but would require diagonals with infinite stiffness. Practical designs for the diagonal would result in some flexibility so an effective length of $0.60L_g$ in Eq. (8) is recommended based on FEA. For beams with lateral end restraint, C_b=1.0 for all loading conditions (SSRC 1966).

For global buckling with pinned end conditions, the slope of the buckled shape at the end supports is $y'=\pi\Delta/L_g$ where Δ is the lateral displacement at midspan (Timoshenko and Gere 1961). Assuming that the end slope is reduced to $0.1y'$ by the warping restraint provided by a partial top flange diagonal systems as shown in Fig. 9(a), the end warping stiffness requirement, M_{ws} (kN m/rad), is

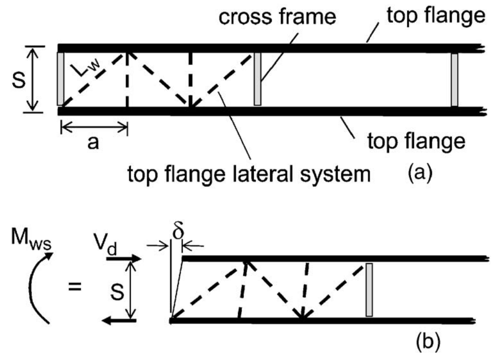

Fig. 9. End warping restraint

$$M_{\mathrm{ws}} = \frac{3(M_u - M_{\mathrm{gl}})L_g}{h_o} \tag{10}$$

where M_u=factored design moment requirement; and M_{gl}=global strength without warping restraint from Eq. (8). The warping moment stiffness can be transformed into a top flange shear stiffness, V_d/δ, [see Fig. 9(b)] by noting that $M_{\mathrm{ws}} \equiv V_d/S$ and the end slope is δ/S. Thus, $V_d/\delta = M_{\mathrm{ws}}/S^2$, where δ=relative end displacement of the two girders. For a single diagonal system, the truss member stiffness area requirement, A_d, is determined from

$$\frac{\Sigma A_d a^2 E}{L_w^3 + S^3} = \frac{M_{\mathrm{ws}}}{S^2} \tag{11}$$

where $\Sigma A_d = mA_d$ with m=number of braced panels; a=panel width; and L_w=diagonal length as shown in Fig. 9(a). Eq. (11) assumes that the struts and diagonals of the top lateral system have the same area.

Second-order analyses established that the shear forces generated in the diagonals of the partial top lateral bracing system are related mainly to the magnitude of the initial out-of-straightness of the compression flange and the ratio of M_u/M_{glw}. However, the development of an exact analytical expression for the brace forces is beyond the scope of this paper. Lacings in built-up columns are designed for a shear component of 2% of the total column force (AISC 2005). Finite-element studies of some sample designs showed that the 2% concept could be applied to the partial top flange diagonal system resulting in the following brace strength requirement:

$$F_d = 0.02\frac{M_u}{h_o}\frac{L_w}{a} \tag{12}$$

where F_d=force in the diagonal. The area required for the diagonal to satisfy Eq. (12) must be compared to the area required to satisfy Eq. (11) to determine whether stiffness or strength controls the member size. If a diagonal bracing system is used in the negative moment regions of continuous girders, the force in the diagonals caused by in-plane bending of the girders (Fan and Helwig 1999), should be added to the brace force requirement from Eq. (12).

A design example illustrating the application of the global buckling equation is given in the Appendix. The use of a partial top flange lateral truss systems to improve the global strength is also considered.

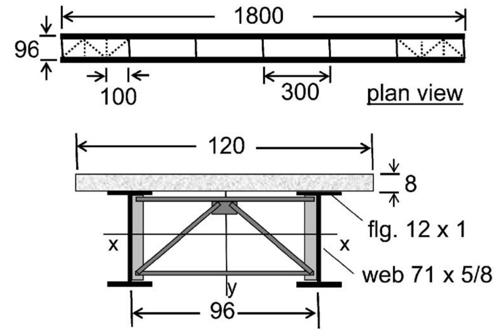

Fig. 10. Example (all dimensions are inches)

Summary and Conclusions

Twin steel girders supporting a concrete slab and interconnected with diaphragms or cross frames are susceptible to global lateral-torsional buckling during construction. Current building and bridge design specifications for lateral buckling are only applicable for single girders. Some bridge failures have been attributed to global buckling. A closed form (global buckling) solution, Eq. (3), was derived for a system subjected to uniform moment. The dominating variables controlling global buckling were the distance between the girders and the strong axis moment of inertia of the individual girders. Finite-element analyses were used to verify the closed form solution and investigate other loading and construction conditions. The following are findings from the finite-element studies of doubly symmetric twin girders:

1. The closed form solution Eq. (3) was verified;
2. The C_b factors for uniform loading (C_b=1.12) and for a concentrated load at midspan (C_b=1.35) for single girders are applicable for the twin girder system;
3. Within practical ranges of design, the size and spacing of the cross frames has little effect on global buckling; and
4. Top flange loading effects are much smaller for twin girder systems than for single girders.

A general global buckling solution, Eq. (8), that is also applicable to singly symmetric girders was developed. Simplified global buckling solutions, Eq. (4) for doubly symmetric and Eq. (9) for singly symmetric sections, were presented. Adjustments to the twin girder solutions for systems with more than two girders were provided. Design recommendations for a partial top-flange lateral system to increase the global buckling strength were given.

All solutions derived in this paper are intended for applications with linear elastic materials. Critical situations will likely involve casting of a concrete slab on the girder system, and the construction stresses will usually be below the elastic limit [$0.7F_y$ in the AISC specification (AISC 2005)]. Without additional work on the inelastic system buckling behavior, engineers should avoid closely spaced girder systems with two or three girders in which the bare steel girder stresses are greater than $0.7F_y$.

Appendix. Design Example (k-in. Units)

Two simply supported steel girders spaced 96 in. apart with a span of 1,800 in. must support a factored moment of 34,700 k in. in each girder during the construction stage. The cross section and other pertinent geometric details are given in Fig. 10. There are five intermediate cross frames spaced at 300 in. Assuming the deck will be poured in one continuous operation, establish the safety of this structure during construction. The yield strength of the steel is 50 ksi, $E=29{,}000$ ksi and $G=E/2.6$. The properties of one girder are $I_x=49{,}700$ in.^{4}, $I_y=289$ in.^{4}, $J=13.8$ in.^{4}, $C_w=375{,}000$ in.^{6}, $S_x=1{,}360$ in.^{3}, and $h_o=72$ in.

Maximum bending stress:

$$M_u/S_x = 34{,}700/1{,}360 = 25.5 \text{ ksi} < 50 \text{ ksi}$$

LTB between cross frames use $L_b=300$ in. with Eq. (1)

$$M_o = \frac{\pi(29{,}000)}{300}\sqrt{\frac{289(13.8)}{2.6} + \frac{\pi^2(289)(375{,}000)}{300^2}} = 35{,}200 > 34{,}700 \text{ k in.}$$

Global LTB: Eq. (9) with $C_b=1.12$, $I_{eff}=I_y$

$$M_{gls} = 1.12\frac{\pi^2(96)(29{,}000)}{(1{,}800)^2}\sqrt{289(49{,}700)} = 36{,}000 < 2 \times 34{,}700 \text{ k in.} \quad NG$$

Check end-restrained global buckling: Eq. (9) with $C_b=1.0$ and $0.6L_g$

$$M_{glw} = \frac{\pi^2(96)(29{,}000)}{(0.6 \times 1{,}800)^2}\sqrt{289(49{,}700)} = 89{,}300 > 2 \times 34{,}700 \text{ k in.} \quad OK$$

Therefore, add three panels of top flange diagonal bracing ($L4 \times 3 \times 3/8$, $A=2.68$ in.^{2}) at each end, $a=100$ in., $L_w=139$ in.

Check brace stiffness: Eqs. (10) and (11)

$$M_{ws} = \frac{3(2 \times 34{,}700 - 36{,}000)1{,}800}{72} = 2{,}500{,}000 \text{ k in./rad}$$

$$A_d = \frac{(2{,}500{,}000)(139^3 + 96^3)}{3(96)^2(100)^2 29{,}000} = 1.13 < 2.68 \text{ in.}^2 \quad OK$$

Check brace force: Eq. (12)

$$F_d = \frac{0.02(2 \times 34{,}700)}{72}\frac{139}{100} = 26.8 \text{ k}$$

The $L4 \times 3 \times 3/8$ can support this load

Notation

The following symbols are used in this paper:

A_b = area of each cross frame member;
A_d = area of top lateral system member;
a = panel spacing of top flange lateral system;
b = distance from centroid to tension flange of singly symmetric section;
C_b = moment diagram modification factor;
C_w = warping constant;
c = distance from centroid to compression flange of singly symmetric section;
E = modulus of elasticity;
F_d = force in diagonal of top lateral system;
G = shear modulus of elasticity;
h_o = distance between flange centroids;
I_{eff} = effective moment of inertia of torsionally braced singly symmetric girder;
I_x, I_y = moment of inertia about horizontal and vertical centroidal axes, respectively;
I_{xs}, I_{ys} = moment of inertia about horizontal and vertical centroidal axes of twin girders;
I_{yc}, I_{yt} = moment of inertia about y-axis of compression and tension flanges, respectfully;
J = torsional constant;
L_b = distance between cross frames;
L_d = diagonal length of cross frame;
L_g = span length;
L_w = diagonal length of top flange bracing system;
M_{cr} = buckling moment of twin girder system;
M_g = global buckling moment for twin girders under uniform moment loading;
M_{gl} = global buckling moment for twin girder systems;
M_{gls} = simplified global buckling moment for twin girder systems;
M_{glw} = global buckling moment with laterally restrained ends;
M_{gs} = simplified global uniform buckling moment;
M_o = lateral-torsional buckling moment of single girder;
M_T = buckling moment of single girder with continuous torsional restraint;
M_u = factored design moment;
M_{wf} = warping moment strength requirement;
M_{ws} = warping moment stiffness;
M_1 = critical moment for girder buckling between cross frames;
m = number of top flange braced panels at each end;
n = number of intermediate cross frames;
S = girder spacing;
t = plate thickness;
w = plate width;
y' = end slope of buckled shape;
β_b = torsional stiffness of single diagonal cross frame;
β_g = in-plane flexibility of twin girders;
β_T = effective torsional stiffness of single cross frame;
$\bar{\beta}_T$ = effective torsional stiffness per unit length;
Δ = lateral displacement at midspan; and
δ = relative top flange end displacement of twin girders.

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