COMPLEX POTENTIAL FORMALISMS FOR FLEXURE OF INHOMOGENEOUS PLATES INCLUDING TRANSVERSE SHEAR DEFORMATION

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The elegance of the Eshelby-Stroh sextic formalism in dealing with equations of plane strain elasticity (e.g. [1]) motivated several investigators to extend its applicability towards the complex potential treatment of classical laminate plate theory (CLPT) equations [2-4]. Moreover, equivalent, though perhaps less elegant, complex variable formulations and applications have also appeared (e.g. [5, 6]) and continue to appear (e.g., [7, 8]) for more than a decade. It can therefore be said that, apart from the numerous, relevant boundary value problem applications that might follow, these efforts have successfully resolved the issue as far as complex potentials formalisms of the CLPT equations are concerned.

CLPT is however adequate for accurate prediction of through thickness averaged displacements, as well as force and moment resultants, of very thin laminates only. This well-known drawback of CLPT led to its replacement with refined, two-dimensional laminate plate theories that take also into consideration the effects of transverse shear deformation. The most popular among them are the laminate extension [9] of the Reissner - Mindlin plate model that assumes uniform distribution of transverse shear strains across the plate thickness, and the more recent theory that assumes parabolic though thickness distribution of those strains [10, 11]. These refined models are certainly more accurate than CLPT as far as the accurate prediction of averaged displacements, force and moment resultants of moderately thick laminates is concerned. A subsequent generalization [12] that includes those theories as particular cases, became however the basis of the development of a two-dimensional model that is further capable of predicting accurate stress distributions through the laminate plate thickness [13].

It then becomes evident that, at least as far as potential boundary value problem applications are concerned, there is much greater scope in using complex potential formalisms in connection with refined than with CLPT equations. The main difficulty in developing such formalisms arises from the fact that, unlike their classical counterparts, the equilibrium equations of refined models do not occur in the form of homogeneous partial differential equations when expressed in terms of their main unknown displacement components. Due to this difficulty, this is the first attempt towards the development of complex potential formalisms in connection with refined plate theories. An initial step that works out such formalisms in connection with the bending problem of shear deformable homogeneous plates and laminates having a special type of inhomogeneity along their thickness direction is about to appear in the literature [14]. This development is based on the equations of the generalized shear deformable plate theory presented in [12] with the adopted type of inhomogeneity being general enough to include the class symmetric laminates as a particular case. Further developments that are based on the most general "equivalent single layer" plate theory available [15] (see also [13]) are also available. These take into consideration the effects of both transverse normal deformation and bending-stretching coupling due to arbitrary through thickness inhomogeneity. It is important to note that corresponding complex variable formalisms concerned with earlier and therefore more popular relevant theories (e.g. [9-11]) can be obtained as particular cases of the present, generalized plate theory formalisms.

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