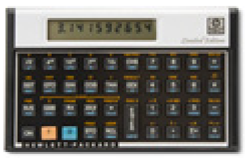
HP Solve

Calculating solutions powered by HP

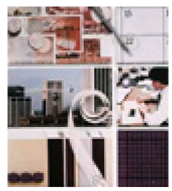
In the Spotlight

» **[30th Anniversary Edition of the HP 12c]**
Performance never goes out of style. Celebrate 30 years of this one of a kind business calculator with a limited edition collector's special.

Your articles

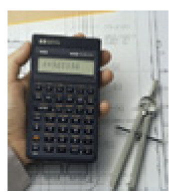


» **The HP 15c Limited Edition calculator**
One of nine machines in the Voyager Series of HP calculators, HP announces a re-release of their legendary HP 15c calculator.

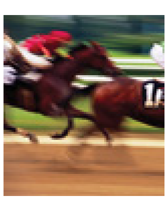


» **The HP-12C, 30 Years and Counting**
Richard J. Nelson and Gene Wright
Read this extensive review of how the HP-12C calculator has withstood the test of professionals and time longer than any other calculator.

» **Calculator Programmability—How Important is it in 2011?**
Richard J. Nelson
In this article, learn if there is an advantage to having a scientific calculator programmable as well as the reasons for programmability.



» **Converting Decimal Numbers to Fractions**
Joseph K. Horn
Joseph explains the continued Fraction Algorithm and provides an RPN HP-15C program that quickly converts any decimal to a fraction to the accuracy set by the FIX display value.



» **Limited Edition HP 15c Execution Times**
Namir Shammas

» **How Large is 10^99?**
Richard J. Nelson
And how important is this

Issue 25
October 2011

Welcome to the twenty-fifth edition of the HP Solve newsletter. Learn calculation concepts, get advice to help you succeed in the office or the classroom, and be the first to find out about new HP calculating solutions and special offers.

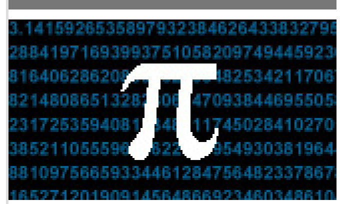
» **Download the PDF version** of newsletter articles.

» **Contact the editor**

From the Editor

Learn more about current articles and feedback from the latest Solve newsletter including One Minute Marvels and a new column, Calculator Accuracy.

Learn more ›



» **Fundamentals of Applied Math Series #8**
Richard J. Nelson

The HP 15c Limited Edition is here and comes with the same features of the original with added speed. How fast, you may ask? This article answers this question by comparing the speeds of the old HP-15C and the new HP 15c LE.

limit? Three examples of the largest possible physical values are used to see if 10^±99 is big or small enough to calculate numbers in the real world.

π is probably the most well-known of all mathematical constants. This overview provides several resources and links exploring why π is such an interesting number.

30th Anniversary Edition of the HP 12c

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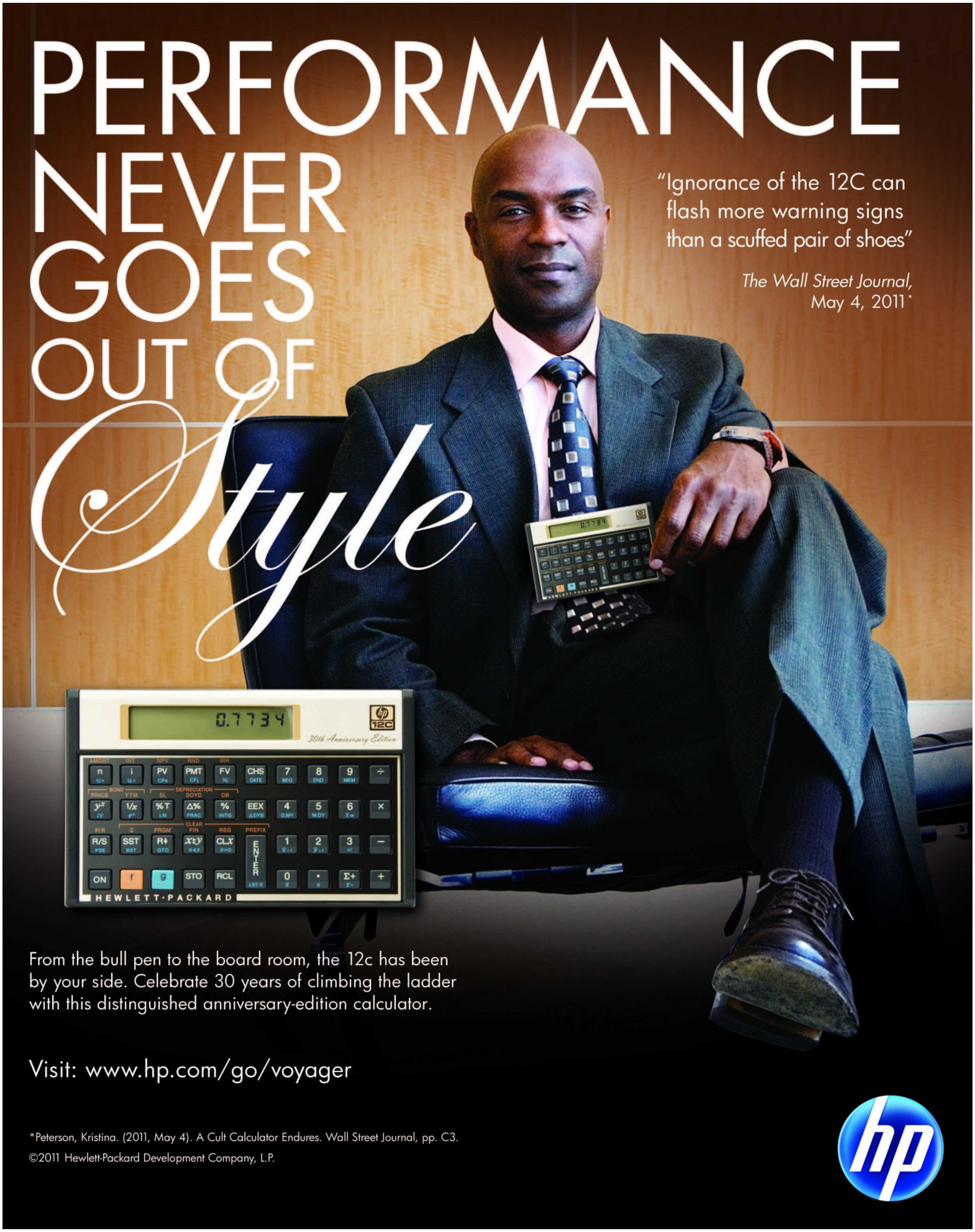
PERFORMANCE NEVER GOES OUT OF Style
"Ignorance of the 12C can flash more warning signs than a scuffed pair of shoes"
The Wall Street Journal, May 4, 2011*
From the bull pen to the board room, the 12c has been by your side. Celebrate 30 years of climbing the ladder with this distinguished anniversary-edition calculator.
Visit: www.hp.com/go/voyager
*Peterson, Kristina. (2011, May 4). A Cult Calculator Endures. Wall Street Journal, pp. C3.
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HEWLETT·PACKARD
0.7734
30th Anniversary Edition

PERFORMANCE NEVER GOES OUT OF *Style*

HP is celebrating the 30th Anniversary of its iconic gold trimmed financial calculator, the HP 12c. With its sophisticated look and feel, this calculator is still in its heyday, withstanding the test of time for professionals in the world of banking, finance, and real estate.

In 1981 HP introduced the 12c---so perfect that very little about this calculator has changed. It was the beginning of the mobile computer and was not only adopted by movers and shakers, but by everyone! Today, people marvel that an electronic brought to market 30 years ago continues to hold value through performance, functionality and durability while new technology reaches obsolescence within 3-6 months.

On May 4, 2011 the Wall Street Journal referred to the HP 12c as the “Cult Calculator” asserting that for finance professionals, “Ignorance of the 12c can flash more warning signs than a scuffed pair of shoes.” This article became HP’s inspiration for the 12c 30th Anniversary ad campaign which will run in numerous financial publications word-wide. The ad portrays a well-dressed, sophisticated business man who has successfully advanced his career from a broker fighting his way through the bull pen on Wall Street to a talented financier in the boardroom of America’s finest corporations—all with the assistance of his trusted companion—the 12c. The WSJ article states: “the 12c is just as commonplace for financial analysts as papers and cell phones” demonstrating that it has become more than just a tool of the trade; it’s a fundamental staple.

Exclusive to the 30th Anniversary, HP is offering this exemplary calculator in a limited edition collector’s special. Each 12c comes in a leather gift box, is embossed with a “30th Anniversary Edition” emblem, and is stamped with a unique production number, making it the perfect gift for business professionals, students, and calculator connoisseurs.

The 12c “30th Anniversary Edition” is available for purchase September 1, 2011 in retail stores and online at: www.hp.com/go/voyager

*Peterson, Kristina. “A Cult Calculator Endures.” *Wall Street Journal*, May 4, 2011: Print
http://online.wsj.com/article/SB10001424052748703841904576257440326458056.html

The HP 15c Limited Edition calculator

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THE LEGENDARY HP 15C IS BACK BY POPULAR DEMAND!
Bring it back!
Regresala al mercado!
Breng het terug!
Bringe es zuruck!
Retourné!
3.141592654
Limited Edition
HEWLETT·PACKARD
While the Voyager Spacecraft journeyed to the Milky Way in 1981, HP explored engineering innovation with the Voyager Calculator Series—whose timeless design made the 15c one of the most esteemed calculators in HP history.
Today, this "blast from the past" is back; celebrate with a limited-edition 15c calculator.
www.hp.com/go/voyager
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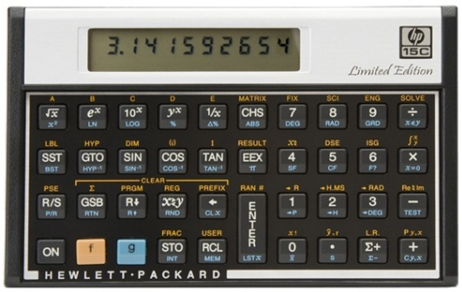
September 2011

THE LEGENDARY 15C--BACK BY POPULAR DEMAND



Introduced to the market in 1982 as a part of the HP Voyager Series, the 15c scientific calculator quickly achieved a loyal following as fans marveled at the calculator's premier performance and functionality. After 7 successful years in the market-place, HP discontinued the 15c and replaced it with multiple line display calculators.

This decision devastated 15c loyalists. Since then, they have beaten their chests loudly for HP to bring it back, creating websites such as, "Bring Back the HP15c".

Their passion for the dearly departed 15c was so strong that over 15,000 people in 175 countries have signed a petition—"HP, bring back the 15c."

With demand on this petition exceeding 84,000 units, bidding wars on Ebay reaching over $500, and tireless forums of fans lamenting the loss of "The long gone but not forgotten" 15c, HP relented—"It's time to resurrect the legendary 15c!"

The 15c PR campaign will commence on September 1, 2011 and be featured in numerous engineering and scientific publications world-wide. The ad is a play on the Voyager Calculator Series and the Voyager Space Mission— both symbols of the technology revolution during the early 1980s. The Voyager Space Shuttle discovers the 15c on the outer edges of the Milky Way, floating further and further away from planet earth and its devotees. The Golden Record, intended to relay messages from human life to extra terrestrial life, communicates the sentiments for the return of the 15c.

Today, fans everywhere can celebrate the return of the HP 15c with this limited-edition collector's special. Each 15c comes in a leather gift box, is embossed with a "Limited Edition" emblem, and is stamped with a unique production number, making it the perfect gift for 15c aficionados!

The 15c Limited-Edition Calculator will be available for purchase September 1, 2011 in retail stores and online at: www.hp.com/go/voyager.

The HP-12C, 30 Years and Counting

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The HP-12C, 30 Years and Counting

Richard J. Nelson and Gene Wright

Introduction

Fig. 1 – HP-12C 30th Anniversary Edition.

HP has a long list of calculator firsts[(1)] and the 1981 HP-12C business, finance, and real estate calculator significantly contributes to the list. The 12C has been continuously made for 30+ years. Made in five countries[(2)], and often in two countries at the same time, HP-12Cs made today look pretty much the same as they did 30+ years ago. The HP-12C calculator has withstood the test of professionals and time longer than any other calculator (and possibly longer than any other consumer electronics product ever made).

The financial HP-12C was introduced September 16th 1981 along with a scientific HP-11C. These two machines were eventually joined by six other similar looking calculators as part of the Voyager series of HP calculators. See Fig. 2 below. Also known as the "Slim Line[(3)] " calculators they were quite different from other calculators in HP's line up because of their smaller size and thinness. Some sellers/reviewers/ websites also refer to the voyager calculators as the "10 Series."

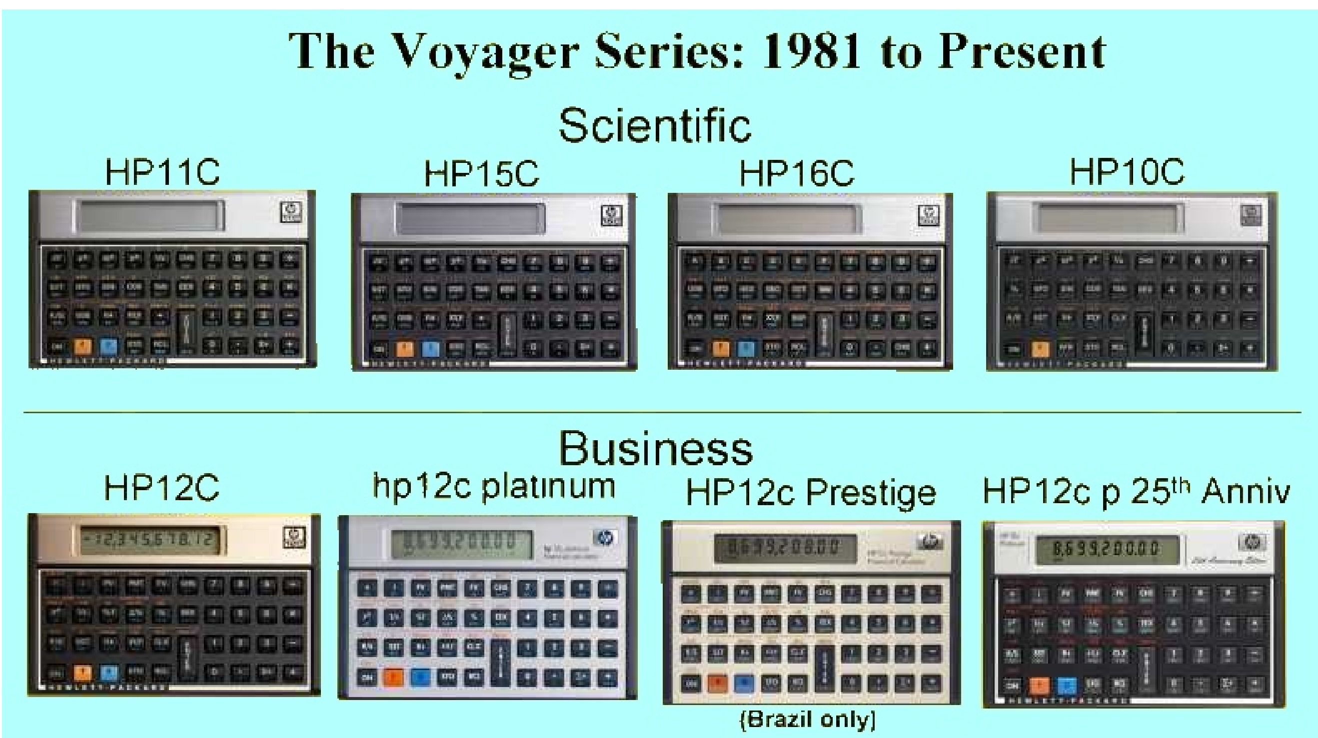


Fig. 2 – The HP-12C 30th Anniversary Edition joins eight other voyager series calculators.

The HP-12C integrates the five Time Value of Money, **TVM**, variables into one solver that provides the easy and very accurate solution of n, i, PV, PMT, and FV using a consistent approach to solving complex compound interest problems.

Why is the HP-12C so popular?

The reasons are many, and which ones you will hear or read about will depend on the knowledge, experience, and background of the reviewer. Here are a few reasons in no particular order.

The HP-12C has a clear readable display: The popularity of the HP-12C was debated at a user's conference in London in 2002. The intent of the one hour discussion was to identify the most important features of the HP-12C that makes it so popular. Six qualities of excellence were given to start the discussion. Everyone had an opinion, but after an hour one quality was mentioned over all others; a clear readable display[4]. The HP-12C uses a 10 digit 7 segment Liquid Crystal Display, LCD, with 0.2 inch high digits.

The HP-12C has a clean uncluttered keyboard: The HP-12C is a tool to be consulted in the "heat of the deal" for the business community. It must not appear intimidating to either the customer or the user. The gold display bezel and the gold frame around the keys give the calculator a timeless quality look. The gold is in contrast to the textured dark brown case. Of course the classical HP calculator keyboard key feel and tactile feedback click is also an important part of the keyboard. Two shift keys, yellow and blue, are used to provide access to the 120+ functions of the calculator.

The HP-12C uses a horizontal format: Most of the calculators HP makes use a vertical format. The Voyager series of calculators, however, broke from the tall thick vertical cases of other models to use a thin horizontal format. This was quickly accepted for two reasons. First, the intended user was often in the office and at a desk. Second, the light weight, 4.1[5] oz., makes it easy to hold in the hand. Holding a machine in your two hands and operating the keyboard with your thumbs (pre-cell phone) is natural for a 12C if used while standing.

Fig. 3 – Button (left) and coin HP-12C cells.

The HP-12C has a long battery life: The HP-12C is famous (as are all Voyager models) for the long life of the battery. The original HP-12C uses three button cells[6] (Eveready A76 alkaline or Eveready 357 silver-oxide) and the current incarnation uses two coin cells (Lithium CR2032). The original "Owner's Handbook and Problem-Solving Guide" dated February 1981, page 216 says, "In 'typical' use, the HP-12C has been designed to operate 6 months or more on a set of alkaline batteries. The batteries supplied with the calculator are alkaline, but silver-oxide batteries (which should last twice as long) can also be used." In today's world of battery operated devices this is an unbelievable understatement. Few battery operated devices run as long as the manufacturer says. Users report multiple (3-5) year battery life.

The HP-12C has a powerful business function set: The HP-12C has a 2,465 year (1582 – 4046) calendar built in with a nice set of date functions. It also has an excellent mix of business and statistical functions with the normal [no trigonometry[7]] math functions. Specific applications such as **IRR** and **NPV** are also included. See Appendix A for the details.

The HP-12C uses RPN: Most business calculator users have no idea what RPN means, but they easily learn the beauty of simply pressing a key to execute a function. You want to add two numbers you press the + key. You could have divided as well because you give the machine the number(s) and then you tell it what to do.

The HP-12C is programmable: See article on the importance of calculator programmability elsewhere in this issue. As with RPN most business users of the HP-12C are not very concerned with its programmability. When they have a number of similar problems to solve, however, they will discover how RPN programmability may be easily learned to "get the work done." The programming capability is limited to 99 program steps and two logic compares (X≤Y? & X=0?). Many of the nice programming features of more capable calculators are missing: absolute value, flags, increment or decrement for looping, indirect addressing, labels, sign, subroutines, etc. This feature is further illustrated with the program examples in appendix C.

The HP-12C is accurate: Most people who use an HP calculator expect that the answer is correct. The situation is that this is not the case in the real world, and HP calculators are renowned for their accuracy. There are two issues of accuracy involved in the HP-12C. First is the normal issue of making calculations accurate rounded to ± 1 count in the 10^{th} place. The second accuracy issue is that of the applications programs, especially the **TVM**. Solutions are iteratively solved and the process may be error prone. The algorithms used are critical and the HP-12C uses the best. The critical feature (and characteristic) of the HP-12C is that a problem solved on the first (1981) calculator will get the same answer on the most recent (2011) calculator. This is quite remarkable in the calculator world – and another first.

What has happened in 30 years?

While the HP-12C looks much the same on the outside it has changed on the inside over the years. One physical specification was previously mentioned, weight. We weighed HP-12C serial number 2620A09672[8] at 117.3 grams (4.1 oz.) with 3 button cells. The 4 oz. weight has been consistent over the 32 years of manufacturer.

Major changes have been made to the electronics circuits. The very first HP-12Cs were wrapped in a black plastic ESD[9] film as shown in Fig. 4.

The original microprocessor is no longer made and a newer microprocessor had to be used. Newer technology is considerably faster and essentially the new microprocessor emulated that of the original so the original ROM code could be used. This is one technology reason the math has been consistent throughout its life. Another is reason is HP's policy of maintaining a standard.

Photo by Geoff Quickfall

Fig. 4 – Early HP-12C circuit board shown wrapped in a black plastic ESD film. .

While the insides have changed and the outside looks much the same there are subtle mechanical (case) changes that have been made over the years. HHC 2011 attendees will be asked to bring their HP-12Cs to the conference so that we may photograph them.

The original HP-12Cs utilized gold plated circuit board traces. This practice was quickly discontinued in favor of more current design practices. An urban legend says that the display bezel was also gold plated. This is not true. Trivia question. There has been an HP calculator model that was gold plated on the outside. What is it?

The HP-12C has been in continuous production for 32 years. Research was made and while this is far from being 100% accurate it illustrates how things have changed over three decades. The HP-12C production was in the countries listed in Table 1 below. This data was obtained from interviews with retired HP personnel, serial numbers from friends and the Internet, and Don O'Rourke of http://internationalcalculator.com/ The serial numbers of HP-12Cs made in China have changed in the last few years and serial number changes and inconsistencies have been observed by the HP User Community. One conclusion that may be made is that HP had a significant overlap of countries whenever a country change was made to insure a continuous supply.

Table 1 – Country of HP-12C Manufacturer with Estimated Dates

1. USA[1]	SN 'A"	September* 1981 to 2003
2. Singapore	SN "S"	September* 1981 to 1996?
3. Brazil[2]	SN "B"	1984 to early 1992.
4. Malaysia (Indonesia)[3]	SN "MY"	Early 1990 to late 1999
5. China	SN "CNA"	1999 to Present (9/2011)

* - *Obviously calculators were made several weeks or months prior to this introduction date. Earliest known (date) 12C SN[(8)] 2031A##### = Made in USA July 4,1980*

Notes:

1. The HP-12C was manufactured in the US and Singapore when introduced.
2. The HP-12C was manufactured in Brazil and because government regulations required a Brazilian component to the machine if it was to be sold there it was moved. Otherwise the source code would have to be provided to the Government.
3. Plant closed due to political instability.

In addition to the technology the competition has changed greatly over the last 30 years and this has influenced the price[(10)] of the HP-12C. The current HP website price is $70, less than half of the original introductory price of $150.

Even business people need to play.

The programmability of the HP-12C provides the capability to key in games programs in addition to complex financial procedures. Numerous games have been programmed. The programs Sum of the digits Game, BlackJack, Slot Machine, and Eleven-Thirty Game, may be found in Appendix C.

A penny for your thoughts

The exceptional accuracy of the HP-12C may be illustrated with a simple problem suggested by William Kahan [(11)]. "A bank retains a legal consultant whose thoughts are so valuable that she is paid for them at the rate of a penny per second, day and night. Lest the sound of pennies dropping distract her, they are deposited into her account to accrete *(accrue)* with interest at the rate of 10% per annum compounded every second. How much will have accumulated after a year (365 days)?" Solve this on your financial calculator and see what answer you get.

Enter the data:
n : = 60 * 60 * 24 *365 = 31,536,000 seconds per year.
i : = 10/n = 0.000 000 317 097 9198 % per second.
PV : = 0
PMT : = -0.01 = one cent per second to the bank.
FV : ?

Pressing [FV] should display one year's accretion, but different financial calculators display different amounts. Here is the result if you use an HP-27, HP-92, HP-37, HP-38, and HP-12C = $ 331,667.0067. As an aside, the new HP 10bII calculates $ 331,667.006691.
Less accurate, non-HP, calculators may get answers like: $ 293,539.16035, $ 334,858.18373, or $ 331,559.383549.

Observations and conclusions

The HP-12C financial calculator has been in continuous production for over 30 years – unchanged! The hardware has changed with improved engineering practices and microprocessor changes, but the exceptional accuracy has been maintained and a problem solved on a 1981 machine will get the same answer as one solved on a 2011 machine. This is reasonable because there is only one correct answer. We have tried to give the reader an overview of this exceptional calculator in this short review and we both look forward to its 35th anniversary.

Additional HP-12C information and support may be found in the appendices.

Appendix A - 1982 HP Catalog Page Showing New HP-12C
Appendix B - 1981 HP Sales Tip Brochure
Appendix C - Four HP-12C Game Programs
Appendix D - HP-12C References

About the Author

Gene Wright is the author of textbook "Quantitative Analysis for Business", a business math textbook using the HP-10BII and HP-12C, available from Amazon.com. He is also a video lecturer for a CFA exam review course. A former teacher at Lipscomb University in Nashville, Tennessee, he now works for a consumer electronics company. Gene has written many articles on HP calculators and serves on the annual HHC committee.

About the Author

Richard J. Nelson has written hundreds of articles on the subject of HP's calculators. His first article was in the first issue of *HP 65 Notes* in June 1974. He became an RPN enthusiast with his first HP Calculator, the HP-35A he received in the mail from HP on July 31, 1972. He remembered the HP-35A in a recent article that included previously unpublished information on this calculator. See http://hhuc.us/2007/Remembering%20The%20HP35A.pdf
He has also had an article published on HP's website on HP Calculator Firsts. See http://h20331.www2.hp.com/Hpsub/cache/392617-0-0-225-121.html.

Notes: The HP-12C, 30 Years & Counting

(1). A list of other HP Calculator firsts may be found at: http://h20331.www2.hp.com/Hpsub/cache/392617-0-0-225-121.html

(2). The countries in historical order are USA, Brazil, Singapore, Malaysia, and China.

(3). The initial specifications of the 12C may be found on page 646 of the 1982 hardbound catalog reproduced in Appendix A. The price was $150. Copies of these catalogs may be found at: http://www.hpmuseum.net/exhibit.php?content=HP%20Catalogues#

(4). A conference reviewer, Jo Vandale of Belgium, wrote in a 9/23/2002 Conference review regarding the HP-12C session, "The screen will be readable and the form factor will be horizontal ... at least for the HP12C successor".

(5). The HP-12C has had many engineering changes to reduce cost and to utilize current electronics technology. These changes have not substantially changed the weight which has been consistently specified by HP as 4 oz.

(6). Battery life is somewhat related to the quality of the manufacturer of the battery. When the button cells eventually need replacement silver button cells are frequently used. See the figure at the right for a comparison.

(7). The UK HPCC publication Datafile has an article and program to add Sin, Cos, Tan and their inverses (in radians) to an HP-12C. The title is HP-12C Tried & Tricky Trigonometrics by Valentin Albillo. It is in the January/February 2002 Issue - V21N1p12.

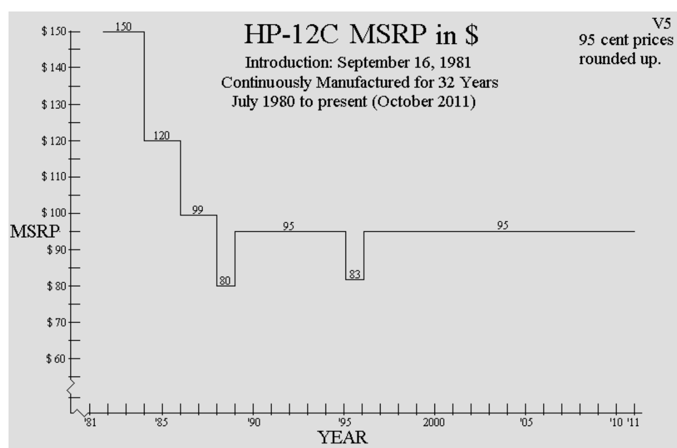
(8). The coded serial number (SN 2620A09672) is decoded as follows. The first two digits are the years since 1960 = 1986, the next two digits is the week of the year = mid April, "A" is the country of manufacturer (USA), and the remaining five digits are sequential for the week = at least 9,672 for that week. The date is not the date of actual manufacturer, but it is led by some number of weeks (4 – 6) for the calculator to be shipped and sold. The reason for this is the warranty period is based (back then) on the serial number – or a copy of your sales receipt. For additional details on decoding HP Calculator serial numbers see: http://www.hpmuseum.org/collect.htm#numbers

(9). ESD is ElectroStatic Discharge and is most commonly experienced when you walk across a carpet (especially one made of synthetic materials) and you get sparked by touching a large metal object. Modern electronics circuits are very sensitive to ESD because a single "spark" will destroy them in microseconds.

(10). You may find the Current HP-12C Data Sheet and pricing at:
http://h10010.www1.hp.com/wwpc/us/en/sm/WF06a/215348-215348-64232-20036-215349-33525.html

Very few calculators have more than a ten year history of being manufactured unchanged. A three decade span is truly exceptional, and the list price of the 12C illustrates that prices may even increase under unusual market conditions. The first 20 years were essentially pre-Internet. Once the Internet became the primary source of information the discount pricing history became more obscure. The data that was found is shown at the right. Early sources were the HP Catalog and all 75 EduCALC catalogs. Both are no longer published.

Even with the price increases in 1989 and 1996 the HP-12C popularity did not change.



HP suggested list price history for the HP-12C.

(11) William Kahan of the University of California Berkeley consulted with HP on the HP-12C and other calculators (HP-15C & HP-34C) to insure algorithmic accuracy. He has a long and productive career in

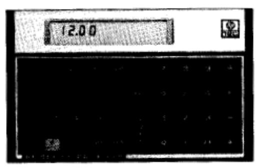
promoting computational accuracy in computers and calculators. See page 15 of Mathematics Written in Sand, Version of 22 November, 1983 at: http://www.cs.berkeley.edu/~wkahan/MathSand.pdf.

Appendix A –1982 HP Catalog Page Showing New HP-12C

COMPUTERS, PERIPHERALS & CALCULATORS

Personal Computation

Models HP-12C, HP-37E, HP-38C



NEW

Why a Professional Calculator?

Proper selection of a calculator depends largely on careful analysis of your current personal and professional needs, plus those you're likely to face in the future. HP calculators have the most needed and useful functions preprogrammed into the calculator, so, most simple problems are solved at the touch of a key. And you can write your own programs or choose from prewritten programs for today's solutions and tomorrow's too.

HP-12C Slim-Line Financial Programmable with Continuous Memory and Special Functions

At four ounces light and half an inch slim, the HP-12C puts more financial solutions in your pocket than ever before. With its special functions, programmability, Continuous Memory, and liquid crystal display, this calculator is ideal for solving most business and financial problems in or out of the office. The HP-12C features basic time and money functions, NPV, IRR, plus a bond function which calculates yield-to-maturity and price. For additional push-button solutions, you can write your own programs, or, take advantage of HP's prewritten software solutions for specific applications.

The HP-12C comes complete with a detailed Owner's Handbook and Problem-Solving Guide; long-life disposable batteries; and a soft carrying case.

HP-12C Specifications

Financial Functions: n, i, PV, PMT, FV, amortization (accumulated interest and remaining balance), simple interest, NPV, IRR, bond yield-to-maturity and price, depreciation (straight-line, declining-balance, sum-of-years' digits), odd days' interest, beginning/end of period selection.
Mathematical Functions: +, −, ×, ÷, y^x, $\sqrt{x}$, 1/x, x^2, LN, e^x, round, integer/fraction truncation.
Statistical Functions: %, Δ%, %T, $\bar{x}$, s, $\hat{y}$, $\hat{x}$, r, summation (n, Σx, Σx^2, Σy, Σy^2, Σxy), factorials.
Calendar Functions: 2000-year calendar, finds number of days between two dates, day of week, future or past date, all on 360- or 365-day calendar basis.
Programming Features: SST, BST, GTO, R/S, pause, two conditional tests (x=0, $x \leq y$).
Clearing Options: CLEAR X, FINANCIAL, STATISTICAL, PREFIX, PROGRAM, REGISTERS.
Memory: five financial registers, four-register stack, last-x register, automatic memory allocation between storage registers and program memory. A maximum of 99 program lines and 7 registers or 8 program lines and 20 storage registers.
Size: 12.7 × 8.0 × 1.5 cm (5 × 3 ⅛ × ⅝ in).

HP-37E Business

The HP-37E is the basic financial calculator for most business and financial problems. In additon to built-in price, percent, and statistical functions, the HP-37E features the basic time and money functions. HP has developed a number of specific application books that address a variety of financial problems.

The HP-37E comes complete with a detailed Owner's Handbook; the informative "Your HP Financial Calculator: An Introduction to Financial Concepts and Problem Solving"; recharger/AC adapter; rechargeable battery pack; soft carrying case; and your choice of one of the optional application books.

HP-37E Specifications

Financial Functions: n, i, PV, PMT, FV, amortization (accumulated interest, payment to principal, remaining balance), begin/end switch for ordinary- or annuity-due problems.
Mathematical Functions: +, −, ×, ÷, 1/x, $\sqrt{x}$, y^x, LN, e^x.
Statistical Functions: %, Δ%, %T, price, $\bar{x}$, s, r, L.R., $\hat{x}$, $\hat{y}$, Σ+, Σ−, summations (n, Σx, Σx^2, Σy, Σy^2, Σxy), factorials.
Clearing Options: CLEAR X, FINANCIAL, ALL.
Memory: 7 storage registers, 5 financial registers, 4-register stack.
Recharger Power Requirements: 90 to 120 Vac or 198 to 242 Vac (50 to 60 Hz).
Size: 30 × 75 × 140 mm (1.2 × 3.0 × 5.6 in).

HP-38C Advanced Financial Programmable with Continuous Memory

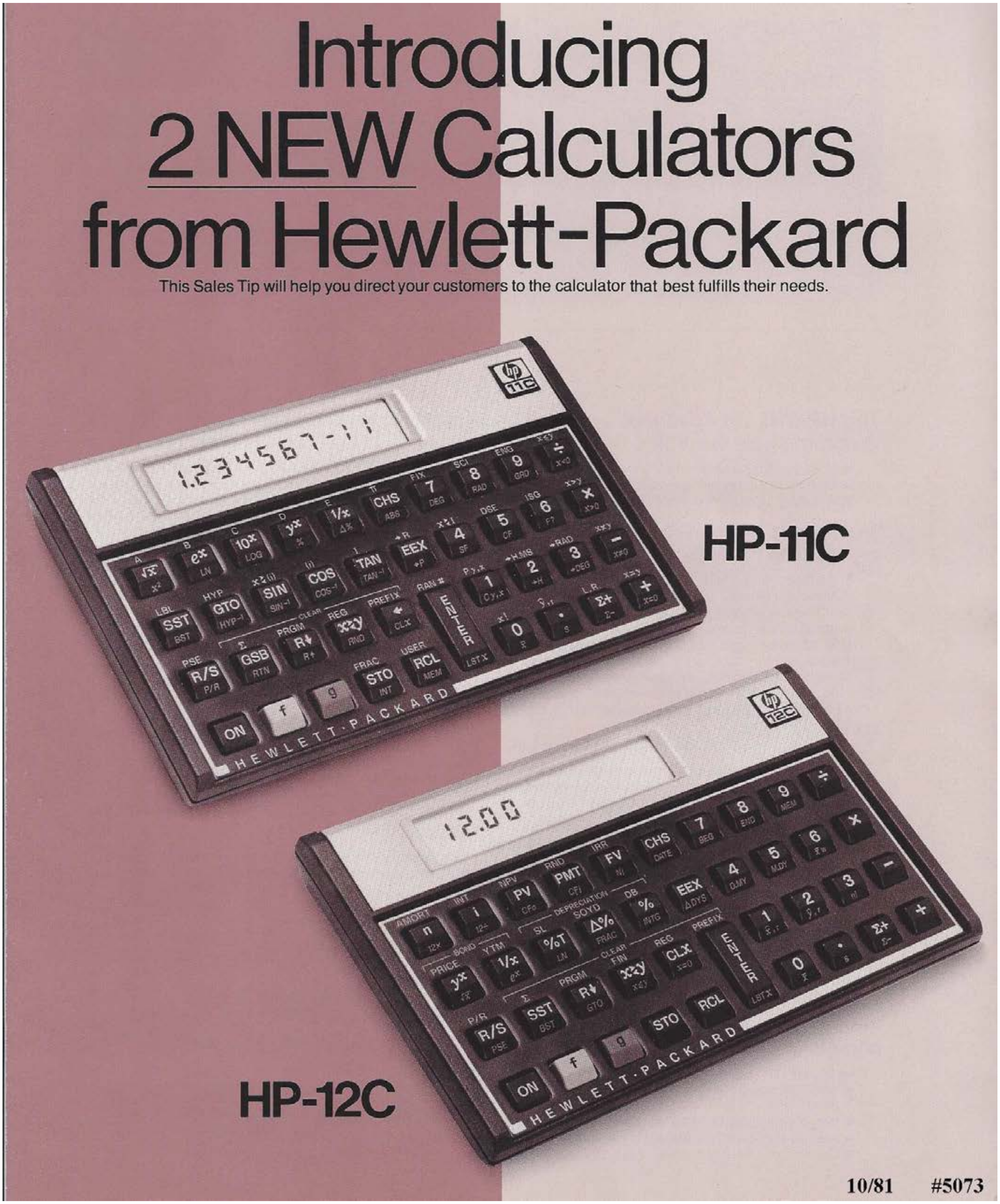
The HP-38C combines a wide array of financial functions with Continuous Memory, which allows you to retain data and programs even when the calculator is turned off. This calculator has the advanced financial capability of discounted cash flow as well as basic time and money functions. You can create your own programs or choose from several application books containing prewritten programs.

The HP-38C comes complete with a detailed Owner's Handbook; the informative "Your HP Financial Calculator: An Introduction to Financial Concepts and Problem-Solving"; recharger/AC adapter; rechargeable battery; quick reference card; soft carrying case; and your choice of one of the optional application books.

HP-38C Specifications

Financial Functions: n, i, PV, PMT, FV, amortization (accumulated interest and remaining balance), simple interest, NPV, IRR, automatic entry for grouped or individual cash flows, begin/end switch for ordinary- or annuity-due problems.
Mathematical Functions: +, −, ×, ÷, 1/x, $\sqrt{x}$, y^x, LN, e^x, round, integer/fraction truncation.
Statistical Functions: %, Δ%, %T, $\bar{x}$, s, r, L.R., $\hat{x}$, $\hat{y}$, summations (n, Σx, Σx^2, Σy, Σy^2, Σxy), factorials.
Calendar Functions: 2000-year calendar, finds number of days between two dates, day of week, future or past date, all with 366- or 365-day calendar basis, M.D.Y.⇄D.M.Y.
Programming Features: SST, BST, GTO, R/S, pause, two conditional tests (x=0, $x \leq y$).
Clearing Options: CLEAR X, FINANCIAL, STATISTICAL, PREFIX, PROGRAM, ALL.
Memory: 5 financial registers, 4-register stack, last-x register. Dynamic memory allocation between storage registers and program memory.
Recharger Power Requirements: 90 to 120 Vac or 198 to 242 Vac (50 to 60 Hz).
Size: 30 × 75 × 140 mm (1.2 × 3.0 × 5.6 in).

Ordering Information	Price
HP-12C	$150.00
HP-37E	$75.00
HP-38C	$150.00
HP-12C Solutions Handbook	$20.00
HP-37E or HP-38C Optional Solutions Books	$5.00


Introducing
2 NEW Calculators
from Hewlett-Packard
This Sales Tip will help you direct your customers to the calculator that best fulfills their needs.
HP-11C
HP-12C
10/81
#5073

Appendix B –1981 HP Sales Tip Brochure - Page 1 of 2

HP-12C
Financial Programmable Calculator

The **HP-12C,** a Financial Programmable Calculator for business-world professionals has powerful **built-in functions** for push-button solutions to:

Business Problems

- solves compound interest (i), net present value (NPV) and internal rate of return (IRR).
- finds bond yield-to-maturity and price.
- calculates amortization schedules.
- determines depreciation schedules.
- calculates odd-days' interest.

Pricing Calculations and Forecasting

- finds percent, percent change, and percent of total.
- finds the mean and weighted mean.
- predicts new values based on known values.

The HP-12C has easy-to-learn **programmability** with:

- ready-to-use program solutions in lending, forecasting, pricing, statistics, and many more.
- and you don't need previous programming experience!

All this capability **fits** comfortably **in your pocket!**

And the HP-12C comes with . . .

- slim-line design — only 4 ounces light, half an inch slim.
- long-life disposable batteries. **No** cord, **No** recharger.
- easy-to-read liquid crystal display with large numbers.
- Continuous Memory to save your programs and data.
- traditional HP quality—built-in Standard of Excellence.
- award-winning owner's handbook, prewritten programs providing ready-made solutions, and a customer support group at the factory that's only a phone call away.

How does this new financial calculator fit into Hewlett-Packard's current financial line?

Below, the financial line is categorized from basic problem-solving capability to advanced problem-solving power.

- **HP-37E** Business
- **HP-38C** Financial Programmable with Continuous Memory
- **HP-12C** Slim-line Financial Programmable with Continuous Memory and Special Functions
- **HP-41** Alphanumeric Full Performance Programmable with Continuous Memory

The HP-12C is a HP-38C plus:

Bonds

- Price
- YTM (Yield to maturity)

Depreciation

- SL (Straight line)
- SOYD (Sum of years' digits)
- DB (Declining balance)

Odd-Days' Interest

HP-12C Sum of the Digits Game

Gene Wright

Taken from the game on page 25 of the HP Digest, Volume 5, 1979. The HP-12C will generate a number between 0 and 99. It will display the sum of the tens place and the ones place. If the number generated were 25, the HP-12C would add the 2 and 5 together and display a 7. The user enters a number to be added to the generated secret number in hopes that when added to it, the new number will equal 99.

If it does, the game is won and the HP-12C displays "e", the number of guesses, and the original secret number. If the user entered number causes the new sum to go over 99, 99 is displayed in fix 9 format, and the previous sum is displayed again for the user to try another, lower guess. If the new number is less than 99, the two digits of the number are added together again and the new sum displayed. The user then enters another number to be added to the secret number.

Keystrokes	Display	Keystrokes	Display	Keystrokes	Display
f P/R		g INTG	20– 43 25	–	41– 30
f CLEAR PRGM	00–	x≷y	21– 34	g x=0	42– 43 35
CLx	01– 35	g FRAC	22– 43 24	g GTO 051	43-43,33,051
STO 0	02– 44 0	1	23– 1	9	44– 9
RCL i	03– 45 12	0	24– 0	9	45– 9
9	04– 9	×	25– 20	f 9	46– 42 9
9	05– 9	+	26– 40	g PSE	47– 43 31
7	06– 7	STO FV	27– 44 15	f 2	48– 42 2
×	07– 20	R/S	28– 31	RCL FV	49– 45 15
g FRAC	08– 43 24	1	29– 1	g GTO 028	50-43,33,028
STO i	09– 44 12	STO + 0	30- 44,40, 0	g e^x	51– 43 22
EEX	10– 26	R↓	31– 33	g e^x	52– 43 22
2	11– 2	RCL n	32– 45 11	f 9	53– 42 9
×	12– 20	+	33– 40	g PSE	54– 43 31
g INTG	13– 43 25	9	34– 9	f 2	55– 42 2
STO PMT	14– 44 14	9	35– 9	RCL 0	56– 45 0
STO n	15– 44 11	g x≤y	36– 43 34	g PSE	57– 43 31
1	16– 1	g GTO 040	37-43,33,040	RCL PMT	58– 45 14
0	17– 0	R↓	38– 33	g GTO 000	59-43,33,000
÷	18– 10	g GTO 015	39-43,33,015		
ENTER	19– 36	x≷y	40– 34		

Enter a decimal seed into i. Re-play does not require a re-seed. Does not require registers cleared beforehand. **Press** R/S and see the sum of the secret two-digit number. **Repeat:** Enter a number to be added to the secret number and press R/S.

Example: 0.123456789 STO i R/S. Display shows 8. Press 95 R/S. Display shows 99.00000000 then 8.00. Guess was too high. Press 12 R/S. Display shows 2. Sum of digits of new number is 2. Press 35 R/S. Display shows 10. Sum of digits of new number is 10. Press 44 R/S. Display shows 2.7182818 (a win!), then 4 (number of guesses) and finally 8 (original number).

HP-12C BlackJack

Gene Wright

Keystrokes	Display	Keystrokes	Display	Keystrokes	Display
f P/R		x≷y	23- 34	RCL 4	47- 45 4
f CLEAR PRGM	00-	RCL 0	24- 45 0	g GTO 000	48-43,33,000
f 0	01- 42 0	g x=0	25- 43 35	R↓	49- 33
STO 6	02- 44 6	g GTO 028	26-43,33,028	g PSE	50, 43 31
CLx	03- 35	g GTO 049	27-43,33,049	STO + 2	51 - 44 40 2
STO 0	04- 44 0	R↓	28- 33	RCL 1	52- 45 1
STO 1	05- 44 1	STO + 1	29 - 44 40 1	RCL 2	53- 45 2
STO 2	06- 44 2	R/S	30- 31	−	54- 30
RCL 5	07- 45 5	RCL 1	31- 45 1	g x=0	55- 43 35
9	08- 9	RCL 3	32- 45 3	g GTO 061	56-43,33,061
9	09- 9	−	33- 30	RCL 1	57- 45 1
7	10- 7	g x=0	34- 43 35	RCL 2	58- 45 2
×	11- 20	g GTO 044	35-43,33,044	g x≤y	59- 43 34
g FRAC	12- 43 24	RCL 1	36- 45 1	g GTO 007	60-43,33,007
STO 5	13- 44 05	RCL 3	37- 45 3	RCL 2	61- 45 2
1	14- 1	g x≤y	38- 43 34	RCL 3	62- 45 3
4	15- 4	g GTO 041	39-43,33,041	−	63- 30
×	16- 20	g GTO 007	40-43,33,007	g x=0	64- 43 35
g INTG	17- 43 25	RCL 6	41- 45 6	g GTO 041	65-43,33,041
g x=0	18- 43 35	CHS	42- 16	RCL 2	66- 45 2
g GTO 007	19-43,33,007	g GTO 045	43-43,33,045	RCL 3	67- 45 3
RCL 7	20- 45 7	RCL 6	44- 45 6	g x≤y	68- 43 34
g x≤y	21- 43 34	STO + 4	45- 44 40 4	g GTO 044	69-43,33,044
g GTO 024	22-43,33,024	g PSE	46- 43 31	g GTO 041	70-43,33,041

This is an HP-12C version of the slot machine game written by Mike Garland and appeared in the V5N4P23 issue of PPC Journal (May 1978). A listing of that game for the HP-25 and instructions on how to play it can be found here:

http://www.rskey.org/gene/calcmuseum/25blkjk.htm

Instructions:

1) Store the initial constants needed by the program: 10, STO 7, 21 STO 3.

2) Enter the initial random number seed (a decimal between 0 and 1) and press STO 5.

3) Enter your starting bankroll and press STO 4.

4) To play a game, press f PRGM, key in your bet and press R/S.

5) Your card will be displayed. Continue pressing R/S until you decide to stay or your total goes over 21.

6) If you bust, press R/S and your bet will be displayed as a negative number and then your balance will be displayed.

7) If you decide to stay, press STO 0, R/S and the machine's cards will be displayed successively. The machine will continue to take cards until it wins or busts.

8) If you win, your bet will be displayed as a positive number and then your balance will be displayed.

9) If you lose, your bet will be displayed as a negative number and then your balance will be displayed.

10) For a new game, go to step 4.

Notes: The machine wins all ties, unless you get a total of 21 on your turn. The machine takes all aces as 1's NOT 11's. You have the option of making your aces (displayed as 1's) into 11's by pressing X <> Y, STO+ 1, when your card is displayed. If you get 21, you win automatically, just press R/S.

Sample Game: Enter the following: 0.123456789 STO 5, 10 STO 7, 21 STO 3, 25 STO 4. Enter 5 for your bet and press f PRGM, then R/S.

A 1 is displayed (your first card). You decide to take this ace as an 11, so press X <> Y, then STO+ 1. Press R/S for the second card. A 2 is displayed (your second card for a total of 13). Press R/S for another card. A 10 is displayed. You busted! Press R/S. Display shows a -5 and a bank of 20 remaining.

Enter 10 for your bet and press R/S. A 3 is displayed. Press R/S. A 10 is displayed for a 2 card total of 13. You decide to stand. Press STO 0 then R/S. Display shows HP's first card is an 8, then HP's second card is a 10, so HP wins! Display shows your -10 bet and then 10 remaining in the bank. Perhaps you can do better? To play again, just press R/S and continue as above.

HP-12C Slot Machine

Gene Wright

This is an HP-12C version of the slot machine game written by Craig Pearce for the HP-25 as found in the February 1976 issue of the PPC Journal. A listing of that game for the HP-25 and instructions on how to play it can be found here: http://www.rskey.org/gene/calcmuseum/25slot.htm

Enter starting bank amount STO 1. Enter a decimal seed and STO 0. Each "spin" costs $0.10. Payoff is $1 for any 0.aaa or 0.aa0 number returned, where "a" is any non-zero digit. A result of 0.000 is worth $10.

Keystrokes	Display	Keystrokes	Display	Keystrokes	Display
f P/R		×	19- 20	g x=0	39- 43 35
f CLEAR PRGM	00-	g INTG	20- 43 25	g GTO 042	40-43,33,042
f 3	01- 42 3	STO 3	21- 44 3	g GTO 051	41-43,33,051
RCL 0	02- 45 0	g LSTx	22- 44 11	RCL 2	42- 45 2
9	03- 9	g FRAC	23- 43 24	g x=0	43- 43 35
9	04- 9	1	24- 26	g GTO 047	44-43,33,047
7	05- 7	0	25- 2	0	45- 0
×	06- 20	×	26- 20	g GTO 048	46-43,33,048
g FRAC	07- 43 24	g INTG	27- 43 25	9	47- 9
g PSE	08- 43 31	STO 4	28- 44 4	STO + 1	48 - 44 40 1
STO 0	09- 44 0	R↓	29- 33	1	49- 1
1	10- 1	−	30- 30	STO + 1	50 - 44 40 1
0	11- 0	g x=0	31- 43 35	•	51- 22
×	12- 20	g GTO 034	32-43,33,034	1	52- 1
g INTG	13- 43 25	g GTO 051	33-43,33,051	STO − 1	53 - 44 30 1
STO 2	14- 44 14	RCL 4	34- 45 4	RCL 1	54- 45 1
g LSTx	15- 43 40	g x=0	35- 43 35	f 2	55- 42 2
g FRAC	16- 43 24	g GTO 042	36-43,33,042	g GTO 000	56-43,33,000
1	17- 1	RCL 2	37- 45 2		
0	18- 0	−	38- 30		

Example: 0.777888999 STO 0, 100 STO 1, R/S. Display shows 0.555 while pausing, then displays 100.90, a winner of $1, less the cost of $0.10 to play. Press R/S. Display shows 0.666 while pausing, then displays 101.80, a winner of $1, less the $0.10 to play. Press R/S. Display shows 0.009 while pausing, then displays 101.70, a winner of $1. Press R/S. Display shows 0.943 while pausing, then displays 101.60, a winner of $1. Play as long as you like!

Appendix C – Four HP-12C Game Programs Cont'd - Page 5 of 5

HP-12C Eleven-Thirty Game

Gene Wright

This is an HP-12C version of the game of Eleven-Thirty on the HP-65. It was written by John Rausch and appeared in the V2N3P28 issue of PPC Journal (March 1975). A listing of that game for the HP-65 and instructions on how to play it can be found here: http://www.rskey.org/gene/calcmuseum/651130.htm

Enter a decimal seed and press STO 4. Store an initial "Pot" by entering the amount and pressing STO 0. Deal the first two numbers by pressing GTO 000 and R/S. The HP-12c will show two numbers between 11 and 30. The numbers will be in the form of XX.YY. Bet any amount you wish that the next number will be between the first two numbers (ties do not count). Enter bet (if you do not wish to bet, enter 0), and press R/S. Display will show the next number with a pause and then your new "Pot" either increased or decreased.

Keystrokes	Display	Keystrokes	Display	Keystrokes	Display
f P/R		0	25– 0	g FRAC	51– 43 24
f CLEAR PRGM	00–	×	26– 20	STO 4	52– 44 04
f 2	01– 42 2	1	27– 1	2	53– 2
RCL 4	02– 45 4	1	28– 1	0	54– 0
9	03– 9	+	29– 40	×	55– 20
9	04– 9	g INTG	30– 43 25	1	56– 1
7	05– 7	RCL 1	31– 45 1	1	57– 1
×	06– 20	x≷y	32– 34	+	58– 40
g FRAC	07– 43 24	g x≤y	33– 43 34	g INTG	59– 43 25
STO 4	08– 44 04	x≷y	34– 34	g PSE	60– 43 31
2	09– 2	STO 2	35– 44 02	RCL 2	61– 45 2
0	10– 0	x≷y	36– 34	g x≤y	62– 43 34
×	11– 20	STO 1	37– 44 01	g GTO 073	63-43,33,073
1	12– 1	RCL 2	38– 45 2	R↓	64– 43 35
1	13– 1	EEX	39– 26	RCL 1	65– 45 1
+	14– 40	2	40– 2	x≷y	66– 34
g INTG	15– 43 25	÷	41– 10	g x≤y	67– 43 34
STO 1	16– 44 01	+	42– 40	g GTO 073	68-43,33,073
RCL 4	17– 45 4	R/S	43– 31	RCL 3	69– 45 03
9	18– 9	f 0	44– 42 0	STO + 0	70- 44,40, 0
9	19– 9	STO 3	45– 44 03	RCL 0	71– 45 0
7	20– 7	RCL 4	46– 45 4	g GTO 000	72-43,33,000
×	21– 20	9	47– 9	RCL 3	73– 45 03
g FRAC	22– 43 24	9	48– 9	CHS	74– 16
STO 4	23– 44 04	7	49– 7	g GTO 070	75-43,33,070
2	24– 2	×	50– 20		

Example: 0.123456789 STO 4, 500 STO 0, g GTO 000 R/S. Display shows 12.14. I don't think the odds are good that the next number will be 13, so enter 0 R/S. Display pauses showing 28 (made a good bet) and then shows the pot of 500. Press R/S. Display shows 16.30. I like these odds for bet the whole pot, 500 R/S. Display pauses showing 23 (I'm rich!) and then shows the pot of 1000. Press R/S. Display shows 12.25. Hmm, bet 200 R/S. Display pauses showing 18 (made a good bet) and then shows the pot of 1200. Press R/S. Display shows 20.21. Hmm, bet 5 R/S. Display pauses showing 11 (oops! wasn't paying attention) and then shows the pot of 1195. Play as long as you like!

Appendix D – HP-12C References - Page 1 of 1

Here is a small sampling of HP and third party support books, manuals, and Internet links.

D1 - http://h10032.www1.hp.com/ctg/Manual/c00363319.pdf
D2 - http://h10032.www1.hp.com/ctg/Manual/c00367122.pdf
D3 - http://dl.dropbox.com/u/10022608/HP-12CRealEstateApplicationsHandbook.pdf
HP-12C Leasing Applications handbook - http://dl.dropbox.com/u/10022608/HP-12CLeasingApplicationsHandbook.pdf

HP has learning module "training guides" that address a wide range of topics. Here is one that explains how to write a small 12C program. - http://h20331.www2.hp.com/Hpsub/downloads/HP12Cprogram.pdf
Here is zip file of all of them. http://h20331.www2.hp.com/Hpsub/downloads/12c.zip

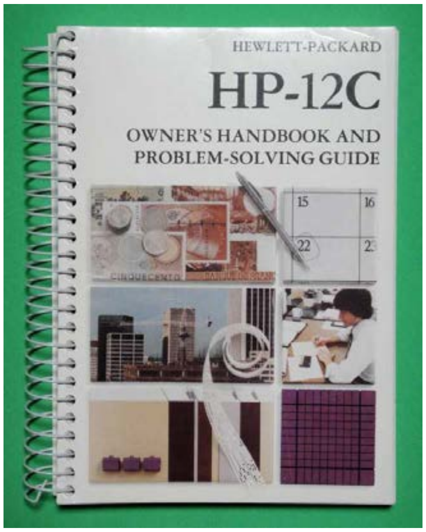


Fig. D1 – Handy sized Owner's Handbook Rev. G 11/85.

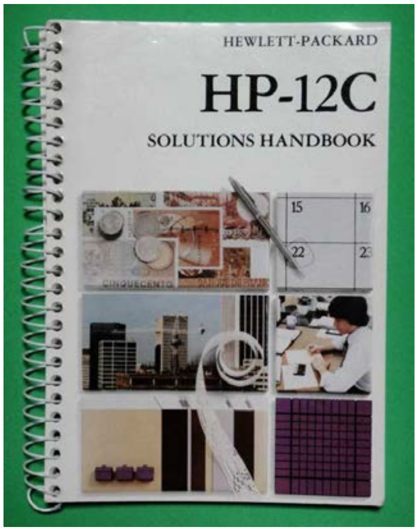


Fig. D2 – Solutions handbook Rev. F 7/87.

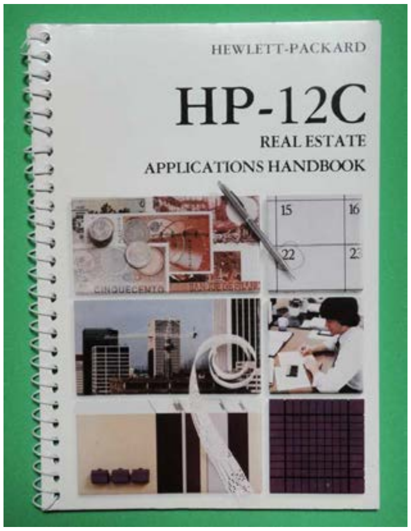


Fig. D3 – Real Estate Applications Handbook Rev. B 3/84.

Early third party support booklets are represented below. Chary is still active in HP-12C support using videos. D3 & D4 - http://hp-12c-calculator.com/?page_id=60

Gene's business math book is at: http://www.amazon.com/Quantitative-Analysis-Business-Statistics-Calculator/dp/1888840382/

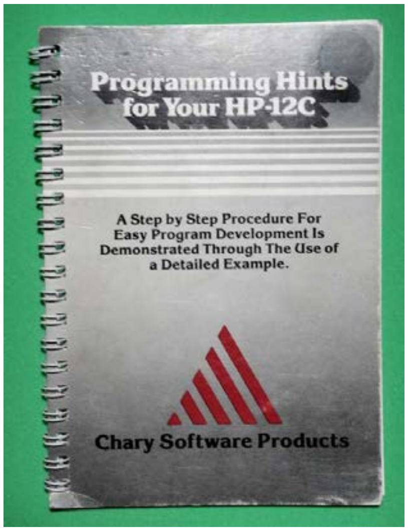


Fig. D3 – Chary Software Programming Hints book Rev. A 12/85

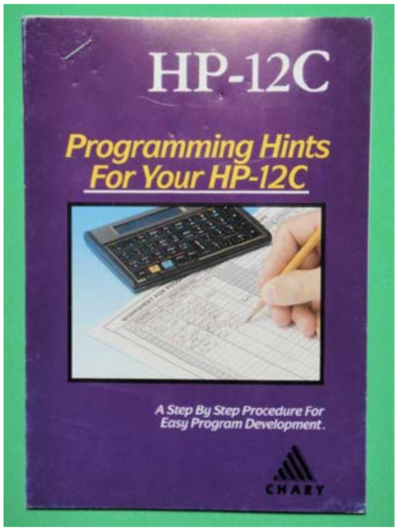


Fig. D4 – Chary Software Programming Hints book Rev. A 8/86

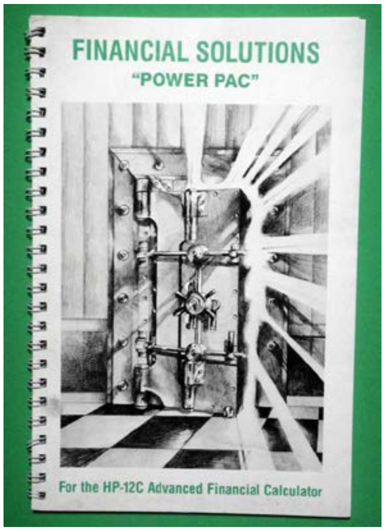


Fig. D6 - Renaissance Publications. Problem examples book © 1985

Calculator Programmability—How Important is it in 2011?

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Calculator Programmability – How Important is it in 2011?

Richard J. Nelson

Introduction

Fig. 1- HP-65A.

When the non-programmable scientific HP-35A calculator first appeared many HP enthusiasts wrote down keystroke lists**(1)** on index cards for the most efficient method of solving problems, especially those that required iterating to a solution. Two years later the programmable HP-65A appeared and tens of thousands of HP calculator users became programmers. This was in the mid 70's and the HP-65A was the first personal computer that gave programmability to everyone, not just to college students or employees of large corporations that could afford room sized computers.

The scientific programming language of the institutional computer was a higher-level language, typically FORTRAN or BASIC. The language of the HP Calculator was a lower level keystroke programming language that was "officially" named FOCAL**(2)** with the peak of the HP-41 popularity of the late 70's and early 80's. The HP-28C, in mid 1980, offered additional programmability in terms of its higher-level RPL programming "language." The vast majority of legacy RPN users didn't like the new language and the number of users who programmed in RPL vs. FOCAL dropped by a factor of 100. The new HP 15c Limited Edition, LE, offers a FOCAL like programming language and it will be used as an example.

Programming, what is it?

Fig. 2 - Binary.

A program is a series of instructions that are usually stored in memory to be "executed" multiple times. The term programming is also applied to the operation of modern electronic devices in terms of setting the various options of how the device works. Programming, as used here, is a list of keystrokes and operations that the calculator executes in the order as programmed into memory. A simple short program that solves a specific task that is used on a daily basis in business or on the production line can save many hours per week in computational time.

A program may also "manage" a data base and serve as a 'look up" mechanism for information such as prices, specifications, or any numeric data in the form of tables or equations.

Programming Advantages

Fig. 3 - Languages

What are the advantages of programmability for the calculator user?

1. Programming provides a quick simple method of solving the same problem over and over again. A single keystroke could replace dozens of keystrokes. An example is calculating the volume of a dozen grapefruit from a quality sampling.

2. A program could solve more complex problems that the user couldn't solve otherwise. The program is written by an expert and used by anyone. An example is a complex statistical calculation such as mean and standard deviation.

3. A program could contain data that isn't conveniently looked up or remembered. Making English Metric conversions is a simple example of this program usage e.g. converting grains to grams or troy ounces to grams.

4. A program could address a very specialized problem that information would be difficult to research. An example is a program that calculates aircraft altitude based on pressure that uses a very obscure equation.
5. A program could implement very complex mathematical methods that require thousands of loops or iterations to arrive at a solution. An example is a random walk calculation for a complex process.
6. A program could apply "trial and error" and data search methods to arrive at a solution. Finding a root of an equation or multiple equations is an example.
7. A program could act as a simple database where pricing information from part number inputs is either looked up or calculated based on quantity or delivery time inputs. An example is part number conversion to a data base number that has the desired price.
8. A simple program could utilize several applications programs that are part of the calculator to arrive at an optimum solution saving keystrokes similar to #1 above. An example is using a symbolic solver, unit management, unit conversion, and a plotting application to see a complex relationship.
9. A program could automatically label and identify inputs and outputs for user convenience. This is especially useful for multiple problems solutions on the stack.
10. Programmability provides a creative applications tool for the user community to develop and share knowledge and techniques for more effective problem solving. High precision arithmetic programs provide an infrequently used tool for those situations that require a high number of significant digits.
11. A creative program could be adaptive and self-learning to the extent that it exceeds the ability of its user. Game playing strategies have been programmed for the HP-65A and later machines so that after many games the machine is able to beat its human competitor.
12. Having a programmable machine provides a research tool for the manufacturer for the next generation of calculators. Many of the built-in applications of more advanced machines were applications programs created by HP's customers. A good example is the HP49 and HP50 models with added applications that were originally programmed by HP's users for the HP48.

Programming online or offline?

Fig. 4 – Connect.

High end scientific or graphing calculators may have an input/output, I/O, capability to connect to a computer - usually it is USB. When this capability is available software is often developed to write calculator programs on the computer. Most users will write the program on the calculator, but more elaborate and complex programming is more effectively done on the computer because of the time it saves. Usually this dividing line is between the amateur who writes an occasional program on the calculator and the professional who develops commercial calculator software on the computer.

HP 15c LE programming

Fig. 5 – HP-15c+ Program Mode.

The HP 15c RPN program instruction set has a respectable collection of programming features. The large number of applications programs, especially those that may be adapted from many other RPN programmable calculators is a substantial advantage the exceptional speed of the HP 15c LE provides an additional advantage of this programmability.

An important marketing aspect of the HP 15c LE is promoting the advantages of its being programmable and providing resources for obtaining programs. See the document titled *HP*-15C LE *Speed Comparison and Reference* provided at HHC MMX (2010) as a resource[(3)]. The HP 15c LE is on the order of 125 to 175 times faster than the original 1982 HP-15C. The "Comparison and Reference" document provides a table of speed comparisons, a list of references for programs, and a Quick Reference Guide for the features and functions of the HP-15C. Also see Appendix A for links to additional resources.

Off line program storage

Fig. 6 – Off line storage is needed.

The greater the number of features, operations, or functions of a calculator the greater is the need for it being programmable. Programming increases the power and capability of the calculator and memory capacity then becomes an issue. How many programs need to be stored and available to the user? When the number of programs increases, another aspect also becomes a vital issue. How may the programs be protected? The HP 15C has continuous memory as indicated by the "C/c" in the model number. There is no off line storage for HP-15C or HP 15c LE programs. This may be viewed by modern calculator users as a serious limitation. The larger the program memory the greater the need for offline storage, e.g. a USB port.

Calculator Programmability Observations and Conclusions

Fig. 7 – Tools.

Modern machines of the last ten years take advantage of lower memory cost (for functions and programs) to keep the cost of the calculator low. The primary advantage of the dedicated calculator is low cost and maximum convenience.

More complex problems are often solved on a personal computer, which tends to be turned on for extended periods, such as a work week of five days. Comparing the calculator solution with a computer solution doesn't yield a clear advantage in turn-on convenience (instant on vs. boot time). Still, a single key press for a programmed solution is nearly impossible to improve and the usefulness of programmability is easily justified in the midrange to the high end models for most of the dozen reasons listed above.

The programming method, however, must be as simple as possible so the average user may easily master it. The HP 15c LE with its FOCAL like keystroke programming is easy to learn. Even if the student is less interested in programming their own machine he or she will still need those programs suitable for the course at hand. Being able to share programs via the Internet and down load them via a USB port will always be a justification to include the programming feature of any serious scientific or graphing calculator.

> The large number of features, memory size, and super speed of the HP 15c makes its programmability an important feature when compared to other calculators - for the 12 programming reasons listed above.

To answer the title question, "Calculator Programmability – How Important is it Today?" I would have to say that it is as important as it ever was. A more modern consideration is having an I/O capable of down loading the programs regardless of the language of the operating system and even if most users are not writing programs themselves.

Notes - Calculator Programmability – How Important is it Today?

1) *The most famous collection of these "index card programs" was written by Lee Skinner of Albuquerque New Mexico who sold them to HP. Readers may know these as the HP-35 Math Pac or the similar HP-45 Math Pac. See page 5 of: http://hhus.us/2007/Remembering%20the%20HP35A.pdf for additional details.*

2) *FOCAL is believed to be what HP calls the Forty-One CAlculator Language. See* ***HP Solve*** *issue 23 for a discussion of FOCAL. Is this an HP Urban legend?*

3) *You may obtain copies of the (must have) Reference DVD at:* http://www.pahhc.org/ppccdrom.htm

Appendix A – HP-15C Programs in the Owner's & Advanced Functions Handbooks

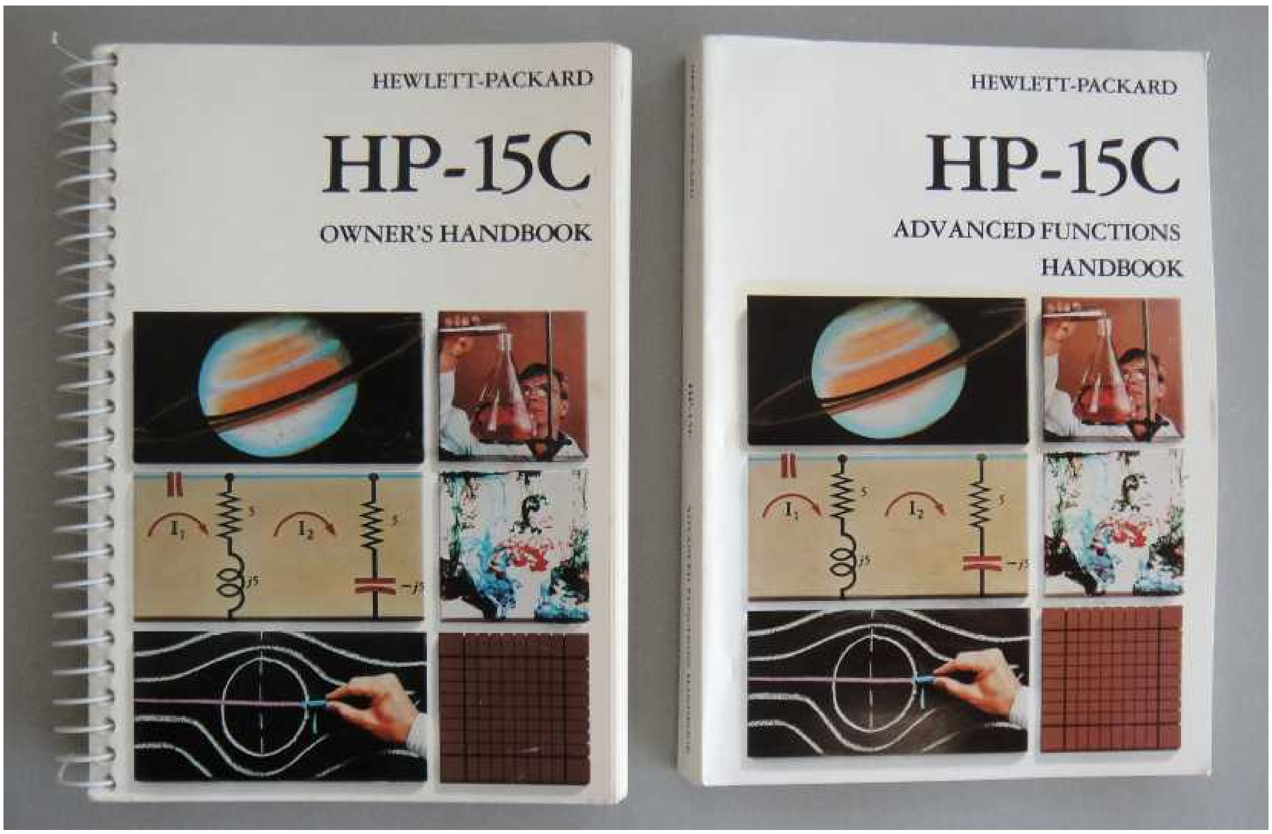

Fig. A1 – Owner's Handbook (initial1982 version) and Advanced Functions handbook (later 1984 version).

Owner's Handbook - 3/82. (00015-90001) http://hp15c.org/hp15c.pdf

1. Example Section 6: Calculating base area, volume, and surface area of cans, page 72,
2. Example Section 6: Calculating $5x^4 + 2x^3$, page 80.
3. Example Section 8: Figure the radio-activity at 3-day intervals until a given limit, page 93.
4. Example Section 8: Calculate payments of an investment for college tuition, page 95.
5. Example Section 8: Converting temperature inputs in degrees Celsius or Kelvin, page 99.
6. Example Section 9: Calculate the slope of the secant line of a parabola, page 103.
7. Example Section 10: Modifies Example #3 to store intervals into registers using DSE, page 113.
8. Example Section 10: Demonstrate FIX displays for all decimal positions, page 114.

Appendix A – HP-15C Programs in the Owner's & Advanced Functions Handbooks – continued.

9. Appendix D: Calculate the shear stress, Q = $3x^2 - 45x^2 + 350$, page 228.
10. Appendix D: Investigate $F(x) = 3 + e^{-|x|/10} - 2e^{x^2 e^{-|x|}}$, Page 231.
11. Appendix D: Use SOLVE initial estimates for #10 above of 10, 1, & 10^{-20}, page 232.
12. Appendix D: Use deflation for all roots of: $60x^4 - 944x^3 + 3003x^2 + 617x - 2890 = 0$, page 234-237.
13. Appendix E: Calculate $J_4(x) = \frac{1}{\pi}\int_0^{\pi} \cos(4\theta - x\sin\theta)\,d\theta$ Page 242.

(1) These are primarily short keystroke routines.

Advanced Functions Handbook - REV B, 8/84. (00015-90011) *http://hp15c.org/hp15cAdvanced.pdf*

Only applications programs are included in this list, excluded are the short keystroke expression evaluation routines.

1. Section 1: Time Value of Money, 108 lines, page 29.
2. Section 1: Discounted Cash flow, 068 lines, page 40.
3. Section 2: Four integrals of four special functions, 083 lines, page 61.
4. Section 3: Calculating the gamma function, 028 lines, page 66.
5. Section 3: Storing and Recalling Complex numbers in a matrix, 025, page 76.
6. Section 3: Calculating nth roots of a complex number, 037 lines, page 78.
7. Section 3: Solve and Equation for its Complex roots, 059 lines, page 82.
8. Section 3: Evaluate Complex integral, 045, page 85.
9. Section 3: Complex potential function. 061, page 90.
10. Section 4: Constructing an identity Matrix, page 119.
11. Section 4: Solves system of equations AX = B for X, then makes residual correction, page 120.
12. Section 4: Solves a system of Nonlinear Equations, pages 123 – 126.
13. Section 4: Solves a large System of Complex Equations, pages 129 – 131.
14. Section 4: Multiple linear regression (Least Squares), pages 135 & 136.
15. Section 4: Weighted least-squares multiple linear regression, pages 143 – 146.
16. Section 4: Eigenvalues of a Symmetric real Matrix, pages 150 – 153.
17. Section 4: Eigenvectors of a Symmetric Real Matrix, pages 155 – 157.
18. Section 4: Optimization (Min & Max) of a real-valued function, pages 163 – 166.

Special note: The Appendix of the Advanced Functions Handbook, pages 172 – 211 provides an excellent discussion of the accuracy of all HP calculator numerical calculations – prior to the RPL generation. See example in Fig. A2. There are four error evaluation and error correcting programs.

1. *Calculating a more accurate $\lambda = \ln(1 + x)$, pages 181 & 182.*
2. *Calculating angle (cosine law) in a triangle, error analysis (very nice), pages 197 – 199.*
3. *Accurate roots to real quadratic equation - real and complex roots, pages 205 & 206.*
4. *Most accurate variation of #3, uses ∑+ & ∑-. Certain calculations are done to 13 places, pages 208 - 210.*

Level 1: Correctly Rounded, or Nearly So

Operations that deliver "correctly rounded" results whose error cannot exceed ½ unit in their last (10th) significant digit include the real algebraic operations [+], [−], [×], [÷], [x^2], [$\sqrt{x}$], [1/x], and [%], the complex and matrix operations [+] and [−], matrix by scalar operations [×] and [÷] (excluding division by a matrix), and [→H.MS]. These results are the best that 10 significant digits can represent, as are familiar constants [π], 1 [e^x], 2 [LN], 10 [LN], 1 [→RAD], and many more. Operations that can suffer a slightly larger error, but still significantly smaller than one unit in the 10th significant digit of the result, include [Δ%], [→H], [→RAD], [→DEG], [$P_{y,x}$], and [$C_{y,x}$]; [LN], [LOG], [10^x], and [TANH] for real arguments; [→P], [SIN^{-1}], [COS^{-1}], [TAN^{-1}], [$SINH^{-1}$], [$COSH^{-1}$], and [$TANH^{-1}$] for real and complex arguments; [ABS], [$\sqrt{x}$], and [1/x] for complex arguments; matrix norms [MATRIX] 7 and [MATRIX] 8; and finally [SIN], [COS], and [TAN] for real arguments in Degrees and Grads modes (but not in Radians mode—refer to Level 2, page 184).

Fig A2 – Error analysis text example page 179.

Converting Decimal Numbers to Fractions

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Converting Decimal Numbers to Fractions

Joseph K. Horn

What fraction is π equal to? Most people would reply "22/7" and of course they would be wrong, since π is an ir*ratio*nal number which means that it cannot be expressed as a *ratio* of integers. But 22/7 is *close* to π. Is there a fraction that is closer?

The answer originally flowed from the following line of brute-force observation: "$\pi \cong 3$. Well, not quite 3, but more precisely $\pi \cong 3\frac{1}{7}$. (Hence 22/7). Well, not quite 7, but $7\frac{1}{15}$, which means that

$\pi \cong 3+\cfrac{1}{7+\cfrac{1}{15}}$. (Hence the better approximation 333/106)." And so on. Following this line of reasoning, closer and closer approximations to π can be generated. The trick is to pick the correct adjustments. There is a method for doing this that avoids brute-force searching. It is called "The Continued-Fraction Algorithm." Here it is.

First you make a list of numbers like this:

A) Write down the integer part (INT) of the number.
B) Subtract the integer part by pressing FRAC.
C) Reciprocate this by pressing 1/*x*.
D) Go to step A. Repeat a few times. Stop if you hit a big number.

This will generate the following list for π: { 3, 7, 15, 1, 293 }. Recognize the first three numbers? They are the denominators printed in red in the large equation for π above. Try using the first *four* numbers from this list instead of just the first *three*; it yields this larger expression:

$$\pi \cong 3+\cfrac{1}{7+\cfrac{1}{15+\cfrac{1}{1}}}=\frac{355}{113}$$

Hence a better approximation for π is 355/113. Now use *all five* numbers from the list:

$$\pi \cong 3+\cfrac{1}{7+\cfrac{1}{15+\cfrac{1}{1+\cfrac{1}{293}}}}$$

This huge expression of nested reciprocals (called a "continued fraction") simplifies to 104348/33215, which is therefore a fraction that is even closer to π than 355/113. It is so close, in fact, that it evaluates on the HP-15 to the same value as the π key.

This procedure can be used to quickly generate a fraction that approximates any decimal number to any desired number of digits. Let's do it by hand one more time to appreciate the method.

What fractions best approximate $\sqrt{\pi}$? First we follow the 4 steps A-D above and get this list: { 1, 1, 3, 2, 1, 1, 6, 1, 28 }. These numbers are called the "partial quotients" of the continued fraction. We then build some continued fractions from the first few partial quotients, thereby deriving better-and-better approximations to $\sqrt{\pi}$, including these three:

$$1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{2}}}=\frac{16}{9}$$

$$1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1}}}}}=\frac{39}{22}$$

$$1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{6}}}}}}=\frac{257}{145}$$

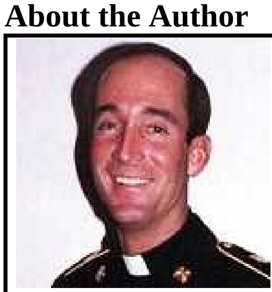
The fractions generated by this method are called the "convergents" to the number being approximated. The infinite set of convergents for $\sqrt{\pi}$ begins thus:

$$\frac{1}{1},\frac{2}{1},\frac{7}{4},\frac{16}{9},\frac{23}{13},\frac{39}{22},\frac{257}{145},\frac{296}{167},\frac{8545}{4821},\ldots$$

Since the continued fraction algorithm is mathematically well-defined, it can be programmed on any programmable HP calculator. Appendix A of this article is an HP-15C program that implements it. Appendix B is an HP 35s program. The HP-15C program only needs 45 steps to convert any decimal number into a fraction with any desired accuracy. Just input the decimal number, set the display FIX to control the accuracy, and GSB A. The numerator appears. Press X<>Y to see the denominator. Optionally press ÷ RCL – 0 to see the difference between the input decimal and the output fraction.

The input that makes the program work the hardest is the golden ratio, $\frac{\sqrt{5}+1}{2}$, because its partial quotients are { 1, 1, 1, 1, 1, 1, 1, … }. The HP-15C program says that the fraction that best approximates the golden ratio to 7 digits is $\frac{6765}{4181}$. What are its other convergents? It has more than any other number! If you write the first several down, you will see the Fibonacci Series hidden in it. Many other Number Theory goodies await discovery along the path of the Continued Fraction Algorithm. Happy exploring!

About the Author



Joseph K. Horn is a high school math teacher in Orange County, California. He has authored many articles related to HP calculators in addition to his book titled, HP-71 Basic Made Easy. He is sometimes known in the HP user community as "The math bug hunter."

Joseph serves on the HP Handheld conference, HHC, committee and is webmaster for the HHC website. hhuc.us

Appendix A - Decimal to Fraction for the HP-15C

Joseph K. Horn

Version 1.2
Converts any positive decimal number to a fraction.
Accuracy is controlled by the current FIX setting.

01*LBL A	42 21 11	16 X<>Y	34	31 1/x	15
02 STO 0	44 0	17 ÷	10	32 STO 1	44 1
03 STO 1	44 1	18 RND	43 34	33 GTO 0	22 0
04 0	0	19 RCL 0	45 0	34*LBL 1	42 21 1
05 ENTER	36	20 RND	43 34	35 RCL 2	45 2
06 ENTER	36	21 TEST 5	43 30 5	36*LBL 2	42 21 2
07 1	1	22 GTO 1	22 1	37 ENTER	36
08 R↑	43 33	23 +	40	38 ENTER	36
09*LBL 0	42 21 0	24 CLx	43 35	39 RCL 0	45 0
10 INT	43 44	25 RCL 2	45 2	40 ×	20
11 R↑	43 33	26 ENTER	36	41 .	48
12 ×	20	27 ENTER	36	42 5	5
13 +	40	28 R↑	43 33	43 +	40
14 STO 2	44 2	29 RCL 1	45 1	44 INT	43 44
15 GSB 2	32 2	30 FRAC	42 44	45 RTN	43 32

Instructions:
(1) Use a FIX setting as desired.
(2) Place decimal number in X.
(3) GSB A ⇨ see numerator.
(4) X<>Y ⇨ see denominator.
Optional: ÷ RCL - 0 ⇨ "error" (difference between input and output)

Example:
Convert π to a fraction accurate to 5 decimal places.
(1) FIX 5
(2) π
(3) GSB A ⇨ 355
(4) X<>Y ⇨ 113
Answer: 355/113

Notes:
- (a) The program uses registers 0 through 2, so make sure that enough register memory is allocated.
- (b) As always, turning off USER mode makes it easier to key in the program.
- (c) When the program finishes, register 0 contains your original input. Registers 1 and 2 are used for temporary storage.
- (d) The Golden ratio, 1.618033989, is the most challenging (takes longest time).

Appendix B - Decimal to Fraction for the HP-35s

Joseph K. Horn

Note: Program execution is the same as for the HP-15C program except it is launched by XEQ A ENTER instead of GSB A.

No attempt was made to optimize the program by using line number addressing instead of labels.

#	**Prgm. Instr.**	**#**	**Prgm. Instr.**	**#**	**Prgm. Instr.**	**#**	**Prgm. Instr.**
1	A001 LBL A	12	B004 ×	23	B015 +	34	C001 LBL C
2	A002 STO A	13	B005 +	24	B016 CLx	35	C002 RCL C
3	A003 STO B	14	B006 STO C	25	B017 RCL C	36	D001 LBL D
4	A004 0	15	B007 XEQ D001	26	B018 ENTER	37	D002 ENTER
5	A005 ENTER	16	B008 x<>y	27	B019 ENTER	38	D003 ENTER
6	A006 ENTER	17	B009 /	28	B020 R↑	39	D004 RCL A
7	A007 1	18	B010 RND	29	B021 RCL B	40	D005 ×
8	A008 R↑	19	B011 RCL A	30	B022 FP	41	D006 0.5
9	B001 LBL B	20	B012 RND	31	B023 1/x	42	D007 +
10	B002 INTG	21	B013 x=y?	32	B024 STO B	43	D008 INTG
11	B003 R↑	22	B014 GTO C001	33	B025 GTO B001	44	D009 RTN

Note: # column numbers do not appear in the display and are included for reference only.

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Limited Edition HP 15c Execution Times

Namir Shammas

Introduction

The famous French writer Voltaire seems to have captured the driving force behind progress when he said "The better is the enemy of the good." Recently, Hewlett-Packard launched the HP 15c Limited Edition (which I will nickname the HP-15C+ in this article) due to popular demand by users of vintage HP calculators. The vintage HP-15C was the second calculator to support the Solve and Integrate features, after the HP-34C, and the first one to support both complex math and matrix operations. The HP-15C has earned many fans that went online and posted a petition for HP to re-launch that machine. After many years of waiting, I am happy to report that HP has listened and indeed responded to these very loyal HP-15C fans. The HP 15c Limited Edition is here and comes with the same features of the original HP-15C, with added speed. I mean this little puppy is fast! How fast, you may ask? This article answers this question by comparing the speeds of the old HP-15C and the new HP 15c.

Speed Test Setup

Like many HP enthusiasts, I heard rumors about the speed of the new HP-15C+. I decided to investigate the speed of the new machine and compare it with the vintage version. My testing includes the following aspects:

1. Each test uses a program that repeats a set of operations for a fixed number of times. Timing (using the stopwatch of an HP-41CX) starts and stops when the test program starts and stops, respectively. To help the setup a test, I use code that initializes memory registers used in the test program.
2. I use simple programs that execute empty loops to obtain the time for these loops. When I perform a test for an operation set, I subtract the empty loop time to obtain the time for the operation set. I calculate the time needed to perform that operation set by dividing the number of test loops by the net operations' time. I get the speed of an operation set.
3. To calculate the speed ratio, I divide the speed of the operation sets performed on the HP-15C+ by that of the HP-15C. This is the target statistic that I am looking for in each test.
4. To test functions that need arguments, I initiaize the tests by storing the arguments in memory registers. The test loops then recall the values from the registers and supply them as arguments to the tested functions. This approach offers uniform timing to obtain the arguments of a function. While most functions need one argument, some like Y^X need two. I have also timed loops that recall one memory register and also two memory registers. I subtract the time for these loops from the time of the loops that recall arguments for built-in functions and then execute these functions.
5. Since the HP-15C and HP-15C+ vary in speed significantly, I use loops that differ in the number of iterations to keep testing within reasonable times and minimize human errors. I estimate human error to be between 0.1 and 0.2 seconds.
6. The DC voltages for the cells in the vintage HP-15C were read by an AVO meter to be 1.505, 1.509, and 1.503 volts. The DC voltages for the two cells of the HP-15C+ were measured as 2.97 and 2.98 volts. The tests were run at room temperature of 70 degrees Fahrenheit.

Speed Test Code

Here is the code for an empty loop that iterates 100 times on the HP-15C:

```
LBL A
EEX
2
STO 0
RTN

LBL B
DSE 0
GTO B
RTN
```

Label A initializes the loop counter stored in register 0. The initial loop counter value depends on the test and the machine version. Label B performs the test to time an empty loop. I use the DSE command because I can count down from numbers greater than 1000. If I use the ISG command instead, I am limited to a maximum of 1000 iterations. On the HP15C+ I use tests programs that loop10000 times. This high loop iteration justifies selecting the DSE command to control the loop iterations.

Here is the test that recalls a value from register 9:

```
LBL B
RCL 9
DSE 0
GTO B
RTN
```

Keep in mind that the above loop recalls a value from register 9 AND also pushes the stack. Most the tests involve operations and/or calculations that push the stack and/or store a value in the LastX register. A few operations, like STO n, do not move the stack and do not alter the Last X register. Math operators, like +, drop the stack and also update the Last X register. Single-argument functions alter the X register in the stack and update the LastX register. The RCL n operations push the stack up but do not affect the LastX register.

Here is a sample code that calculates the sine of a 0.6 radians, stored in register 9:

```
LBL A
RAD       # switch to radians
.
6
STO 9     # store 0.6 in memory register 9
EEX
2
STO 0     # initialize the loop counter for the HP-15C
RTN

LBL B
RCL 9     # recall argument value
SIN       # calculate the sine
DSE 0
GTO B     # end of the test loop
RTN
```

Label A initializes the test by switching to radians, storing 0.6 in memory register 9, and setting up the loop counter. Label B performs the test by repeatedly recalling the value of 0.6 from memory register 9 and executing the sine function. The above code is very similar to versions for the other built-in functions. In the case of a using function with two arguments I use code that looks like the following one:

```
LBL A
Π
STO 9     # store value for first argument
STO 8     # store value for second argument (same as first)
EEX
2
STO 0     # initialize loop test counter for the HP-15C
RTN

LBL B
RCL 9     # recall first argument
RCL 8     # recall second argument
Y^X       # use built-in function
DSE 0
GTO B
RTN
```

Label A initializes the arguments by storing them in registers 8 and 9, and then initializes the test loop counter. Label B performs the test by repeatedly recalling the two arguments from registers 9 and 8, and then using the built-in function Y^X.

Testing the Σ+ operation is a bit different. The test repeats the process of adding 100 random values for Y and X in the statistical registers. The test uses the following code:

```
LBL A
EEX
2
STO
0         # initialize the loop counter for the HP-15C
RTN

LBL B
CLEAR Σ   # clear the statistical summations
EEX
2
STO 1     # set counter for the inner loop
LBL 0     # start inner loop
RAN #     # generate random Y
RAN #     # generate random X
Σ+        # add to statistical summations
DSE 1     # test end of inner loop
GTO 1     # end of inner loop
DSE 0     # test end of outer loop
GTO B     # end of outer loop
RTN
```

Label A initializes the main loop in label B. This label has an inner loop that starts at label 0. The first lines after label B initialize the statistical summations and the counter for the inner loop. The inner loop generates the random values for Y and X, and then adds them to the statistical summations. The outer loop repeats the process of generating random numbers and adding them to the statistical registers.

Special Speed Test Code

Most of the tests I conducted use programs that are similar to the ones I presented in the last section. These programs are aimed at simple operations and the evaluation of built-in functions. In this section, I present the code for more complex tests. These tests are:

- Calculating e^π using a Taylor expansion polynomial. The computations add the values of different Taylor expansion terms to obtain the exponential value. When the program rounds the value of a term to zero (based on the current number of displayed digits), the iteration stops. I tested calculating e^ π in FIX 7 and FIX 4 display modes.
- Calculating the integral of 1/x from 1 to 100.
- Calculating the root of the function e^x-3*x^2 between 3 and 4.
- Converting decimals to fractions. This test is based on a program written by Joseph Horn . The test uses the value of the golden ratio, since it requires more calculations than many other common numbers with fractions, like π or e. The test program uses the FIX 9 display mode.

The code in this section is aimed at the new HP-15C+. The code for the vintage HP-15C differs only in the values stored in the loop counter (in register 0), which is set to 10.

The code for calculating e^π using Taylor approximation is:

```
LBL A
FIX 7     # can set to other values
Π         # π
STO 1
EEX
3
STO 0     # initialize the loop counter for the HP-15C+

RTN

LBL B
0
STO 2     # counter
STO 3     # sum
1
STO 4     # x^counter
LBL 1     # start of inner loop
RCL 4
RCL 2
X!
/
STO+ 3
RND       # round term
```

```
x=0?       # term rounds to zero?
GTO 2
1
STO+ 2     # counter += 1
RCL 1
STO* 4     # x^counter
GTO 1      # end of inner loop
LBL 2
RCL 3      # recall e^π
DSE 0
GTO B
RTN
```

Here is the code for testing the Integrate function:

```
LBL A
1
STO 9      # store first argument
EEX
2
STO 8      # store second argument
EEX
3
STO 0      # initialize the loop counter for the HP-15C+
RTN

LBL B
RCL 9      # recall lower limit for integration
RCL 8      # recall upper limit for integration
∫ 0        # integrate function in label 0
DSE 0
GTO B
RTN
LBL 0      # custom function to integrate
1/X
RTN
```

Here is the code for testing the Solve function:

```
LBL A
3
STO 9
4
STO 8
EEX
3
STO 0      # initialize the loop counter for the HP-15C+
RTN

LBL B
```

```
RCL 9     # recall first value to define solution interval
RCL 8     # recall second value to define solution interval
SOLVE 0   # solve function in label 0
DSE 0
GTO B     # end of loop
RTN
LBL 0     # custom function to solve
e^X
LSTX
X^2
3
*
-
RTN
```

Here is the code for the decimal to fraction program (version 1.1) that Joseph Horn wrote:

```
LBL A
STO 0
STO 1
0
ENTER
ENTER
1
R^
LBL 0
INT
R^
*
+
STO 2
GSB 2
X<>Y
/
RND
RCL 0
RND
TEST 5
GTO 1
CLx
+
CLx
RCL 2
ENTER
ENTER
R^
RCL 1
FRAC
1/X
```

```
STO 1
GTO 0
LBL 1
RCL 2
LBL 2
ENTER
ENTER
RCL 0
*
.
5
+
INT
RTN
```

I am including the above version, because that's the one I used for testing, and in case Joseph Horn further improves his code. Here is the code I added to Joe's program to test the decimal to fraction calculations:

```
LBL C
FIX 9
STO 4      # store number (e.g. golden ratio)
EEX
3
STO 3      # initialize the loop counter for the HP-15C+
RTN

LBL B
RCL 4
GSB A      # call Joe Horn's program in LBL A
DSE 3
GTO B
RTN
```

LBL C initializes the test. Enter the value for the number with a fractional part and then press the key C. In the case of the golden ratio, enter 1.6180339887 and then press the key C. Label B starts the test by repeatedly recalling the value with a fractional part from register 4 and then invoking Joe's code in label A as a subroutine. The test code uses register 3 as a loop counter since Joe's code already uses registers 0, 1, and 2.

Results

Table 1 shows the speed comparison summary. The table has the following columns:

1. The **Test** column specifies the test conducted.
2. The **HP-15C** column lists the speed for the various operation sets for the HP-15C.
3. The **HP-15C+** column lists the speed for the various operation sets for the HP-15C+.
4. The **Speed Ratio** column displays the ratio for the speeds of the HP-15C+ divided by the speeds of the HP-15C.

Tables 2 and 3 show the detailed timings for the HP-15C and HP-15C+, respectively.

Table 1. List of timing tests.

Test	*HP-15C Result Ops/sec*	*HP-15C+ Result Ops/sec*	*Speed Ratio*
Empty loop 100 iterations	3.06		
Empty loop 10000 iterations		473.93	
Empty loop 10 iterations	2.57		
Empty loop 1000 iterations		398.41	
STO 9	13.84	2475.25	178.84
RCL 9	12.49	2252.25	180.29
RCL 9, RCL 8	6.20	1119.82	180.57
RCL 9, RCL 8 for 10 iters	6.21	1176.47	189.41
Roll entire stack down using four Rv commands	3.58	595.24	166.04
+	10.47	1572.33	150.24
-	10.52	1666.67	158.42
*	8.58	924.21	107.72
/	6.64	750.19	113.02
997, SQRT	6.58	734.21	111.53
997, X^2	8.61	1112.35	129.14
π, 1/X	6.94	766.87	110.43
π, π, Y^X	0.97	105.85	108.68
SIN(0.6 RAD)	1.18	123.32	104.18
COS(0.6 RAD)	1.25	133.05	106.52
TAN(0.6 RAD)	1.89	211.60	111.76
ASIN(0.6)	1.48	173.04	116.68
ACOS(0.6)	1.41	163.85	115.83
ATAN(0.6)	2.26	263.09	116.23
SINH(0.6)	1.63	182.68	111.86
COSH(0.6)	3.58	227.38	63.56
TANH(.6)	1.24	131.35	106.24
ASINH(π)	1.18	122.46	104.00
ACOSH(π)	1.25	132.82	105.92
ATANH(0.6)	2.32	284.09	122.36
45, 45, ->P	2.45	303.21	123.89
π/4, 63,->R	0.96	98.84	102.48
1.3333, ->H	9.44	1412.43	149.58
1.5591666667, ->HMS	11.57	1805.05	155.96
LN(997)	3.65	426.44	116.67
LOG(99□)	2.74	290.61	105.99
EXP(2.123)	2.67	308.83	115.87
10^2.123	2.69	305.62	113.78
π, X!	0.51	59.21	116.53
Random number	7.59	859.85	113.28
2 Random numbers	3.76	430.29	114.44

Test	*HP□15C Result Ops/sec*	*HP-15C+ Result Ops/sec*	*Speed Ratio*
FRAC of π	13.23	2141.33	161.88
INT of π	13.61	2227.17	163.70
Integrate 1/x from 1 to 100	3.04E-02	3.63	119.71
Solve (e^x-3*x^2) for guesses 3 and 4	8.43E-02	10.95	129.97
Adding pairs of rand□m numbers in stat summations	1.78	205.76	115.76
Calculate e^π using FIX 7	3.75E-02	5.75	153.44
Calculate e^π using FIX 4	4.76E-02	7.37	154.83
Fractions for golden ratio in FIX 9	1.52E-02	2.47	161.83

Looking at Table 1, you can notice the following speed ratios for the different categories of operators, functions, and programs:

- The STO and RCL operations show the highest speed ratio gain which is around 180.
- The stack roll down operation shows a high speed ratio of about 166.
- The addition and subtraction operators show speed ratios that average 154.33.
- The multiplication and division operators show speed ratios that average 110.
- The square root, square, reciprocal, and raising to power operations have speed ratios that average 114.95.
- The trigonometric functions and their inverses have speed ratios that average 111.87.
- The hyperbolic functions and their inverses have speed ratios that average 102.32.
- The conversion to polar coordinate has a speed ratio of 122, while the conversion to rectangular coordinates has a speed ratio of 102.
- The Hour and HMS conversion functions have speed ratios that average 152.77.
- The common log, natural log, and their inverse functions have speed ratios that average 113.08.
- Random number generation has a speed ratio of 113.28.
- The INT and FRAC functions have speed ratios that average 162.79.
- The Integral and Solve functions have speed ratios that average 124.84.
- Adding random pairs of X and Y values to statistical summations is an operation with a speed ratio of 115.76
- The programs that calculate e^π have speed ratios that average 154.13.
- Joe Horn's program that converts decimals to fractions has a speed ratio of 161.83.

Looking at the above results you can see that speed ratios of 150 and higher appear in register operations, rolling down the stack, addition, subtraction, the INT function, the FRAC function, the program that calculates e^ π (using the Taylor approximation), and the decimal to fraction program. The two programs in the list show high speed ratios, perhaps, because they do not use functions that exhibit lower speed ratios (like the trigonometric, hyperbolic, and logarithmic functions.) This conclusion agrees with the relatively lower speed ratio of the statistical summation test. The listing for that test shows that it uses two RAN# commands (which show speed ratios of about 113). The speed ratio for the statistical summation test is very close to the speed ratio of the RAN# command. This observation tells me that the RAN# is the slowest part of the statistical summation test—akin to a bottleneck, if you like.

The results of the speed comparison have eliminated my own initial naïve expectation that the new HP-15C+ would have the exact same gain in speed across the board--for each and every operation. The results show more gain in certain operations than others. Nevertheless, even in the lower speed ratios, we are still talking about a new machine that is at least one hundred times faster than its predecessor. You can say that HP put a tiger in the new CPU! If the late actor Jackie Gleason was a dedicated HP-15C fan he would have said about the new machine, “How sweet it is!”

> The HP-15C Limited Edition shows speed ratios, compared to the vintage HP-15C, in the range of 102 to 189. Even at the lower end of this range, the new HP-15C offers very impressive speed!

Table 2. Timing Results for the HP-15C

Test	*Raw Time (seconds)*	*Shift (seconds)*	*Adjusted Time (seconds)*	*Loops*	*Result Ops/sec*
Empty loop 100 iterations	32.705	0	32.705	100	3.06
Empty loop 10 iterations	3.89	0	3.89	10	2.57
STO 9	39.93	32.705	7.225	100	13.84
RCL 9	40.71	32.705	8.005	100	12.49
RCL 9, RCL 8 for 100 iterations	48.83	32.705	16.125	100	6.20
RCL 9, RCL 8 for 10 iterations	5.5	3.89	1.61	10	6.21
Roll entire stack down	60.6	32.705	27.895	100	3.58
+	42.26	32.705	9.555	100	10.47
-	42.21	32.705	9.505	100	10.52
*	44.36	32.705	11.655	100	8.58
/	47.77	32.705	15.065	100	6.64
997, SQRT	55.9	40.71	15.19	100	6.58
997, X^2	52.32	40.71	11.61	100	8.61
π, 1/X	55.11	40.71	14.4	100	6.94
π, π, Y^X	151.5	48.83	102.67	100	0.97
SIN(0.6 RAD)	125.19	40.71	84.48	100	1.18
COS(0.6 RAD)	120.77	40.71	80.06	100	1.25
TAN(0.6 RAD)	93.53	40.71	52.82	100	1.89
ASIN(0.6)	108.14	40.71	67.43	100	1.48
ACOS(0.6)	111.4	40.71	70.69	100	1.41
ATAN(0.6)	84.89	40.71	44.18	100	2.26
SINH(0.6)	101.94	40.71	61.23	100	1.63
COSH(0.6)	91.03	40.71	50.32	180	3.58
TANH(.6)	121.59	40.71	80.88	100	1.24
ASINH(π)	125.64	40.71	84.93	100	1.18
ACOSH(π)	120.46	40.71	79.75	100	1.25
ATANH(0.6)	83.78	40.71	43.07	100	2.32

Test	Raw Time (seconds)	Shift (seconds)	Adjusted Time (seconds)	Loops	Result Ops/sec
45, 45, ->P	89.69	48.83	40.86	100	2.45
π/4, 63,->R	152.51	48.83	103.68	100	0.96
1.3333, ->H	51.3	40.71	10.59	100	9.44
1.5591666667,->HMS	49.35	40.71	8.64	100	11.57
LN(997)	68.07	40.71	27.36	100	3.65
LOG(997)	77.18	40.71	36.47	100	2.74
EXP(2.123)	78.23	40.71	37.52	100	2.67
10^2.123	77.94	40.71	37.23	100	2.69
π, X!	237.53	40.71	196.82	100	0.51
Random number	45.88	32.705	13.175	100	7.59
2 Random numbers	59.3	32.705	26.595	100	3.76
FRAC of π	48.27	40.71	7.56	100	13.23
INT of π	48.06	40.71	7.35	100	13.61
Integrate 1/x from 1 to 100	334.97	5.5	329.47	10	3.04E-02
Solve (e^x-3*x^2) for guesses 3 and 4	124.16	5.5	118.66	10	8.43E-02
Adding pairs of random numbers in stat summations	115.56	59.3	56.26	100	1.78
Calculate e^π using FIX 7	270.83	3.89	266.94	10	3.75E-02
Calculate e^π using FIX 4	214.07	3.89	210.18	10	4.76E-02
Fractions for golden ratio in FIX 9	660.23	3.89	656.34	10	1.52E-02

Table 3. Timing results for the HP-15C+

Test	Raw Time (seconds)	Shift (seconds)	Adjusted Time (seconds)	Loops	Result Ops/sec
Empty loop 10000 iterations	21.1	0	21.1	10000	473.93
Empty loop 1000 iterations	2.51	0	2.51	1000	398.41
STO 9	25.14	21.1	4.04	10000	2475.25
RCL 9	25.54	21.1	4.44	10000	2252.25
RCL 9, RCL 8 for 10000 iterations	30.03	21.1	8.93	10000	1119.82
RCL 9, RCL 8 for 1000 iterations	3.36	2.51	0.85	1000	1176.47
Roll entire stack down	37.9	21.1	16.8	10000	595.24
+	27.46	21.1	6.36	10000	1572.33
-	27.1	21.1	6	10000	1666.67
*	31.92	21.1	10.82	10000	924.21
/	34.43	21.1	13.33	10000	750.19

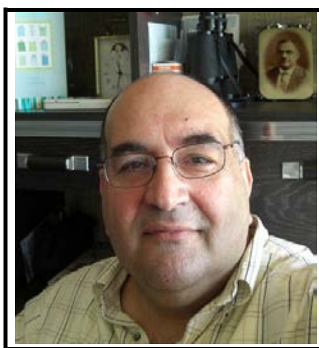
Test	*Raw Time (seconds)*	*Shift (seconds)*	*Adjusted Time (seconds)*	*Loops*	*Result Ops/sec*
997, SQRT	39.16	25.54	13.62	10000	734.21
997, X^2	34.53	25.54	8.99	10000	1112.35
π, 1/X	38.58	25.54	13.04	10000	766.87
π, π, Y^X	124.5	30.03	94.47	10000	105.85
SIN(0.6 RAD)	106.63	25.54	81.09	10000	123.32
COS(0.6 RAD)	100.7	25.54	75.16	10000	133.05
TAN(0.6 RAD)	72.8	25.54	47.26	10000	211.60
ASIN(0.6)	83.33	25.54	57.79	10000	173.04
ACOS(0.6)	86.57	25.54	61.03	10000	163.85
ATAN(0.6)	63.55	25.54	38.01	10000	263.09
SINH(0.6)	80.28	25.54	54.74	10000	182.68
COSH(0.6)	69.52	25.54	43.98	10000	227.38
TANH(.6)	101.67	25.54	76.13	10000	131.35
ASINH(π)	107.2	25.54	81.66	10000	122.46
ACOSH(π)	100.83	25.54	75.29	10000	132.82
ATANH(0.6)	60.74	25.54	35.2	10000	284.09
45, 45, ->P	63.01	30.03	32.98	10000	303.21
π/4, 63,->R	131.2	30.03	101.17	10000	98.84
1.3333, ->H	32.62	25.54	7.08	10000	1412.43
1.5591666667,->HMS	31.08	25.54	5.54	10000	1805.05
LN(997)	48.99	25.54	23.45	10000	426.44
LOG(997)	59.95	25.54	34.41	10000	290.61
EXP(2.123)	57.92	25.54	32.38	10000	308.83
10^2.123	58.26	25.54	32.72	10000	305.62
π, X!	194.44	25.54	168.9	10000	59.21
Random number	32.73	21.1	11.63	10000	859.85
2 Random numbers	44.34	21.1	23.24	10000	430.29
FRAC of π	30.21	25.54	4.67	10000	2141.33
INT of π	30.03	25.54	4.49	10000	2227.17
Integrate 1/x from 1 to 100	277.73	2.51	275.22	1000	3.63
Solve (e^x-3*x^2) for guesses 3 and 4	93.81	2.51	91.3	1000	10.95
Adding pairs of random numbers in stat summations	92.94	44.34	48.6	10000	205.76
Calculate e^π using FIX 7	176.48	2.51	173.97	1000	5.75
Calculate e^π using FIX 4	138.26	2.51	135.75	1000	7.37
Fractions for golden ratio in FIX 9	408.09	2.51	405.58	1000	2.47

References

1. *HP-15C Owner's Handbook*, June 1987, Hewlett-Packard.

2. *HP-15C Advanced Functions Handbook*, August 1982, Hewlett-Packard.

About the Author



Namir Shammas is a native of Baghdad, Iraq. He resides in Richmond, Virginia, USA. Namir graduated with a degree in Chemical Engineering. He received a master degree in Chemical engineering from the University of Michigan, Ann Arbor. He worked for a few years in the field of water treatment before focusing for 17 years on writing programming books and articles. Later he worked in corporate technical documentation. He is a big fan of HP calculators and collects many vintage models. His hobbies also include traveling, music, movies (especially French movies), chemistry, cosmology, Jungian psychology, mythology, statistics, and math. As a former PPC and CHHU member, Namir enjoys attending the HHC conferences. *Email him at:* nshammas@aol.com

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How Large is 10^{99}?

Richard J. Nelson

Introduction

The dynamic range of many scientific calculators (entry level) is specified as -9.999999999 x 10^{-99} for the smallest number to 9.999999999 x 10^{99} for the largest number if it is a ten digit machine. This was the range of the first scientific calculator the 1972 HP-35A. How large or small is the number represented by $10^{\pm 99}$?

As a number 10^{99} is 1 followed by 99 zeros. **10^{99} = 1,000,000,000,000,000,000,000,000,000,000,000,000, 000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000.**

Add another zero and the number has a special name, googol (not Googel). Ten raised to the power of googol is called a googolplex. This number is impossible to write out.

In the early 1970's scientific calculators were often called electronic slide rules and this $10^{\pm 99}$ range was simply astounding. It was so unbelievable that the literature of the time often explained that all known values in the universe would easily be covered by numbers within the $10^{\pm 99}$range. Certainly all known physical constants were not known beyond six or seven significant figures so the HP-35A was what would be called today "The Killer Ap" of its day. Let's explore a few real world "numbers" just to see how meaningful $10^{\pm 99}$ really is.

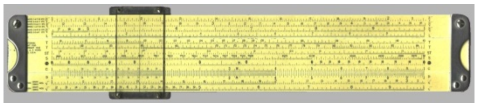

Fig. 1 – Typical slide rule of the 1960's.

Students of my generation could make astounding calculations in their heads because they used a slide rule for their daily numerical problem solving**(1)**. They had to do this because the slide rule could not keep track of the magnitude of the "answer."

An answer might be 2.75, 0.000275, or 2.75 x 10^{13}. Two problems had to be solved, one for the significant digits and one the power of ten of the answer. The power of ten part was done in your head. The student quickly learned to express all problems with the numbers in scientific notation. Let's use the example of the problem of multiplying 321 x 57,943.

Mentally the problem is expressed as 3 x $10^{\mathbf{2}}$ multiplied by 6 x $10^{\mathbf{4}}$. 3 x 6 = 18. You add exponents if they are multiplied so the answer is 18 x 10^6 or in normal form 1.8 x 10^7. The correct answer from my calculator is 18,599,703 or 1.8 x 10^7 as an approximation. Teachers often lament that students all too often just mindlessly accept the answer they see in the calculator display without their having any real world sense for the number. This is the reason you should always "sketch out the problem" just as a technician or engineer must always sketch out a circuit he or she is wiring no matter how simple it may be – even a battery, switch, resistor, and LED series circuit. To do otherwise is just plain sloppy work (prone to human error) no matter how brilliant you may be.

Example 1: How big is the universe?

The idea here is to find something really big. Let's calculate the volume, in cubic inches, of the (theoretically) known universe which is, by the current Big Bang cosmology**(2)** theory, estimated to have a radius of about 45.7 billion light years. In scientific notation this is 4.57 x 10^{10} light years. Assume that the universe is spherical in shape. The volume of a sphere = $4\pi r^3/3$.

A light year is the distance light travels in one year. Since we need to calculate the answer in cubic inches we need to know the distance in inches that light travels in one year. I will use the HP48/49/50 type of calculator to solve this problem because it has all the constants I need built in.

a. The speed of light is: 186,282.397051 miles per second ($1.86282397051 \times 10^5$).

b. The length of a year is: 365.2421198781 days or 31,556,925.9747 seconds. ($3.652421198781 \times 10^2$ or $3.155556,925.9747 \times 10^7$ seconds)

c. Distance light travels in one light year: 186,282.397051 mi/s x 31,556,925.9747 s = $5.87849981413 \times 10^{12}$ miles per year.

d. Distance light travels in one year in feet: $5.87849981413 \times 10^{12}$ mi. x 5,280 ft/mi = $3.10384790186 \times 10^{16}$ feet per year.

e. Distance light travels in one year in inches: $3.10384790186 \times 10^{16}$ x 12 in/ft = $3.72461748223 \times 10^{17}$ inches per year.

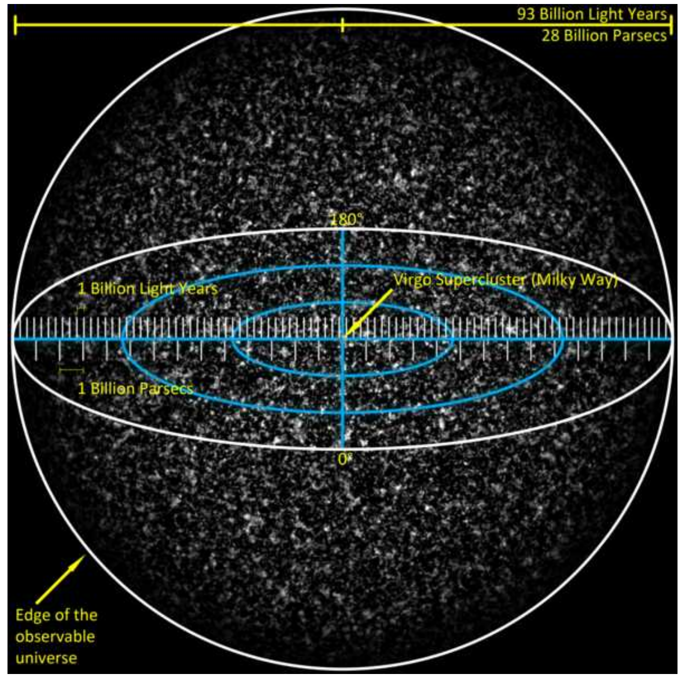

Fig. 2 – Universe[1] model showing estimated dimensions.

The Universe volume in cubic inches = $4*\pi(3.72461748223 \times 10^{17})^3/3$ $= 2.16438064428 \times 10^{53}$ cubic inches.

The distance light travels per year in mm = $3.72461748223 \times 10^{17}$ in/lt yr. x 25.4 mm/in = $9.4605284052 \times 10^{18}$ mm/lt yr.

The Universe volume in cubic millimeters $= 4*\pi(9.4605284052 \times 10^{18})^3/3 = 3.54678441413 \times 10^{57}$ mm^3.

Only 10^{57}? That number is quite small compared to 10^{99}.

Example 2: How many water molecules could there be in the Universe?

Let's calculate how many atoms there are in a 45.7 billion light year Universe that is completely composed of water - neglecting black holes, etc.


Fig. 3 – Earth is covered 71% with water.

The number of atoms of ANY substance in a volume is: # of atoms = N*d*v/ (Molecular Weight).

Where: N = Avogadro's number = 6.0221367×10^{23} atoms/mole.
d = Density = 1g/cc.
v = volume in cc.

From your chemistry classes you may remember that a mole of any substance has exactly the same number of atoms (or molecules) no matter what the substance is made of. Water is made up of 2 hydrogen atoms and one oxygen atom. Hydrogen has an atomic weight of 1 and oxygen has an atomic weight of 16. Water, H_2O, has a molecular weight of 18.

A cubic centimeter of water is 1 gram. The number of moles is 1/18 = $5.55555555556 \times 10^{-2}$ moles.

The number of molecules is therefore:

$6.0221367 \times 10^{23} * 5.55555555556 \times 10^{-2} = 3.3456315 \times 10^{22}$ molecules per cc. The number of atoms is 3 times larger because each molecule has three atoms, so there are $1.00368945 \times 10^{23}$ atoms of water per cubic centimeter, cc, of water.

The answer from example 1 calculated the universe as $3.54678441413 \times 10^{57}$ mm^3. This value converted to cm^3 is divided by 1,000 mm^3/cm^3 which = $3.54678441413 \times 10^{54}$ cm^3 in the Universe.

The number of water molecules to fill the Universe is $3.54678441413 \times 10^{54}$ cm^3 x $1.00368945 \times 10^{23}$ molecules /cm^3.= $3.55987009789 \times 10^{77}$ H_2O molecules.

Only 10^{77}? That number is still quite small compared to 10^{99}.

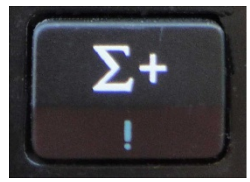

Fig. 4 – Factorial key.

Fig. 5 – Stat key.

Exception I am limiting my search for a large number to real world values specifically omitting "calculated" mathematics values such as the number of piano pieces that could be written using each of the 88 keys on the piano once = 88! = $1.85482642257 \times 10^{134}$. See Fig 4 and Fig 5 for HP-35s keys that may be used to calculate functions that result in very large numbers. Another exception for our reality number search is Graham's number**[(3)]** which cannot be expressed using the conventional notation of powers, and powers of powers..

Example 3: How old is the Universe in nanoseconds?

The age of the Universe is usually based on the Hubble constant which is a measure of the current expansion rate of the universe. Many astronomers are working to measure the Hubble constant using different techniques. Until recently, the best estimates ranged from 65 km/sec/Megaparsec to 80 km/sec/Megaparsec, with the best value being about 72 km/sec/Megaparsec. In more familiar units, astronomers believe that age of the Universe is between 12 and 14 billion years. Let's use 14 billion years.

From example 1.

b. The length of a year is: 365.2421198781 days or 31,556,925.9747 seconds.

14 billion is 1.4×10^{10} yrs x $3.1556{,}9259747 \times 10^{7}$sec./yr. = $4.41796963646 \times 10^{17}$ = Universe age in seconds.

If the big bang theory theorizes the early events in one nanosecond increments how old is the Universe in nanoseconds (light travels about one foot in one nanosecond)? There are 10^9 nanoseconds in one second.

The age of the Universe in nanoseconds is approximately $4.41796963646 \times 10^{26}$ nanoseconds.

Only 10^{26}? That number is almost tiny compared to 10^{99}.

Can the reader think of any physical reality object that requires its value to be expressed as a numeric value that cannot be calculated by an HP scientific or graphing calculator?

Obversations and Conclusions

How Large is 10^{99}?

Our largest samplings of "real world" numbers[4] are:

1. The Universe volume in cubic millimeters = 3.5×10^{57}. ($3.54678441413 \times 10^{57}$).
2. The number of water molecules to fill the Universe = 3.6×10^{77} ($3.55987009789 \times 10^{77}$).
3. The age of the Universe in nanoseconds is approximately 4.4×10^{26}. ($4.41796963646 \times 10^{26}$ ns.)

These numbers are quite small compared to 10^{99}.

Even if we assume that we have missed a few numbers by a factor of a thousand (10^3) or ten thousand (10^4) none of our real world examples come close to 10^{99}.

Our current generation of calculators have an even wider dynamic range with the upper limit of $10^{\pm 499}$. As mentioned with our piano key example it is easy to generate very large or very small numbers mathematically. The new question is, "How Large is 10^{499}?"

It is certainly larger than any reality numbers you may think about. Any factorial calculation greater than 253 ($253! = 5.17346099264 \times 10^{499}$) will cause overflow, but when it comes to "pure" mathematics a finite state machine like a scientific calculator is essentially useless. The important question to ask is:

What are realistic dynamic range limits for a scientific or graphing calculator?

Epilog

Modern graphing calculators have morphed into symbolic math machines. The student asks: Is the hand held calculator adequate?

This topic was examined by Wlodek Mier-Jedrzejowicz in a series of talks at , , **and** titled "Would Einstein Have Used One?", "Could Einstein have Used One?", and "Should Einstein Have Used One? The whole range of mathematics is addressed with a review of what advanced mathematics is calculator solvable today and what could or should be added in the future. From the 2009 presentation;

"In the second presentation I asked if Einstein could have done some General Relativity work on an HP50g, even if this were to be very difficult. Say he was shipwrecked on a deserted island with only an HP50g for company, could he have used it? Fundamentally, the Computer Algebra System on an HP50g is just about powerful enough to do some symbolic tensor calculus, but I doubt that Einstein or anyone else would happily do so. Instead, I suggested that if any General Relativity were to be done on an HP50g, the programming would have to be done first in some other language than RPL. If Einstein were to use an HP50g he could do so, but he would have to learn to program in an appropriate language, as well as learn to use Tensor Calculus, which took him some effort. He would need some other computer to write the programs first, then translate them into code to run on the HP50g, and transfer it to the HP50g. So he could just about have used an HP50g on his desert island, but with considerable difficulty."

Finishing up Wlodek said;

"This means that if E wants to work with tensors on an HP calculator, then he or she has to get someone to write the programs for them. This was discussed on comp.sys.hp48 two years ago, but at that time no-one had done it yet. I just wonder if anyone will ever have the time or the wish to do so. And that is the answer to "Should E use one?" They won't know until they try! So, they should find someone willing to write the necessary programs for an HP50g! I'm not volunteering unless I get a substantial university grant to do it. If I do, I'll let you know next year. Otherwise I might go back to programming on my HP-41 and give a talk on that.

I'd like to finish by thanking everyone who has made suggestions for these presentations and at them – especially Roger Hill, Richard Nelson and Bill Butler who pointed out to me, just as I was finishing writing this, that HP50g tensor software is now available from http://www.heuson-software.de/ "

Notes

(1) *See* ***HP Solve*** *issue 21 for an article, Calculating before Calculators, that describes the various pre-calculator methods of performing numerical calculations.*

http://h20331.www2.hp.com/hpsub/downloads/Newsletters_HP_Calculator_eNL_12_Dec_2010_v1.pdf

(2) *This number is from: http://en.wikipedia.org/wiki/Observable_universe .*

(3) *See http://www-users.cs.york.ac.uk/susan/cyc/g/graham.htm for an explanation of Graham's number and other large numbers and how they are named and expressed.*

(4) *All of the numbers given in this article are silly in terms of the number of digits provided. They are given as shown in your 12 digit calculator display as a means of verifying that you have pressed the correct keys and you have the correct answer.*

About the Author

Richard J. Nelson is a long time HP Calculator enthusiast. He was editor and publisher of *HP-65 Notes*, *The PPC Journal*, *The PPC Calculator Journal*, and the *CHHU Chronicle.* He has also had articles published in *HP65 Key Note* and *HP Key Notes*. As an Electronics Engineer turned technical writer Richard has published hundreds of articles discussing all aspects of HP Calculators. His work may be found on the Internet and the HCC websites at: hhuc.us He proposed and published the PPC ROM and actively contributed to the UK HPCC book, *RCL 20*. His primary calculator interest is the User Interface.

From the Editor

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From The Editor – Issue 25

Fall is a time for the HP Handheld Conference, and active and serious HP calculator users have been registering in exceptional numbers to attend HHC 2011 in San Diego the last weekend in September. Since this issue of ***HP Solve*** is a bit early for the obvious reasons of new product announcements a Conference report will have to wait for the next issue, #26.

HP News

Photo by Jake Schwartz

Fig. 1 – Lower left, 2^{nd} from left HPs Laura Harich. Lower right 2^{nd} from front HP 12c author Gene Wright.

August 30, 2011, NYC @ Harry's Cafe & Steak. HP hosted an HP 12c 30^{th} Anniversary Celebration for the financial press. This event also served to announce the re-release of the popular HP-15C now known as the HP 15c Limited Edition.

Because so many of the press are "electronic" these days a two day lead was considered adequate for the official announcement date of September 1, 2011 for both machines. Laura Harich, HP Calculator Marketing (CA) and Mike Hockey HP PR (TX) provided support for the event. Gene Wright, well known financial, author and ***HP Solve*** contributor (TN) attended as did Jake Schwartz another well known ***HP Solve*** contributor (NJ). See photo above. Aside from answering questions and providing personal background information the group viewed an HP video which covered the long history of the HP 12c including an interview with one of its developers, Dennis Harms.

There was even a business card drawing for one of each of the two new products. Radiris Diaz of CuteGeek.com won the HP 12c 30^{th} Anniversary Edition and Stacy Cowley of CNNMoney.com won the HP 15c Limited Edition.

The press event was planned in advance of the storm, but everything dried out just in time for everyone to have a great evening sharing stories of how the finance world has been impacted by the over 30 years of HP-12C usage.

Community News

As I mentioned in the last issue, the HP user Community is participating in the "designing" of two new calculators. If you think that it is an easy task to design a calculator you soon realize that the devil is in the details. One of the most challenging aspects is accuracy - a hallmark of HP calculators. Another aspect is the documentation.

Suppose you are King and you make all the decisions. You have the factory standing by to start the new model on a specific date. If you don't release the new design the factory will (instead) make another model (HP-12C's are always needed 🙂) and the new model production will have to be moved to another date – many months later. Now you have to make an important change at the last minute. What do you do? Do you delay the machine and miss an important sales period such as Christmas or back to school? In some situations the change could ripple through several teams (to change the hardware and/or the documentation, etc.) to extend the introduction a full year. The market radically changes in one year.

Another topic being presented at HHC 2011 is a new HP calculator restoration book by Geoff Quickfall. ***HP Solve*** readers will remember that Geoff was interviewed in ***HP Solve*** issue 12, and the book was previewed in issue 19, page 23. Geoff is also a Beta HP-41CL user/documentator so he is busy indeed - and he will have a lot to talk about at HHC 2011. hhuc.us

Another very exciting project repurposes one of HP's business calculators (HP 20b or HP 30b) to create a new calculator dubbed the WP 34s as mentioned in this column in issue 24. The WP 34s calculator continues to develop with features added nearly every week. Originally it had only 506 program steps, but through very clever firmware programming, it now has 8 flash banks of 506 steps that can hold user callable programs, utilities, or subroutines. This is quite capable indeed. One of the areas not addressed by built-in functionality of the WP 34s is matrix math. Jake Schwartz and Gene Wright filled in this functionality by adapting the 30 year old PPC ROM matrix subroutines M1 through M5 (and a couple of others) and the PPC ROM application program RRM and now the WP 34s has a matrix handling program that can solve a system of 6 equations, find the inverse of a 6x6 matrix, or the determinant of an 8x8 matrix. By the way, the WP 34s is fast. Thanks HP. A 7x7 determinant was computed in just under 2 seconds. Wow. There was one fundamental difference, however. The HP-41 SIGN function was "defined" differently, and they had to add two steps for each time it was used in the routines. The HP-41 returns -1, +1, & +1 if the number is -, 0, or +. The WP 34s returns -1, 0, +1 under the same conditions. If you were King what decision would you make? It makes you wonder what the HP-41 developers were thinking. Was there some hardware restriction?

Here is the content of this issue.

S01 – The HP-12C *30th Anniversary Edition* is Announced This one of a kind business calculator has the unique distinction of being manufactured longer than any other calculator – July 4 1980 to present inclusive is 32 years. See additional article related to the HP-12C.

S02 – The HP-15C Limited Edition, LE, is Announced. This is a re-release of the famous HP-15C, one of (now, including the 12C 30th anniversary machine) nine machines in the Voyager Series of HP calculators. The 15C LE enjoys the exceptional accuracy of the improved algorithms of the Voyager series.

S03 – The HP-12C, 30 Years and Counting by Gene Wright and me. This is a short review of the very popular business calculator with new information related to the manufacturing history and the MSRP pricing over the 32 years of its being manufacturered.

S04 – Calculator Programmability – How Important is it in 2011? Is there an advantage to having a scientific calculator programmable? What are the reasons for programmability? This article examines the subject and concludes with: The large number of features, memory size, and super speed of the HP 15 LE makes its programmability an important feature when compared to other calculators - for the 12 programming reasons listed.

S05 – Converting Decimal Numbers to Fractions by Jospeh Horn. Joseph explains the continued Fraction Algorithm and provides an RPN HP-15C program that quickly converts any decimal to a fraction to the accuracy set by the FIX display value. The program listing provides program line numbers for easier conversion to other RPN calculators.

S06 – Limited Edition HP-15C Execution Times by Namir Shammas. Namir examined the new incarnation of the 15C LE and compared the execution times for many of the instructions, programs, and built-in applications with those of the original 15C. The results were surprising because the expectation was that it would be faster by some factor. This turned out to be very untrue. The speed advantage actually ranged from 102 to 180 times faster. A total of 44 speed ratios were measured, the total is 5,659.06 and the average is 128.615. Saying that that the HP-15C LI is about 128 times faster is a good average value to use.

S07 – How Large is 10^{99}? Ten digit calculators usually have an exponent range of ±99. Exactly how large is this number and how does it compare with the real universe? Three examples of the largest possible physical values are used to see if $10^{\pm 99}$ is big or small enough to calculate numbers in the real world. Twelve digit machines provide an exponent of $10^{\pm 499}$. How important is this limit?

S08 – Regular Columns This is a collection of news items and repeating/regular columns. A new column, Calculator Accuracy, starts with this issue.

- **From the editor.** This column provides feedback and commentary from the editor.
- **One Minute Marvels.** This OMM provides a routine to determine when a Friday the 13^{th} occurs. The incredible calendar built into the 48/49/50 series of machines makes this a very easy task. Tables are provided for the number of F13's from 1600 to 3299. Details on how this was done on request.
- **Calculator Accuracy.** This is an important topic for HP calculator users to explore and use to be better calculator users. No calculator is 100% accurate – yet.

S09 – #8 in the Math Review Series: Mathematical Constants – π This constant is probably one of the most well-known of all mathematical constants. Here is an overview of this interesting number with lots of links for further astonishment and study.

That is it for this issue. I hope you enjoy it. If not, tell me!

Also tell me what you liked, and what you would like to read about.

X < > Y,

Richard

Email me at: hpsolve@hp.com

HP 48 One Minute Marvel – No. 12 – Friday the 13^{th}

One Minute Marvels, OMMs, are short, efficient, unusual, and fun HP 48 programs that may be entered into your machine in a minute or less. These programs were developed on the HP 48, but they will usually run on the HP 49 and HP 50 as well. Note the HP48 byte count is for the program only.

One of the really powerful functions of the HP-48/49/50 series of RPL machine is the built in 8,419 year (October 15, 1582 to December 31, 9999) calendar. Using the powerful date commands all kinds of powerful date routines may be written. Some people enjoy the fun of a special date in the United States (other countries have similar days and dates, in Columbia and Spain it is Tuesday the 13^{th}) when Friday falls on the 13^{th} of the month.

Every year has at least one Friday the 13^{th}. What is the maximum number possible in one year? The following 14 years from 1900 to 2000 have three Friday the 13^{ths}: 1903, 1914, 1925, 1928, 1931, 1942, 1953, 1956, 1959, 1970, 1981, 1984, 1987, 1998. Now that we are well into the 21^{st} century you may easily determine how many years will have three Friday the 13ths in this century.

Friday the 13^{th} (Joseph K. Horn)

Key in a year and **'FRI13'** will return the dates of Fridays that occur on the 13^{th} of the month.

'FRI13' << 1000000 / 1.13 + 13 FOR **d** 6.133 **d** DDAYS 7 MOD NOT **d** IFT NEXT >>

16 commands, 88.0 Bytes, # 670Bh. Timing: 1987⇒ 2.131987, 3.131987, 11.131987 in 0.148_sec.

What is the answer to the question of the occurrences of Friday The 13th for various centuries? I made the tables below using **'FRI13'**. See the observations below the tables.

Table 1 — 17th Through 33rd Century (traditional)
Friday The 13th Occurrences

#	Century	Period	Total	Occurrences Per Year		
				Once	Twice	Thrice
1.	17th	1601 - 1700	172	43	42	15
2.	18th	1701 - 1800	172	43	42	15
3.	19th	1801 - 1900	173	42	43	15
4.	20th	1901 - 2000	171	43	43	14
5.	21st	2001 - 2100	172	43	42	15
6.	22nd	2101 - 2100	172	43	42	15
7.	23rd	2201 - 2200	173	42	43	15
8.	24th	2301 - 2300	171	43	43	14
9.	25th	2401 - 2400	172	43	42	15
10.	26th	2501 - 2500	172	43	42	15
11.	27th	2601 - 2600	173	42	43	15
12.	28th	2701 - 2700	171	43	43	14
13.	29th	2801 - 2800	172	43	42	15
14.	30th	2901 - 2009	172	43	42	15
15.	31st	3001 - 3000	173	42	43	15
16.	32nd	3101 - 3100	171	43	43	14
17.	33rd	3201 - 3200	172	43	42	15

Table 2 — 17th Through 33rd Century (popular)
Friday The 13th Occurrences

Century	Period	Total	Occurrences Per Year		
			Once	Twice	Thrice
17th	1600 - 1699	172	43	42	15
18th	1700 - 1799	172	43	42	15
19th	1800 - 1899	172	43	42	15
20th	1900 - 1999	172	42	44	14
21st	2000 - 2099	172	43	42	15
22nd	2100 - 2199	172	43	42	15
23rd	2200 - 2299	172	43	42	15
24th	2300 - 2399	172	42	44	14
25th	2400 - 2499	172	43	42	15
26th	2500 - 2599	172	43	42	15
27th	2600 - 2699	172	43	42	15
28th	2700 - 2799	172	42	44	14
29th	2800 - 2899	172	43	42	15
30th	2900 - 2999	172	43	42	15
31st	3000 - 3099	172	43	42	15
32nd	3100 - 3199	172	42	44	14
33rd	3200 - 3299	172	43	42	15

Observations:

1. **Occurrences per century.** Depending on how you wish to define a century - I called one traditional and the other popular - the total number of occurrences of F13 per century vary. The popular belief of the recent new millennium starting on January 1st 2,000 picks up the cycle "out of sync" so that there are not 172 Friday The 13ths for each century.

2. **The calendar repeats every 4 centuries.** Roger Hill participated in a contest to print calendars and he investigated them. I asked him for his thoughts. His comment is: "One thing to note is that, for the presently-used Gregorian calendar, you only have to look at a 400-year interval, say 1600-1999, because the calendar repeats every 400 years." The total number of Friday The 13ths per cycle is 688.

3. **Having three Friday The 13ths in one year occurs 15% of the time.** Roger observes: "I did some research on Friday the 13ths (or F13's for short) a long time ago and (I think) figured out the probability of having one, two or three in a year chosen at random. I don't remember what I got now, but with a little luck I might be able to find it."

   "Another thing is that, in any year, the 7 months from May through November (inclusive) always start on 7 different days, so there will always be one and only one F13 in that 7-month interval. For non-leap years, the ones that have three F13's are the ones in which February 13 and March 13 are on a Friday, which happens in about 1 year out of 7. The criterion is different for a leap year, but again it happens in about 1 leap year out of 7. Hence the 14 or 15 per century that you found." I did not pick up that bit of information from the tables.

4. **Having one or two Friday The 13ths in one year is about equal probability.** This is obvious from the tables. Roger notes: "An interesting fact about the Gregorian calendar is that, while you would expect a year chosen at random to be equally likely to start on any day of the week, over the

400 year period of repetition it turns out that some days of the week are very slightly favored over others. For example, the probability that a year chosen at random starts on a Sunday is very close to 1/7 but not exactly. I've forgotten the exact results, but (again with a little luck) I could probably find them again. (It was almost 30 years ago when I fooled around with all this...)

Calculator Accuracy – Part 1 - Calculator Forensics

Introduction

When we pick up a calculator and make a calculation we place a great deal of faith that the numbers resulting from the calculation are correct. Hewlett Packard created the first scientific calculator with the HP-35A and as part of the specifications of that machine there was an accuracy statement. For the basic math functions of +, -, x , ÷, and √ the answer is supposed to be within one count in the last place, typically the 10^{th} (or 12^{th}) digit). Transcendental functions were slightly less accurate.

Function accuracy, however, is only one part of the accuracy "specification." If the calculator has built in applications such as complex numbers, solve, integrate, or matrices the accuracy specification is even less. In some cases surprisingly, perhaps three digits, less. The average user will seldom consult the User's Guide for an accuracy statement – if there is one – to know how reliable, how accurate, all those digits are. After all, the calculator designers should know what they are doing.

Customers needing to buy a calculator won't spend very much time delving into the details of calculator accuracy other than to hear about the reputation of a particular manufacturer.

The calculator programmers that microcode the internal math routines spend their time trying to reduce the number of instructions and to optimize them to run as fast as possible. They cannot be expected to be numerical analysts in terms of mathematical accuracy. They just implement the algorithms they are given.

In order to determine the accuracy of the number in the display, with its limited fixed number of digits, you must know the correct answer to several digits greater than that produced by the function. Casual calculator users just want to know how the various models compare, and they have devised various "tests" to compare machines.

Comparison testing

One such scientific model test has been proposed by Mike Sebastian. He has a website dedicated to what he calls Calculator Forensics[(1)]. Below is the "forensics" data for HP calculators. The results in Tables 1 and 2 are for his single forensics trigonometry function test. Some of the data is current data not found in Mikes 2007 table[(2)]. The tables also add a Rating column and an Error column for comparison purposes. The models are arranged in increasing accuracy order.

Here is the test problem. In degrees mode calculate arcsin (arccos (arctan (tan (cos (sin (9)))))) algebraically, and 9 SIN COS TAN ARCTAN ARCCOS ARCSIN in RPN. A rating is suggested for comparison purposes with the higher the value the better. The idea is that the function-inverse is supposed to return the input, in this case 9 degrees. Supposedly the more accurate calculator (overall?) is the one that is closest to 9. All calculators in the tables use BCD arithmetic except as noted + in table 1.

Table 1 - Mike Sebastian's Calculator Forensics Results for 10D HP Models

Calculators	Result	Rating*	Error
HP-35A with bug	9.002 983 113	2	2.98311×10^{-3}
HP-45A, 65A,	9.004 076 644	2	4.07664×10^{-3}
HP-35A bug corrected	9.004 076 901	2	4.076901×10^{-3}
HP-21, 25/C,	9.004 076 649	2	4.076649×10^{-3}
HP-55A	9.004 076 898	2	4.076898×10^{-3}
HP-41C/Cv/CX, 10C, 11C, 15C, 27, 29C, 30E, 32E, 33E, 34C, 67A, 91A, 97A/S	9.000 417 403	3	4.1743×10^{-4}
HP 9g+, 30s+	9.000 000 000	9	0+

** Simply the number of zeros or nines following the decimal point. The higher the better. The rating is one less than the exponent of the error.*
+ Uses 80 bit microprocessor binary arithmetic which provides at least 23 decimal digits.

Table 2 - Mike Sebastian's Calculator Forensics Results for 12D HP Models

Calculators	Result	Rating*	Error
HP-19BII, 20b. 20S, 22S, 27S, 28C/S, 30b, 32S, 32SII, 38G, 39G, 42S, 48G/GII/G+/GX, 48S, 49g/g+, 50g, 71B, 75C/D,	8.999 998 642 67	5	-1.35733×10^{-6}
HP-6s/6s Solar	8.999 996 370 4	5	-3.6296×10^{-6}
HP 33S, 35s	8.999 999 860 01	5	-1.3999×10^{-6}
HP 9s	8.999 998 643 82	5	-1.35618×10^{-6}
HP 10s	9.000 000 342 383	6	3.4239×10^{-7}
WP 34s (V1444)	9.000 000 000 03	10	3×10^{-11}

** Simply the number of zeros or nines following the decimal point. The higher the better. The rating is one less than the exponent of the error.*

Observations and conclusions

If you are familiar with HP's calculator models you will notice that as you go down the table the model numbers are more recent. This demonstrates that HP's calculators keep getting more accurate with time. Table one values (ten digit machines) are all slightly high. Table two values (12 digit machines) are "all" (96%) slightly low. Another "obvious" conclusion you can make is that various series of models produce the same results. The only calculators that return 9 are the 80 bit (lowest cost) machines.

Calculator Accuracy - Part 2

Inherent to calculator accuracy is the topic we will discuss in issue 26 – Guard Digits. The correct use of guard digits is what sets HP calculators apart from the others.

Notes: Calculator Accuracy – Part 1

(1). Mike makes no claim as to any mathematical (analysis) basis for his Calculator Forensics problem he uses to arrive at a calculator comparison number. The "9" for example was chosen to be a single digit (for ease of keying in) far from zero which is always troublesome for trigonometry functions. Obviously he wanted to give the inaccurate machines more leeway. His website is at: http://www.rskey.org/~mwsebastian/miscprj/forensics.htm

(2). Thanks to Palmer Hanson, Joseph Horn, and Jake Schwartz for contributing data.

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Mathematical Constants – π

Richard J. Nelson

Introduction – What is a mathematical constant?

A mathematical constant is a special number, usually a real number that is especially interesting and useful to mathematicians. Constants[(1)] arise in many different areas of mathematics and two especially well known constants are Euler's number *e*, and Pi, π. *e* was discussed in Math Review #6, ***HP Solve*** issue 23.

π is well known and every high school student learns that π is the ratio of the circumference of any circle to its diameter. Fig. 1 shows 24 digits of π. While π is a mathematical constant it is also found in many of the formulas of Physics and Electronics.

Albert Einstein suggested that the course length of looping rivers, from source to end, compared to a straight course has a ratio which approaches π.

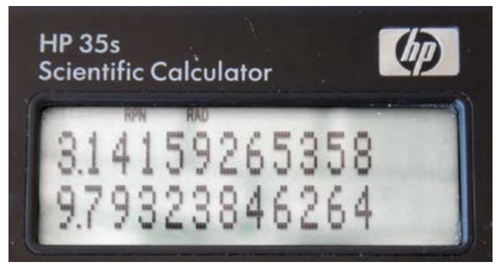

Fig. 1 – HP-35s Display showing π.

Then there is the π joke. (Forwarded to me by Roger Hill Hill, PPC ROM Manual contributor, HP calculator author and enthusiast, meet Roger at HHC 2011).

Question: How can you tell if a person is a mathematician, a physicist, an engineer, or a salesman?
Answer: Ask him what π is.

The mathematician will say, "It is the ratio of the circumference of a circle to the diameter".
The physicist will say, "It's 3.14".
The engineer will say, "It's about 3".
The salesman will say, "It's normally 3, but I can get it to you for 2.6".

Discovering π

π has a very long colorful and interesting history. Table 1 is a brief portion of the Chronological Table taken from A History of π by Petr Beckmann, see page 196 shown as reference 1 of Appendix B.

Table 1 – The Early History of π

Date	**π Status**
Circa 2000 BC	Babylonians use $\pi = 3\ 1/8$.
Circa 2000 BC	Egyptians use $\pi = (16/9)^2 = 3.1605$.
12^{th} century BC	Chinese use $\pi = 3$.
Circa 500 BC	I Kings vii, 23 implies $\pi = 3$.
Circa 440 BC	Hippocrates of Chios squares the lune.
Circa 434 BC	Anaxagoras attempts to square the circle.
Circa 430 BC	Antiphon enunciates the principle of exhaustion.
Circa 420 BC	Hippias discovers the quadratix.
Circa 335 BC	Dinostratos uses the quadratix to square the circle.

Date	π Status
3rd century BC	Archimedes establishes 3 10/71 < π < 3 1/7 and π = 211875: 67441 = 3.14163.
2nd century AD	Ptolemy uses π = 377/120 = 3.14166…
3rd century AD	Chung Hing uses $\pi = \sqrt{10}$ = 3.16…
236 AD	Liu Hui uses π = 157/50 = 3.14
5th century	Tsu Chung-Chi establishes 3.1415926 < π < 3.1415927

The relationship of the circle circumference to its diameter has been known for about 4,000 years, but the first use of the π symbol was by William Jones in 1706 because it was the first letter of the Greek word for "perimeter" or "periphery." The great mathematician Leonhard Euler used it in 1737 and it has been accepted since. π is a lower case greek letter pronunced "pie" that is not capitalized if it begins a sentence because the upper case greek letter, Π, is used for designating the product of a sequence. If C is the circumference of the circle and D is the diameter of the circle, π is defined as shown below.

$$\pi = \frac{C}{D}$$

Measuring π

You may determine a desired number of digits of π in one of three ways. You may measure it, you may calculate it, or you may look it up on the internet. I really wanted to make an attempt to measure π, but the daily summer 107 degrees here in the Sonoran desert discouraged me. Instead, let's perform a thought measurement. Here are the materials I was going to use.

1. Three metal stakes (12" nails).
2. 210 feet of white polypropylene cord.
3. 110 feet of bailing wire.
4. 100 foot measuring tape.
5. Bicycle (front wheel and fork). I would have to borrow one.
6. Masking tape.
7. Carpenters marking pencil and ball point pen.
8. Large flat level smooth layout area at least 220 x 110 feet. (half of a 200 foot diameter circle.)

Here is the measurement procedure. The goal is to make all measurements to a resolution of 1/8 inches.

A. Remove the front fork from the bicycle.
B. Wrap a piece of masking tape around the tire and wheel and mark a line down the outside perpendicular to the ground.
C. Attach one end of the wire to the frame near the wheel axle.
D. Move all of your materials to the layout area and pound in two stakes about 220 feet apart and stretch the white polypropylene cord to serve as the diameter of the circle.
E. Pound in the third stake in the center of the white cord to mark the center of the circle.
F. Position the wheel 100 feet from the center (an assistant is most helpful here) and make a loop to be placed over the center stake.
G. Holding the wheel with the wire tight (it should not stretch) rotate and align the mark on the rim with the white cord.
H. Keeping the wire tight walk the wire around the semicircle keeping the wheel perpendicular to the surface counting the number of times the tape comes in contact with the surface. When you are

near the other end of the white diameter cord you will usually have less than one full rotation left. Finish that last rotation and place a second piece of tape on the wheel to indicate the last portion of the rotation. Write down the number of full rotations, gather your materials and head home.

I. In the car port use a long straight edge and draw a line on the concrete with a cross line at one end.

J. Use your wheel and align the first tape mark with the cross line and roll the wheel down the line one revolution and make another cross line. This is the second measurement (the first was the radius of the circle at 100 feet). Repeat the rotational measurement for the length to the second tape.

K. Calculate π.

You must account for the diameter of the center stake in your measurements.

Let's assume the radius (wire length) is 100 feet or 1200 inches. The diameter is 2400 inches. After doubling the thought wheel measured total length (measured circumference x revolutions + extra partial revolution) the circumference is 7,539 3/4hs (7,539.75 inches). I realize that making these measurements to a resolution of 1/8th inches takes great skill and is very difficult. Making the division we have π = 3.1415625.

The theoretical circumference with a 100.00000 feet diameter is 7,539.82236862 inches. Let's assume an error of ¼ inch too high for the circumference and ¼ inch too low for the diameter for a maximum error on the high side. The calculated theoretical value for π is 7,540.07236862 divided by 2,400.25 = 3.14136959426. The difference (high error) with 12 digits of π is 0.00022305933. This means that we can easily depend on three accurate decimal digits or 3.141 using our measurement method. If our work is really practiced and careful we might hope for 3.1415.

CONCLUSION: Physically measuring to determine π using normal measuring methods is not very productive.

Calculating π

Fig. 2 - HP35s π

Most students don't even think of the numerical value of π because it is a keystroke constant of most HP scientific and graphing calcualtors. This most popular and very easily entered irrational and transcendental number is found on the HP 35s shifted COS key as shown in Fig. 2.

All scientific and graphing calcualtors have π internally stored to a value much greater than what the display will show. This is required to perform the trig functions to their specified accuracy. The number of digits of π stored in the HP-48/49/50 calculators, as a constant, is 31.

Here is a technique suggested by Joseph K. Horn to obtain the digits of π that are used by your HP calculator for the sin function. Using RAD mode type in as many digits of π as you wish up to the limit of the display. Do not round, use the truncated value. Suppose I only remember that π is 3.14159 and I have an HP-15C handy. After typing in the 6 digits I remember (let's pretend that the π key is broken) I press the SIN key. The display will show: 0.000002654. Use FIX 9. Now multiply the number by 10000 to see 0.026535900. That is all the useful digits we can get, but we now have 12 accurate *rounded* digits of π.

Calculators use more digits than they display for their calculations and these extra digits are called guard digits. HP calculators discard the guard digits (other calculator manufacturers usually do not) when the answer is calculated to give you a “what you see is what you get” result. To demonstrate this press the π key to see 3.141582654. If the calculator saved the guard digits you could subtract 3 and multiply by 10 to see the next guard digit. For HP calculators this will be zero. Subtract 1 and multiple by 10 again. Now the last two digits are zero.

For 10 digit calculators the guard digits are usually two. For 12 digit calculators there are typically three guard digits.

Now let’s use the “SIN π Digits” technique using the HP 35s. Keying in the maximum number (12) of truncated digits, 3.14159265358, and taking the SIN we see 0.000000000001. Multiplying this by 10^{12} we will see 9.79323846264. We now have 24 accurate digits of π if we append these digits to our 12 input digits. This is shown in Fig. 1. No matter what you do you can’t get more digits. Checking the normal π key for guard digits (the only other technique we have to squeeze out extra digits of a calculation) all we get are 12 digits, 3.14159265359 which is the correctly rounded value of π to 12 digits.

Exactly what this means is unclear. We know that the Tangent function is the most sensitive to errors at the extremes and that π is used internally by HP for the Tangent function to 31 digits in the later (1990) RPL calculators. Below is a table of the current machines with a few historical models for reference.

Table 2 – Digits of π That May be Extracted From Your HP Calculator

Sin π Digits (RAD)	**HP Model(s)** [π is given below for reference]
(12, 14, 24, 28) 3.14159 26535 89793 23846 26433 83279 50288 41971 69399	
12	HP 9s, 10C, 10s, 11C, 15C, 15C Limited Edition, 41C/CV/CX
14	HP 300s
24	HP 9g*, 20b, 30b, 33s, 35s, 39gs. 40gs, 48/49/50,
28	HP 20b/30b repurposed as WP-34s

* = *Garbage digits after the indicated number of correct digits obtained by looking at the guard digits.*

Palmer Hanson offers an explanation on how the technique works.

Using the old trigonometry identity: sin(A + B) = sin A cos B + cos B sin A

For our problem A = π, and B = a small angle

Then sin(π + B) = sin π cos B + cos π sin B

and since sin π = 0 and cos π = -1 *(in radian mode)*

sin(pi + B) = -sin B

and for small B, good enough for what we are doing here

sin(pi + B) = -B

For pi + B = 3.141492653, (10D truncated value of π) B is negative so sin(π + B) = +B

For pi + B = 3.141592654, (10D rounded value of π) B is positive so sin(pi + B) = -B

This illustrates why it is best (simpler) to use a truncated input for Sin and to view the result in scientific notation or multiply by E^{10} or E^{12} depending on the number of display digits. The result will then be as shown in fig. 1.

π has been calculated to more than 5 trillion digits and memorized to 67,890 digits[(5)] by Lu Chao a 24-year old chinese graduate student as recognized by the Guinness book of records. It took 24 hours and 4 minutes to speak them – without error! See **Exploring π** below to learn how this is done.

Calculating the decimal digits of π is a popular HP calculator programming challenge and various authors have written programs to calculate the first 1,000 digits of π for the calculators of the 70's and 80's. Often the various machines were compared for speed by using the number of hours it would take to produce the first 1,000 π digits. This number was used because of memory restrictions. RPL calculators of the 90's and later have math libraries that calculate hugh numbers of the digits of π**.**

Calculating an approximation for π may be also be done by converting the decimal value desired to a fraction using Joseph's program found elsewhere in this issue. Using his program running on the HP 35s results in the values shown in Table 3. The input is 3.14159265359. The accuracy is set by the FIX mode which is one less than the Digits column.

Table 3 – Fractions That Evaluate to π

Desired Accuracy	Digit s	Nume-rator	Denom -inator	Error
3	1	3	1	-0.14159265358
3.1	2	22	7	0.00126448927
3.14	3	22	7	0.00126448927
3.142	4	333	106	0.00008321963
3.1416	5	355	113	0.00000026676
3.14159	6	355	113	0.00000026676
3.141593	7	355	113	0.00000026676
3.1415927	8	103,993	33,102	-0.00000000058
3.14159265	9	103,993	33,102	-0.00000000058
3.1415922654	10	104348	33,215	0.00000000033
3.1415926536	11	1,146,408	364,913	0.00000000003
3.14159265359	12	1,146,408	364,913	0

If you use the program you will have two choices for the 12 digits. You may use a truncted input with the twelfth digit 8 or you may use a rounded input with the twelfth digit 9. Joseph says the the rounded value must be used because that is the mathematical basis for the continued fractions algorithm. If you use 3.14159265358 as the input several fractions will change and the error will be off by one digit.

Mathematicians have been calculating (approximating) π by ratios since the beginning and Beckmann provides a table of these values as shown in Table 4.

Table 4 – Historical Fractions That Evaluate to π

Partial N & D values from Beckmann, page 171, Fig. A1

N	D	12D Divided Value	Error
			3. 14159 26535 89793 23846 26433 832
3	1	3.00000000000	-0.14159265359
22	7	3.14285714286	0.00126448927
333	106	3.14150943396	-0.00008321963
355	113	3.14159292035	0.00000026676
103,993	33,102	3.14158265301	-0.00000000058
104,348	33,215	3.14159265392	0.00000000033
208,341	66,317	3.14159265347	0.00000000012
312,689	99,532	3.14159265362	0.00000000003
833,719	26,5381	3.14159265358	0.00000000001
1,146,408	364,913	3.14159265359	0
4,272,943	1,360,120	3.14159265359	0
1,725,033	5,419,351	3.14159265359	0

Exploring π

The Internet provides a wealth of information about π. For a list of formulas that use π see note (6). You may download millions of digits if you are interested and a text editor (Word) may be used to explore the digits as you wish. Appendix A shows 10,000 digits. If you copy the desired digits into Word you may use the word counting, find, and the find and replace functions to discover a few of the properties of this irrational number. Highlight the digits before the ones of interest and use the word count function to determine the location in the π string.

One question to ask is the distribution of the digits. If the digits 0 through 9 were evenly distributed there would be 1,000 of each in 10,000 digits. I searched the first 10,000 digits for each digit and replaced it with the same digit. Word returns the number of replacements made which is the count of each digit. Table 5 shows the results. This is a trivial observation and Bailey[(2)] (1988) found that the first 30 million digits are very uniformly distributed. I wonder how many hours it would take to use Word to make a similar table for 30 million digits.

Table 5 – Digit Occurances in First 10,000 Digits of π

Digit	Occurrences	Digit	Occurrences
1	1,026	6	1,020
2	1,021	7	970
3	975	8	948
4	1,012	9	1,014
5	1,046	0	968

The highest digit count = 1,046, the lowest = 968, and the mean (of the occurances) is 1000. The standard deviation, σx, (population) = 30.3743312684.

One of the interesting observations usually mentioned is the occurance of six consecutive 9's starting at the decimal position of 762**[3]** (1-1/2 inches from the left on line six of Appendix A). Consecuctive digits of five or more do not occur for 0 to 9. My four digit house address occurs twice in the first 10,000 digits. The series of the odd digits 13579 does not appear, but the even digits 24680 appear twice; 17th line and the 6th line from the bottom. These are highlighted in yellow in Appendix A. For the reader interested in exploring the digits of π there is a serious dedicted website designed to do this. See Note (4). One advantage of searching your own word file is finding multiple occurances of a string.

Are the digits of π random**[7]**? Speculation is that they are. Should the digits of π be used as an encoding key? Probably not because they are so "common."

How may you memorize long strings that contain the digits of π? Usually this is done using a "poem" or specifically, a piem. Here is one devised by Sir James Jeans.

How I want a drink, alcoholic of course, after the heavy lectures involving quantum mechanics.

The number of letters in each word is a digit of π. A more refined and very long (thousands of words) piem starts:

Poe^{3}, E^{1}.
Near^{4} a^{1} Raven^{5}
Midnights^{9} so^{2} dreary^{6}, tired^{5} and^{3} weary^{5},
Silently^{8} pondering^{9} volumes^{7} extolling^{9} all^{3} by^{2}-now^{3} obsolete^{8} lore^{4}.
During^{6} my^{2} rather^{6} long^{4} nap^{3} – the^{3} weirdest^{8} tap^{3}!
An^{2} ominous^{7} vibrating^{9} sound^{5} disturbing^{0} my^{2} chamber's^{8} antedoor^{8}.
"This^{4}", I^{1} whispered^{9} quietly^{7}, "I^{1} ignore^{6}".

This verse provides the first 42 digits of π. - Mike Keith, First verse of Near a Raven *(with respects to Edgar Allan Poe).*

Randomly π

The French scientist Georges Buffon (1707 – 1788) conducted what he called his needle experiment to calculate π which caused quite a stir in his day. Here is how it is described. If we have a uniform grid of parallel lines, unit distance apart and if we drop a needle of length $k < 1$ on the grid, the probability that the needle falls across a line is $2k/\pi$.

The temptation to repeat this experiment is strong and in 1901 Lazzerine is reported to have found that $\pi = {}^{355}/_{113} = 3.1415929$ after 34,080 tosses. This correct six decimal digit value had been previously determined by Chinese mathematician Zu Chongzhi (429 – 501). It is obvious that "working backwards" you could stop the number of tosses to achieve the desired accuracy and the 34,080 number seems to indicate this.

The connection of randomness to π is strong. Note (8) provides a more modern approach to the fun of using random numbers to predict π. These methods, however, are very very inefficient.

Observations and Conclusions

π is probably the most well-known mathematical constant. The idea that the ratio of the circumference of any circle to its diameter is a constant has been known for over 4,000 years. Determining the value of π however, is an ongoing exercise because it is an irrational and transcendental number that never repeats and never ends.

Memorizing the value of π is a popular pastime for young students and one of the easiest ways to remember π is to think of the first three odd numbers, 1, 3 & 5. Write them down twice, 113355 and divide the last three by the first three to get 3.141592^9205 on a 12 digit calculator. You used six digits to get the answer and your result is correct to a truncated six decimal digits.

The review provides several resources and links to websites that are especially useful. Note (1) shows a list of mathematical constants that will or have been discussed in this math review series. The next one just may be the most interesting number you could put into a calculator display.

An appendix (A) includes a single page of the first 10,000 digits of π with a few of my observations of "interesting" sequences including the Feynman point, note (3), which occurs at decimal digit 762. Word (or other word processor) may be used to search through and discover various series of digits from a down loaded file of digits from the Internet. One website, note (4), is especially designed to search 200 million digits of π very very quickly. Reading through the techniques and methods used for this project provides an interesting perspective on the fun aspect of π,

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Notes for Mathematical Constants – *π*

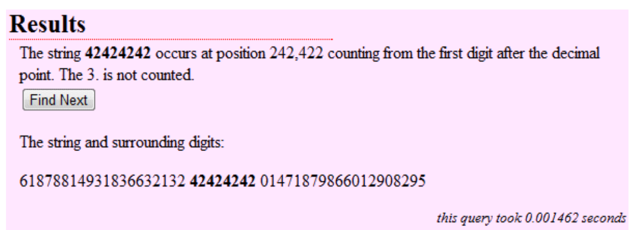
(1). A selected short list of the first ten of the more common 47 mathematical constants from Wikipedia is shown below. # 1 – 3, & 5 – 8 have been discussed in previous math review articles. #10 will be discussed in Issue 26.

#	Symbol	Value	Name	Field	N	First Described	# of Known Digits
1	0	= 0	Zero	Gen	R	c. 7th–5th century BC	N/A
2	1	= 1	One, Unity	Gen	R		N/A
3	i	$= \sqrt{-1}$	Imaginary unit	Gen, Ana	A	16th century	N/A
4	π	≈ 3.14159 26535 89793 23846 26433 83279 50288	Pi, Archimedes' constant or Ludolph's number	Gen, Ana	T	by c. 2000 BC	5,000,000,000,000[30]
5	e	≈ 2.71828 18284 59045 23536 02874 71352 66249	Napier's constant, or Euler's number, base of Natural logarithm	Gen, Ana	T	1618	100,000,000,000
6	$\sqrt{2}$	≈ 1.41421 35623 73095 04880 16887 24209 69807	Pythagoras' constant, square root of 2	Gen	A	by c. 800 BC	137,438,953,444

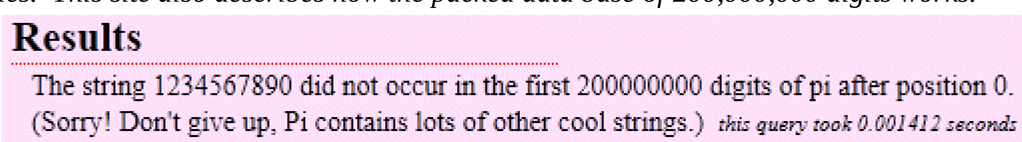
#	Symbol	Value	Name	Field	N	First Described	# of Known Digits
7	√3	≈ 1.73205 08075 68877 29352 74463 41505 87236	Theodorus' constant, square root of 3	Gen	A	by c. 800 BC	
8	√5	≈ 2.23606 79774 99789 69640 91736 68731 27623	square root of 5	Gen	A	by c. 800 BC	1,000,000
9	γ	≈ 0.57721 56649 01532 86060 65120 90082 40243	Euler–Mascheroni constant	Gen, NuT		1735	14,922, 244,771
10	φ	≈ 1.61803 39887 49894 84820 45868 34365 63811	Golden ratio	Gen	A	by 3rd century BC	100,000, 000,000

(2). See a much larger table of the distribution of π digits at: Weisstein, Eric W. "Pi Digits." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/PiDigits.html This source discusses many interesting patterns that occur in the decimal digits of π. A list of references is included.

(3). From the (2) source. "The sequence 9999998 occurs at decimal 762 (which is sometimes called the Feynman point; Wells 1986, p. 51). This is the largest value of any seven digits in the first million decimals.



(4). The best website to search the digits of π to find your birth date etc. is: http://www.angio.net/pi/piquery You will also find a lot of trivia regarding the many observations made on the trivia of what has been found in the first 200 million digits Topics such as Self-locating Strings, The Meaning of Life (42), Repeating Patterns, How Many digits of π are required to find a birthday (60,872), and Loop sequences are described. Above right and below are two search examples. This site also describes how the packed data base of 200,000,000 digits works.



(5). See http://en.wikipedia.org/wiki/Pi for additional information about π.

(6). A very nice list of formulas involving π may be found at: http://en.wikipedia.org/wiki/List_of_formulae_involving_%CF%80

(7). Here is a 2005 research study announcement that addresses using the digits of π as a source of random numbers. The ScienceDaily article also provides some insights as to the methods used to determine randomness. "Our work showed no correlations or patterns in pi's number set – in short, pi is indeed a good source of randomness," Fischbach said. "However, there were times when pi's performance was outdone by the RNGs."

"These tests are simple to reproduce with a desktop computer. All you need is time," he said. "It took us almost a year of work to crunch these numbers. We have included the program we used in the paper if anyone would like to try doing the analysis with a larger number set. I hope someone will because pi shows up in security systems,

cryptography and other places that have nothing to do with circles. That's part of what gives it a fascination that will not go away." *http://www.sciencedaily.com/releases/2005/04/050427094258.htm*

(8). Here is a link for the results of a more modern approach to using random numbers to calculate π. *The method uses Python for the calculations.*
http://www.stealthcopter.com/blog/2009/09/python-calculating-pi-using-random-numbers/

About the Author

Richard J. Nelson has written hundreds of articles on the subject of HP's calculators. His first article was in the first issue of *HP 65 Notes* in June 1974. He became an RPN enthusiast with his first HP Calculator, the HP-35A he received in the mail from HP on July 31, 1972. He remembered the HP-35A in a recent article that included previously unpublished information on this calculator. See http://hhuc.us/2007/Remembering%20The%20HP35A.pdf
He has also had an article published on HP's website on HP Calculator Firsts. See http://h20331.www2.hp.com/Hpsub/cache/392617-0-0-225-121.html.

Appendix A – First 10,000 Digits of π – Page 1 of 1.

3.
14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798214808651328230664709384460955058223172535940812848111745028410270193852110555964462294895493038196442881097566593344612847564823378678316527120190914564856692346034861045432664821339360726024914127372458700660631558817488152092096282925409171536436789259036001133053054882046652138414695194151160943305727036575959195309218611738193261179310511854807446237996274956735188575272489122793818301194912983367336244065664308602139494639522473719070217986094370277053921717629317675238467481846766940513200056812714526356082778577134275778960917363717872146844090122495343014654958537105079227968925892354201995611212902196086403441815981362977477130996051870721134999999837297804995105973173281609631859502445945534690830264252230825334468503526193118817101000313783875288658753320838142061717766914730359825349042875546873115956286388235378759375195778185778053217122680661300192787661119590921642019893809525720106548586327886593615338182796823030195203530185296899577362259941389124972177528347913151557485724245415069595082953311686172785588907509838175463746493931925506040092770167113900984882401285836160356370766010471018194295559619894676783744944825537977472684710404753464620804668425906949129331367702898915210475216205696602405803815019351125338243003558764024749647326391419927260426992279678235478163600934172164121992458631503028618297455570674983850549458858692699569092721079750930295532116534498720275596023648066549911988183479775356636980742654252786255181841757467289097777279380008164706001614524919217321721477235014144197356854816136115735255213347574184946843852332390739414333454776241686251898356948556209921922218427255025425688767179049460165346680498862723279178608578438382796797668145410095388378636095068006422512520511739298489608412848862694560424196528502221066118630674427862203919494504712371378696095636437191728746776465757396241389086583264599581339047802759009946576407895126946839835259570982582262052248940772671947826848260147699090264013639443745530506820349625245174939965143142980919065925093722169646151570985838741059788595977297549893016175392846813826868386894277415599185592524595395943104997252468084598727364469584865383673622262609912460805124388439045124413654976278079771569143599770012961608944169486855584840635342207222582848864815845602850601684273945226746767889525213852254995466672782398645659611635488623057745649803559363456817432411251507606947945109659609402522887971089314566913686722874894056010150330861792868092087476091782493858900971490967598526136554978189312978482168299894872265880485756401427047755513237964145152374623436454285844479526586782105114135473573952311342716610213596953623144295248493718711014576540359027993440374200731057853906219838744780847848968332144571386875194350643021845319104848100537061468067491927819119793995206141966342875444064374512371819217999839101591956181467514269123974894090718649423196156794520809514655022523160388193014209376213785595663893778708303906979207734672218256259966150142150306803844773454920260541466592520149744285073251866600213243408819071048633173464965145390579626856100550810665879699816357473638405257145910289706414011097120628043903975951567715770042033786993600723055876317635942187312514712053292819182618612586732157919841484882916447060957527069572209175671167229109816909152801735067127485832228718352093539657251210835791513698820914442100675103346711031412671113699086585163983150197016515116851714376576183515565088490998985998238734552833163550764791853589322618548963213293308985706420467525907091548141654985946163718027098199430992448895757128289059232332609729971208443357326548938239119325974636673058360414281388303203824903758985243744170291327656180937734440307074692112019130203303801976211011004492932151608424448596376698389522868478312355265821314495768572624334418930396864262434107732269780280731891544110104468232527162010526522721116603966655730925471105578537634668206531098965269186205647693125705863566201855810072936065987648611791045334885034611365768675324944166803962657978771855608455296541266540853061434443185867697514566140680070023787765913440171274947042056223053899456131407112700040785473326993908145466464588079727082668306343285878569830523580893306575740679545716377525420211495576158140025012622859413021647155097925923099079654737612551765675135751782966645477917450112996148903046399471329621073404375189573596145890193897131117904297828564750320319869151402870808599048010941214722131794764777262241425485454033215718530614228813758504306332175182979866223717215916077166925474873898665494945011465406284336639379003976926567214638530673609657120918076383271664162748888007869256029022847210403172118608204190004229661711963779213375751149595015660496318629472654736425230817703675159067350235072835405670403867435136222247715891504953098444893330963408780769325993978054193414473774418426312986080998886874132604721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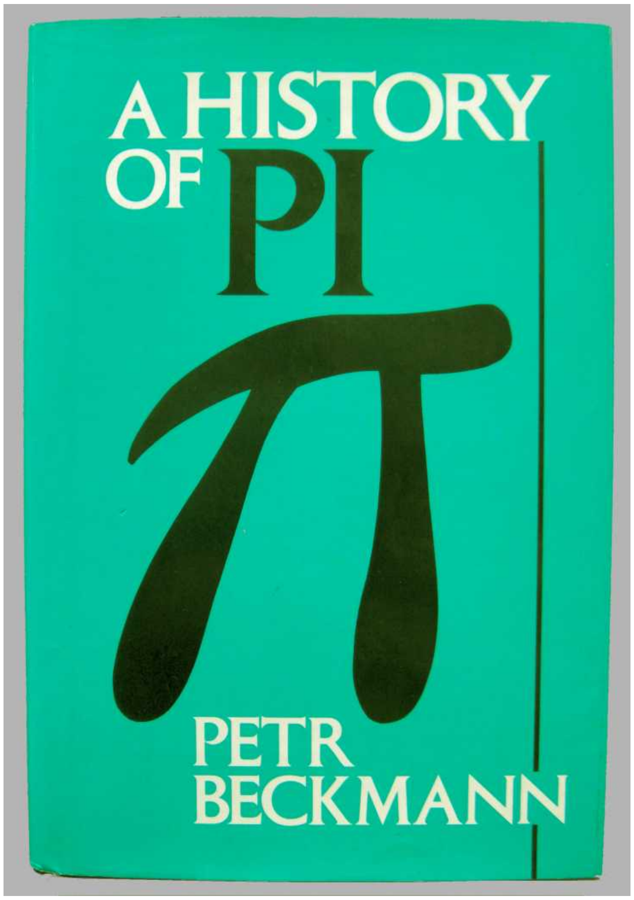


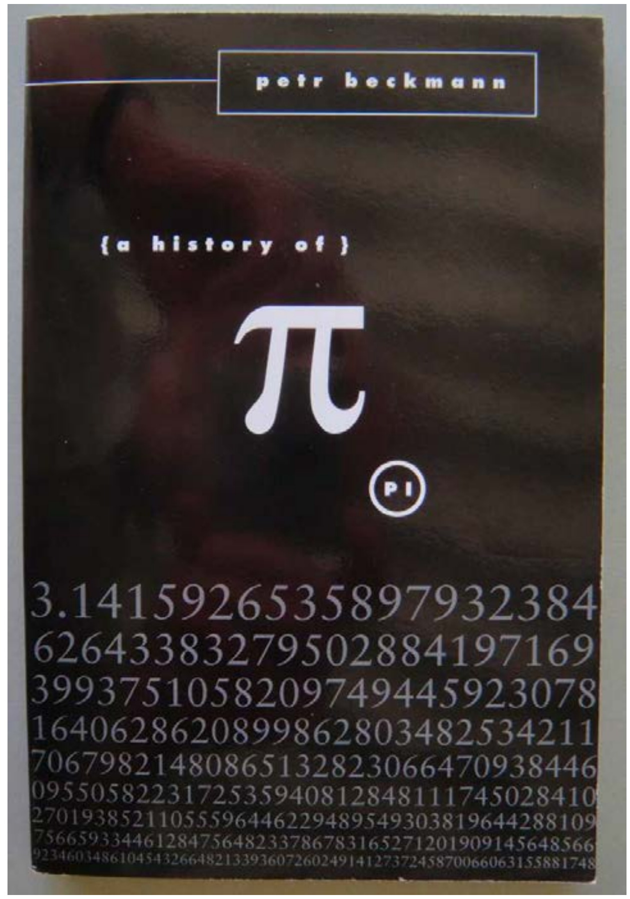
Appendix B – References – Page 1 of 1.

1. The most famous book about π was written by Dr. Petr Beckmann, (November 13, 1924, Prague, Czechoslovakia - August 3, 1993, Boulder, Colorado) a research scientist, statistician, and physicist who has authored 14 books and more than 60 scientific papers, mostly on probability theory. A prolific author he founded Golem Press in 1967 to publish nine of his books. The covers of <u>A History of Pi</u>, 200 pages, ISBN 0-911762-12-4, are shown in Figs. A1 & A2.



RJN photo.

Fig. B1 – 1971 hardbound Golem Press Edition.



RJN photo

Fig. B2 – 1971 softbound Golem Press Edition.

2. A second good reference, but expensive, is <u>Pi: A source book</u> (Hardcover) by Lennart Berggren, Peter Borwwdin, and Jonathan Borwein. The book a source book and is not necessarily a book on the history of π or a compendium of computation methods. <u>Pi: A source book</u> is a part of the π lore and it is more acedemic in nature by its Canadian authors.

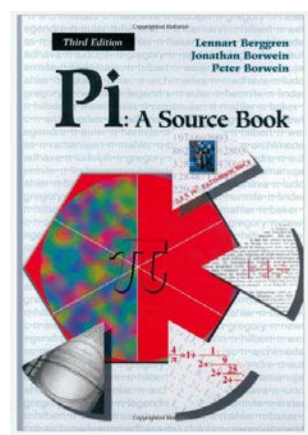


Fig. B3 – 2004 hardbound Edition.