Class ⇒ B.Sc.(Hons.) Part I
Subject ⇒ Chemistry
Paper ⇒ 1A (Physical chemistry)
Chapter ⇒ Gaseous state (Group-A)
Topic ⇒ Derivation of kinetic gas equation.

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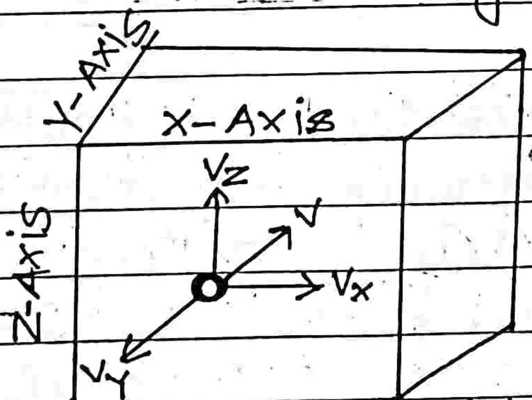
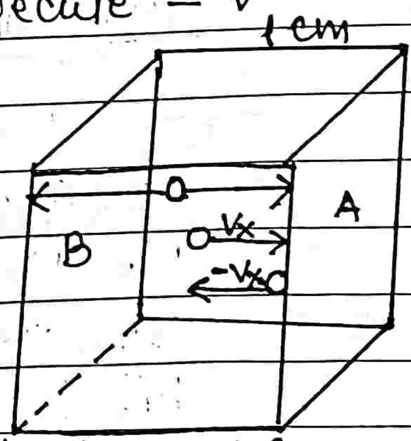
Derivation of Kinetic gas equation

Let us consider a certain mass of gas enclosed in a cubic box at a fixed temperature.

The length of each side of the box = 1 cm
The total no. of gas molecules = n
The mass of one molecule = m
The velocity of a molecule = v



Resolution of velocity v into components v_x, v_y and v_z.



Molecular collisions along X-axis.

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I The kinetic gas equation is derived by the following steps.

1. Resolution of velocity v of a single molecule along x, y and z Axes ⇒

According to the kinetic theory, a molecule of a gas move with velocity v in any direction and resolved into the components V_x, V_y and V_z along x, y and z axes.

$$V^2 = V_x^2 + V_y^2 + V_z^2$$

Now, we consider the motion of a single molecule moving with the component velocities independently in each direction.

2. The no. of collisions per second on face A due to one molecule. ⇒

Let us consider a molecule moving in x direction between opposite faces A and B. It will strike the face A with velocity V_x and Rebound with velocity $-V_x$. To hit the same face again, the molecule must travel l cm to collide with classmate

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the opposite face B and then again l cm to return to face A. Therefore,

The time between two collisions of face A $= \frac{2l}{V_x}$ seconds

The no. of collisions per second on face A $= \frac{V_x}{2l}$

3. The total change of momentum on all faces of the box due to one molecule only.

Each impact of the molecule on the face A causes a change of momentum (Mass x velocity).

The momentum before the impact $= mV_x$

The momentum after the impact $= m(-V_x)$

∴ The change of momentum $= mV_x - (-mV_x)$
$= 2mV_x$

But the no. of collisions per second on face A due to one molecule $= \frac{V_x}{2l}$

∴ The total change of momentum per second on face A caused by one molecule $=$

$$2mV_x \times \left(\frac{V_x}{2l}\right) = \frac{mV^2_x}{l}$$

The change of momentum on both the opposite faces A and B along x-axis would be double i.e.,

$$\frac{2mv_x^2}{l}$$

Similarly,

The change of momentum along Y axis $= \frac{2mv_y^2}{l}$

The Change of momentum along Z axis $= \frac{2mv_z^2}{l}$

∴ The overall change of momentum per second on all faces of the box will be

$$= \frac{2mv_x^2}{l} + \frac{2mv_y^2}{l} + \frac{2mv_z^2}{l}$$

$$= \frac{2m}{l}(v_x^2 + v_y^2 + v_z^2)$$

$$= \frac{2mv^2}{l} \quad (\because v^2 = v_x^2 + v_y^2 + v_z^2)$$

4. Total change of Momentum due to impact of all the Molecules on all faces of the Box ⇒

Let N molecules in the box each of which is moving with a different velocity v_1, v_2 and v_3 respectively.

∴ The total change of momentum due to impact of all the molecules on all faces of the Box $= \frac{2m}{l}(v_1^2 + v_2^2 + v_3^2 + \cdots)$

Multiplying and dividing by n we get

$$= \frac{2mN}{l}\left(\frac{v_1^2 + v_2^2 + v_3^2 + \cdots}{n}\right)$$

$$= \frac{2mNu^2}{l}$$

Where u^2 is the mean square velocity.

5. Calculation of Pressure from change of momentum; Derivation of kinetic gas equation ⇒

The change in momentum per second is called force.

$$\therefore \text{Force} = \frac{2mNu^2}{l}$$

But, Pressure $= \frac{\text{Total force}}{\text{Total Area}}$

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$$P = \frac{2mNu^2}{l} \times \frac{1}{6l^2}$$

$$= \frac{1}{3} \frac{mNu^2}{l^3}$$

Since l^3 = volume of the cube V

$$\therefore P = \frac{1}{3} \frac{mNu^2}{V}$$

or $$PV = \frac{1}{3} mNu^2$$

This is the fundamental equation of the kinetic molecular theory of gases. It is called the kinetic gas equation.