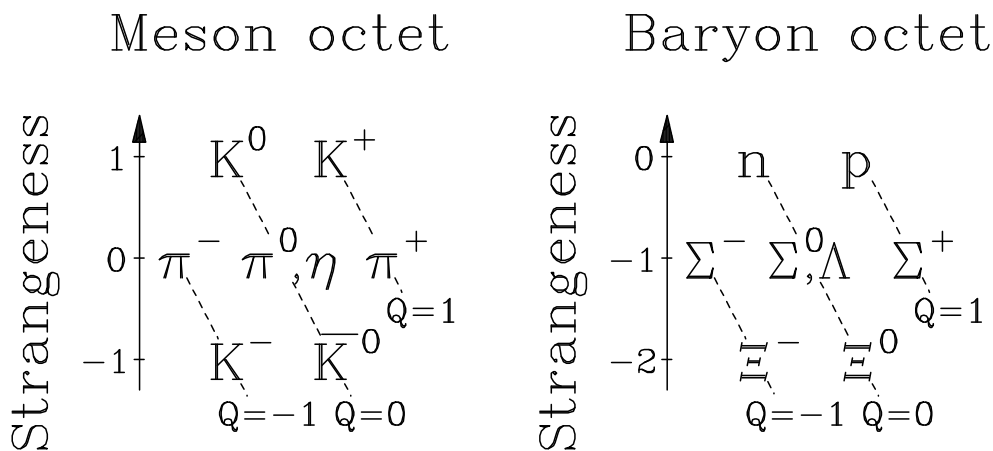


THE EIGHTFOLD WAY ¹

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The Eightfold Way is the name coined by Murray Gell-Mann (1961) to describe a classification scheme of the elementary particles devised by him and Yuval Ne'eman (1961). The name, adopted from the Eightfold Path of Buddhism, refers to the eight-member families to which many sets of particle belong.

In the 1950s Gell-Mann and Kazuo Nishijima invented a scheme to explain a “strange” feature of certain particles; they appeared to be easily produced in cosmic-ray and accelerator reactions, but decayed slowly, as if something were hindering their decays. These particles were assumed to carry a property known as *strangeness* which would be preserved in production but could be changed in decays. Two examples of plots of electric charge Q (in units of the fundamental charge $|e|$) versus strangeness for particles most of which were known in the late 1950s are the following:



Mesons include the π particles, known as *pions*, whose existence was proposed by Hideki Yukawa in 1935 to explain the strong nuclear force, and the K particles (also known as kaons), discovered in cosmic radiation in the 1940s.

¹ Enrico Fermi Institute Report No. 01-41, hep-ph/0109241, to be published in *Macmillan Encyclopedia of Physics, Supplement: Elementary Particle Physics*, edited by John S. Rigden, Jonathan Bagger, and Roger H. Stuewer (Macmillan Reference USA, New York, 2002).

Pions and kaons weigh about one-seventh and one-half as much as protons, respectively. *Baryons* (the prefix *bary-* is Greek for *heavy*) include the proton p, the neutron n, and heavier relatives Λ (lambda), Σ (sigma), and Ξ (xi), collectively known as *hyperons* and discovered in the 1940s and 1950s. The rationale for these families was sought through *symmetries of the strong interactions*.

According to the Gell-Mann–Nishijima scheme, reactions in which these particles are produced must have equal total strangeness on each side. For example, a K^0 and a Λ can be produced by the reaction

$$\pi^- (S = 0) + p (S = 0) \rightarrow K^0 (S = 1) + \Lambda (S = -1) \quad .$$

This scheme thus explained another curious feature of the “strange particles”: They never appeared to be produced singly in reactions caused by protons, neutrons, and π mesons.

In the 1930s, Werner Heisenberg and others had recognized that the similarities of the proton and neutron with respect to their nuclear interactions and masses could be described by a quantity known as *isotopic spin*. This quantity, called *isospin* for short, is analogous to ordinary spin with the proton’s isospin pointing “up” and the neutron’s pointing “down.” Mathematically, isospin is described by a *symmetry group*, i.e., a set of transformations which leaves interactions unchanged, known as SU(2). The “2” refer to the the proton and neutron.

Families whose members are related to one another by SU(2) transformations can have any number of members, including the two-member family to which the proton and neutron belong. Collectively, p and n are known as *nucleons*, and denoted by the symbol N. Isospin predicts that certain sets of particles with different charges (e.g., K or Σ) should have similar masses and strong interactions, as is observed.

Shoichi Sakata (1956) proposed that mesons were composed of the proton

p, the neutron n, the lambda Λ , and corresponding antiparticles, with binding forces so large as to overcome most of their masses. Thus, for example, the K^+ would be $p\bar{\Lambda}$. (The bar over a symbol denotes its antiparticle; electric charges and strangeness of antiparticles are opposite to those of the corresponding particles.) The remaining known baryons (the Σ and Ξ) had to be accounted for in more complicated ways. The Sakata model had the symmetry known as SU(3), where “3” referred to p, n, and Λ .

Gell-Mann and Ne’eman recognized that if electric charge were to be part of the SU(3) description, particles whose electric charges were integer multiples of $|e|$ could belong only to certain families. The simplest of these contained one, eight, and ten members. Other families, such as those containing three and six members, would have fractionally-charged members, and fractional charges had never been seen in nature. Both the mesons and the baryons mentioned above would then have to belong to eight-member families. The baryons fit such a family exactly, leading Gell-Mann to call his scheme the “Eightfold Way.” In addition to the known K mesons and π mesons shown in the figure for mesons, there would have to be an eighth meson, neutral and with zero strangeness. This particle, now called the η (eta), was discovered in 1961.

A consequence of the Eightfold Way for describing mesons and baryons was that their masses M could be related to one another by formulae proposed by Gell-Mann and by Susumu Okubo (1962):

$$\text{Mesons : } 4M(K) = M(\pi) + 3M(\eta) \quad ;$$

$$\text{Baryons : } 2[M(N) + M(\Xi)] = M(\Sigma) + 3M(\Lambda) \quad .$$

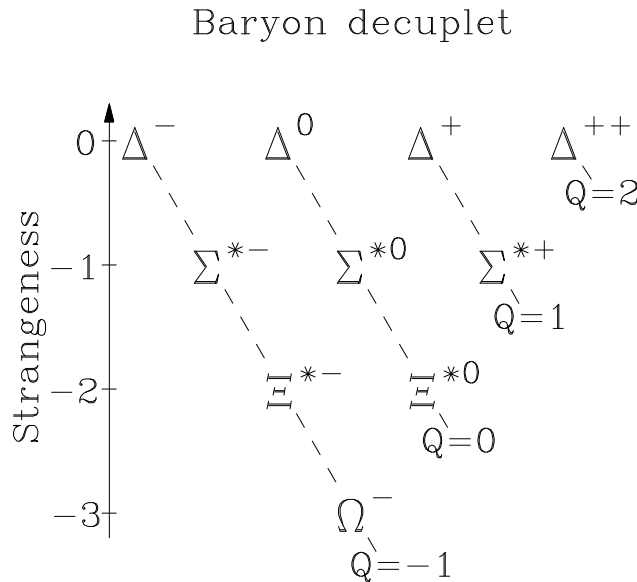
These formulae, particularly the one for baryons, were obeyed quite well. More evidence for SU(3) soon arrived from another experimental discovery.

Certain baryons known as Δ (delta), Σ^* (sigma-star), and Ξ^* (xi-star) appeared to fit into a ten-member family (a *decuplet*), which would be completed

by a not-yet-observed particle known as the Ω^- (omega-minus). The mass of the Ω^- could be anticipated within a few percent because the Gell-Mann–Okubo mass formula for these particles predicted

$$M(\Omega^-) - M(\Xi^*) = M(\Xi^*) - M(\Sigma^*) = M(\Sigma^*) - M(\Delta) \quad .$$

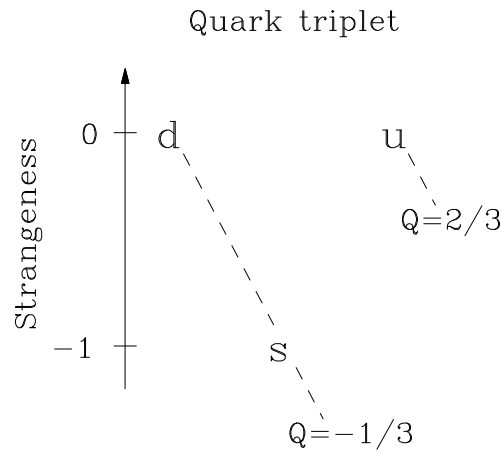
An experiment at Brookhaven National Laboratory (Barnes *et al.*, 1964) detected this particle with the predicted mass through a decay that left no doubts as to its nature. The figure shows its place in the baryon decuplet:



An early application of the Eightfold Way, building upon suggestions by Gell-Mann and Maurice Lévy (1960) and by Gell-Mann (1962), was made by Nicola Cabibbo (1963) to certain decays of baryons, which showed that SU(3) symmetry could be used to describe not only the existence and masses of particles but also their interactions.

Underlying the success of the Eightfold Way and the symmetry group SU(3) is the existence of fundamental subunits of matter, called *quarks* by Gell-Mann (1964) and *aces* by their co-inventor, George Zweig (1964). These objects can

belong to a family of three *fractionally-charged* members u (“up”), d (“down”), and s (“strange”):



The fact that fractionally-charged objects have not been seen in nature requires quarks to combine with one another in such a way as to produce only integrally-charged particles. This is one successful prediction of the theory of the strong interactions, *quantum chromodynamics* (QCD). Baryons are made of three quarks, while mesons are made of a quark and an *antiquark* (with reversed charge and strangeness). For example, the Δ^{++} is made of uuu ; the Δ^- is made of ddd ; the Ω^- is made of sss ; and the K^+ is made of $u\bar{s}$.

The $SU(3)$ symmetry described here refers to the “flavor” of quarks (u, d, s). Other flavors of quarks — *charm* (c), *bottom* (b), and *top* (t) — were discovered subsequently. They are much heavier than u, d, and s, and so do not fit easily into a generalization $SU(n)$ of $SU(3)$ with $n > 3$. The approximate $SU(3)$ symmetry of particles containing u, d, and s quarks remains a useful guide to properties of the strong interactions, since the masses of these quarks are small compared with the mass of the proton. A separate $SU(3)$ symmetry, associated with quantum chromodynamic degrees of freedom, describes “colors” of quarks. Each flavor of quark can exist in three colors (see the article on quantum chromodynamics).

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