International Journal of Advance Research in Science and Engineering
Vol. No.6, Issue No. 02 , February 2017
www.ijarse.com
IJARSE
ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

EULERIAN GRAPHS AND ITS APPLICATIONS

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ABSTRACT

A connected graph G is Eulerian if there exists a closed trail containing every edge of G. Such a trail is an Eulerian trail. Note that this definition requires each edge to be traversed once and once only. In this article we discuss the notions of a graph, Eulerian graph and certain application problems that involve Eulerian graphs, starting from the problem of Konigsberg seven bridges to the current problem of DNA fragment assembly.

Keywords*: *Chinese postman problem, DNA, Eulerian graph, Konigsberg seven bridges

I INTRODUCTION

Eulerian graphs A connected graph G is Eulerian if there exists a closed trail containing every edge of G. Such a trail is an Eulerian trail. Note that this definition requires each edge to be traversed once and once only, A non-Eulerian graph G is semi-Eulerian if there exists a trail containing every edge of G. Problems on N Eulerian graphs frequently appear in books on recreational mathematics [1]. A typical problem might ask whether a given diagram can be drawn without lifting one's pencil from the paper and without repeating any lines (Fig1).

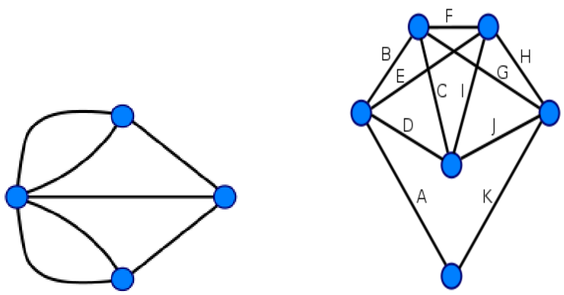


Fig.1: Every vertex of this graph has an even degree. Therefore, this is an Eulerian graph. Following the edges in alphabetical order gives an Eulerian circuit/cycle.

II APPLICATIONS

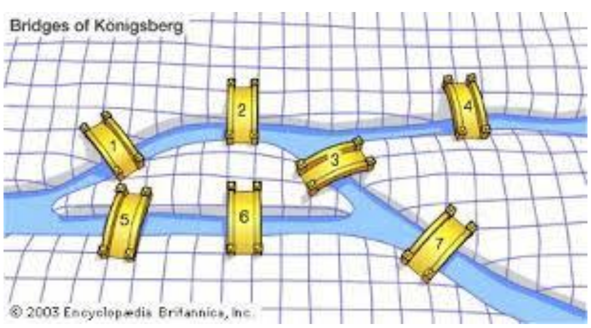
Eulerian graphs can be used to solve many practical problems like Konisberg Bridge problem. They can also be used to by mail carriers who want to have a route where they don't retrace any of their previous steps. Euler graphs are

IJARSE
ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

also useful to painters, garbage collectors, airplane pilots and all world navigators [3]. We discussed some of the applications of Euler graphs.

2.1 The Problem of Seven Bridges

The year 1736 when Euler solved the problem of seven bridges of Königsberg is taken to mark the birth of graph theory[4]. The seven bridges problem is a well known problem that can be stated as follows: The Pregel river in the town of Königsberg divided it four land regions with a central island. These four land regions were connected by seven bridges as shown in the diagram in Figure 2 where the land areas are denoted by the letters A, B, C, D. It is said that the people of the town used to ponder over the question of whether it is possible to start in a land area and make a walk through the bridges visiting each bridge exactly once and returning to the starting point. In solving this problem of "Seven bridges", Euler developed an abstract approach which is considered to be related to the "geometry of position". Represented in terms of a graph Eg (Figure 2), the four land areas A, B, C, D are the vertices and the seven bridges are the edges of the graph. The vertices A and B are joined by multiple edges (two edges), so that the graph Eg is a multigraph! The "Seven bridges problem" can be expressed in graph-theoretic terms as follows: Is there an Eulerian circuit in the graph Eg? The answer from Euler's 1736 paper to this question is NO! . This is stated as an important theorem in the study of Eulerian graphs. Theorem on Eulerian graphs: A connected graph with two or more vertices is an Eulerian graph (ie. has an Eulerian circuit) if and only if each vertex of the graph has even degree. Note that the necessary part of the theorem is based on the fact that, in an Eulerian graph, every time a circuit enters a vertex through an edge it exits the vertex through another edge, thus accounting for an even degree two at the vertex. At the starting vertex, the circuit initially exits and finally enters when the circuit is completed. In the graph Eg in the Seven bridges problem, all the four vertices have odd degree. So, it cannot have an Eulerian circuit



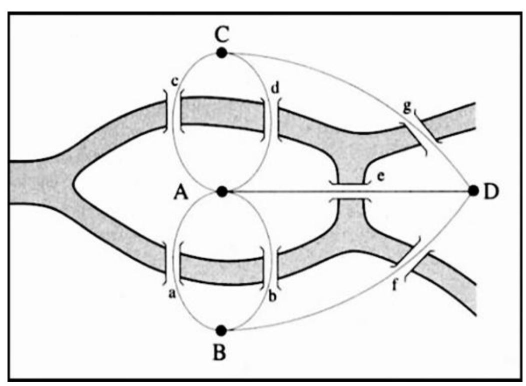


Fig-2: Seven Bridges

2.2 Diagram Tracing and Eulerian graph

A popular old game that entertains children runs as follows: Can you trace with a pencil a diagram of points (with the small circles representing the points) and lines as shown in Figures 3a and 3b? The condition is that the diagram is to be traced beginning at a point and on completion end at the same point but the pencil should not be lifted till the

International Journal of Advance Research in Science and Engineering
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www.ijarse.com

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ISSN (O) 2319 - 8354
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diagram is completely traced and a line in the diagram should not be retraced (i.e. can be traced only once). A curious child will certainly try to find the answer by trial and error and will arrive at the conclusion after sometime that it is not possible in diagram in Figure 3b but it is possible in the diagram in Figure 3a. The question of whether it is possible to trace such a diagram, can be quickly answered if the concept of Eulerian graph is known. Indeed the diagram in Figure 3a is Eulerian whereas the diagram in Figure 3b is not. If e = xy is an edge in a graph, then x is called the start vertex and y, the end vertex of e. A path P (or u-v path P) in a graph is a sequence of edges so that the end vertex of an edge in the sequence is the start vertex of the next edge in the sequence and the path begins in the vertex u and ends in the vertex v. If vertices u and v are the same, then the path is called a circuit (some call it cycle). For example in the graph in Figure 3c, (a,b)(b,c)(c,b)(b,c)(c,d) is a path and (a,b)(b,c)(c,b)(b,c)(c,a) is a circuit. Vertices and/or edges can be repeated in a path or in a circuit. (A path is called a walk by some authors. Due to the diversity of people who use graphs for their own purpose, the naming of certain concepts has not been uniform in graph theory). For example in the graph in Figure 3c, (a,b)(b,c)(c,e)(e,d)(d,c)(c,a) is an Eulerian circuit and hence the graph in Figure 3c is an Eulerian graph

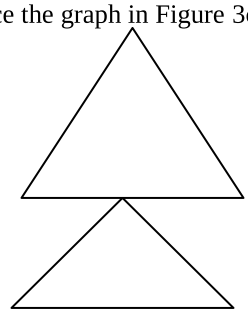

Fig-3(a) Diagram that can be traced

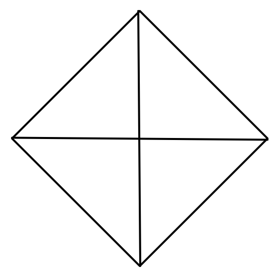

Fig-3(b) Diagram that cannot be traced

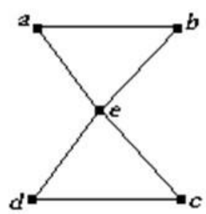

Fig 3 c: An Eulreian Graph

2.3 Floor Designs and Eulerian graphs

Traditional interesting floor designs, known as "kolam", are drawn as decorations in the floor and in large sizes and in interesting shapes, during festivals and weddings, with the drawing done with rice flour or rice paste especially in South India[1]. Generally, in drawing a kolam, first a suitable arrangement of dots is made and then lines going around the dots are drawn. An example kolam is shown in Figure 4a where the dots are ignored. One of the types of kolam, is known as 'kambi kolam' (the Tamil word kambi meaning wire). In this type, the kolam is made of one or more of such 'kambi's . The kolam in Figure 4a is of this type and is made of a single 'kambi' whereas the kolam in

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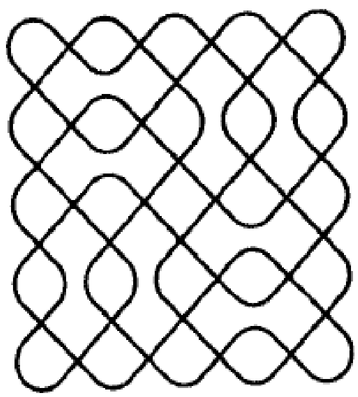
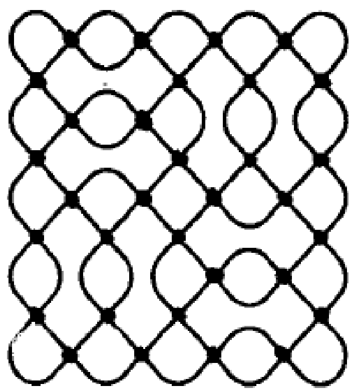
www.ijarse.com
IJARSE
ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

Figure 4c is made of three 'kambi's. A kolam drawing can be treated as a special kind of a graph with the crossings considered as vertices and the parts of the kambi between vertices treated as edges. The only restriction is that unlike in a graph, these edges cannot be freely drawn as there is a specific way of drawing the kolam. The single kambi kolam will then be an Eulerian graph with the drawing starting and ending in the same vertex and passing through every edge of the graph only once. In Figure 4b, the kambi kolam of Figure 4a is shown as a graph with vertices (indicated by small thick dots) and edges and this graph is Eulerian as every vertex is of degree 4. Note that the graph of kambi kolam (with more than one kambi) in Figure 4c will also be Eulerian but the drawing of the kolam in the way it will be done by the kolam practitioners will not be giving rise to the Eulerian circuit. On the other hand in the case of single kambi kolam it is of interest to note that the drawing of the kolam will give a tracing of an Eulerian circuit in the corresponding graph.





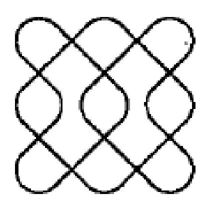


Fig 4:a Single kambi kolam **fig 4b: graph of kolam** **Fig-4c: kolam made of 3 kambis**

2.4 Chinese Postman Problem

All vertices of a graph need not be of even degree and so a graph may not be Eulerian. But if a graph has all except two vertices of even degree then it has an Eulerian path which starts at one of the odd vertices and ends at the other odd vertex. A graph having an Eulerian path but not an Eulerian circuit is called semi-Eulerian. For example in the graph in Figure 8, (a,b)(b,c)(c,d)(d,b)(b,e)(e,d)(d,f) is an Eulerian path and hence the graph in Figure 8 is semi-Eulerian[3].

In a graph Gp that models streets and street corners in a town, with the street corners as the vertices and the streets as the edges, someone starting in a street corner and walking along the streets one after another can end up in a street corner, giving rise to a path in the graph. Note that walking along a street more than once from one corner to another, corresponds to repeating an edge or a vertex in the graph. Suppose that a postman has to deliver letters to the residents in all the streets of a village. Assume that the village is small enough for the postman to be assigned this task every day. If the graph Gp that represents the streets and street corners as mentioned above, is semi-Eulerian so that there is an Euler path in the graph Gp, then this path gives rise to a route following which the postman can start in a street corner, deliver letters going through every street exactly once. This is a desirable

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www.ijarse.com
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situation for the postman. But if no such Euler path exists in Gp, then the postman may have to repeat some of the streets. This problem is called the Chinese postman problem in honour of a Chinese mathematician Meigu Guan who proposed this problem.

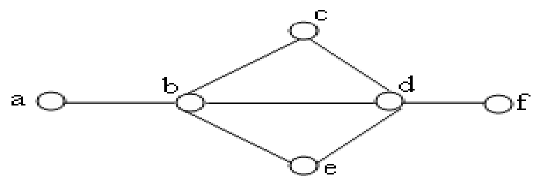

Fig. 8: A semi-Eulerian graph

2.5 DNA fragment assembly

DNA (deoxyribonucleic acid) is found in every living organism and is a storage medium for genetic information. A DNA strand is composed of bases which are denoted by A (adenine), C (cytosine), G (Guanine) and T (thymine). The familiar DNA double helix arises by the bondage of two separate strands with the Watson-Crick complementarity (A and T are complementary; C and G are complementary) leading to the formation of such double strands. DNA sequencing and fragment assembly is the problem of reconstructing full strands of DNA based on the pieces of data recorded. It is of interest to note that ideas from graph theory, especially Eulerian circuits have been used in a recently proposed approach to the problem of DNA fragment assembly. We do not enter into the details but only mention that this brings out the application of graph theory in the field of bioinformatics[1].

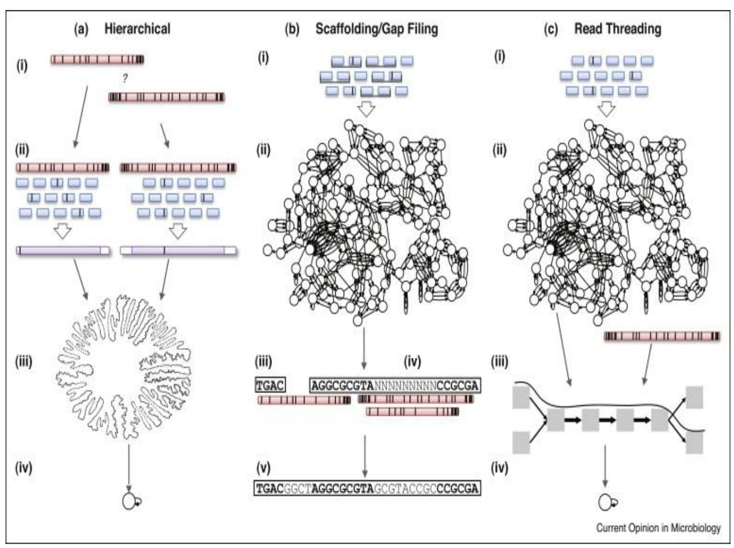

Fig-9: DNA Fragment Assembly

IJARSE
ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

III CONCLUSION

The main aim of this paper is to explain Eulerain graphs and its various applications. We discussed the applications of Eulerian theory to solve Konigsberg seven bridges problem, Chinese postman problem, in floor designs and DNA fragment assembly generally. There are many applications of Eulerian theory in VLSI design, real life and where graphs have found their use.

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