Making the ‘rebound effect’ more useful for performance evaluation of thermal retrofits of existing homes: defining the ‘energy savings deficit’ and the ‘energy performance gap’

JUST SOLUTIONS, CAMBRIDGE, UK. WORKING PAPER 2013-3

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Abstract

Concern has recently intensified regarding increases in the consumption of energy services that often follow energy efficiency improvements, a phenomenon widely called the ‘rebound effect’. However, while some economists have precisely defined this as a metric, much discussion in academic and policy literature is imprecise, leading to confusion and miscommunication. This is especially so regarding direct ‘rebound effects’ in thermal retrofits of existing homes. This study surveys common usages of the term ‘rebound effect’ in domestic heating, identifying three main metrics, which employ different mathematical forms and therefore give different results, but are often lumped together. It defines these as the ‘classic’ rebound effect; the ‘energy savings deficit’, and the ‘energy performance gap’. It then applies these to an empirical case study of three recently retrofitted 30-apartment buildings in Germany. It finds that each metric gives different results for identical situations, ranging from 2.0% to 29.9% for one building, 15.7% to 56.8% for the second, and 43.7% to 272.9% for the third. This may be one reason so-called ‘rebound effect’ results from various studies are so disparate. Nevertheless, specific uses are identified for each of the three metrics, provided their precise definitions are made clear.

Key words: Rebound effect; thermal retrofits; energy savings deficit; energy performance gap; prebound effect

1. Introduction

There is increasing concern about the effectiveness of energy efficiency upgrades in bringing about the level of savings in energy consumption these upgrades aim for. Savings are frequently found, in practice, to be less than those predicted in calculations, and the shortfall is often discussed under the heading of the ‘rebound effect’ [1,2]. Concern about this phenomenon has now become global, with bodies such as the EU Commission, the UN, and national government departments initiating discussion on how to understand it better and what steps can be taken to reduce it (e.g. [3-7].

The phenomenon is seen to occur on micro-and macro-levels of the economy and across a very wide range of energy services [8]. On the micro-level it is thought to occur both directly, where energy service consumption increases in the sector where energy efficiency upgrades have taken place, and indirectly, where money freed up through efficiency improvements in one area leads to consumption increases in other areas. This paper considers direct micro effects only, and in relation to thermal retrofits of existing homes. Further, while most current discussion on the direct micro rebound effect in this sector is concerned with its magnitude and ways to reduce it, this paper focuses on the *terminology and communication* of issues and findings to do with the rebound effect in this sector. It uses a case study of thermal retrofits in three apartment buildings to illustrate the points discussed.

It is important to note that this paper does not seek to distinguish between various possible causes of rebound effects in buildings, or what proportion of these might be caused by user behaviour, technological failure, poor mathematical modelling of expected efficiency gains, etc. These are all important subjects for research and discussion, but this paper confines itself to a narrow focus on the terminology and communication of micro rebound effects in retrofitting homes, and the various current methodologies used to quantify these effects.

In broad terms, the 'rebound effect' quantifies the proportion by which the consumption of energy services increases as a result of an energy efficiency upgrade, and usually in relation to the proportionate increase in energy efficiency. However, there are other formulations of these energy or energy-related consumption increases in both academic and policy literature. This can lead to confusion and cross-purposes in communicating empirical findings, and in the field of thermal retrofits of homes the numbers produced by these diverse methods can be misunderstood and severely misinterpreted (for a discussion of economy-wide rebound effect issues needing clarification see [9].

Further, there is a range of different actors who have a stake in knowing how well a thermal retrofit project has met its energy saving targets: national energy policymakers; economists interested in wider energy and price related consumer behaviour issues; the local investors who pay for the retrofits; the householders who pay the fuel bills; and landlords who might wish to increase the rent to compensate for their investments in efficiency upgrades. When different stakeholders put numbers to the 'rebound effect', other stakeholders need to know precisely what they mean.

Hence this paper aims to clarify some key issues in valuating and communicating energy savings shortfalls with respect to thermal retrofits. In doing so we survey the field of current conceptualisations of these shortfalls and propose a more robust set of definitions, suggesting where each has its specific usefulness and offering empirical case study examples. Hence Section 2 reviews formulations of the phenomenon broadly termed the 'rebound effect' and similar concepts, in relation to home heating, then proposes three distinct metrics which define the phenomenon rigorously, so as to communicate it for distinct purposes and sets of actors. Section 3 tests this nomenclature in an empirical case study of 90 newly retrofitted apartments in three buildings in southern Germany, and discusses the results. Section 4 concludes.

For clarity of expression we will use the following nomenclature:

- **Demand** (symbol D): calculated energy consumption required to provide 100% energy services in a given dwelling.
- **Consumption** (symbol E): actual energy consumption occurring in a given dwelling.
- Both these metrics are given in kilowatt-hours per square meter of useful floor area per year (**kWh/m^{2}a**).
- Consumption here means **primary** energy consumption, and refers to combined energy consumption for space heating and hot water. 'Primary' energy consumption means the total energy consumption that is required to provide the (heating) services in the apartments. For example, for electricity it includes generating losses, losses in the lines, and the performance factors of heating systems such as heat pumps. For gas it includes the energy required for extraction and transportation. This differs from the **metered** energy consumption in the apartments, which includes only the actual quantity of energy consumed for useful heating.
- A **thermal retrofit** is here defined as any significant heating energy efficiency improvement, which may include insulation, new windows, water pipe lagging, a new heating system, new water heating system, or improved ventilation controls.

2. The 'rebound effect' and its cousins in recent literature related to thermal retrofits

Contemporary discussion of the 'rebound effect' identifies its roots in the 'Jevons Paradox' [10], which posits that increasing efficiency in obtaining useful work from coal would increase, rather than decrease, the quantity of coal consumed. This notion was revived by Kazzoom [11] and labeled the 'Kazzoom-Brookes postulate' by Saunders [12]. Whatever the actual origin of the term 'rebound effect' , by 2000 it was being widely used [13] to describe and quantify shortfalls large or small in expected energy savings, while 'backfire' had become the preferred term for increases in consumption such as Jevons had suggested (see articles in special edition of *Energy Policy* 2000, 28(6-7)).

By the mid-2000s a particular, formal definition of the 'rebound effect', expressed in [2], was being widely adopted among economists, while other definitions were being used by others.

Table 1 gives a selection of works to illustrate this diversity, which can be broadly summarized as follows.

Following Berkhout et al. [14], Sorrell and Dimitropoulos [2] define the rebound effect as a partial derivative, namely a small proportionate change in demand for useful work S, also called 'services', as a ratio of the associated proportionate change in energy efficiency ε, as in equation (1):

$$R_{\varepsilon}\,(S) = \frac{\partial S}{S} \Big/ \frac{\partial \varepsilon}{\varepsilon} = \frac{\partial S}{\partial \varepsilon} \cdot \frac{\epsilon}{S} \qquad (1)$$

'Useful work' is not the same thing as 'energy consumed'. For car travel, for example, useful work is the number of kilometers travelled, or alternatively person-kilometers or weight-kilometers, but not the quantity of fuel consumed. For thermal retrofits, useful work is the energy services taken, such as heating in rooms, rather than the quantity of energy consumed. This immediately raises difficulties in using this definition for thermal retrofit evaluation, as we need a proxy for 'energy services', which could include: room temperature; air quality; evenness of radiant heat from indoor surfaces; or some metricized combination of these. This issue is discussed further, below and in Section 3.

Since partial (and other) derivatives are based on small changes, and because energy efficiency is inversely proportional to the cost of energy services, a rigorously calculated rebound effect within the above definition can in some circumstances be the negative of the price elasticity of demand for energy services:

$$\eta_{\varepsilon}\,(S) = \frac{\partial S \cdot P}{\partial P \cdot S} \qquad (2)$$

Some authors use this formulation for estimating the rebound effect for home heating (e.g. Guertin et al. [15]). Nesbakken [16] also uses this formulation but refrains from calling it the 'rebound effect'.

A further variation is to compare the proportionate change in *energy* consumption (as distinct from energy *services* consumption) with the proportionate change in energy efficiency, again as a partial differential:

$$R_{\varepsilon}\,(E) = \frac{\partial E}{E} \Big/ \frac{\partial \varepsilon}{\varepsilon} = \frac{\partial E}{\partial \varepsilon} \cdot \frac{\epsilon}{E} \qquad (3)$$

Where E = energy consumption. This is sometimes used as an alternative for (1) above, if energy 'services' are difficult to define. It can be easily shown (see [2]) that for very small changes in efficiency the following relationship between $R_{\varepsilon}(S)$ and $R_{\varepsilon}(E)$ holds:

$$R_{\varepsilon}(E) - R_{\varepsilon}(S) = -1 \qquad (4)$$

This means that, if these metrics are used mathematically correctly, *all* the increase in energy efficiency is accounted for: some of it goes to increase the services taken, while the remainder of it goes to reduce the energy consumed.

Equation (4) assumes that the relationship between energy consumed and energy services taken is:

$$S = \varepsilon \cdot E \qquad (5)$$

Equation (4) is then used to translate $R_\varepsilon(E)$ into $R_\varepsilon(S)$. A further assumption lies behind equation (5), namely that the energy efficiency ε is known. When engineers give energy efficiency changes for thermal retrofits, they are basing these figures on assumptions as to what level of consumption E is required to provide 100% energy services S. Hence if ε is known, the level of 'services' S for any particular value of E has already been assumed. In Germany, for example, a building's energy performance rating (EPR) is the annual energy consumption per square metre of useful living area (kWh/m^{2}a), that would be required to provide 100% energy services in that particular building, given its thermal characteristics. The calculation methodology for this is published by the German Institute of Standards *(Deutsches Institut für Normung* - www.din.de/), in document DIN V 4108-6. When engineers say that a building's energy efficiency has been increased by X% due to a retrofit, they mean it requires proportionately less energy consumption E to achieve energy services S of 100%. The definition of '100% services' is embedded in the DIN formulae, namely, a year-round indoor temperature of 19°C and a ventilation rate of 0.7 volumes per hour. Hence we are already making assumptions about energy services when we use $R_\varepsilon(E)$ to derive values for $R_\varepsilon(S)$. The assumptions are embedded in ε, the parameter that links these two formulations.

A further problem is that for thermal upgrades, $R_\varepsilon(E)$, like $R_\varepsilon(S)$, is also limited by being a partial differential. The symmetry between these two formulations, given in equation (4), breaks down when anything larger than an infinitesimal change in energy efficiency is being considered. We will return to this point in our discussion of results, in Section 3.

Before this definition of the rebound effect became de-facto standardized as a partial differential, Haas and Biermayr [17] used an entirely different rebound effect definition for home heating, which is often preferred by engineers: the shortfall in the quantity of energy saved, as a proportion (or percentage) of the expected energy savings. They used four different empirical methods, each based on its own dataset, to estimate the value of (this version of) what they called the 'rebound effect' for space heating in Austria, and got reasonably convergent results. Formally expressed, their metric is:

$$RE(E) = \frac{F}{\Delta D}$$

where F = shortfall in energy savings; ΔD = savings expected if the design is successful.

This definition avoids the difficulties of the energy services-efficiency definition (above), though it needs to be used with caution because of its less rigorous mathematical basis. Since it does not utilize demand curve calculus, it cannot be interchanged with price elasticities, nor with measures of change in energy services, even for very small changes in energy efficiency.

Nevertheless, we will argue, this definition offers considerable advantages to certain actors and stakeholders, so we give it the label of the **'energy savings deficit'** (ESD) and will return to it in later discussion. For clarity of discussion we will, however, confine the term 'rebound effect' to the former formulation. Hence we write:

$$ESD(E) = \frac{F}{\Delta D} \qquad (6)$$

Making this distinction clear will enable us to avoid the confusion of definitions such as in [18], which lists various author's findings, including those of Sorrell and Dimitropoulos [2] and Haas and Biermeyer [17] as if they are talking about the same thing.

Further alternative definitions abound. Yun et al. [19] and Jenkins et al. [20] come close to the first definition (what we are calling the 'rebound effect', above), but it is unclear whether their comparator is *energy* change or energy *services* change. The report for the EU Commission [4], keeps this vaguer still, appearing to vacillate between energy and energy services for one part of the definition, and absolute change and proportional change for another part. Houden and Chapman [21] use the term 'rebound effect' as a general descriptor, offering no specific definition, but compare percentage fuel consumption and emission changes before and after retrofits and for fuel switching.

Two different definitions appear to circulate in UK government literature. The Energy Savings Trust [22] gives a vague definition of using more energy (not energy services) than expected after an efficiency upgrade, while formulations in publications of the Department of Energy and Climate Change come close to the 'energy savings deficit' (above), often using the term 'comfort benefits' for an increase in energy services consumption [3,23].

Contrasting with all these is the implied definition in Hens et al.'s [24] study of Belgian home heating behavior. Here the 'rebound effect' is the percentage difference between calculated energy rating and actual consumption, for all homes, including older, energy inefficient buildings that have not been retrofitted. This has the ironic effect of assigning large 'rebound effects' to old buildings that have not even been retrofitted and are consuming well below their calculated ratings. As Sunikka-Blank and Galvin [25] point out, this is effectively the opposite of the rebound effect, identical to those authors' definition of the 'prebound effect'.

Table 1. Summary of variation in Retrofit Rebound Effect Formulations. 'ESD' refers to the 'energy savings deficit'.

Work	**How rebound effect is defined for retrofits (or new builds)**	**Where**	**Comments**
Sorrell and Dimitropoulos (2008)	**Energy efficiency** elasticity of the demand for useful work (i.e. **energy services**)	general	Uses the most common definition found in micro-economics
Yun et al. (2013)	percentage change in **energy consumption** [but seems to be referring to energy **services** rather than energy itself] following a percentage change in the technological efficiency or price of a service	China	Also has good summary of others & review of methods
Guertin et al. 2003	Price elasticity of demand for space heating **energy services** (-0.38)	Canada	Calculated from differences in consumption etc.
Madlener and Hauertmann (2011)	**Energy efficiency** elasticity of demand for **energy**	Germany	Alternative definition from Sorrell and Dimitropoulos (2008)
Jenkins et al. (2011)	Take-back in **energy** demand, but some confusion with energy services	Inter-national	
Maxwell & McAndrew (2011)	Increase in **consumption** due to environmental efficiency interventions that can occur through a price reduction (p. 6)	EU wide	Some confusion, e.g. p. 29, where an increase in consumption (energy?)seems to be conflated with having higher indoor temperatures (an energy service)
EST (Energy Savings Trust)	Using more **energy** than expected after an efficiency upgrade	UK	Popular web leaflet
Berkhout et al. (2000)	Increase in **consumption services** due to price reduction due to efficiency upgrade – cf Yun … and Sorrell…)	Netherlands etc	Mostly descriptive
Nesbakken, (2001)	Does not use term 'rebound effect' but short-run elasticity as change in heating energy **consumption** with price change.	Norway	
Howden-Chapman et al. (2009)	'Rebound effect' used as a **general descriptor**. No specific definition, but percentage fuel consumption changes are compared for before and after retrofits and for fuel switching	New Zealand	
Hens et al., (2010)	Direct rebound is valued by dividing the difference between calculated reference consumption and the normalized measured consumption by that reference: - i.e. how Sunikka-Blank and Galvin (2012) define the **Prebound effect**.	Belgium	Gives the inverse relationship to the EPG, i.e. 100-EPG.
DECC (2012)	The percentage **shortfall in the savings** that are predicted for the retrofit – i.e. the same as the **ESD**	UK	See esp. fig 3.1 p. 19
DECC (2008)	**'comfort benefits'**: the fraction of **energy savings** taken in increased comfort benefits (p. 90), or this combined with the shortfall due to technical failings – similar to **ESD**	UK	Similar to ESD
Haas and Biermayr 2000	Shortfall in savings as percentage of expected savings (but they call it 'rebound effect'	Austria	Same as ESD but they call it 'rebound effect'
Ouyang et al. (2010)	Blended: use classic and also ESD as if interchangeable.	Global	
Demanuele et al. (2010)	Difference between calculated and actual post-retrofit (or new build) performance in school buildings	UK	Expressed as 'gap' and 'discrepancy'
Tronchin and Fabbri (2001)	Difference between calculated and actual post-retrofit (or new build) performance	Italy	Expressed as 'gap' in energy performance; quantified as % of design rating.

A different approach uses the terms 'gap', 'discrepancy' or 'energy performance gap' for the difference between calculated and actual post-retrofit performance, expressed either in absolute terms or as a percentage of the calculated energy rating [26,27]. Unlike the metrics outlined above, this metric is not a function of pre-retrofit performance or rating, and can be used with both new builds and retrofits. Because of its usefulness in comparing and communicating energy over-consumption where no pre-retrofit figures are available or appropriate, we adopt this definition as the **'energy performance gap'** (EPG), formally defined as:

$$EPG(E) = \frac{V}{D} \qquad (7)$$

Where V = over-consumption; D = design consumption

In summary, we suggest the terms:

Rebound effect = proportionate change in energy services consumption as a proportion (or percentage) of the proportionate change in energy efficiency (following [2]) as in equation (1); alternatively expressed as *energy efficiency elasticity of energy services consumption.* Its mathematical correlate is the *energy efficiency elasticity of energy consumption,* as in equation (3).

We note that the concept of an 'energy-efficiency elasticity' might be difficult to grasp for readers not used to the economics literature from which it was originally derived. It simply means that, for a small detectable increase in energy efficiency there is a corresponding change in the quantity of 'services' (e.g. indoor temperature). For example, if a 1% increase in energy efficiency is associated with a 0.4% increase in energy services, this is said to be an energy efficiency elasticity of energy services of 0.4, or 40%.

To keep strictly to this definition, the relationship is meaningful only for very small changes in efficiency, as it is mathematically modeled as a partial differential. However, thermal retrofits produce very large increases in energy efficiency. Since the use of (partial) derivatives for large changes is a clear violation of mathematical rules, we offer a quasi-correct version of the rebound effect definition which we call the **'one-step'** rebound effect, as:

$$RE_{\varepsilon}\,(S) = \frac{\Delta S \cdot \epsilon}{\Delta \varepsilon \cdot S} \qquad (8)$$

The correlate of this, for energy consumption, becomes:

$$RE_{\varepsilon}\,(E) = \frac{\Delta E \cdot \epsilon}{\Delta \varepsilon \cdot E} \qquad (9)$$

We offer these quasi-definitions merely to be able to illustrate how the results for a one-step rebound effect calculation would compare and contrast with those for a correctly configured elasticity calculation and with the other metrics, together with some of the difficulties that ensue when the rebound effect is used for thermal retrofits.

Energy savings deficit (ESD) = shortfall in energy savings F as a proportion (or percentage) of expected energy savings ΔD, following [17] but avoiding calling this the 'rebound effect' – recalling equation (6):

$$ESD(E) = \frac{F}{\Delta D}$$

Energy performance gap (EPG) = overconsumption V as a proportion (or percentage) of design consumption D – recalling equation (7):

$$EPG(E) = \frac{V}{D}$$

We now explore how these metrics fare in a case study of a recent thermal retrofit project.

3. Using the definitions in a retrofit case study

3.1 Case study description

The case study is a set of three buildings in southern Germany (location withheld to preserve residents' anonymity), which we label Building 1, Building 2 and Building 3, each consisting originally of 30 identical and mirror-image apartments. These 1950s buildings were comprehensively retrofitted in the mid-2000s. Prior to the retrofits, their energy demand for space heating and hot water was 320kWh/m^{2}a. This was calculated according to the German Institute of Standards' DIN V 4108-6 (see above), which is used as the basis for determining energy ratings for the thermal building regulations, which are given in the *Energieeinsparverordnung* [28]. It assumes this quantity of heating energy is required to provide 100% of the energy services needed for year-round comfortable indoor living, i.e. 19°C in all rooms, with ventilation of one full exchange of air every one-and-a-half hours.

We use this definition of 100% energy services here without critical comment, merely to enable the analysis to proceed. Whether or not a home needs 19°C in all rooms, all year round, with the generous level of ventilation noted here, is a social and policy question which is beyond the scope of this investigation.

The retrofit of Building 1 was designed to achieve a heating energy demand of 50kWh/m^{2}a, and its metered consumption over a year of running was 51kWh/m^{2}a. This building used district heating and conventional radiators. Building 2 used district heating and under-floor heating, and its insulation and window U-values were slightly lower (better) than those of Building 1. Its design consumption was 37kWh/m^{2}a, but its metered consumption was 58 kWh/m^{2}a. Building 3 used heat pumps, and one-third of its apartments used under-floor heating, one-third ceiling radiant heating, and one-third ventilation heating. Its U-values were a further step lower (better) than for Building 2. Its design consumption was 23.6kWh/m^{2}a, but its metered consumption was 88kWh/m^{2}a. The large discrepancy between design and actual consumption in Building 3 is believed to be due to technical failings in the heat pumps and difficulties experienced by the occupants in controlling their heating, and is being researched separately. These buildings make a suitable case study for this investigation because of the range of discrepancies between design and actual consumption, while they are geometrically identical and had identical thermal properties prior to retrofitting.

Throughout the entire year 2012 (and for a year previously) a number of sensors in each of the 90 apartments gave readings for a range of parameters, including space and water heating consumption. These findings are used in this analysis. We consider the year 2012 because this is sufficiently long after the retrofits were completed for occupants to have become familiar with their new heating and ventilation systems and insulation.

In order to precisely define the metrics that we are considering, four parameters were defined (the subscripts $_B$ and $_A$ denote 'before' and 'after' the retrofit).

D_B = the (calculated) heating energy demand before retrofitting (note that German engineers call this the *Bedarf* and use the nomenclature q for it, but we use D for consistency of nomenclature throughout).

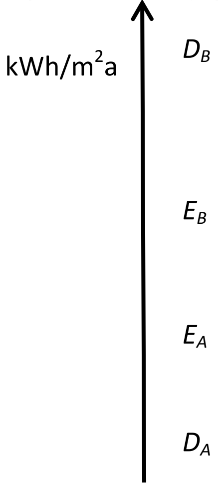
E_B = the actual (measured or metered) heating energy consumption before retrofitting (called the *Verbrauch* in Germany)

D_A = the (calculated) design heating energy demand of the retrofit

E_A = the actual (measured) heating energy consumption after the retrofit

In most retrofit cases these parameters scale in the order shown in Figure 1.

Figure 1. Typical scaling of variables for calculating rebound effect, energy savings deficit and energy performance gap.



It is essential to distinguish between D_B, the calculated pre-retrofit consumption, and E_B, the actual pre-retrofit consumption, which is usually the lower of the two. Here we note the finding of Sunikka-Blank and Galvin [25], that there is a consistent difference between these two in German (and French, Dutch, Belgian and British) homes, with E_B 30% lower than D_B on average. These authors label this gap the 'prebound effect' and define it as the percentage by which actual consumption falls below calculated consumption, a metric we will return to later in our analysis. Recently the German Energy Agency *(Deutsche Energie-Agentur* – DENA) has confirmed this average gap to be 30% [30]. Hence the large pre-retrofit difference between building demand and actual consumption, found in these buildings, is not unusual in Germany.

For the buildings we are considering, as a whole, the values of these variables are given in Table 2. We use the figure 171kWh/m^{2}a for the pre-retrofit consumption for all three buildings, as this is their average E_B and there were changes of occupation between the three buildings before and after the retrofits.

Table 2. Pre-retrofit demand and consumption; post-retrofit design and achieved demand; and post-retrofit consumption.

	Building 1	Building 2	Building 3
D_B (kWh/m2a)	320	320	320
E_B (kWh/m2a)	171	171	171
E_A (kWh/m2a)	51	58	88
D_A (kWh/m2a)	50	37	23.6

3.2 Rebound effect

Considering equations (1) and (5), the 'demand for useful work' here is the energy services taken, and for this we use, as a proxy, the proportion of calculated consumption actually used. For example, if a household is consuming 150 kWh/m^{2}a in a dwelling with an energy demand rating of 200 kWh/m^{2}a (prebound effect = 25%), the energy services are 75%. Hence for all three of our case study buildings the pre-retrofit energy services are 171/320 = 0.534 or 53.4% (a prebound effect of 46.6%).

To illustrate the difficulties with the one-step rebound effect we look closely at Building 2. Here the post-retrofit energy services are 58/37 = 1.57 or 157%. Recalling that energy efficiency is defined as the reciprocal of the design demand, the design energy efficiency increase, as a proportion of its original value, is:

$$\frac{\frac{1}{37}-\frac{1}{320}}{\frac{1}{320}} = 7.65 \ \text{ or } 765\%$$

For all three buildings the pre-retrofit energy efficiency is 1/320 = 0.00313. Note that there is no absolute definition of thermal ‘efficiency’ for a building, but this does not affect our calculations because in every instance we are dealing with proportionate changes in energy efficiency rather than absolute values. As an arbitrary base we assume here that a building that requires 1kWh/m^{2}a of primary energy to provide 100% energy services is 100% efficient.

For Building 2 the post-retrofit energy efficiency is 1/37 = 0.0270.

Hence we calculate the *energy services* one-step rebound effect from equation (8):

$$RE_{\varepsilon}\,(S) = \frac{\Delta S\cdot\epsilon}{\Delta\varepsilon\cdot S} = \frac{(1.57-0.534)}{(0.0270-0.00313)}\cdot\frac{0.00313}{0.534} = 0.254 \text{ or } 25.4\%$$

This would imply that 25.4% of the energy efficiency increase has been used to increase energy services, while the remaining 74.6% has been used to reduce energy consumption.

However, when we calculate the one-step *energy* rebound effect we get:

$$RE_{\varepsilon}\,(E) = \frac{\Delta E\cdot\epsilon}{\Delta\varepsilon\cdot E} = \frac{(58-171)}{(0.0270-0.00313)}\cdot\frac{0.00313}{171} = -0.087 \ \text{ or } -8.7\%$$

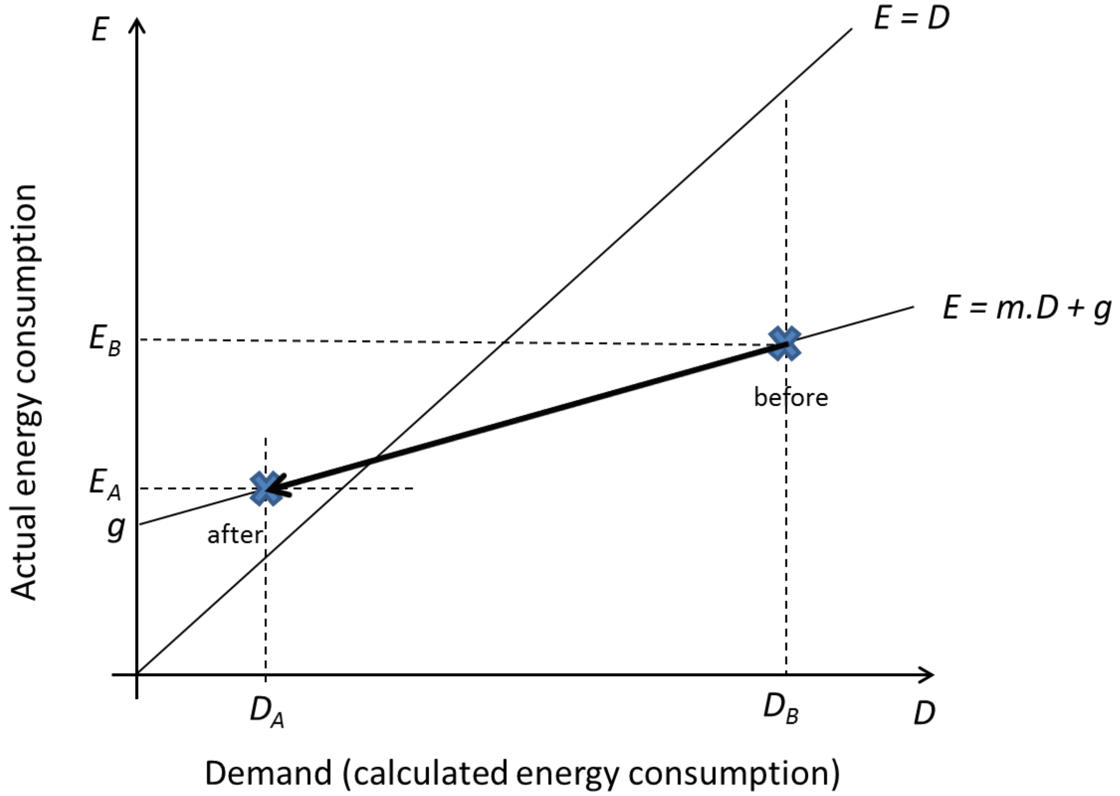
This would imply that 8.7% of the energy efficiency upgrade has been used to reduce energy consumption, while the remaining 91.3% has been used to increase the energy services consumption. The results are incommensurate. Clearly the relationship $RE_{\varepsilon}\,(E) - RE_{\varepsilon}\,(S) = -1$ does not hold true, since

$$-0.087 - 0.254 \ = \ -0.341 \ \neq -1$$

These results are incommensurate because using the rebound effect over large changes in energy efficiency violates the rules of differential calculus, which work only for infinitesimal changes in the parameter values. Galvin [31] has shown that this problem can be solved by considering the rebound effect of an energy efficiency upgrade from ε_1 *to* ε_2 as the average rebound effect of an infinite number of infinitesimally small energy efficiency upgrades over the full range ε_1 *to* ε_2. In practice this involves integrating the rebound effect as a function of ε over this range and dividing the definite integral by $\varepsilon_1 - \varepsilon_2$. When this is done, the relation $RE_{\varepsilon}\,(E) - RE_{\varepsilon}\,(S) = -1$ (equation 4) holds true.

Consider the retrofit upgrade depicted schematically in Figure 2.

Figure 2. Modelled relationship between calculated heating energy consumption *D* and actual heating energy consumption *E* before and after a thermal retrofit (subscript *B* = 'before'; *A*= 'after').



Prior to the retrofit the consumption/demand position is given at the point marked 'before' in Figure 2, and after the retrofit by the position marked 'after'. The energy-efficiency increase can be considered to drive the consumption/demand ratio leftward along the line $E = m{\cdot}D + g$. In the case of Building 2 (m is the gradient of the straight line from 'before' to 'after'; g is the intercept of this line with the vertical axis):

$$m = \frac{(171 - 58)}{(320 - 37)} = 0.399; \qquad g = 171 - 0.399 \times 320 = 43.32$$

The consumption/demand relation therefore moves along the line

$$E = 0.399D + 43.32 \qquad (10)$$

Recalling from Section 2 that energy efficiency is here defined as the reciprocal of calculated energy demand ($D=\varepsilon^{-1}$), we can express actual consumption E in terms of energy efficiency:

$$E = 0.399\varepsilon^{-1} + 43.32 \qquad (11)$$

Hence from (3) we can derive a relation for the energy rebound effect for every point along this line:

$$R_\varepsilon(E) = \frac{\partial E}{\partial \varepsilon} \cdot \frac{\epsilon}{E} = \frac{-0.399\varepsilon^{-1}}{0.399\varepsilon^{-1} + 43.32} \qquad (12)$$

Substituting D for ε^{-1} gives us the energy rebound effect as a function of the demand:

$$R_\varepsilon\,(E) = \frac{-0.399D}{0.399D + 43.32} \qquad (13)$$

By substituting $S = E/D$ it can easily be shown that

$$R_\varepsilon\,(S) = \frac{43.32}{0.399D + 43.32} \qquad (14)$$

and therefore the relation $R_\varepsilon(E) - R_\varepsilon(S) = -1$ (equation 4) holds true in this case.

Equation (14) may be easily integrated:

$$I = \int R_\varepsilon\,(S)dD = \frac{43.32}{0.399} \cdot \ln(0.399D + 43.32) + C \qquad (15)$$

Hence the average value of $R_\varepsilon\,(S)$ over the full range of the retrofit is given by:

$$R_\varepsilon\,(S_{ave}) = \frac{1}{320 - 37} \cdot \int_{37}^{320} R_\varepsilon\,(S)dD = 0.414 \; or \; 41.4\%$$

Although equation (13) can also be integrated, the result is a converging infinite series, and the persistence of zero-value terms in parts of the denominator make it difficult to incorporate into a computer programme. We therefore find the definite integral of the energy rebound effect directly, using a Reimann series:

$$I = \int_{D_1}^{D_2} f(D)dD = \sum\nolimits_{n=1}^{N} \{f(D) \cdot p + 0.5p \cdot [f(D + p) - f(D)]\}$$

where $p = \frac{D_2 - D_1}{N}$

and $R_\varepsilon\,(E) = I/(D_2 - D_1)$

This algorithm was embedded in a programme using Visual Basic, as this language can be programmed to produce Excel spreadsheets with printouts of results. The programme was further developed to give average energy rebound effects for energy efficiency increases of any magnitude up to the value in the actual retrofit, starting from the building's pre-retrofit position and tracking the line in equation (10).

The result for the full retrofit of Building 2 was an energy rebound effect $R_\epsilon(E)$ = -0.586 or -58.6%. Again this gives results for $R_\epsilon(E)$ and $R_\epsilon(S)$ that satisfy the criterion of equation (4).

These results mean at 41.4% of the energy efficiency upgrade was taken back in increased energy services (e.g. warmer rooms), while 58.6% went to reducing the building's energy consumption.

The results were checked by transforming equation (13) into the form $R = \left(aD^b + c\right)^{-1}$, which [31] shows can be integrated to give the infinite (but convergent) series:

$$\int R\,dD = \sum_{n=1}^{\infty} \frac{(n-1)!\,(ab)^{n-1} D^{(n-1)b+1} (aD^b + c)^{-n}}{[nb+1][(n-1)b+1][(n-2)b+1]\ldots[0\times b+1]} \quad (15)$$

This was also programmed in Visual Basic and produced the same results, to an accuracy of 6 figures, i.e. error < ∓0.0001%. However, the Reimann series is preferred because its mathematical form is less demanding on computer capacity.

These results compare with the cruder, one-step rebound effect figure calculated in Section 3.1 of $R_\epsilon(S)$= 25.4% (or 91.3% if calculated starting with $R_\epsilon(E)$), illustrating how inaccurate the one-step version of the rebound effect is.

For Building 1 the energy consumption/demand relation is

$$E = 0.444D + 30.2 \quad (16)$$

This gives rebound effects $R_\epsilon(S)$ = 0.298 (29.8%) and $R_\epsilon(E)$ = -0.702 (-70.2%). This means that in this building 29.8% of the energy efficiency upgrade was taken back in increased energy services while 70.2% went to reducing the building's energy consumption.

For Building 3 the consumption/demand relation is

$$E = 0.274D + 83.17 \quad (17)$$

In this case $R_\epsilon(S)$ = 0.660 (66.0%) and $R_\epsilon(E)$ = -0.340 (-34.0%), meaning that 66.0% of the energy efficiency upgrade was taken back in increased energy services while 34.0% went to reducing the building's energy consumption.

Comparing these findings, we see the unexpected result that the least ambitious retrofit gives the lowest rebound effect, 29.8% (as well as the lowest consumption in absolute terms), while the most ambitious retrofit gives the highest, 66.0%. While this is not always the case, with these three buildings the proportion of the energy efficiency increase that was 'taken back' – either as increased energy services or as a result of technical failings – tends to correlate with the level of ambition in the retrofit. One important factor that could be influencing this is that the complexity of the heating systems and user controls was greatest in the most ambitious retrofit (Building 3) and greater than usual for Building 2, while Building 1 had conventional radiators with familiar thermostatic adjustment valves. Some of the rebound effect could have been caused by households not being able to get control of the more complex systems (a social science based study is planned, to explore this possibility).

A further important point is that Building 1 showed a significant rebound effect of 29.8%, even though its consumption was only 2% above the design standard. While this makes mathematical sense in terms of the classical definition of the 'rebound effect' (see above), it can be misleading and confusing as it seems to imply that the building did not perform as intended. This is one of the key reasons we need to consider other formulations of the so-called 'rebound effect', the first of which we have called the 'energy savings deficit'.

3.3 Energy savings deficit

The energy savings deficit (ESD) is defined here as the shortfall in savings, after a retrofit, as a proportion of the expected savings. This was the definition Haas and Biermeyer [17] used for the rebound effect, while Druckman et al. [32] define the indirect rebound effect in these terms but in relation to take-back of energy savings through abatement actions rather than through energy efficiency upgrades.

We will define 'expected savings' rigorously, as the numbers depend very much on whose expectations of what are being considered. The savings we consider, for our definition of the ESD, are of *actual* energy consumption ΔE rather than the reduction in demand ΔD. As we note above, [25] and [30] found a consistent difference between the demand D and the actual average energy consumption $\hat{E}$ for each specific value of D in the German housing stock. For homes with D higher than around 100kWh/m^{2}a $\hat{E}$ was lower than D, and this gap increased as D increased. Figure 2 reflects this schematically, and it holds true in general terms for Buildings 1, 2 and 3. Pre-retrofit consumption E is 46.6% below demand D, a result these authors call a 'prebound effect' of 46.6%. As we also schematise in Figure 2, after their retrofits the buildings are 'low energy buildings', in which E and D tend to cross over.

So for Building 1 the expected savings were 171 – 50 = 121kWh/m^{2}a, while the shortfall in savings was 51 – 50 = 1kWh/m^{2}a. This gives an energy savings deficit:

$$ESD = \frac{1}{126} = 0.0079 = 0.79\%$$

For Building 2 we have:

$$ESD = \frac{21}{134} = 0.157 = 15.7\%$$

For Building 3 we have:

$$ESD = \frac{64.4}{147.4} = 0.437 = 43.7\%$$

We note the considerable differences between these figures and those derived above for rebound effects. Building 1 has a very small energy savings deficit because actual post-retrofit consumption is very close to design consumption. But its rebound effect is close to the German average [33], in that about 30% of the energy efficiency increase is taken back for increased comfort or because of technical failings. At the other extreme, for Building 3 the *ESD* and rebound effect are of comparable magnitude, probably because the rebound effect is increased by the same technical failings or human-technical interface failings that cause such a high *ESD.*

This is clearly a further reason that so-called 'rebound effects' found in empirical studies are so disparate. Authors calculating rebound effects using the *ESD* methodology are talking about quite different things from those using the 'rebound effect' methodology we outline in Section 3.2 above.

From a policy perspective the energy savings deficit (*ESD*) could be a useful metric for energy planning. It gives a direct measure of the shortfall in energy savings as a result of the retrofit, compared to what was planned and expected. For Building 2, for example, this is 15.7%. Despite technical failings, which no doubt disappoint the engineers involved, the deficit is small in relative terms, which could bring some comfort to designers and policy actors. For Building 1 the *ESD* is very small, which can be taken to mean the retrofit was entirely successful (despite a 30% 'rebound effect').

This metric might also be useful for energy planning. If we can survey a representative sample of retrofits in a given housing estate or class of buildings, we should be able to obtain an average figure for the *ESD* (say 5%) for a particular type of retrofit. This would indicate that, if we expect savings of, say, 500MWh through technical upgrades of existing homes in a given year, we are likely to achieve a level of savings 5% less than this, i.e. 475MWh. Nevertheless, projection into the future would be reliable only if future retrofits contained the same mix of pre-retrofit prebound effects and post-retrofit energy demand ratings as that of our sample, due to the mathematical limitations of the *ESD* as a metric. An apartment with an expected saving of 100 kWh/m^{2}a and a savings shortfall of 20 kWh/m^{2}a has the same *ESD* (20%) as an apartment with an expected saving of 200 kWh/m^{2}a and a savings shortfall of 40 kWh/m^{2}a, but this does not imply that the behavioural and technical determinants are the same in both cases. Both the behavioural and technical issues in upgrading an apartment building with a pre-upgrade demand of 200kWh/m^{2}a are likely to be different from those of upgrading an apartment building with a demand of 100kWh/m^{2}a.

If, however, the mixes are approximately the same, using the *ESD* would save us from misjudgements and miscommunications where performance is measured by various versions of the rebound effect, since it is not energy service excesses but actual energy savings deficits that cause excess CO2 emissions and frustrate energy security goals. We note, also, that the ESD is smaller than the rebound effect in all three buildings, a finding that policymakers and investors might find encouraging.

3.4 Energy performance gap

Recalling equation (7), we define the energy performance gap (EPG) as the excess consumption as a fraction of the design consumption. For Building 1 this is:

$$EPG(E) = \frac{V}{D} = \frac{51 - 50}{50} = 0.02 = 2.0\%$$

As for the *ESD,* the *EPG* for Building 1 is small, since post-retrofit consumption only slightly higher than the design demand. For Building 2 we have:

$$EPG(E) = \frac{58 - 37}{37} = 0.568 = 56.8\%$$

This is considerably higher than the ESD and significantly higher than the rebound effect for this building, the reverse of the case for Building 1. For Building 3 we have:

$$EPG(E) = \frac{88 - 23.6}{3723.6} = 2.729 = 272.9\%$$

This is over 4 times as high as this building's rebound effect, and indicates that there is something seriously wrong with this building's performance. The rebound effect $R\varepsilon(S)$ masks this, as it takes as its starting point the pre-retrofit relationship between actual and theoretical consumption. Since post-retrofit energy consumption is lower than pre-retrofit, the rebound effect is less than 100%, whereas the *EPG* is derived independently of pre-retrofit figures and measures only the post-retrofit performance in relation to what it was designed to be.

An advantage of the *EPG* is that it can be used when pre-retrofit consumption and/or demand are unknown, or indeed when dealing with new builds, where these parameters do not apply. It gives engineers and planners a straightforward figure for the performance result of their building (with both occupant and technical determinants in play) compared to the design performance.

A further advantage is that it can help social scientists identify individual households that might need help in managing their heating consumption in their new or retrofitted homes. For these buildings the apartment by apartment pre-retrofit consumption figures were not applicable, as there were occupancy changes during retrofitting for most apartments, but the individual *EPGs* could be worked out because the pre-retrofit information is not necessary for this. These results are displayed for Buildings 2 and 3 in Figures 3 and 4, with apartment numbers randomly assigned so as to enhance anonymity of households (apartment by apartment consumption figures for Building 1 were not available).

For apartments in Building 2 the *EPGs* range from -43.4% to 257.4%, and for Building 3 from -39.2% to 664.7%. These ranges show the diversity of consumption patterns among the apartments. Further, in Building 2 the 6 highest consuming apartments consume 50% of the building's heating energy, while a similar percentage is consumed by the highest 6 in Building 3.

Figure 3. Energy performance gap for apartments in Building 2

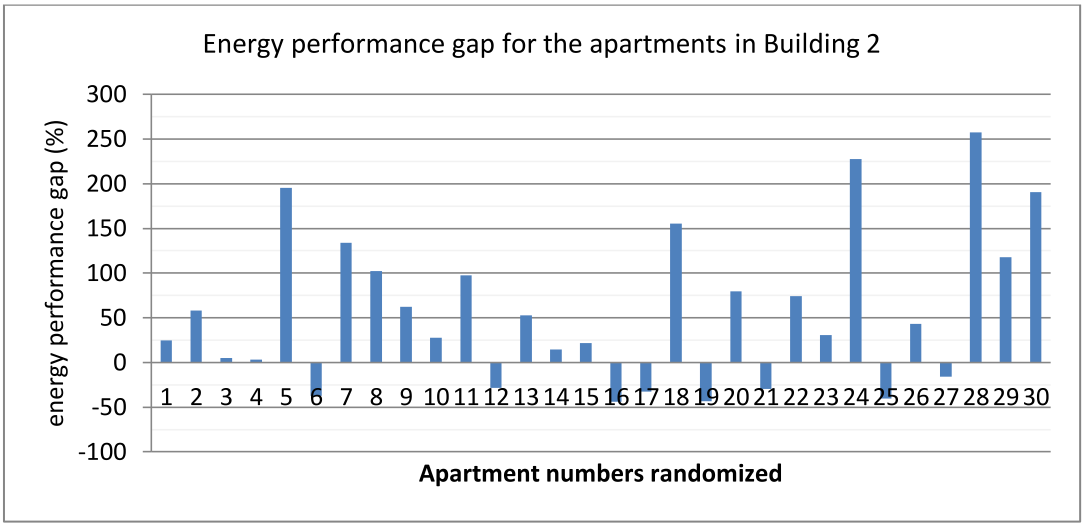
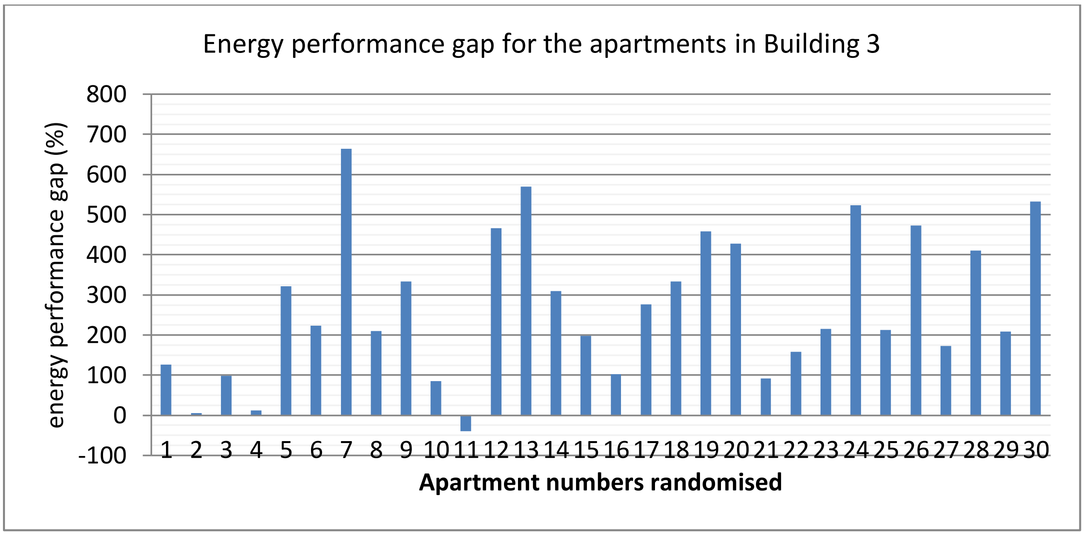


Figure 4. Energy performance gap for apartments in Building 3



We note once again, however, that these are not 'rebound effect' figures in the sense of our definition of that term. The apartment showing an *EPG* of 664.7% does not have a rebound effect $R\varepsilon(S)$ of 664.7%. Its post-retrofit consumption is 158kWh/m^{2}a, so its rebound effect will be less than 100% provided its pre-retrofit consumption was greater than this – which is highly likely, since the average pre-retrofit consumption was 171kWh/m²a and this is a high consuming household. We note, however, that the average heating consumption in German homes is around 149kWh/m²a [33], and to have an apartment consuming more than this in a building designed to consume 23.6kWh/m²a is a cause for concern.

3.5 Summary of the three metrics

The results for all the above metrics, for Buildings 1, 2 and 3, are displayed in Figures 5, 6 and 7.

Figure 5. Results for different methodologies for 'rebound effects', Building 1.

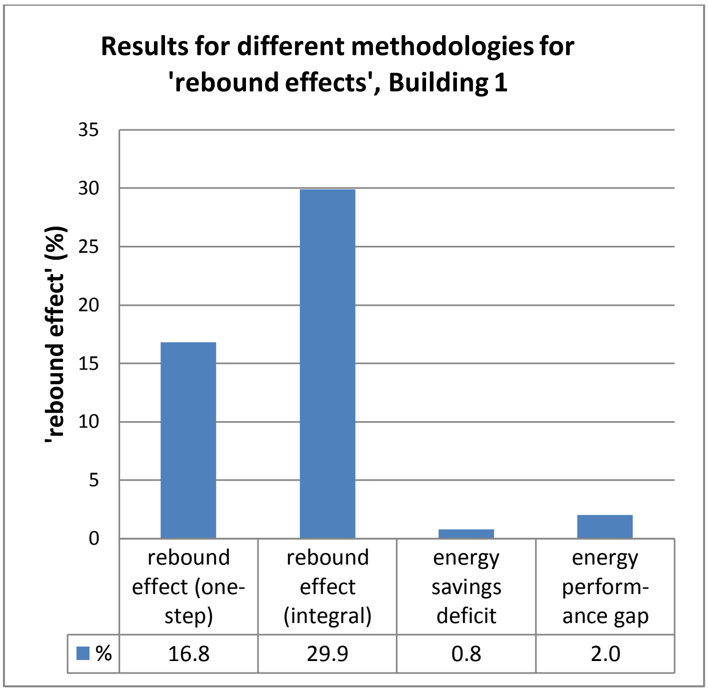


Figure 6. Results for different methodologies for 'rebound effects', Building 2

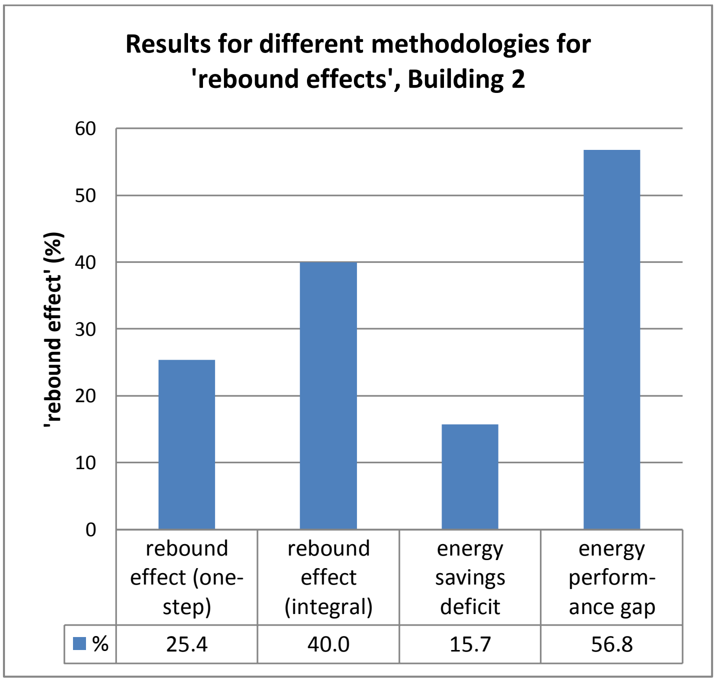
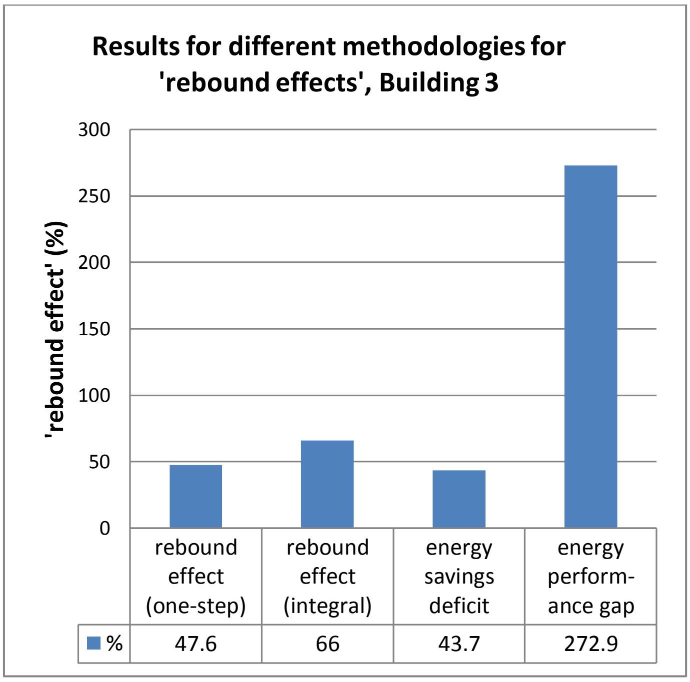


Figure 7. Results for different methodologies for 'rebound effects', Building 3



These results show the wide disparity of values for what are frequently called 'rebound effects' for the same building. This must clearly be one reason why the results of rebound effect studies are so diverse. Unless the metric is carefully defined, reporting that a building or upgrade gives a particular rebound effect is like reporting that the cost of 1kg of food in Asia is 60 local currency units. We need to know which food and what currency is being talked about.

However, if the various metrics are carefully defined they each have important uses. The classic rebound effect $R\varepsilon(S)$ can be used for estimating proportionate changes in energy service take as a consequence of proportionate changes in energy efficiency. Even here, however, it is misleading to use this in a one-step fashion for large changes, since its mathematical integrity only holds together for infinitesimal changes. If changes are large, such as in thermal retrofits, the calculus needs to be properly developed with integration along the energy efficiency/energy services consumption path. This is especially so if we are using energy consumption rather than energy services as our starting point, as we saw in Section 3.2.

The second metric, the energy savings deficit (ESD), gives us a direct and simple measure of how well our energy saving aims have been achieved, without reference to the size of the energy efficiency increase. It is useful for engineering assessments of retrofits, for energy planning, and as a first indication of possible problems with consumer adaptation to their new thermal technology.

The third metric, the energy performance gap (EPG), is useful for engineering assessments, as above, and can also identify over-consumption patterns directly, as no pre-retrofit parameters are needed for it. For this same reason it is an appropriate metric for performance assessments of new builds.

4. Conclusions

This paper has offered a way of clarifying academic, policy and practical discussion and communication on the so-called 'rebound effect' as applied to consumption after thermal retrofits of existing homes. It has identified perennial difficulties in the use of metrics such as the 'rebound effect' and associated concepts for evaluating these retrofits' effectiveness, and defined a set of more clearly defined metrics which might improve interpretation and communication of results.

We found that the term 'rebound effect', at least in relation to domestic heating consumption, means different things in different publications. These include energy efficiency elasticities of energy services (or of energy); comparisons of energy savings shortfall with expected energy savings; comparison of actual post-retrofit (or new build) consumption with design consumption; and comparison of actual consumption with energy demand where there is no retrofit or new build. We also found that common usage of the first of these metrics often violates the mathematical integrity of its definition, so that results do not adequately reflect what is claimed, especially for situations of large increases in energy efficiency such as in thermal retrofits. Nevertheless, we found that the flaws in this 'one-step' approach can be remedied by a mathematically robust approach involving integration of the energy services (or energy)/energy efficiency curve.

We suggested a more careful set of definitions of metrics that are commonly called the 'rebound effect', especially for dealing with thermal retrofits. These are:

1. The 'classic' rebound effect: the energy efficiency elasticity of demand for energy services, and its mathematical corollary, the energy efficiency elasticity of demand for energy, properly integrated along the consumption/demand line pertaining to the energy efficiency upgrade.

2. The energy savings deficit (ESD): the shortfall in expected energy savings as a proportion of the expected energy savings.

3. The energy performance gap (EPG): the over-consumption of energy as a proportion of the design energy demand rating.

We noted that for the first two of these metrics it is essential to distinguish between the pre-retrofit actual consumption and the pre-retrofit calculated demand, a point highlighted in [25] with reference to the 'prebound effect'.

We tested the schema of the three different metrics with a case study of three residential buildings, of 30 apartments each, which have been retrofitted from a calculated demand of

320kWh/m^{2}a to design demands of 50, 37 and 23.6kWh/m^{2}a respectively. We found large differences between the results for each of the metrics, for each building taken as a whole. For Building 1 these ranged from an *ESD* of 0.8% to an integrated rebound effect of 29.9%. For Building 2 they ranged from an *ESD* of 15.7% to an *EPG* of 56.8%, with an integrated rebound effect of 40.0%. For Building 3 the range was 43.7% for the *ESD* to 272.9% for the *EPG*, with an integrated rebound effect of 66.0%.

We suggested one reason for the large range in results for rebound effect studies for domestic heating is the different ways rebound effects are calculated. We contend that it is pointless to compare one set of rebound effect results with another unless the metrics being used are defined precisely identically. There is no formula for translating each metric into another. In Building 1 the integral rebound effect is 37 times as high as the *ESD*; Building 3 is the opposite, with the *ESD* over 4 times as high as the integral rebound effect.

Nevertheless, it is suggested that each of the three metrics, properly used, has important uses. The choice of metric will depend on what precisely one wishes to compare, and this may be determined by whether one is a policy planner, engineer, landlord, tenant, social scientist or economist, to name but a few possibly actors.

While this paper has used three empirical retrofit case studies to illustrate the discrepancies in the metrics, it has avoided wider questions as to why these 'rebound effects' happen. They may be the result of post-retrofit user behaviour changes, difficulties users have operating their new heating controls, failings in the retrofit technology, or inadequate mathematical modelling to calculate pre-and post-retrofit theoretical consumption demand. All these issues need continuing research. Research is also needed to relate rebound effects in buildings to rebound effects in other sectors such as private motorized transport, and manufacturing industry. It is hoped that the clearer definitions offered in this paper may provide more useful ways of linking the effects of energy efficiency upgrades from one of these sectors to the other.

Acknowledgement

The author wishes to thank the German Federal Ministry for the Economy and Technology (Bundesministerium für Wirtschaft und Technologie), Projekt-Träger-Jülich, EnEff:Stadt, and the E.ON-Energy Research Centre, RWTH-Aachen University, for the funding and logistical support that enabled this paper to be produced; and an anonymous housing corporation for strategic assistance.

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