

# Software for solution of 3D problems of electrical survey by transient electromagnetic field using finite elements method

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**Abstract - In this article the software which allows the modeling of 3D non-stationary electromagnetic field with the use of parallelepiped and hexahedral elements is suggested. The example of the practical task is given.**

## I. INTRODUCTION

Now the prevailing electrical survey methods based on coaxial profiling are gradually giving way to the most economical methods of 3D electrical survey. These methods sometimes have more determinative capacity than coaxial profiling. However, the success of the application of these methods is defined by the corresponding mathematical apparatus for 3D modeling.

The difficulties arising from solving a 3D problem are mostly connected with the accuracy and the rate the solution is obtained as well as the presetting a calculation domain and a finite element mesh in it.

In this article the software is suggested. Firstly, this software makes it possible to get the 3D problem solution with high accuracy and rate, thanks to the special mathematical statement based on the separation of the fields, and, secondly, it is rather straightforward to use due to the automatically rebuilding finite element meshes being included.

When modeling 3D non-stationary electromagnetic fields for the solution of various geoelectrical tasks the parallelepiped meshes are more convenient to use as the finite element meshes, because most often the form of calculating domain represents the horizontal-layer area, which includes the parallelepiped 3D objects. In many cases the use of parallelepiped objects is quite enough when solving the problem during the projecting phase as well as for solving the interpretation problems. Nevertheless, in practice the situations when it is necessary to model the electromagnetic fields in the area which includes inclined structures, are very frequent. These structures can represent, for example, pinching-out of layers, coal-beds, veins of gold etc.

Therefore, the finite elements meshes are built with the use of both parallelepiped and hexahedral finite elements. Besides, the irregular rarefaction meshes are used. These meshes include multinode parallelepipeds and hexahedrons, which, in turn, makes it possible to greatly decrease the computational expenses.

In this article the problem of the 3D non-stationary electromagnetic fields modeling by the use of parallelepiped and hexahedral finite elements is considered. The example of the application of the suggested software is given.

## II. MATHEMATICAL MODEL

For the modeling of non-stationary electromagnetic field the method based on the separation of the field on the normal and abnormal constituent is applied. Here the normal constituent of the field describes the field of horizontal-layer area and axisymmetrical objects. Abnormal constituent of the field describes the field induced by the 3D inhomogeneities. This separation allows the use of far more rough meshes in the 3D case but for all that it ensures the high accuracy of the solution obtained.

The following mathematical model is used for description of the abnormal 3D constituent of electromagnetic fields [1]:

$$-\frac{1}{p_0} \operatorname{div} \operatorname{grad} \bar{A}^a + q \frac{d\bar{A}^a}{dt} + q \operatorname{grad} V^a = (q^n - q) \bar{E}^n, \quad (1)$$

$$-\operatorname{div} \left( q \operatorname{grad} V^a + q \frac{d\bar{A}^a}{dt} \right) = \operatorname{div} \left( (q - q^n) \bar{E}^n \right). \quad (2)$$

Here  $\bar{A}^a$  and  $V^a$  are vector and scalar potential of abnormal 3D constituent of electromagnetic fields,  $\bar{E}^n$  is electric intensity of the normal constituent,  $q$  is electrical conductivity of the area with the 3D objects,  $q^n$  is

electrical conductivity of the axisymmetrical area,  $p_0$  is permeance of the vacuum,  $\Gamma$  is the border of the calculation area. At that,  $q$  coincides with  $q^n$  in the subdomain where the 3D objects are absent and in the parts of calculation area which corresponds 3D inhomogeneities  $q$  is different from  $q^n$ . The electrical intensity  $\bar{E}^n$  of the normal constituent of the field can be defined with the help of the solution of the following axisymmetrical problem [1]:

$$-\frac{1}{p_0} \operatorname{div} \operatorname{grad} A_f^n + \frac{1}{p_0 r^2} A_f^n + q \frac{dA_f^n}{dt} = J_f,$$

$$E_f^n = -\frac{dA_f^n}{dt}.$$

### III. APPLICATION OF HEXAHEDRAL FINITE ELEMENTS

As it was mentioned above, depending on the form of the investigated 3D objects the parallelepiped and hexahedral finite elements are used in a finite element mesh. Parallelepiped elements are rather easy to use and only the construction of finite element approximation on the parallelepiped with additional nodes can present some difficulty. The corresponding problem is considered in detail in the article [2]. Let's consider the features of the construction of the finite element approximation on the hexahedral finite elements.

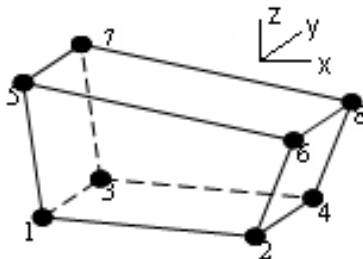


Fig. 1. Hexahedral element

Let's present the calculation area in terms of the union of disjoint hexahedron  $D_m$ . Such a hexahedron  $D_m$  is obtained by means of the parallel shift of some quadrangle  $d_m$  which lies in the plane  $y = const$  along axis  $Oy$ .

Let the local numbering of the nodes be set on the  $D_m$  as it is shown in Fig. 1. The node with local number  $i$  has the coordinates  $(x_i, y_i, z_i)$ . The local basic functions look like  $V_k(x, y, z) = T_j(x, z)Y_i(y)$ , where the triplets of indexes  $(k, j, i)$  set the following membership:

$$I = \left\{ (1,1,1), (2,2,1), (3,1,2), (4,2,2), (5,3,1), (6,4,1), (7,3,2), (8,4,2) \right\}.$$

Here  $Y_1(y) = (y_3 - y)/h_y$ ,  $Y_2(y) = (y - y_1)/h_y$ ,  $h_y = y_3 - y_1$ ,  $T_j(x, z)$  are the functions, which are defined on the quadrangle  $d_m$ . Let's consider the construction of the functions  $T_j(x, z)$ . The quadrangle  $d_m$  is mapped onto square  $E = [0,1] \times [0,1]$  by means of one-to-one map:

$$x = (1-s)(1-r)x_1 + s(1-r)x_2 + (1-s)rx_5 + sr x_6, \quad (3)$$

$$z = (1-s)(1-r)z_1 + s(1-r)z_2 + (1-s)rz_5 + srz_6. \quad (4)$$

The bilinear functions  $v_j, j = \overline{1,4}$  is constructed on square  $E$ . Then the basic functions  $T_j(x, z)$  are defined with the help of the expression  $T_j(x, z) = v_j(s(x, z), r(x, z))$ . Here  $s(x, z), r(x, z)$  is the solution of the system (3)-(4). Let's note, that only one function  $T_j(x, z)$  corresponds to each node of quadrangle  $d_m$ . The function  $T_j(x, z)$  is equal to 1 in node  $j$  and equal to 0 in other nodes. Therefore, only one function  $V_k$  corresponds to each node of the hexahedron and is equal to 1 in this node and equal to 0 in other nodes.

The elements of the local matrix of finite element  $D_m$  depends only on the elements of the local matrix of finite element  $d_m$  and 1D finite element  $(y_1, y_3)$ . Once the calculation of the local matrix of 1D finite element is obvious we consider only the calculation of local matrix of finite element  $d_m$ .

The main difficulty consists in the fact that analytical form of the functions  $T_j(x, z)$  represents a rather awkward expression. Therefore, we will change variables (3)-(4) when building the local matrix. This allows us to pass over from the calculation of integral over quadrangle  $d_m$  to the calculation of integral over square  $E$ , where the functions  $T_j(x, z)$  are presented in a rather simple way.

The elements of local stiffness matrix of finite element  $d_m$  are the following

$$Z_{ij} = \iint_{d_m} \operatorname{grad} T_i \operatorname{grad} T_j dx dz = \int_0^1 \int_0^1 \frac{dT_i}{dx} \frac{dT_j}{dx} |J| ds dr + \int_0^1 \int_0^1 \frac{dT_i}{dz} \frac{dT_j}{dz} |J| ds dr. \quad (5)$$

Here  $J$  is a Jacobian of transformation (3)-(4).

Inasmuch as  $\frac{dT_j}{ds}$  and  $\frac{dT_j}{dr}$  are linear functions of coordinates  $s$  and  $r$ , then for the elements of stiffness matrix the analytical form can be obtained.

However, firstly, these formulas have a rather awkward form and, secondly, they are computational unstable when the quadrangle  $d_m$  verges towards the form of a trapezium. Therefore, the integrals in (5) are better to calculate numerically by the Gauss method. This method makes it possible to avoid the above considered problems.

The elements  $M_{ij} = \iint_{d_m} T_i T_j dx dz$  of the local mass matrix and the elements  $G_{ij}^x = \iint_{d_m} \frac{dT_i}{dx} T_j dx dz$  and  $G_{ij}^z = \iint_{d_m} \frac{dT_i}{dz} T_j dx dz$  needed for approximation of the component  $q \text{grad} V^a$  (see (1)) can be calculated analytically.

V. SOFTWARE

The above described mathematical apparatus was realized in the GEOPREP software which consists of the following main modules.

a. The preprocessor module which is shown in Fig. 2. This module involves the setting of the geoelectrical model which in its turn includes the setting of the horizontally stratified medium parameters (thickness and conductivity of the stratum), parameters of the 3D objects which have a parallelepiped or hexahedral form (conductivity and coordinates of the object) and parameters of the transmitter and receivers. The user can also set an arbitrary form of the impulse of the transmitter loop and point the required time span. The objects can be set and corrected (moved or transformed) by the mouse or by setting the accurate data on the right panel.

b. The calculated modules are the building of the finite element mesh, the assembling of finite element matrixes and decision program. Several levels of accuracy are provided. The characteristics of the first level of accuracy are the following:

- calculation of the EM field of the 3D objects takes about 10-15 minutes; in this case the error is approximately 5% in the total field;
- calculation of the EM field of the axisymmetrical objects takes about 3-5 seconds; in this case the error is approximately 2% in the total field.

The highest level of the accuracy provides the calculations with the error not more than 1%, but in this case the calculation time is about 3-4 hours and about 1 minute for 3D and axisymmetrical objects respectively.

c. The postprocessor module is shown in Fig. 3. The result of the program work represents the time-area emf distribution in the receivers given by the user. This module is intended for the review of the area summary

emf distribution at any instant of time, for the review of the area abnormal emf distribution and for the review of the time-dependence emf in any receiver.

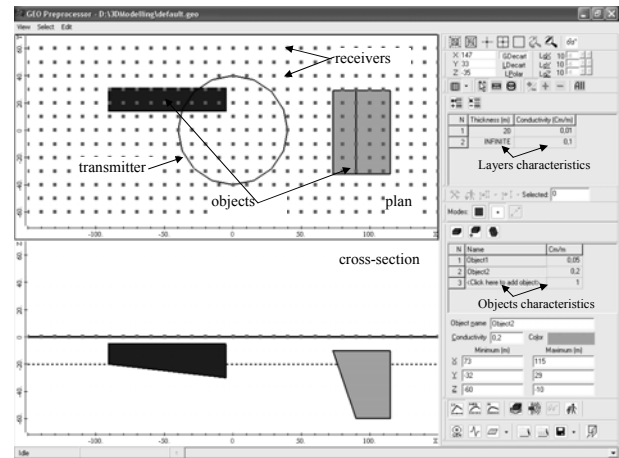


Fig. 2. The window of preprocessor of the software GEOPREP

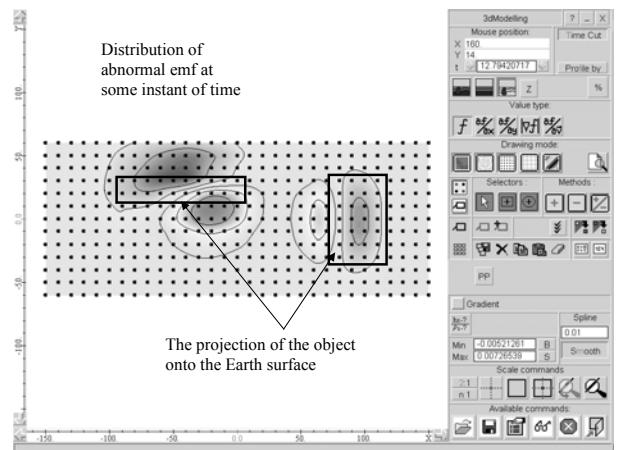


Fig. 3. The window of postprocessor of the software GEOPREP (Distribution of the EMF at some time section)

Thus, the suggested GEOPREP software can be applied for solution of the following problems:

- a) field operation design;
- b) working out of requirements and testing of measuring apparatuses;
- c) data interpretation. The above given characteristics of this complex allow the use of 3D calculations directly in data interpretation process, which, in its turn, consists of the following stages:
  - matching the horizontally stratified medium parameters (1D inversion with parametrization of the geoelectrical cross-section);
  - separating the abnormal field;
  - matching the 3D prospecting objects in the abnormal zones.

VI. EXAMPLE OF CALCULATION OF ELECTROMAGNETIC FIELD IN THE AREA WHICH INCLUDES AN INCLINED 3D OBJECT

Here we have an example of application of the software for calculating the effect of inclined well-conductive object on the electromagnetic field.

The investigated object is placed into a layer with the conductivity equal to 0.01 mho/m and is covered with the poorly-conductive layer whose conductivity is equal to 0.0067 mho/m. The object represents a hexahedron with the square of the upper base equal to 5000m<sup>2</sup> and the square of the low base equal 7000m<sup>2</sup>. The conductivity of the object is 0.1 mho/m, and its thickness is 70m. The view of geoelectrical model is shown in Fig. 4.

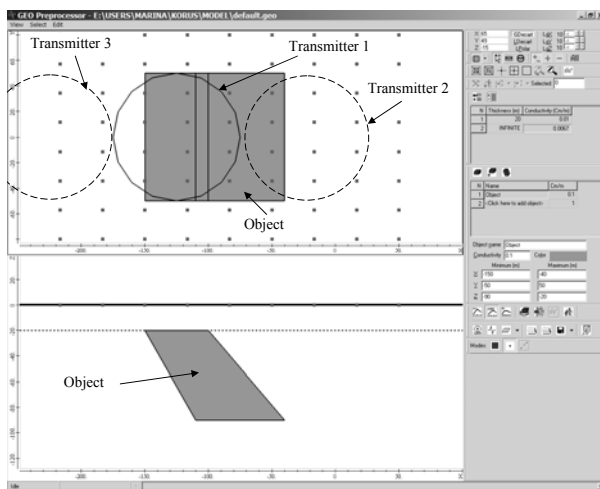


Fig. 4. Geoelectrical model

We show the distribution of the emf on the Earth surface for this model for three different positions of the transmitter loop. In Fig. 5a and 5b one can see the distributions of the emf at the instant of 30 microseconds after switching off the current in the transmitter loop for the area without the object (normal field) and for the area which includes the object (summary field). In this case the centre of the transmitter loop coincides with the projection of the centre of the upper base of the object onto the Earth surface. In Fig. 5 one can see, firstly, that the object intensifies the field, and, secondly, that the distribution of the field is unsymmetric, because the object has an inclined form.

In Fig. 6a and 6b one can see the distributions of the emf in cases when the transmitter loop is placed at the equal distance on the left and on the right from the projection of the centre of the upper base of the object onto the Earth surface. The differences in the distribution of the signal are also explained by the incline of the object.

VI. CONCLUSION

The developed software allows the modeling of transient electromagnetic field in the area which includes 3D inhomogeneities and it can be used during the projecting phase of the electromagnetic investigation as well as at the stage of interpretation.

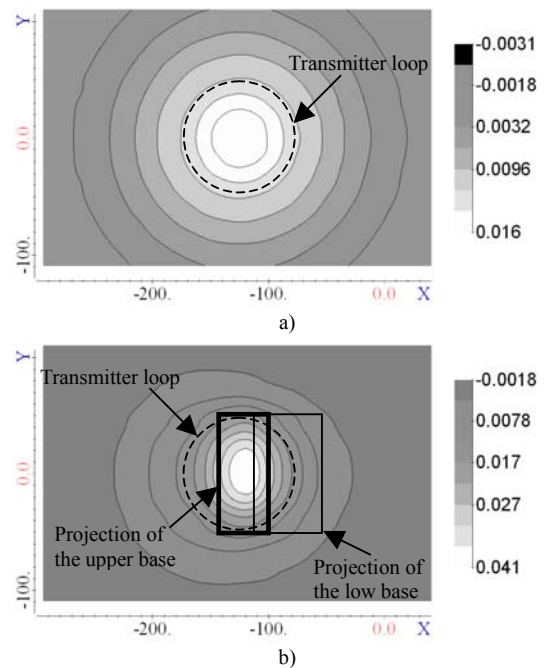


Fig. 5. Emf distribution for the center position of loop:  
a) Emf distribution for the area without object;  
b) Emf distribution for the area which includes object

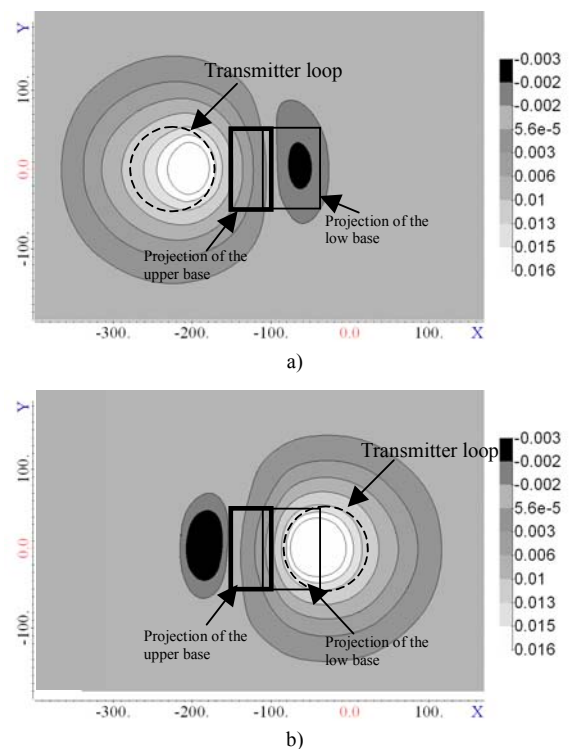


Fig. 6. Emf distribution for right and left position of loop  
a) left position;; b) right position

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