ON THE ONE-DIMENSIONAL CONSOLIDATION BY THREE-DIMENSIONAL DEHYDRATION, WITH SECONDARY COMPRESSION TAKEN INTO CONSIDERATION

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I. INTRODUCTION

Determination of coefficients relating to soil consolidation needs much care in the design and execution of ground improvement works with rapid consolidation effect by three-dimensional dehydration. And there have been many problems in the methods to determine the horizontal and the vertical coefficients of consolidation permeabilty of soil. In most cases it is assumed to make analysis easier that the horizontal coefficient of consolidation permeability and the vertical one are the same. But the assumption is not reasonable from the viewpoint of soil stratification process. It is said that in some cases the horizontal coefficients of permeability of soil related to the consolidation coefficients are several times greater than the vertical ones. So soil may not be regarded simply as isotropic.

On the other hand as to soil consolidation, not only primary consolidation but also secondary compression must be taken into consideration. Especially in case of weak ground, secondary compression plays an important role which cannot be ignored. The author analysed the mechanism of consolidation by three-dimensional dehydration, taking secondary compression into consideration, and investigated the method to determine coefficients of consolidation.

II. DERIVATION OF THE BASIC EQUATION (A model of the consolidation mechanism)

(1) Amount of dehydration

With a small element shown in Fig. 1, i_A and i_B may denote hydraulic gradients at faces A and B, u may denote pore water pressure, and γ_ω unit weight of water. Then one may get,

$$i_A=-\frac{1}{\gamma_\omega}\frac{\partial u}{\partial r}$$

$$i_B=-\frac{1}{\gamma_\omega}\left(\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial r^2}dr\right)$$

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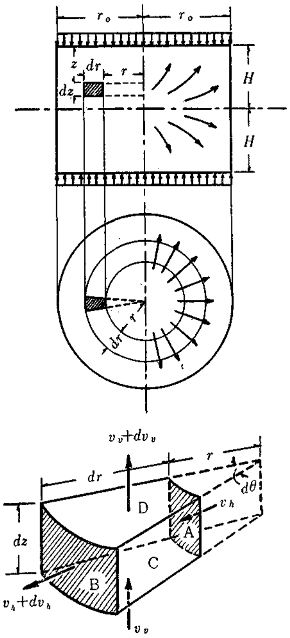

Fig. 1.

Applying the Darcy's law to this element, the amount of water lost per unit time by the horizontal flow is expressed as

$$\varDelta Q_h=k_h i_B B-k_h i_A A$$
$$\fallingdotseq -\frac{k_h}{\gamma_\omega}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)dV$$

where, A : area of the face A,
B : area of the face B,
dV : the volume of the element.

In the same way as before, the rate of water lost by the vertical flow is

$$\varDelta Q_v=k_v i_D D-k_v i_C C=-\frac{k_v}{\gamma_\omega}\frac{\partial^2 u}{\partial z^2}dV$$

where, C : area of the face C,
D : area of the face D.

Then one may get the total amount of dehydration of the element per unit time, as

$$\varDelta Q_h+\varDelta Q_v$$
$$=-\left\{\frac{k_h}{\gamma_\omega}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial u}{\partial r}\right)+\frac{k_v}{\gamma_\omega}\frac{\partial^2 u}{\partial z^2}\right\}dV$$

(2) Amount of deformation

Deformation of soil by consolidation consists of elastic deformation and permanent deformation (plastic deformation). The latter is clarified by Dr. Ishii's Theory. As his original is hardly available now, the author explains the theory and its development here.

At first, (the amount of elastic deformation)/(load) is called the rate of elastic deformation, and is symbolized as v, while permanent deformation of soil particles occurs due to permanent deformation movement of soil particles, and (the amount of plastic deformation)/(load) is called the rate of permanent deformation, symbolized as r.

Then on gets,

$$\varepsilon=\varepsilon_e+\varepsilon_c=(v+r)\bar{p} \quad \cdots\cdots(2.1)$$

where ε is the total amount of consolidation, ε_e is eleastic strain ε_c is permanent strain, and $\bar{p}$ is intensity of load.

When the permanent deformation begins, the friction resistance arises among soil particles, the friction resistance increases according to the increase of friction area as the distance among particles becomes smaller, and it continues until the resistance and the outer force are balanced. Based on this idea, permanent deformation of soil particles is thought to "occur at the rate proportional to the amount of permanent deformation to occur", then

$$\frac{\partial \varepsilon_c}{\partial t}=\eta(r\bar{p}(t)-\varepsilon_c)$$

where η is the creep coefficient.

$$\frac{\partial \varepsilon_c}{\partial t}+\eta\varepsilon_c=\eta\, r\bar{p}(t) \quad \cdots\cdots(2.2)$$

Rearrangement of the equation making use of an operator $\partial/\partial t=p$ gives

$$(p+\eta)\varepsilon_c=\eta\, r\bar{p}(t)$$

As a particular solution of the homogeneous equation obtained by making the right side of the equation (2.2) zero is $\varepsilon_c=e^{-\eta t}$, the general solution of the equation (2.2) is, by Duhamel's rule,

$$\varepsilon_c(t)=r\int_0^t \eta\, e^{-\eta(t-\tau)}\bar{p}(\tau)d\tau$$
$$=r\int_0^t\left[-\frac{\partial}{\partial\tau}(1-e^{-\eta(t-\tau)})\right]\bar{p}(\tau)d\tau$$

where τ is a parameter of time.

So one obtains

$$\varepsilon=v\bar{p}+r\int_0^t\left[-\frac{\partial}{\partial t}(1-e^{-\eta(t-\tau)})\right]\bar{p}(\tau)d\tau$$

Therefore the amount of deformation of the element per unit time, considering both elastic and plastic deformation, is

$$\frac{\partial\varepsilon}{\partial t}\cdot dV=\left[v\frac{\partial\bar{p}}{\partial t}+r\frac{\partial}{\partial t}\times\int_0^t\bar{p}(\tau)\left\{-\frac{\partial}{\partial t}(1-e^{-\eta(t-\tau)})\right\}d\tau\right]dV$$

This is rearranged making use of an operator,

$$\frac{\partial\varepsilon}{\partial t}\cdot dV=\left(v\frac{\partial\bar{p}}{\partial t}+r\frac{\eta p}{p+\eta}\bar{p}\right)dV$$
$$=-\left(vpu+r\frac{\eta p}{p+\eta}u\right)dV$$

With saturated clay the amount of deformation is thought to equal the amount of dehydration. So the basic equation is obtained from the both abovementioned amount by making the one equal the other.

$$\frac{k_h}{r_\omega}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial u}{\partial r}\right)+\frac{k_v}{r_\omega}\frac{\partial^2 u}{\partial z^2}$$
$$=vpu+r\frac{\eta p}{p+\eta}u$$

III. DEHYDRATION ONLY BY THE HORIZONTAL FLOW

The basic equation of dehydration only by the horizontal flow is obtained by making $\partial u/\partial z=0$, on the above equation.

$$\frac{k_h}{r_\omega}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial u}{\partial r}\right)=vpu+r\frac{\eta p}{p+\eta}\cdot u$$

Analysis of this equation gives the equation of u, pore water pressure.

Operating $p+\eta$ in both sides, and rearrangement with p gives

$$vp^2u+\left\{\eta(v+r)u-\frac{k_h}{r_\omega}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)\right\}p$$
$$-\frac{k_h}{r_\omega}\eta\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)=0$$

Assuming that u at any time t is the product of a function only of r, $\psi(r)$, and one only of t, $\phi(t)$, namely $u_r=\psi(r)\cdot\phi(t)$, and writing that $1/\psi(r)\cdot(\psi''(r)+1/r\cdot\psi'(r))=-M^2$,

$$\phi''(t)+\left\{\frac{\eta}{v}(v+r)+C_hM^2\right\}\phi'(t)$$
$$+C_h\,\eta\,M^2\,\phi(t)=0 \quad \cdots\cdots(3.1)$$

$$\psi''(r)+\frac{1}{r}\psi'(r)+M^2\,\psi(r)=0 \quad \cdots\cdots(3.2)$$

in which, $C_h=\dfrac{k_h}{vr_\omega}$

These are two independent equations.

The equation (3.1) is a linear differential equation of the second order and the first degree, and its general solution is

$$\phi(t)=C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t}$$

in which

$$\lambda_1,\lambda_2=\frac{1}{2}\left[-\frac{\eta}{v}(v+r)-\frac{k_h}{vr_\omega}M^2\right.$$
$$\left.\pm\sqrt{\left\{\frac{\eta}{v}(v+r)+\frac{k_h}{vr_\omega}M^2\right\}^2-\frac{4k_h}{vr_\omega}\eta M^2}\right] \quad \cdots\cdots(3.3)$$

The equation (3.2) is Bessel's equation of order 0, and its general solution is

$$\psi(r)=C_3J_0(Mr)+C_4Y_0(Mr)$$

where $J_0(Mr)$ is the Bessel function of first kind of order 0 and $Y_0(Mr)$ is the Bessel function of second kind of order 0. Since $u=\phi(t)\psi(r)$, the general solution of the basic equation is

$$u_r=\{C_1e^{\lambda_1t}+C_2e^{\lambda_2t}\}\{C_3J_0(Mr)+C_4Y_0(Mr)\}$$

Next, the following boundary conditions are introduced to this general solution.

(1) $0<t<\infty$, at $r=0$ $\frac{\partial u}{\partial r}=0$

The differentiated value of u_r with respect r is

$$\frac{\partial u_r}{\partial r}=-M\{C_1e^{\lambda_1t}+C_2e^{\lambda_2t}\}\times\{C_3J_1(Mr)+C_4Y_1(Mr)\}$$

where $J_1(Mr)$ is the Bessel function of first kind of order 1, and $Y_1(Mr)$ is that of second kind of order 1.

Since $J_1(0)=0$ and $Y_1(0)=-\infty$ One gets, $C_4=0$.

Therefore

$$u_r=C_3J_0(Mr)(C_1e^{\lambda_1t}+C_2e^{\lambda_2t})$$

(2) $0\leqslant t\leqslant\infty$, at $r=r_0$ $u_r=0$

where, r_0 is the radius of the consolidated cylindrical soil section.

This gives $J_0(Mr_0)=0$

Letting m_i be the zero point of the Bessel function of first kind of order 0, $Mr_0=m_i$ $M=\frac{m_i}{r_0}$

Therefore

$$u_r=\sum_{i=1}^{\infty}J_0\left(\frac{m_i}{r_0}r\right)(B_1e^{\lambda_1t}+B_2e^{\lambda_2t})\quad\cdots\cdots\cdots(3.4)$$

where, $B_1=C_1C_3$, $B_2=C_2C_3$

(3) $0\leqslant r\leqslant r_0$ at $t=0$ $u_r=K$

where, K is the intensity of load.

This gives

$$K=\sum_{i=1}^{\infty}(B_1+B_2)J_0\left(\frac{m_i}{r_0}r\right)$$

Multiplication of both sides by $J_0(m_i/r_0\cdot r)$ and integration gives

$$B_1+B_2=\frac{2K}{m_iJ_1(m_i)}\quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(3.5)$$

(4) at $t=0$, $\varepsilon_c=0$

where, ε_c is the permanent strain and is expressed by the following equation.

$$\varepsilon_c(t)=r\int_0^t\left[-\frac{\partial}{\partial\tau}(1-e^{\eta(t-\tau)})\right]\bar{p}(\tau)d\tau$$
$$=r\eta\int_0^te^{-\eta(t-\tau)}\bar{p}(\tau)d\tau$$

Substitution of $\bar{p}=K-u_r$ in the above equation gives

$$\varepsilon_c(t)=r\eta\int_0^te^{-\eta(t-\tau)}\left\{\Sigma\frac{2K}{m_iJ_1(m_i)}J_0\left(\frac{m_i}{r_0}r\right)-\Sigma J_0\left(\frac{m_i}{r_0}r\right)(B_1e^{\lambda_1t}+B_2e^{\lambda_2t})\right\}d\tau$$

The coefficient of $J_0(m_i/r_0\cdot r)$ is

$$r\eta\int_0^te^{-\eta(t-\tau)}\left\{\frac{2K}{m_iJ_1(m_i)}-(B_1e^{\lambda_1t}+B_2e^{\lambda_2t})\right\}d\tau$$

The values at $t=0$ are zero, and can be omitted, and applying the condition that $\varepsilon_c=0$ at $t=0$,

$$\frac{2K}{m_iJ_1(m_i)}=\eta\left(\frac{B_1}{\lambda_1+\eta}+\frac{B_2}{\lambda_2+\eta}\right)\cdots\cdots\cdots\cdots(3.6)$$

Next, B_1 and B_2 are calculated from the equation (3.5) and (3.6) as

$$B_1=\frac{2K}{m_iJ_1(m_i)}\cdot\frac{\lambda_2(\lambda_1+\eta)}{\eta(\lambda_2-\lambda_1)}$$

$$B_2=\frac{-2K}{m_iJ_1(m_i)}\cdot\frac{\lambda_1(\lambda_2+\eta)}{\eta(\lambda_2-\lambda_1)}$$

Introducing these to the equation (3.4),

$$u_r=\sum_{i=1}^{\infty}\frac{2K}{m_iJ_1(m_i)}J_0\left(\frac{m_i}{r_0}r\right)\times\left[\frac{1}{\eta(\lambda_2-\lambda_1)}\{\lambda_2(\lambda_1+\eta)e^{\lambda_1t}-\lambda_1(\lambda_2+\eta)e^{\lambda_2t}\}\right]$$

IV. DEHYDRATION ONLY BY THE VERTICAL FLOW

The basic equation of dehydration only by the vertical flow is obtained by making $\partial u/\partial r=0$, namely

$$\frac{k_v}{r_\omega}\cdot\frac{\partial^2u}{\partial z^2}=vpu+r\frac{\eta p}{p+\eta}\cdot u$$

Operating $(p+\eta)$ in both sides, transposition and rearrangement with respect to p give

$$vp^2u+\left\{\eta(v+r)u-\frac{k_v}{r_\omega}\cdot\frac{\partial^2u}{\partial z^2}\right\}p-\frac{k_v}{r_\omega}\eta\frac{\partial^2u}{\partial z^2}=0$$

Assuming that pore water pressure u at any time t is the product of a function only z, $\psi(z)$, and a function only of t, $\phi(t)$, and making $1/\psi(z)\cdot\psi''(z)=-N^2$,

$$v\phi''(t)+\left\{\eta(v+r)+\frac{k_v}{r_\omega}N^2\right\}\phi'(t)+\frac{k_v}{r_\omega}\eta N^2\phi(t)=0\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(4.1)$$

$$\psi''(z)+N^2\psi(z)=0\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(4.2)$$

Here are obtained two independent equations.

The equation (4.1) is the linear differential equation of second order and first degree, and its general solution is

$$\phi(t)=C_1e^{\lambda_1't}+C_2e^{\lambda_2't}$$

where,

$$\lambda_1',\lambda_2'=\frac{1}{2}\left[-\frac{\eta}{v}(v+r)-\frac{k_v}{vr_\omega}N^2\pm\sqrt{\left\{\frac{\eta}{v}(v+r)+\frac{k_v}{vr_\omega}N^2\right\}^2-\frac{4k_v}{vr_\omega}\eta N^2}\right]\cdots\cdots\cdots\cdots\cdots(4.3)$$

The equation (4.2) is the linear differential equation of second order and first degree, and its general solution is

$$\psi(z)=C_3\cos Nz+C_4\sin Nz$$

Since $u=\psi(z)\phi(t)$, the general solution of the basic equation is

$$u_z=(C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t})(C_3\cos Nz+C_4\sin Nz)$$

To this general solution, the following boundary conditions are introduced.

(1) $0\leqq t\leqq\infty$ at $z=0$, $u=0$

This condition means $C_3=0$

Therefore

$$u_z=C_4\sin Nz\,(C_1\,e^{\lambda_1' t}+C_2\,e^{\lambda_2' t})$$

(2) $0\leqq t\leqq\infty$ at $z=2H$, $u=0$

where, $2H$ is the thickness of the consolidated layer. From this condition, $\sin 2NH=0$

Then $2NH==m\pi$ $(m=1,2,3\cdots)$ namely, $N=m\pi/2H$

$$u_z=\sum_{m=1}^{\infty}(B_1e^{\lambda_1' t}+B_2e^{\lambda_2' t})\sin\frac{m\pi}{2H}z\quad\cdots\cdots(4.4)$$

(3) $0\leqq z\leqq 2H$, at $t=0$, $u_z=K$

where, K is the intensity of load.

Therefore

$$K=\sum_{m=1}^{\infty}(B_1+B_2)\sin\frac{m\pi}{2H}z$$

Multiplication of the both sides by $\sin\cdot m\pi/2H\cdot z$ and integration give

$$B_1+B_2=\frac{2K}{m\pi}(1-\cos m\pi)=\frac{4K}{n\pi}\cdots\cdots\cdots(4.5)$$

where, $n=1,3,5\cdots$

(4) at $t=0$, $\varepsilon_c=0$

where, ε_c is the permanent strain and is expressed by the following equation.

$$\varepsilon_c(t)=r\eta\int_0^t e^{-\eta(t-\tau)}\bar{p}(\tau)d\tau$$

Substitution of $\bar{p}=K-u_z$ in the above equation gives

$$\varepsilon_c(t)=r\eta\int_0^t e^{-\eta(t-\tau)}\left\{\Sigma\frac{4K}{n\pi}\sin\frac{n\pi}{2H}z-\Sigma(B_1e^{\lambda_1' t}+B_2e^{\lambda_2' t})\sin\frac{n\pi}{2H}z\right\}d\tau$$

The coefficient of $\sin\frac{n\pi}{2H}z$ is

$$r\eta\int_0^t e^{-\eta(t-\tau)}\left\{\frac{4K}{n\pi}-(B_1e^{\lambda_1' t}+B_2e^{\lambda_2' t})\right\}d\tau$$

The values at $t=0$ are zero, and can be omitted, and applying the condition that $\varepsilon_c=0$ at $t=0$

$$\frac{4K}{n\pi}=\eta\left(\frac{B_1}{\lambda_1'+\eta}+\frac{B_2}{\lambda_2'+\eta}\right)\cdots\cdots\cdots\cdots\cdots\cdots(4.6)$$

Now B_1 and B_2 are calculated from the equation (4.5) and (4.6) as

$$B_1=\frac{4K}{n\pi}\cdot\frac{(\lambda_1'+\eta)\lambda_2'}{(\lambda_2'-\lambda_1')\eta}$$

$$B_2=-\frac{4K}{n\pi}\cdot\frac{(\lambda_2'+\eta)\lambda_1'}{(\lambda_2'-\lambda_1')\eta}$$

Introducing these to the equation (4.4),

$$u_z=\sum_{n=1,3,5}^{\infty}\frac{4K}{n\pi}\left[\frac{1}{\eta(\lambda_2'-\lambda_1')}\times\{\lambda_2'(\lambda_1'+\eta)e^{\lambda_1' t}-\lambda_1'(\lambda_2'+\eta)e^{\lambda_2' t}\}\right]\times\sin\left(\frac{n\pi}{2H}\cdot z\right)$$

V. THREE-DIMENSIONAL DEHYDRATION

As shown in II, the basic equation in three-dimensional dehydration by the synthetic flow of the vertical and horizontal flows is

$$\frac{k_h}{r_w}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial u}{\partial r}\right)+\frac{k_v}{r_w}\cdot\frac{\partial^2 u}{\partial z^2}=vpu+r\frac{\eta p}{p+\eta}u$$

This equation can be solved by analyzing the dehydration to that only by the vertical flow and that only by the horizontal flow. When external load is borne by the synthetic flow of the vertical and horizontal flows, pore water pressure is, as shown by Carrillo's, expressed as follows.

$$u=K\,u_r'\,u_z'$$

where, K is external load, u_r' is the value of u_r when $K=1$, and u_z' is the value of u_z when $K=1$. Furthermore, letting $\bar{u}_r$, and $\bar{u}_z$ be the mean value of them with respect to r or z. The mean value of $\bar{u}$ with respect to r and z, respectively, there is a relation

$$\bar{u}=K\bar{u}_r\bar{u}_z$$

Next, $\bar{u}_r$ and $\bar{u}_z$ are calculated.

$$\bar{u}_r=\frac{2}{r_0^2}\int_0^{r_0}u_r'rdr=\sum_{i=1}^{\infty}\frac{4}{m_i^2}\left[\frac{1}{\eta(\lambda_2-\lambda_1)}\times\{\lambda_2(\lambda_1+\eta)e^{\lambda_1 t}-\lambda_1(\lambda_2+\eta)e^{\lambda_2 t}\}\right]$$

$$\bar{u}_z=\frac{1}{2H}\int_0^{2H}u_z'dz=\sum_{i=1,3,5}^{\infty}\frac{8}{(n\pi)^2}\left[\frac{1}{\eta(\lambda_2'-\lambda_1')}\times\{\lambda_2'(\lambda_1'+\eta)e^{\lambda_1' t}-\lambda_1'(\lambda_2'+\eta)e^{\lambda_2' t}\}\right]$$

Then the mean pore water pressure by the synthetic flow, $\bar{u}$, is obtained as follows.

$$\begin{aligned}\bar{u}&=K\bar{u}_r\cdot\bar{u}_z\\&=K\,\Sigma\frac{4}{m_i^2}\cdot\Sigma\frac{8}{(n\pi)^2}\left[\frac{1}{\eta^2(\lambda_2-\lambda_1)(\lambda_2'-\lambda_1')}\right.\\&\quad\times\{\lambda_2\lambda_2'(\lambda_1+\eta)(\lambda_1'+\eta)e^{(\lambda_1+\lambda_1')t}\\&\quad-\lambda_2\lambda_1'(\lambda_1+\eta)(\lambda_2'+\eta)e^{(\lambda_1+\lambda_2')t}\\&\quad-\lambda_1\lambda_2'(\lambda_2+\eta)(\lambda_1'+\eta)e^{(\lambda_2+\lambda_1')t}\\&\quad\left.+\lambda_1\lambda_1'(\lambda_2+\eta)(\lambda_2'+\eta)e^{(\lambda_2+\lambda_2')t}\}\right]\end{aligned}$$

VI. THE AMOUNT OF CONSOLIDATION SETTLEMENT

The amount of consolidation settlement S by the synthetic flow at time t is

$$S=\int_0^{2H}\varepsilon\,dz=\int_0^{2H}(\bar{p}v+\varepsilon_c)dz$$
$$=\int_0^{2H}dz\left[Kv+r\,\eta\int_0^t Ke^{-\eta(t-\tau)}d\tau\right]$$
$$-\int_0^{2H}dz\left[\bar{u}v+r\,\eta\int_0^t \bar{u}e^{-\eta(t-\tau)}d\tau\right] \quad \cdots\cdots\cdots\cdots\cdots(6.1)$$

Then terms in the equation are calculated.

$$r\,\eta\int_0^t Ke^{-(t-\tau)}d\tau=r\,\eta\,Ke^{-\eta t}\left|\frac{e^{\eta\tau}}{\eta}\right|_0^t=r\,K$$

$$r\,\eta\int_0^t \bar{u}e^{-\eta(t-\tau)}d\tau$$
$$=re^{-\eta t}\int_0^t \eta\,\bar{u}e^{\eta\tau}d\tau$$
$$=rK\,\Sigma\frac{4}{m_i^2}\,\Sigma\frac{8}{(n\pi)^2}\left[\frac{1}{\eta(\lambda_2-\lambda_1)(\lambda_2'-\lambda_1')}\right.$$
$$\times\left\{\frac{\lambda_2\lambda_2'(\lambda_1+\eta)(\lambda_1'+\eta)}{\lambda_1+\lambda_1'+\eta}e^{(\lambda_1+\lambda_1')t}\right.$$
$$-\frac{\lambda_2\lambda_1'(\lambda_1+\eta)(\lambda_2'+\eta)}{\lambda_1+\lambda_2'+\eta}e^{(\lambda_1+\lambda_2')t}$$
$$-\frac{\lambda_1\lambda_2'(\lambda_2+\eta)(\lambda_1'+\eta)}{\lambda_2+\lambda_1'+\eta}e^{(\lambda_2+\lambda_1')t}$$
$$\left.\left.+\frac{\lambda_1\lambda_1'(\lambda_2+\eta)(\lambda_2'+\eta)}{\lambda_2+\lambda_2'+\eta}e^{(\lambda_2+\lambda_2')t}\right\}\right]\cdots(6.2)$$

$$\bar{u}v=vK\,\Sigma\frac{4}{m_i^2}\,\Sigma\frac{8}{(n\pi)^2}\left[\frac{1}{\eta^2(\lambda_2-\lambda_1)(\lambda_2'-\lambda_1')}\right.$$
$$\times\{\lambda_2\lambda_2'(\lambda_1+\eta)(\lambda_1'+\eta)e^{(\lambda_1+\lambda_1')t}$$
$$-\lambda_2\lambda_1'(\lambda_1+\eta)(\lambda_2'+\eta)e^{(\lambda_1+\lambda_2')t}$$
$$-\lambda_1\lambda_2'(\lambda_2+\eta)(\lambda_1'+\eta)e^{(\lambda_2+\lambda_1')t}$$
$$\left.+\lambda_1\lambda_1'(\lambda_2+\eta)(\lambda_2'+\eta)e^{(\lambda_2+\lambda_2')t}\}\right]\cdots\cdots(6.3)$$

Applying the binomial theorem to the term of $\sqrt{\ }$ in the equation (3.3)

$$\lambda_1=-\frac{C_h\,\eta\left(\frac{m_i}{r_0}\right)^2}{\frac{v+r}{v}\eta+C_h\left(\frac{m_i}{r_0}\right)^2}-\frac{\left[C_h\,\eta\left(\frac{m_i}{r_0}\right)^2\right]^2}{\left[\frac{v+r}{v}\eta+C_h\left(\frac{m_i}{r_0}\right)^2\right]^3}-\cdots\cdots$$

$$\lambda_2=-\frac{v+r}{v}\eta-C_h\left(\frac{m_i}{r_0}\right)^2+\frac{C_h\,\eta\left(\frac{m_i}{r_0}\right)^2}{\frac{v+r}{\nu}\eta+C_h\left(\frac{m_i}{r_0}\right)^2}+\frac{\left[C_h\,\eta\left(\frac{m_i}{r_0}\right)^2\right]^2}{\left[\frac{v+r}{v}\eta+C_h\left(\frac{m_i}{r_0}\right)^2\right]^3}+\cdots\cdots$$

where, $C_h=\frac{k_h}{v\gamma_\omega}$, $M=\frac{m_i}{r_0}$

Now investigation is made with the case where the creep coefficient η is very small compared with the coefficient of permeability and creep continues very long. Namely,

$$\eta\ll C_h\left(\frac{m_i}{r_0}\right)^2$$

Then

$$\lambda_1\doteqdot-\eta-\frac{\eta^2}{C_h\left(\frac{m_i}{r_0}\right)^2}$$
$$\lambda_2\doteqdot-C_h\left(\frac{m_i}{r_0}\right)^2$$

In the same way,

$$\eta\ll C_v\left(\frac{n\pi}{2H}\right)^2$$

Then

$$\lambda_1'=-\eta-\frac{\eta^2}{C_v\left(\frac{n\pi}{2H}\right)^2}$$
$$\lambda_2'=-C_v\left(\frac{n\pi}{2H}\right)^2$$

Applying these results to coefficients in the equation (6.3),

$$\bar{u}.v\doteqdot vK\sum_{i=1}^{\infty}\frac{4}{m_i^2}e^{-C_h\left(\frac{m_i}{r_0}\right)^2t}\times\sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2}e^{-C_v\left(\frac{n\pi}{2H}\right)^2t}$$

Then as to coefficients in the equation (6.2),

$$r\eta\int_0^t\bar{u}e^{-\eta(t-\tau)}d\tau\doteqdot rKe^{-2\eta t}$$

Then the amount of settlement S in this case is calculated according to the equation (6.1).

$$S=2\,HK(v+r)-2\,HKv\sum_{i=1}^{\infty}\frac{4}{m_i^2}e^{-C_h\left(\frac{m_i}{r_0}\right)^2t}\times\sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2}e^{-C_v\left(\frac{n\pi}{2H}\right)^2t}-2\,HKre^{-2\eta t}$$

And the final amount of settlement S_∞ due to consolidation is calculated by making $t\to\infty$.

$$S_\infty=2\,HK(v+r)$$

Degree of consolidation in this case is

$$U=\frac{S}{S_\infty}=\frac{v}{v+r}\left(1-\sum_{i=1}^{\infty}\frac{4}{m_i^2}e^{-C_h\left(\frac{m_i}{r_0}\right)^2t}\times\sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2}e^{-C_v\left(\frac{n\pi}{2H}\right)^2t}\right)+\frac{r}{v+r}(1-e^{-2\eta t})$$

where,

$$U_\nu=\frac{v}{v+r}\left(1-\sum_{i=1}^{\infty}\frac{4}{m_i^2}e^{-C_h\left(\frac{m_i}{r_0}\right)^2t}\times\sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2}e^{-C_v\left(\frac{n\pi}{2H}\right)^2t}\right)$$

means the primary consolidation rate, and

$$U_r=\frac{r}{v+r}(1-e^{-2\eta t})$$

means the secondary compression rate.

Next, investigation is made with the case where the creep coefficient is very large compared with the coefficient of permeability and creep is finished in a very short time.

Namely,

$$\eta\gg C_h\left(\frac{m_i}{r_0}\right)^2$$

Then

$$\lambda_1 \fallingdotseq -\frac{v}{v+r} C_h\left(\frac{m_i}{r_0}\right)^2$$

$$\lambda_2 \fallingdotseq -\frac{v+r}{v}\eta$$

In the same way,

$$\eta \gg C_v\left(\frac{n\pi}{2H}\right)^2$$

$$\lambda_1' \fallingdotseq -\frac{v}{v+r} C_v\left(\frac{n\pi}{2H}\right)^2$$

$$\lambda_2' \fallingdotseq -\frac{v+r}{v}\eta$$

Applying these results to coefficients in the equation (6.3),

$$\bar{u}v = vK\sum_{i=1}^{\infty}\frac{4}{m_i^2}e^{-\frac{v}{v+r}C_h\left(\frac{m_i}{r_0}\right)^2 t} \times \sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2}e^{-\frac{v}{v+r}C_v\left(\frac{n\pi}{2H}\right)^2 t}$$

Then as to coefficients in the equation (6.2),

$$r\eta\int_0^t \bar{u}e^{-\eta(t-\tau)}d\tau \fallingdotseq rK\sum_{i=1}^{\infty}\frac{4}{m_i^2}e^{-\frac{v}{v+r}C_h\left(\frac{m_i}{r_0}\right)^2 t} \times \sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2}e^{-\frac{v}{v+r}C_v\left(\frac{n\pi}{2H}\right)^2 t}$$

Then the amount of settlement S is, according to the equation (6.1),

$$S = 2HK(v+r) - 2HK(v+r)\sum_{i=1}^{\infty}\frac{4}{m_i^2}e^{-\frac{v}{v+r}C_h\left(\frac{m_i}{r_0}\right)^2 t} \times \sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2}e^{-\frac{r}{v+r}C_v\left(\frac{n\pi}{2H}\right)^2 t}$$

Degree of consolidation U in this case is

$$U = 1 - \sum_{i=1}^{\infty}\frac{4}{m_i^2}e^{-\frac{v}{v+r}C_h\left(\frac{m_i}{r_0}\right)^2 t} \times \sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2}e^{-\frac{r}{v+v}C_v\left(\frac{n\pi}{2H}\right)^2 t}$$

This equation means that the delay of creep is negligible in the case where η is so large that the permanent deformation occurs in a moment, and so it becomes the same equation of degree of consolidation as in the case where only the primary consolidation is taken into consideration.

VII. DETERMINATION OF THE CONSOLIDATION CONSTANTS

In the previous section the equation to calculate the degree of consolidation, with the secondary compression and creep taken into consideration, was obtained.

As the equation shows, both the primary consolidation part, namely the elastic deformation part due to consolidation, and the secondary compression part, namely the plastic deformation part due to permanent deformation movement of soil particles, occur at the beginning of consolidation, constituting the amount of consolidation. However, at the early stage of consolidation the amount of consolidation is mainly attributed to the primary consolidation. Therefore, in the U-log t curve obtained by the consolidation test, the primary consolidation rate U_v is considered to mainly form the curve at the early stage of consolidation, and at the stage near the finish of the primary consolidation, the secondary compression largely appears in the curve.

Thus determination of the consolidation constants, such as elastic deformation rate v plastic deformation rate r, consolidation coefficients C_v and C_h, and creep coefficient η, etc. is possible by analyzing the U-log t curve.

The primary consolidation rate is

$$U_v = \frac{v}{v+r}\left(1 - \sum_{i=1}^{\infty}\frac{4}{m_i^2}e^{-m_i^2\alpha T_v} \times \sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2}e^{-\left(\frac{n\pi}{2}\right)^2 T_v}\right)$$

where $T_h = \frac{C_h}{r_0^2}\cdot t$ and $T_v = \frac{C_v}{H^2}\cdot t$, which are the time factor for the horizontal flow and for the vertical flow, respectively and $\alpha = \frac{T_h}{T_v} = \frac{C_h}{C_v}\left(\frac{H}{r_0}\right)^2$.

In this way the primary consolidation rate by the synthetic flow is expressed by a function only of T_v.

Furthermore, as to the secondary compression rate,

$$U_r = \frac{r}{v+r}(1 - e^{-T'})$$

where, $T' = 2\eta t$, which is the time factor with respect to creep. Therefore the secondary compression rate can be expressed by a function only of T'

Now the U_v-log T_v curve and the U_r-log T' curve drawn under the conditions,

$$\frac{v}{v+r} = 0.025,\ 0.050,\ 0.075, \cdots\cdots 1.0$$

$$n = 1,\ 3,\ 5, \cdots\cdots$$

$$T_v = 0.001,\ 0.002,\ 0.003, \cdots\cdots 5.0$$

$$\alpha = 0$$

and

$$\frac{r}{v+r} = 0.025,\ 0.050,\ 0.075, \cdots\cdots 1.0$$

$$T' = 0.001,\ 0.002,\ 0.003, \cdots\cdots 10$$

are shown as follows.

(1) Determination of $\frac{v}{v+r}$ and $\frac{r}{v+r}$

$\frac{v}{v+r}$ and $\frac{r}{v+r}$, coefficient values in the U_v-log T_v and U_r-log T' curves, can be determined on the basis of the U_v-log T_v and U_r-log T' curves at $\alpha = 0$.

Now explanation is made with the method of using these figures. First, in order to determine the constants of consolidation by three-dimensional dehydration, the U-log t curve for dehydration only by the vertical flow using the manufactured three-dimensional dehydration consolidometer is obtained.

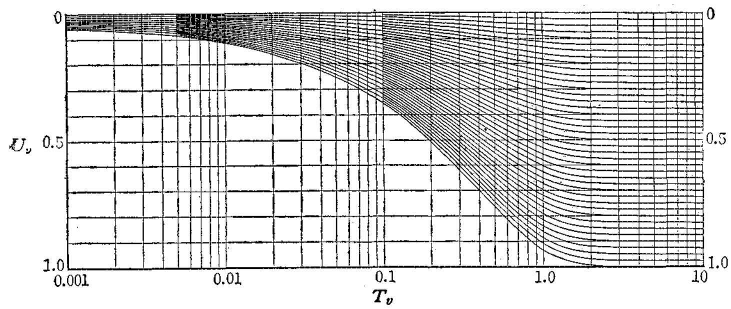

Fig. 2 The U_v-log T_v ruled curve for determination of the primary consolidation constants.

In Expression, $U_v=\frac{v}{v+\gamma}\left(1-\sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2}e^{-\left(\frac{n\pi}{2}\right)^2 T_v}\right)$

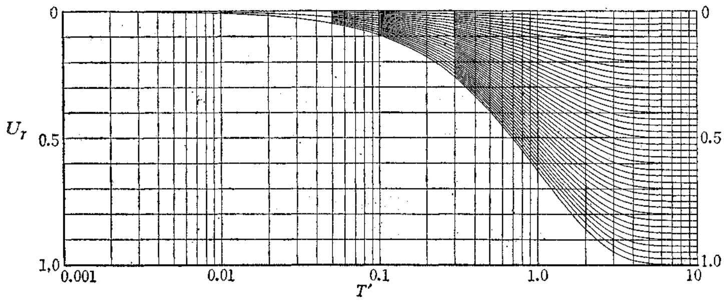

Fig. 3 The U_γ-log T' ruled curve for determination of the secondary compression constants.

In Expression, $U_\gamma=\frac{\gamma}{v+\gamma}(1-e^{-T'})$

As the U-log t curve comprises the U_v and U_γ parts, the curve must be analyzed into the U_v and U_γ parts, by using the figures.

The U-log t curve is drawn on a sheet of tracing paper according to the same scale as in the figure of the U_v-log T_v ($\alpha=0$) curve (semi-log scale), which sheet is put on the figure of U_v-log T_v ($\alpha=0$) curve.

The sheet of tracing paper is moved right and left with such care that the t-axis of the sheet is in accordance with the T_v-axis of the figure of the U_v-log T_v ($\alpha=0$) curve until the U_v-log T_v ($\alpha=0$) curve that fits the curve on the sheet well is found in the figure. Then the value of $\frac{v}{v+\gamma}$ corresponding to the very U_v-log T_v ($\alpha=0$) curve gives the the ratio of the elastic deformation rate to the total

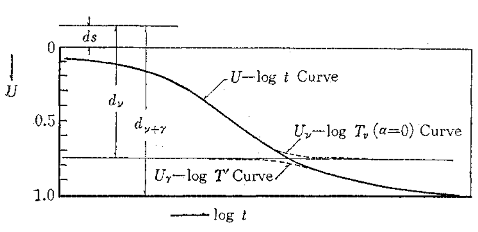

Fig. 4 Analytical sketch for U-log t curve

deformation rate of soil. Furthermore, using the above procedure, v and γ are obtained as follows.

In the Fig. 4, to let d_s, d_v and $d_{v+\gamma}$ be the amount of settlement at the moment of loading, the amount of settlement until the finish of the primary consolidation, and the amount of settlement until the finish of the total consolidation, respectively, then the amount of strain due to the primary consolidation part ε_v and the amount of strain due to the secondary compression part ε_γ are

$$\varepsilon_v=\frac{d_v-d_s}{z}$$

$$\varepsilon_\gamma=\frac{d_{v+\gamma}-d_v}{z}$$

Since $\varepsilon_v=v.\Delta p$ and $\varepsilon_\gamma=\gamma\Delta p$, one obtains

$$v=\frac{d_v-d_s}{z\Delta p}$$

$$\gamma=\frac{d_{v+\gamma}-d_v}{z\Delta p}$$

where z is the thickness of the layer immediately after loading, and Δp is increase of load.

Next, in order to determine the value of α, the figure of the U_v-log T_v curve are drawn.

The conditions are

$\frac{v}{v+\gamma}=1$

$n=1,\ 3,\ 5\cdots\cdots$

$m_i=2.405,\ 5.520,\ 8.654,\ 11.792,\ 14.931\cdots\cdots$

(where m_i is the value satisfying the equation $J_0(m_i)=0$ obtained by the boundary condition (2) in the section Ⅲ, which is the zero point of the Bessel function of first kind of order 0)

$\alpha=0,\ 1,\ 2,\ 3\cdots\cdots$

$T_v=0.001,\ 0.002,\ 0.003\cdots\cdots5.0$

(2) Determination of C_v and C_h

C_v and C_h can be determined by making use of the figure of the U_v-log T_v curve ($\alpha=0$, 1, 2, ……, 5).

Now the method of using the figure and determining the values is explained. The U-log t curve is obtained for dehydration only by the vertical flow using the three-dimensional dehydration consolidometer, which curve is put on the figure of the U_v-log T_v curve $\left(\frac{v}{v+\gamma}=1\right)$ and fitted it the curve at $\alpha=0$. Then the ratio of any value of t on the

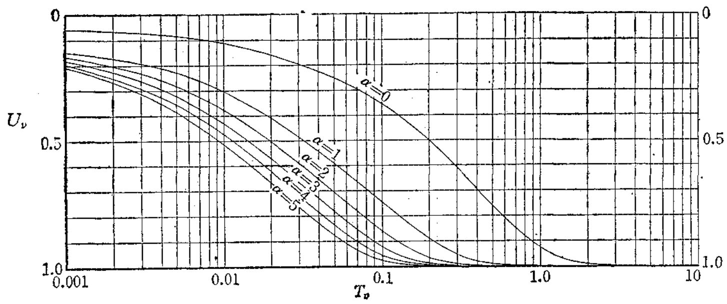

Fig. 5 The U_v-log T_v Curve for determination of α

$$U_v=\frac{v}{v+r}\left(1-\sum_{i=1}^{\infty}\frac{4}{m_i^2}e^{-m_i^2\alpha T_v}\cdot\sum_{i=1,3,5}^{\infty}\frac{8}{(n\pi)^2}e^{-\left(\frac{n\pi}{2}\right)^2T_v}\right)$$

sheet of tracing paper to the value of T_v under it in the figure of the U_v-log T_v curve $\left(\frac{v}{v+r}=1\right)$ is always constant.

Namely, letting t_1, and t_2 be the values of t on the sheet of tracing paper which fit to the values of T_{v_1} and T_{v_2} in the figure of the U_v-log T_v curve $\left(\frac{v}{v+r}=1\right)$, respectively, one obtains

$$\log T_{v_1}-\log T_{v_2}=\log t_1-\log t_2$$

Therefore,

$$\frac{T_{v_1}}{t_1}=\frac{T_{v_2}}{t_2}=\frac{T_v}{t}$$

From this value of $\left(\frac{T_v}{t}\right)$, the coefficient of vertical consolidation C_v is obtained.

$$C_v=\frac{T_v}{t}H^2$$

(H is a half of the thickness of the sample forthe three-dimensional dehydration consolidometer)

Furthermore in case of using the consolidometer generally used at present (the thickness of the sample ($2S=2$ cm)) to determine C_v,

$$C_v=\left(\frac{T_v}{t}\right)'S^2=\frac{T_v}{t}H^2$$

$$\frac{T_v}{t}=\left(\frac{T_v}{t}\right)'\left(\frac{S}{H}\right)^2$$

So the value of T_v/t when the thickness of the sample is $2H$ must previously be obtained.

Then the primary consolidation part of the U-

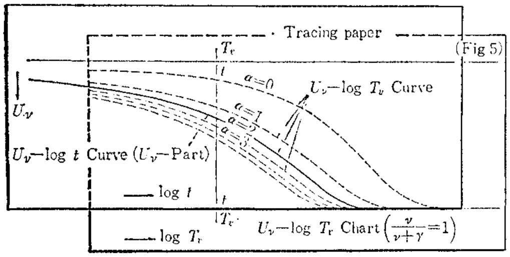

Fig. 6 A figure explaining how to analyze the constant α.

log t curve for three-dimensional dehydration by using the three-dimensional dehydration consolidometer is drawn on a sheet of tracing paper as previously described accoriding to the same scale as in the figure of the U_v-log T_v curve $\left(\frac{v}{v+r}=1\right)$, which sheet is put on the figure of the U_v-log T_v curve.

Then t-axis on the sheet of tracing paper is adjusted to T_v-axis in the figure of the U_v-log T_v curve so that the values of t and T_v corresponding to each other may have the same ratio as the (T_v/t) value obtained by the method previously described. And the curve just fitting to the U-log t curve on the sheet of tracing paper is found on the figure of the U_v-log T_v curve $\left(\frac{v}{v+r}=1\right)$. Then the value of α of the very curve relates the coefficient of vertical consolidation C_v to the coefficient of horizontal consolidation C_h as follows.

$$\alpha=\left(\frac{H}{r_0}\right)^2\frac{C_h}{C_v}$$

Therefore,

$$C_h=\alpha\left(\frac{r_0}{H}\right)^2C_v$$

or

$$C_h=\alpha\left(\frac{T_v}{t}\right)r_0^2$$

Thus the coefficient of horizontal consolidation of soil is obtained.

On the other hand, the secondary compression part is expressed

$$U_r=\frac{r}{v+r}(1-e^{-2\eta t})=\frac{r}{v+r}(1-e^{-T'})$$

$\frac{r}{v+r}$ is the proportion of the amount of secondary compression to the amount of total deformation, and remains constant if the sample and load are fixed. With the U_r-log T' curve, $\frac{r}{v+r}$ shows the point where the curve intersects the vertical axis, and determines the position with respect to the horizontal axis, and the shape of the curves in the figure are substantially the same.

Next, the method of determination of η using the figure of the U_r-log T' curve is described.

The U-log t curve (on the sheet of tracing paper) previously drawn for determination of the values of consolidation constants concerning the U_v part is put on the figure of the U_v-log T' curve, and the sheet is moved right and left with such care that the line dividing the U-log t curve into the U_v part

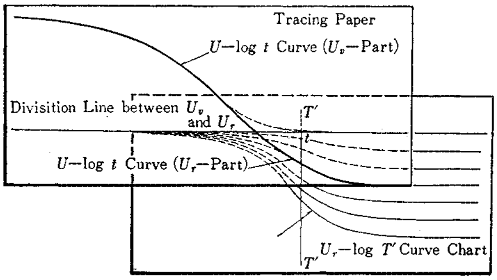

Fig. 7 A figure explaining how to analyze the constant η.

and the U_r part is in accordance with the T'-axis of the figure of the U_r-log T' curve, until the U_r-log T' curve just fitting the U-log t curve (the U_r part) is found in the figure.

Then from the value of T' of any point in the figure of the U_r-log T' curve and the value of t of the point on the U-log t curve corresponding to the above-described point, η is calculated as follows.

$$\eta=\frac{T'}{2t}$$

By the method above described, the consolidation constants can be determined, in the case of three-demensional dehydration of soil, with creep taken into consideration.

VIII. AN APPLICATION EXAMPLE

On the analysis of the constants by the consolidation test of three-dimensional dehydration.

In this paragraph the analytical example of the consolidation constants of the soil is explained, which was by the method of the analysis of the consolidation due to three-dimensional dehydration.

1) The synopsis of the testing method

The weak soil, collected from central zone of the reclamation land of the Lake Hachiro H-1 spot 0.8~1.59 m in depth, was used as a sample. At the experiment the fixed ring type consolidometer (a sample, $2H=2$ cm thick, $2r_0=6$ cm in diameter), which is generally used to determine the values of v, r, C_v and η of the soil, was used to make the consolidation test due to dehydration of only the verticl direction.

The sample was put to the consolidation test on the four stages of loading, ie 0.1, 0.2, 0.4, 0.8 kg/cm^2 after it was fully saturated with water.

Next, in order to determine C_h by the consolidation test due to three-dimensional dehydration, the sample, fully saturated with water, was tested on the three stages of loading, ie 0.1, 0.2 and 0.4 kg/cm^2, using the consolidometer due to three-dimensional dehydration which was produced by way of the experiment as its apparatus.

Furthermore, the author examined the applicability of the consolidation coefficients due to three-dimensional dehydration, including the creep coefficient in this case.

(2) The analysis of the consolidation test result

i) The value of v and r

At first we suppose the amount of final settlement including the secondary compression to be 1.0, by the consolidation data of the case due to dehydration of only the vertical direction.

And we write U-log t curve on a tracing paper graduated on the same scale as the graph of U_v-log T_v ruled curve. (Fig. 2 cf.)

We devide it into two parts of the primary consolidation and the secondary compression by overlapping U-log t curve on U_v-log T_v ruled curve.

Thus the values of $\frac{v}{v+r}$ and $\frac{r}{v+r}$ have been determined.

Using these values and the next equation, we can obtain the values of v and r on this stage of loading. (Fig. 4 cf.)

$$v=\frac{d_v-d_s}{\Delta p(2H)}=\frac{d_{v+r}-d_s}{\Delta p(2H)}\left(\frac{v}{v+r}\right)$$

$$r=\frac{d_{v+r}-d_v}{\Delta p(2H)}=\frac{d_{v+r}-d_s}{\Delta p(2H)}\left(\frac{r}{v+r}\right)$$

In Fig. 8 is illustrated U-log t curve obtained from the result of the consolidation test on this sample.

The calculation of v and r is given in Table 1.

The changes of $\frac{v}{v+r}$, $\frac{r}{v+r}$, v and r towards

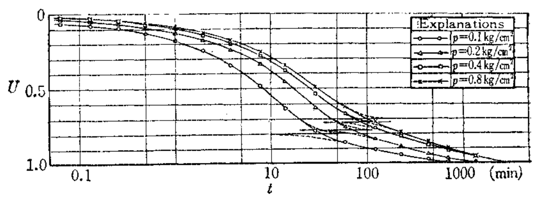

Fig. 8 U-log t curves for one-dimensional dehydration

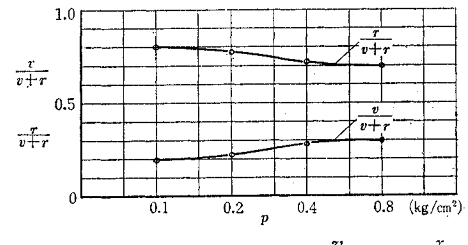

Fig. 9 The variation-curves of $\frac{v}{v+r}$ and $\frac{r}{v+r}$ by load strength per unit area

Table 1. The calculation table for v and γ.

p (kg/cm²)	Δp (kg/cm²)	$2H$ (cm)	$d_{v+\gamma}$ (1/100 mm)	d_s (1/100 mm)	$\frac{v}{v+\gamma}$	$\frac{\gamma}{v+\gamma}$	v (cm²/kg)	γ (cm²/kg)
0.1	0.1	2.000	141.0	13.0	0.800	0.200	5.12×10^{-1}	1.28×10^{-1}
0.2	0.1	1.859	125.0	1.7	0.775	0.225	5.14×10^{-1}	1.49×10^{-1}
0.4	0.2	1.734	187.0	1.5	0.725	0.275	3.88×10^{-1}	1.47×10^{-1}
0.8	0.4	1.547	226.0	2.4	0.700	0.300	2.53×10^{-1}	1.08×10^{-1}

Table 2. The calculation table for C_v

p (kg/cm²)	$2H_n$ Thickness of a sample just after loading (cm)	$\overline{H}=H_n+H_{n+1}$ (cm)	$\overline{H}^2$	$\left(\frac{T_v}{t}\right)$ (1/min)	C_v (cm²/min)
0.1	2.000	0.965	0.931	0.0350	3.26×10^{-2}
0.2	1.859	0.898	0.806	0.0180	1.45×10^{-2}
0.4	1.734	0.820	0.672	0.0128	8.60×10^{-3}
0.8	1.547	0.717	0.514	0.0128	6.58×10^{-3}

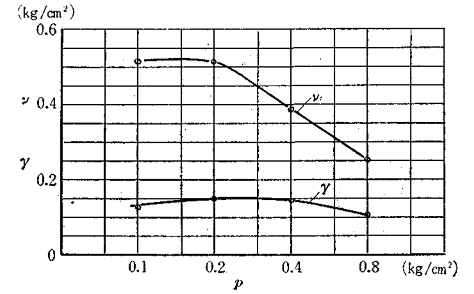

Fig. 10 The variation-curves of v and γ by load strength per unit area

load strength per unit area p are illustrated in Fig. 9 and Fig. 10.

ii) The determination of the value of C_v

We overlap the primary consolidation part of U-log t curve due to dehydration of only the vertical direction on U_v-log T_v ruled curve, and we find out the value of T_v on U_v-log T_v ruled curve which is located under spontaneous t of U-log t curve. We can obtain the value of C_v from the next equation, using the values of t and T_v.

$$C_v=\frac{T_v}{t}\cdot(\overline{H})^2$$

where $\overline{H}$ is a half of the average height of the material corresponding to each stage of load strength per unit area of consolidation.

Table 2 presents a calculation table.

iii) The determination of the value of C_h

Now we write U-log t curve including secondary compression, by using the ruled scale of the consolidation and divide it into two parts of primary consolidation and secondary compression by the method mentioned in the paragraph i).

In Fig. 11 is illustrated the curve.

We write U_v-log t curve about this primary consolidation part divided and we can obtain the value of C_h from the next equation when we obtained the value of α in Fig.-α (Fig. 5) from the time-difference between this curve and U_v-log t curve due to dehydration of only the vertical direction.

$$C_h=\alpha\left(\frac{r_0}{\overline{H}}\right)^2\cdot C_v$$

where, $\overline{H}$ is a half of the average height of the sample corresponding to each stage of load strength per unit area of consolidation and r_0 is a half of a diameter of the sample with $r_0=3$ cm.

Next, the author explains the method of obtaining the value of α in the equation.

At first, as the consolidation sample due to dehydration of only the vertical direction is $2H=2$ cm thick, U_v-log t curve has to be compared with the one corrected to $2H'=6$ cm thick which is the same as the consolidation sample due to three-dimensional dehydration.

Now, the value of $\left(\frac{T_v}{t}\right)'$ in U_v-log t curve in the case of $2H'=6$ cm is,

in the case of $p=0.1$ kg/cm²

$$\left(\frac{T_v}{t}\right)'=\left(\frac{T_v}{t}\right)\left(\frac{2H}{2H'}\right)^2$$

$$=0.0350\times\left(\frac{2.0}{6.0}\right)^2=0.0039 \text{ 1/min.}$$

similarly,

in the case of $p=0.2$ kg/cm²

$$\left(\frac{T_v}{t}\right)'=0.0180\times\left(\frac{2.0}{6.0}\right)^2=0.0020 \text{ 1/min.}$$

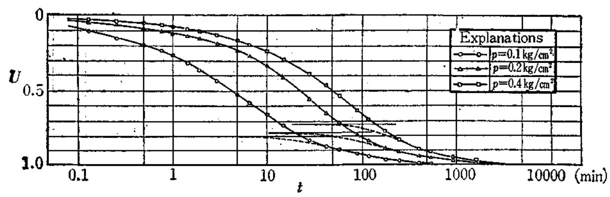

Fig 11 U-log t curves for three-dimensional dehydration

Table 3 The calculation table for C_h

p (kg/cm²)	C_v (cm²/min)	$2H_n$ Thickness of a sample just after loading (cm)	$\bar{H}=H_n+H_{n+1}$ (cm)	$\frac{r_0}{\bar{H}}$	$\left(\frac{r_0}{\bar{H}}\right)^2$	α	C_h (cm²/min)
0.1	3.26×10^{-2}	6.00	2.895	1.036	1.073	4.5	1.57×10^{-1}
0.2	1.45×10^{-2}	5.58	2.695	1.113	1.239	2.0	3.59×10^{-2}
0.4	8.60×10^{-3}	5.20	2.460	1.219	1.486	1.0	1.28×10^{-2}
0.8	6.58×10^{-3}	4.64	—	—	—	—	—

in the case of $p=0.4\,\text{kg/cm}^2$

$$\left(\frac{T_v}{t}\right)'=0.0128\times\left(\frac{2.0}{6.0}\right)^2=0.0014 \quad 1/\text{min}.$$

Thus we can obtain U_v-log t curve due to dehydration of only the vertical direction in the case of $2H'=6$ cm, sliding off in time-dimension U_v-log t curve in the case of $2H=2$ cm so as to obtain the value of $\left(\frac{T_v}{t}\right)'$ above mentioned.

Next we write this curve and U_v-log t curve due to three-dimensional dehydration in the same figure. It is illustrated in Fig. 12.

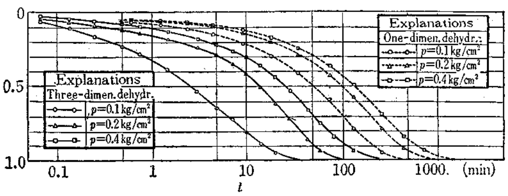

Fig. 12 U_v-log t curves (the case of $2H=6$ cm)
Notes : The above U_v−log t curves (dotted lines) show converted values of $2H=2$ cm to $2H=6$ cm, and the U_v−log t curves (full lines) are experimental curves in $2H=6$ cm.

Now we put the figure on Fig.-α and overlap U_v-log t curve due to the primary consolidation on the curve of $\alpha=0$ in Fig.-α and find out the value of α on the curve in Fig.-α on which U_v-log t curve due to three-dimensional dehydration overlaps in the figure.

We can obtain the value of C_h using the value of α as is shown in Table 3.

iv) The determination of the value of η.

At first, overlapping the secondary compression part of U-log t curve, obtained from the consolidation test due to dehydration of only the vertical direction, on U_γ-log T' ruled curve, we find out the value of T' on U_γ-log T' ruled curve located under spontaneous t on U-log t curve. By using these values, we can obtain the value of η from the equation $\eta=\frac{T'}{t}$.

The values of η in each stage of load strength per unit area are given in Table 4.

By applying the same way to the secondary compression part of U-log t curve obtained from the consolidation test due to three-dimensional dehydration, the value of η can be obtained from the equation $\eta=\frac{1}{2}\left(\frac{T'}{t}\right)$. The result is shown in Table 5.

Table 4 The calculation table for η
(the case of one-dimensional dehydration)

p (kg/cm²)	t (min)	T'	η (1/min)
0.1	100	0.56	5.6×10^{-3}
0.2	300	0.87	2.9×10^{-3}
0.4	500	0.85	1.7×10^{-3}
0.8	500	0.75	1.5×10^{-3}

($2H=2$ cm)

Table 5 The calculation table for η
(the case of three-dimensional dehydration)

p (kg/cm²)	t (min)	T'	η (1/min)
0.1	100	1.00	5.0×10^{-3}
0.2	200	0.84	2.1×10^{-3}
0.4	500	1.25	1.3×10^{-3}

($2H=6$ cm)

The value of η, obtained from the consolidation test due to three-dimensional dehydration, was a little smaller than that of the test due to dehydration of only the vertical direction.

This seems to arise from an experimental error but the two values are fairly coincident with each other.

In this report the author has explained the result of the consolidation test due to the one-dimensional and three-dimensional dehydration on the weak soil of central zone of the reclamation land of the Lake Hachiro and presented a simple way to get these consolidation constants and has shown these values.

The values of C_h and η, obtained from the result of the consolidation test due to three-dimensional dehydration, are fairly coincident with the values of the usual supposition.

Thus the author has presented a way to gain the precise consolidation constants by trying not only the consolidation test due to the one-dimensional dehydration which is usually adapted but also that due to three-dimensional dehydration.

IX. CONCLUSION

Here are reported analysis of consolidation in three-dimensional dehydration with the secondary compression taken into consideration, the method of determination of consolidation constants with this case, and further its simplified graphic solutions and their applications on the actual subjects.

In determination of the horizontal consolidation-permeability coefficient, which has long been a problem in many cases, the author treated the flow as a radial synthetic flow of a vertical and a horizontal one, well grounded on the actual state, making use of a newly manufactured three-dimensional dehydration consolidometer designed by one's idea.

Furthermore the author described the mechanism of consolidation with the secondary compression comprised, the influence of which is remarkable generally in the weak ground.

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