Section 3.5
Recovery Systems: Parachutes 101

Material taken from: Parachutes for Planetary Entry Systems
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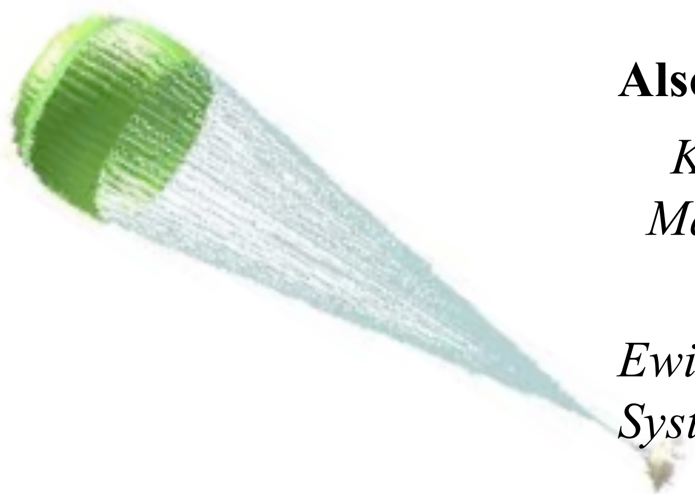
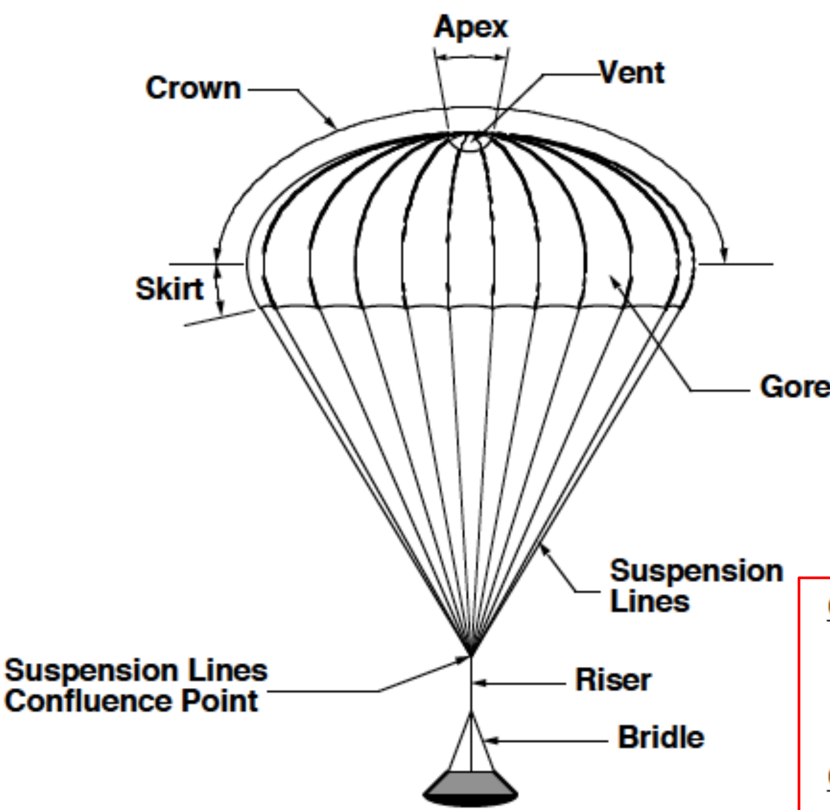


Also, Images from:

Knacke, T. W.: Parachute Recovery Systems Design Manual, Para Publishing, Santa Barbara, CA, 1992.
and

Ewing, E. G., Bixby, H.W., and Knacke, T.W.: Recovery System Design Guide, AFFDL-TR-78-151, 1978.

Basic Terminology



<u>Nominal Area</u>, S_0

- Area based on canopy constructed surface area
- Includes vent area and other open areas (e.g., gap area in a DGB parachute)
- Often (but not always!) used as reference area for aerodynamic coefficients

<u>Nominal Diameter</u>, D_0

- Fictitious diameter based on S_0:

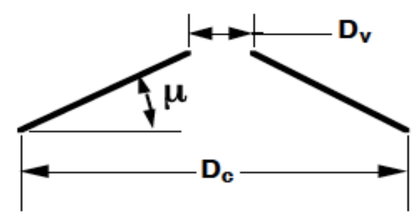
$$D_0 = \sqrt{\frac{4S_0}{\pi}}$$

- Often (but not always!) used as reference length for aerodynamic coefficients and other calculations

<u>Constructed Diameter</u>, D_c

- Maximum diameter of the parachute (measured along the gore radial seam) of the parachute

<u>Conical Parachute Base Angle</u>, μ



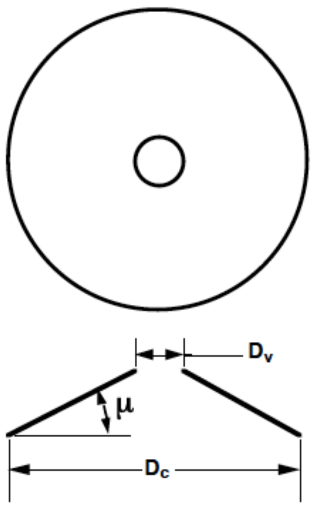
<u>Vent Diameter</u>, D_v

<u>Vent Area</u>, S_v

- Constructed area of the vent
- Although related, the vent area and vent diameter (D_v) are not always related by the simple relationship between the area and diameter of a circle (see following example for a conical parachute)
- S_v is typically ~1% of S_0

Basic Terminology (2)

Example: Conical Parachute



$$S_0 = \lambda \frac{D_c^2}{4}\sqrt{1+\tan^2\mu}$$

$$D_0 = \sqrt{\frac{4S_0}{\lambda}}$$

$$S_v = \lambda \frac{D_v^2}{4}\sqrt{1+\tan^2\mu}$$

$$\lambda_g = \frac{S_v}{S_0}$$

For our purposed conical and elliptical parachutes are same thing"

Projected Area, S_p

- Projected area of the inflated parachute
- Sometimes used as reference area for aerodynamic coefficients in parachutes for which it is difficult to define S_0 (e.g., Guide Surface parachutes)

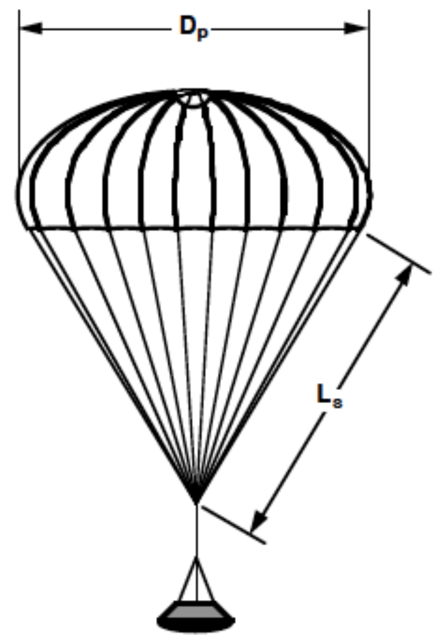
Projected Diameter, D_p

- Maximum projected diameter of the parachute based on S_p:

$$D_P = \sqrt{\frac{4S_P}{\pi}}$$

Suspension Line Length, L_s

- Typically L_s/D_0 = 1 to 2

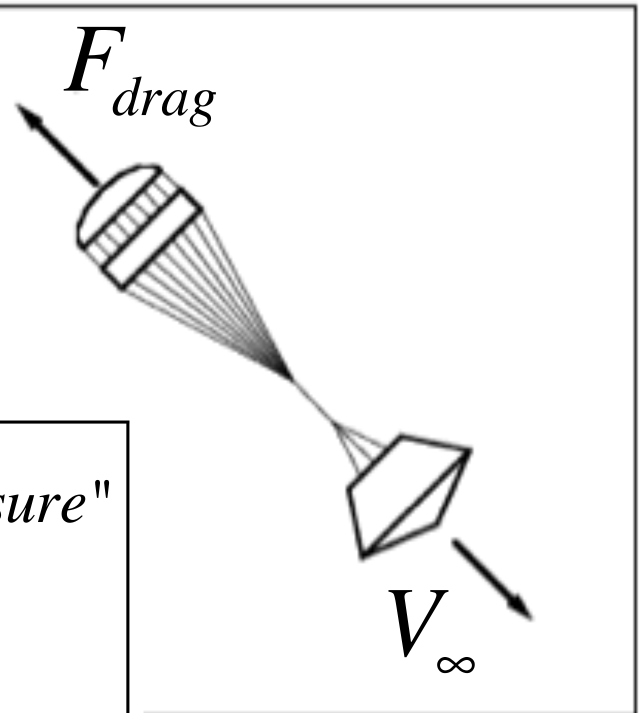


Basic Terminology (3)

Drag

Drag - Force parallel to the free-stream velocity,

Assuming quasi steady-state conditions (e.g., parachute is fully inflated) the parachute drag force F_P can be calculated from:



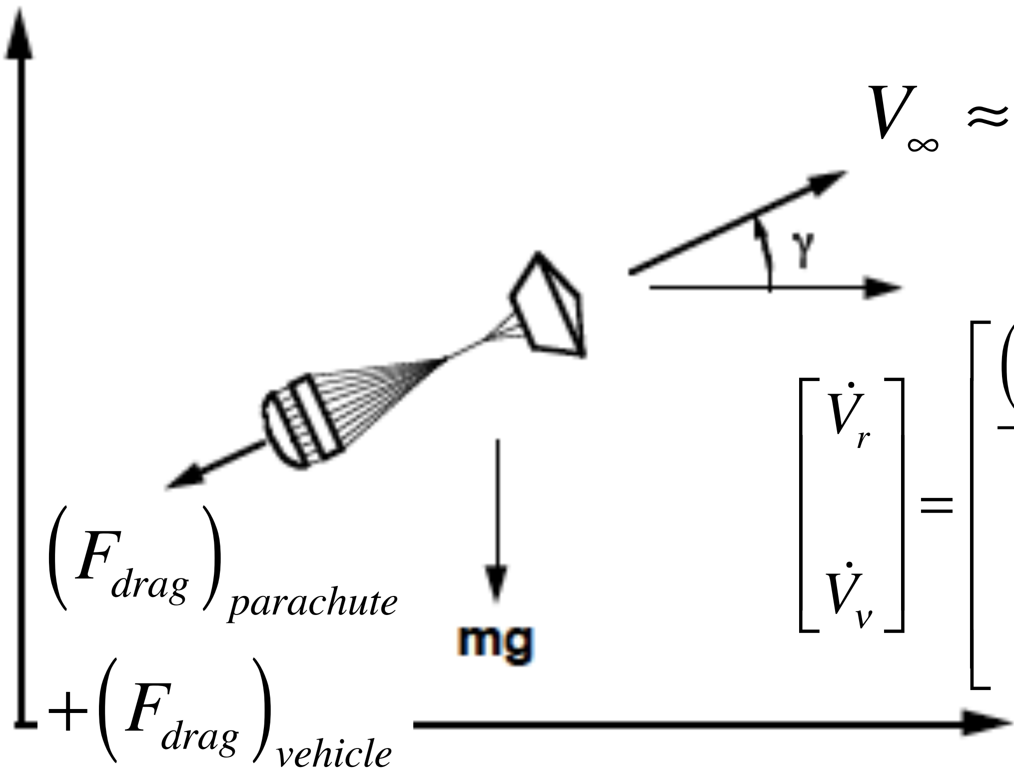
$$F_{drag} = \bar{q} \cdot C_D \cdot S_0$$

$$\rightarrow \begin{array}{l} \bar{q} = \frac{1}{2}\rho \cdot V_\infty^{\ 2} \ \text{"incompressible dynamic pressure"} \\ C_D = \text{"drag coefficient"} \\ C_D \cdot S_0 = \text{"drag area"} \end{array}$$

Basic Terminology (4)

In general, under parachute, 2-DOF equations of motion are ….
(ignore centrifugal & Coriolis forces)

$$V_\infty \approx \sqrt{V_r^2 + V_v^2}$$



$$\begin{bmatrix} \dot{V}_r \\ \dot{V}_v \end{bmatrix} = \begin{bmatrix} \dfrac{\left(F_{drag}\right)_{parachute} + \left(F_{drag}\right)_{vehicle}}{m} \cdot \sin\gamma - g \\ -\dfrac{\left(F_{drag}\right)_{parachute} + \left(F_{drag}\right)_{vehicle}}{m} \cdot \cos\gamma \end{bmatrix}$$

Vehicle decelerates very rapidly in horizontal direction

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Basic Terminology (4)

"Terminal Velocity" .. Equilibrium velocity where parachute + vehicle are no longer accelerating

$$\begin{bmatrix} \dot{V}_r \\ \dot{V}_v \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \gamma \approx 90^\circ \\ \left(F_{drag}\right)_{parachute} + \left(F_{drag}\right)_{vehicle} = m \cdot g \end{bmatrix}$$

$$\rightarrow D_0^{parachute} = 58.0589_{cm}$$

$$\left(\frac{1}{2}\rho \cdot V^2_{\text{terminal}}\right) \cdot \left[\left(C_D \cdot S\right)_0^{parachute} + \left(C_D \cdot S\right)_{ref}^{vehicle}\right] = M_{vehicle} \cdot g$$

$$V^2_{\text{terminal}} = \frac{2M_{vehicle} \cdot g / \rho}{\left(C_D \cdot S\right)_0^{parachute} + \left(C_D \cdot S\right)_{ref}^{vehicle}}$$

Parachute Types

Parachute Types

Solid Textile Parachutes

- Parachutes with canopies fabricated mainly from cloth materials
- Typically these types of parachutes have no openings other than the vent
- Relatively easy to manufacture

We'll be using solid parachutes

Guide Surface Parachute

Slotted Textile Parachutes

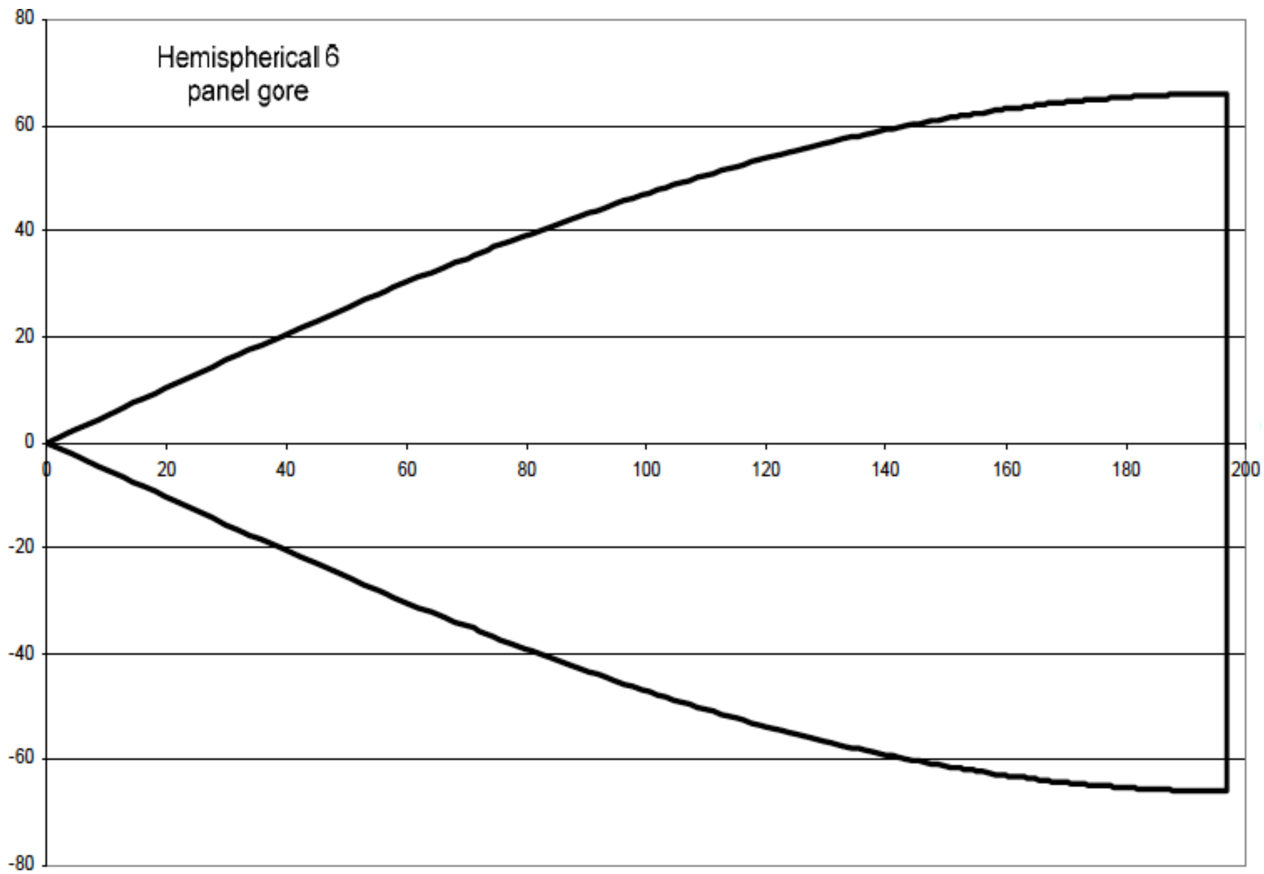
- Parachutes with canopies fabricated from either cloth materials or ribbons
- These types of parachutes have extensive openings through the canopy in addition to the vent
- Can be expensive to manufacture
- Most common parachute type used in planetary exploration missions

Galileo Ribbon Parachute

MER DGB Parachute

Parachute Shapes

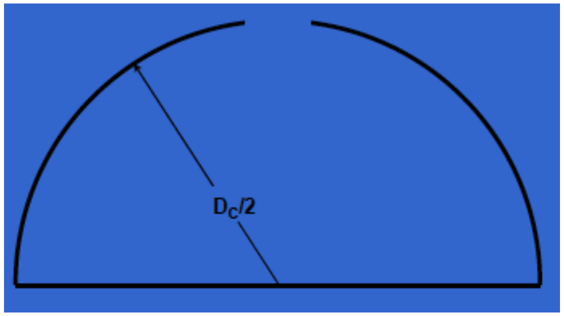
- **Hemispherical parachute:**
  - Deployed canopy takes on the shape of a hemisphere.
  - Three dimensional hemispherical shape divided into a number of 2-D panels, called gores



Gore pattern for 6 gore 252mm dia hemispherical parachute

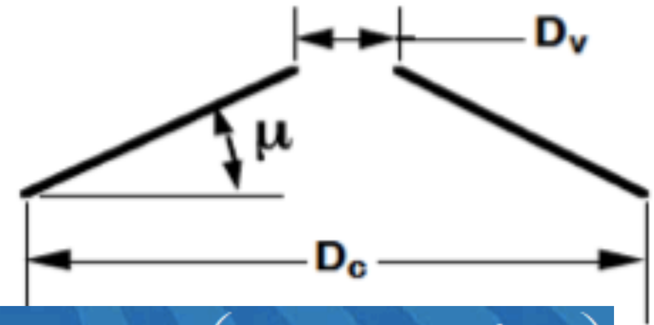
- Angle subtended on the left hand side of the pattern is 60 degrees

- When all six gores are joined they complete the 360 degree circle.



Parachute Shapes (2)

- **Conical Parachute**
  - 2-D Canopy shape in form of a triangle



- For conical parachutes $D_C = D_O\sqrt{\cos\mu}$ $\left(\mu = \text{cone}\ \frac{1}{2}\angle\right)$

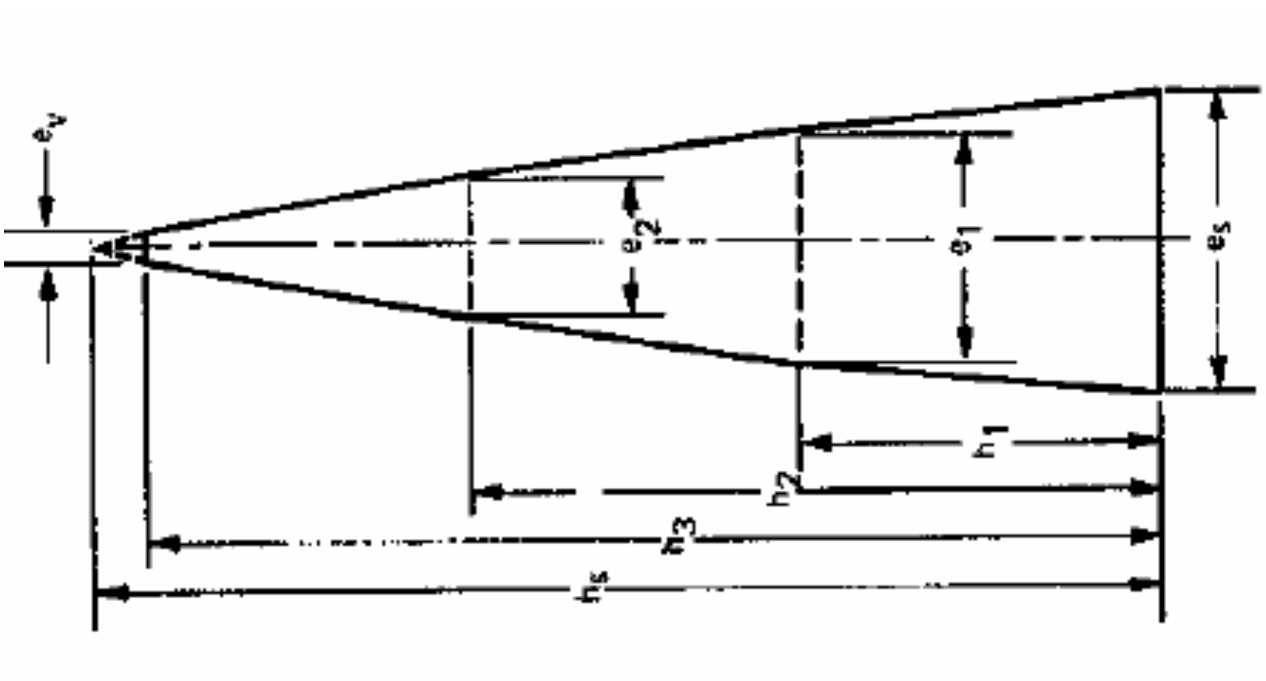
- For 10° conical parachute:
  - $D_O = 1.008\ D_C$

- For 20° conical parachute:
  - $D_O = 1.03\ D_C$

- For 30° conical parachute:
  - $D_O = 1.07\ D_C$

Parachute Shapes (3)

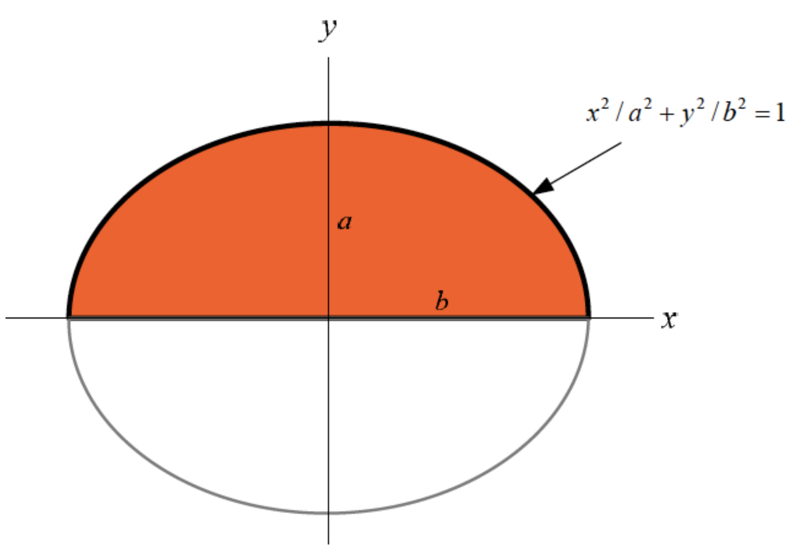
- **Conical Parachute Gore Shape**
  - 2-D Canopy shape in form of a triangle

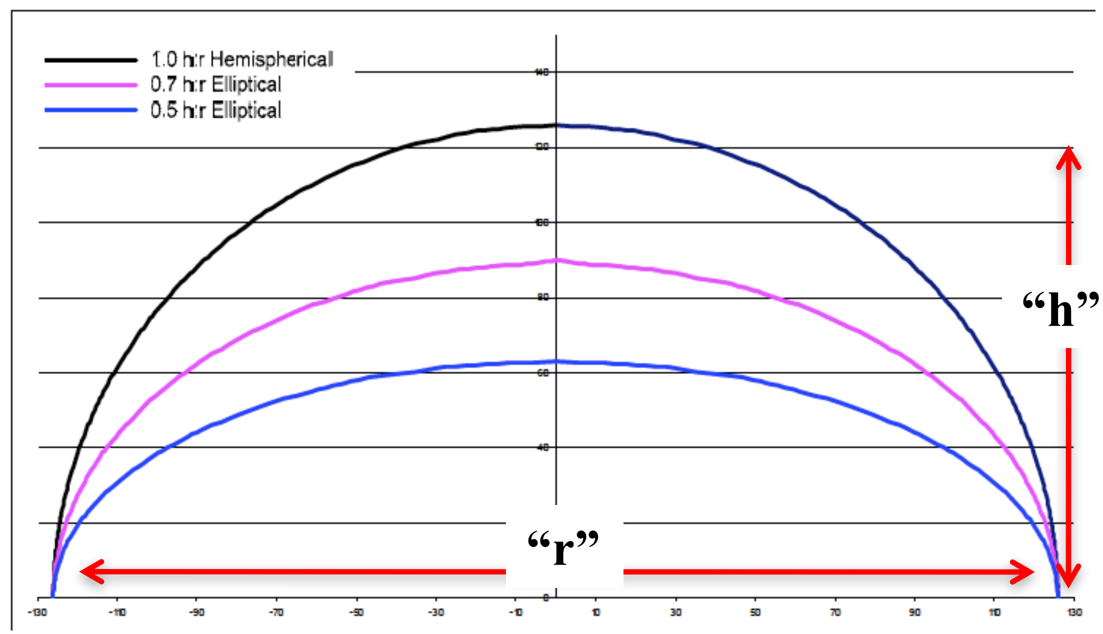


- ***Higher drag coefficient than hemispherical parachutes, but also less stability***

Parachute Shapes (4)

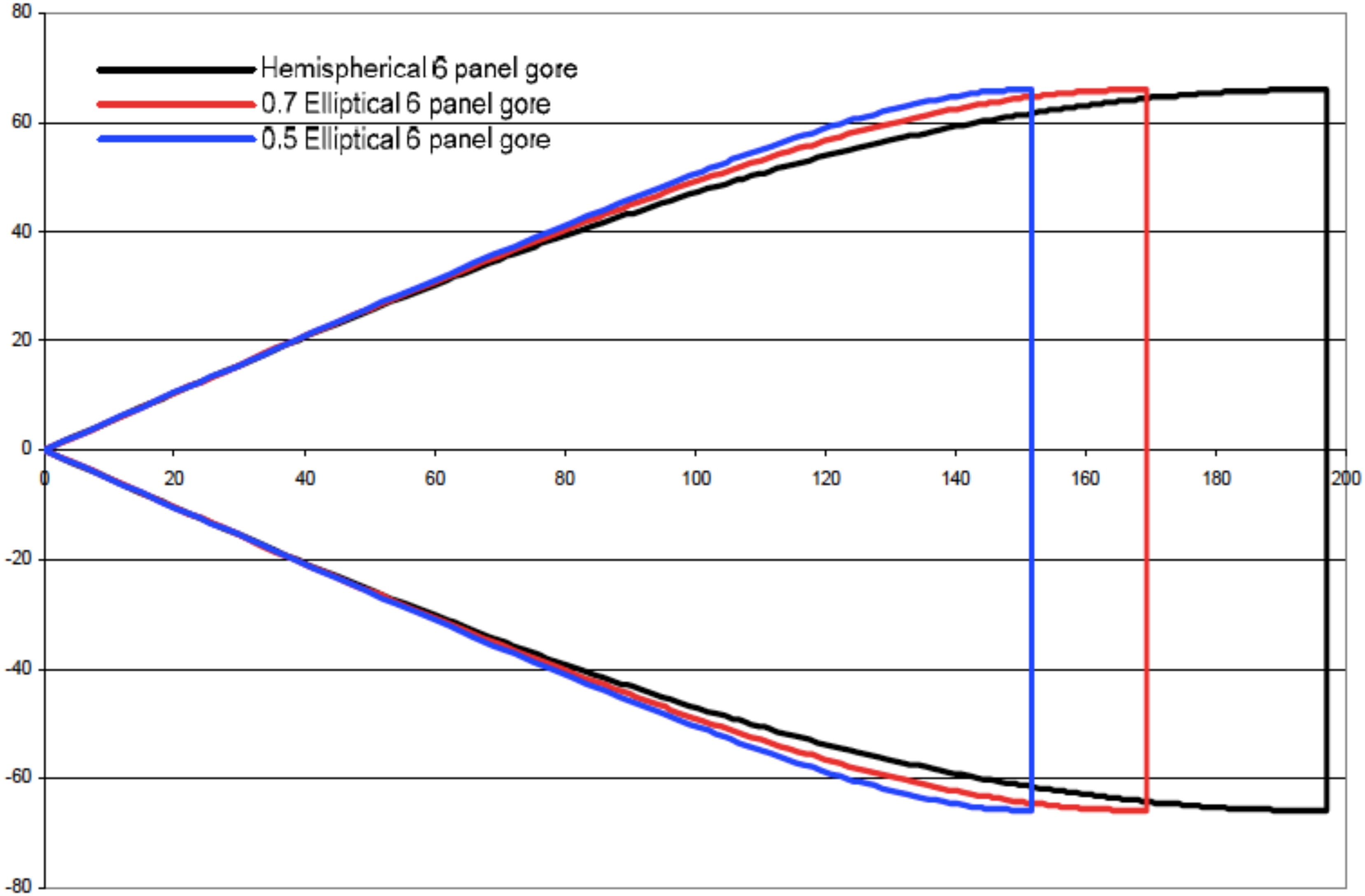
- **Elliptical parachute:**
  - Parachute where vertical axis is smaller than horizontal axis
  - A parachute with an elliptical canopy has essentially the same CD as a hemispherical parachute, but with less surface material





Canopy profile for different height / radius ratios

Parachute Shapes (5)



Comparison of gore shapes for different height : radius ratios

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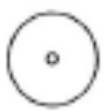
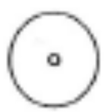
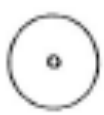
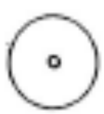
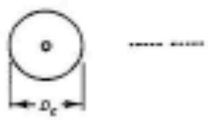
Parachute Types (2)

Solid Textile Parachutes

Type	Constructed Shape: Plan	Constructed Shape: Profile	Constructed Shape: $\frac{D_c}{D_o}$	Inflated Shape $\frac{D_p}{D_o}$	Drag Coef. C_{D_o} Range	Opening Load Factor C_X (Inf. Mass)	Average Angle of Oscillation	General Application
Flat Circular			1.00	.67 to .70	.75 to .80	~1.8	±10° to ±40°	Descent
Conical			.93 to .95	.70	.75 to .90	~1.8	±10° to ±30°	Descent
Bi-Conical			.90 to .95	.70	.75 to .92	~1.8	±10° to ±30°	Descent
Tri-Conical			.90 to .95	.70	.80 to .96	~1.8	±10° to ±20°	Descent
Hemispherical			.71	.66	.62 to .77	~1.6	±10° to ±15°	Descent

Parachute Types (3)

Slotted Textile Parachutes

Type	Constructed Shape: Plan / Profile	Constructed Shape D_c/D_o	Inflated Shape D_p/D_o	Drag Coef. C_{D_o} Range	Opening Load Factor C_X (Inf. Mass)	Average Angle of Oscillation	General Application
Flat Ribbon		1.00	.67	.45 to .50	~1.05	0° to ±3°	Drogue, Descent, Deceleration
Conical Ribbon		.95 to .97	.70	.50 to .55	~1.05	0° to ±3°	Descent, Deceleration
Conical Ribbon (Varied Porosity)		.97	.70	.55 to .65	1.05 to 1.30	0° to ±3°	Drogue, Descent, Deceleration
Ribbon (Hemisflo)		.62	.62	.30* to .46	1.00 to 1.30	±2°	Supersonic Drogue
Ringslot		1.00	.67 to .70	.56 to .65	~1.05	0° to ±5°	Extraction, Deceleration
Ringsail		1.16	.69	.75 to .90	~1.10	±5° to ±10°	Descent
Disc-Gap-Band		.73	.65	.52 to .58	~1.30	±10° to ±15°	Descent

*Supersonic

Example Calculation: Drogue Chute Terminal Velocity

$$h_{apogee} = h_{agl} + h_{\substack{launch\\site}} =$$

$$\left(1609.23 + 240\right)_{meters} \simeq\approx 1850_{meters}$$

$$\rho_{apogee} = 1.0218_{\frac{kg}{m^3}}$$

$$g = \frac{\mu}{r^2} = \frac{3.9860044 \times 10^5{}_{\frac{km^3}{\sec^2}}}{\left(6371 + 1.85\right)^2{}_{km^2}} = 9.815_{\frac{m}{\sec^2}}$$

$$Maximum\ mass\ at\ apogee: m_{apogee} = m_{launch} - m_{fuel} = \left(14.188 - 1.76\right) = 12.428_{kg}$$

$$m_{apogee} \cdot g = 12.428 \cdot 9.815 = 121.975_{Nt}$$

Example Calculation: Drogue Chute Terminal Velocity (2)

- Descent rate under drogue, 50-100 ft/sec
- Go with minimum value ~ 15.24 m/sec (50 ft/sec)

"Vehicle Drag Area" ..
Rocket is broken into two pieces

$$\left(C_D \cdot S_0\right)_{vehicle} \approx 2 \cdot \left[\left(C_D\right)_{rocket} \cdot \left(A_{ref}\right)_{rocket}\right] = \left(2 \cdot 0.35 \cdot 0.01589\right) \approx 0.0111_{m^2}$$

"Double up" nominal rocket drag area

Example Calculation: Drogue Chute Terminal Velocity (3)

- Parachute Drag Coefficient

- Elliptical Parachute .. Take median value

Solid Textile Parachutes

Type	Constructed Shape: Plan	Profile	$\frac{D_c}{D_o}$	Inflated Shape $\frac{D_p}{D_o}$	Drag Coef. C_{D_o} Range	Opening Load Factor C_x (Inf. Mass)	Average Angle of Oscillation	General Application
Conical			.93 to .95	.70	.75 to .90	~1.8	±10° to ±30°	Descent
Hemispherical			.71	.66	.62 to .77	~1.6	±10° to ±15°	Descent

$$\left(C_D\right)_{chute} \approx 0.76 \pm 0.115$$

Example Calculation: Drogue Chute Terminal Velocity (4)

- Calculate required chute area:

$$V^2_{\text{terminal}} = \frac{2 M_{vehicle} \cdot g / \rho}{\left(C_D \cdot S\right)_0^{parachute} + \left(C_D \cdot S\right)_{ref}^{vehicle}}$$

$$\rightarrow \left(S_0\right)_{parachute} = \frac{\dfrac{m \cdot g}{\frac{1}{2}\rho \cdot V_{terminal}^{\;\;2}} - \left(C_D \cdot S_0\right)_{vehicle}}{\left(C_D\right)_{parachute}} =$$

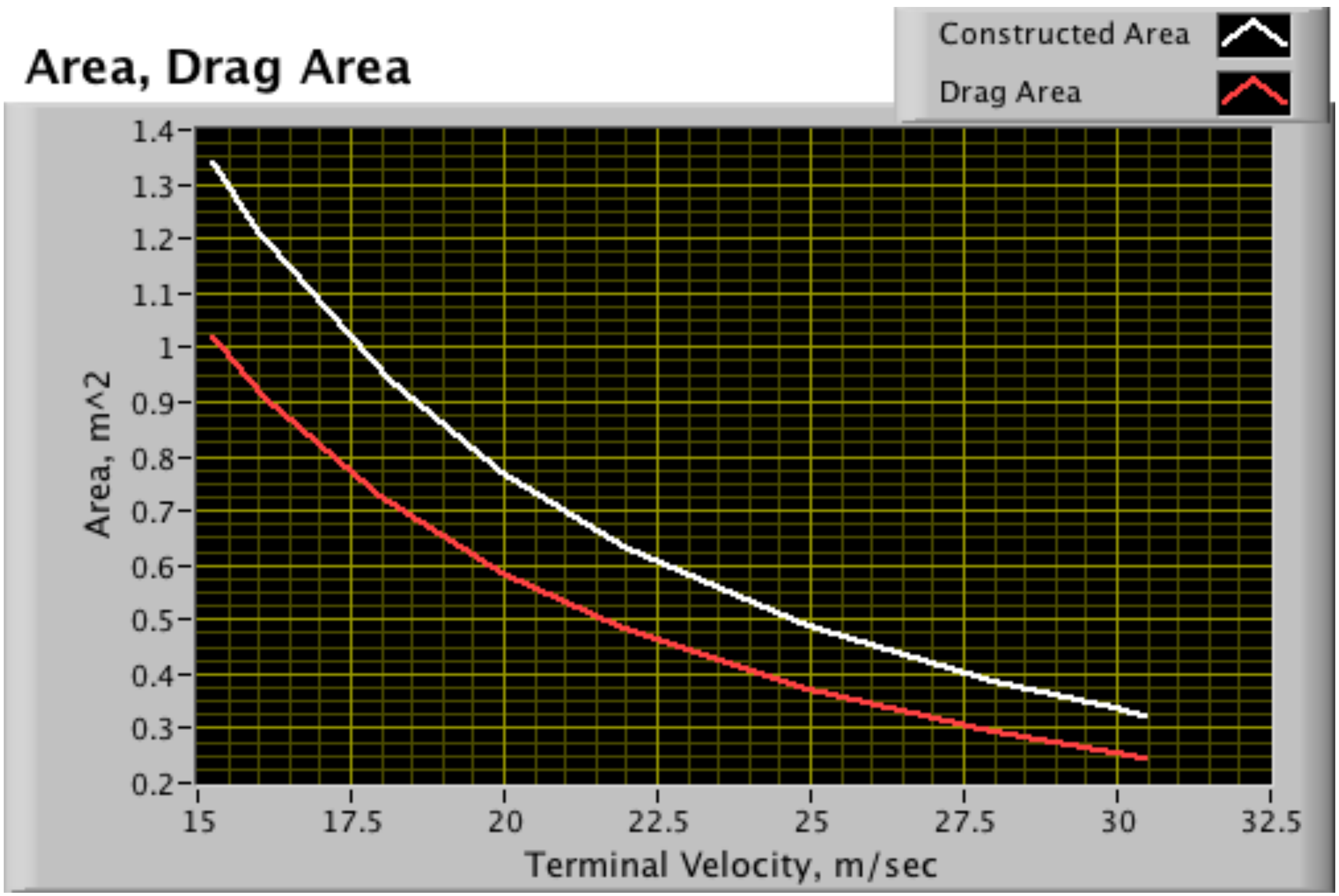
$$\frac{\dfrac{121.965}{\left(\frac{1}{2} 1.0218 \cdot 22.86^2\right)} - 0.0111}{0.76}$$ **1.3378 m²**

$$D_0 = \sqrt{\frac{4 \cdot \left(S_0\right)_{parachute}}{\pi}} =$$

$$\left(\frac{4 \cdot 1.33783}{\pi}\right)^{0.5} \frac{39.37}{12}$$ **= 4.28 ft**

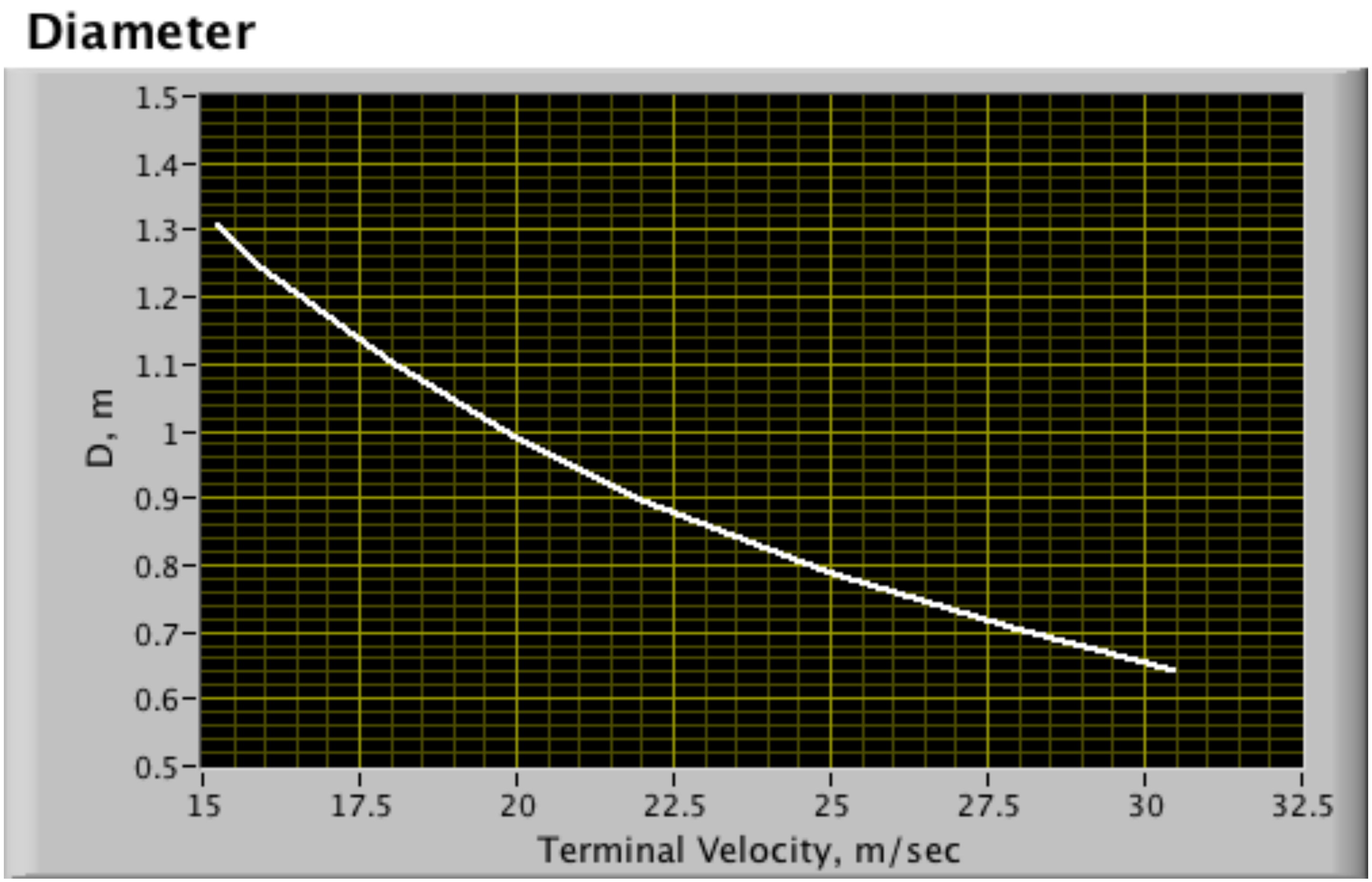
Example Calculation: Drogue Chute Terminal Velocity (5)

Drag Chute Areas Versus Terminal Velocity



Example Calculation: Drogue Chute Terminal Velocity (5)

Drag Chute Diameter Versus Terminal Velocity



Parachute Opening Loads

Largest Tensile Load on Vehicle … often the Ultimate Design Load Driver

Accurate calculation of opening loads are critical for:

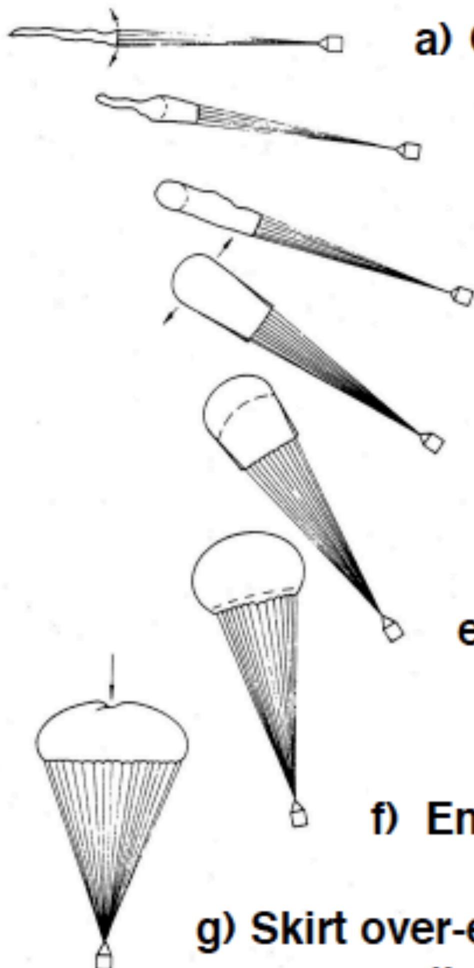
- Stress analysis of parachute
- Stress analysis of entry vehicle
- Calculating acceleration of payload
- Specification of on-board accelerometers

Three opening loads analysis methods are discussed here:

- Pflanz's Method ← *Design Tool*
- Inflation Curve Method ← *Verification Tool (Direct Simulation)*
- Apparent Mass Method

Parachute Opening Loads (2)

Inflation Process



a) Opening of canopy mouth (start of inflation)

b) Air mass moves along canopy

c) Air mass reaches crown of canopy

d) Influx of air expands crown

e) Expansion of crown resisted by structural tension and inertia

f) Enlarged inlet causes rapid filling

g) Skirt over-expanded, crown depressed by momentum of surrounding air mass

Parachute Opening Loads (3)

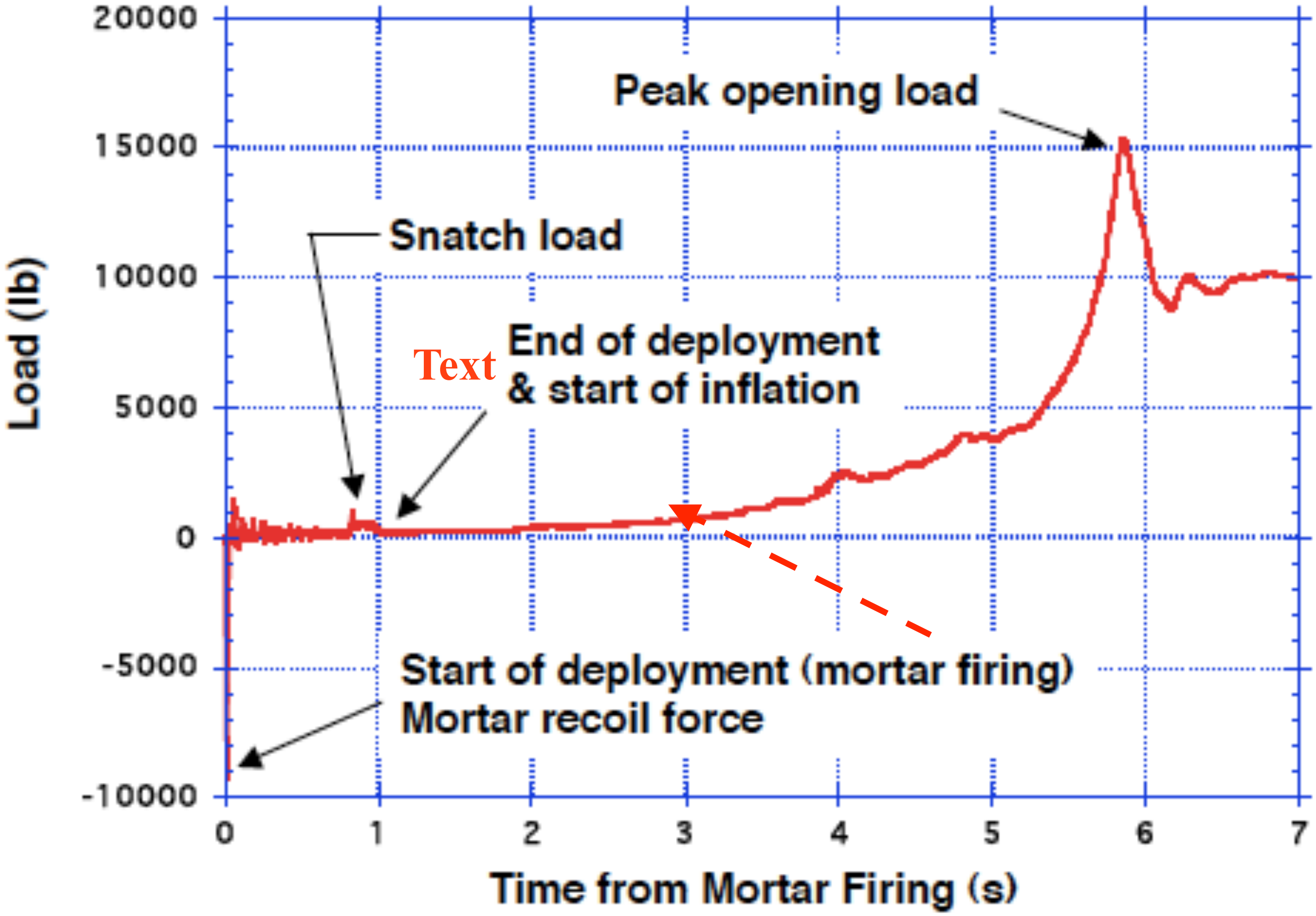
- At subsonic speeds, inflation is often modeled as occurring over a constant number of parachute diameters (i.e., multiples of D_0) for a given parachute type
- Parachute is "scooping" a given volume of air to inflate
- For the most part, experimental data supports this assumption
- Thus if inflation occurs at a constant velocity, V, the inflation time, t_{Inf}, can be estimated from:

$$t_{\mathrm{inf}} = n \cdot \frac{D_0}{V_1^{k}} \rightarrow \begin{matrix} n = \text{canopy fill constant} \\ k = \text{decceleration exponent} \end{matrix}$$

where n depends on the parachute type and geometry (typically $n_{Inf} \sim 6$ to 15)

- If V varies significantly during inflation, the equations of motion must be integrated to obtain the inflation time for a given inflation distance

Parachute Opening Loads (4)



Parachute Opening Loads (5)

Infinite-Mass Inflation

- If inflation is of the infinite mass type there will be little deceleration and reduction in the dynamic pressure during inflation
  - Peak opening load will occur at full inflation
- Infinite-mass inflation can happen when inflation occurs so rapidly that there is no time for significant deceleration of the entry vehicle during inflation
- Parachute inflation in thin atmospheres at supersonic speeds is often of the infinite mass type -> Mars!
- Infinite-mass inflation is difficult to obtain at subsonic speeds at low Earth altitudes - this presents a challenge to the qualification of supersonic parachutes at low Earth altitudes
- To obtain infinite-mass inflation at low Earth altitudes:
  - Payload mass must be large or,
  - Test must be conducted in a wind tunnel

Parachute Opening Loads (6)

Finite-Mass Inflation

- If the payload has "finite-mass," there will be substantial deceleration and reduction in the dynamic pressure during the inflation
  - Peak opening load <u>will not</u> occur at full inflation

- This is the typical situation when parachutes are inflated at low Earth altitudes

- It is more difficult to accurately predict the opening loads in a finite-mass inflation

Pflanz's Method

• **Pflanz' (1942):**

-introduced analytical functions for the drag area

(finite mass inflation approximation)

• **Simple, frst-order, design book type method**

-Requires least knowledge of the system compared to other methods

-Assumes no gravity acceleration – limits application to shallow fight path angles at parachute deployment

- Neglects entry vehicle drag

- Yields only peak opening load

- Allows for finite mass approximation

• **Doherr (2003) extended method to account for gravity and arbitrary fight path angles**

Pflanz's Method (2)

(finite mass inflation approximation)

$$F_{peak} = \bar{q}_1 \cdot \left(C_D \cdot S\right)_0 \cdot C_x \cdot X_1 \rightarrow \begin{bmatrix} \bar{q}_1 \rightarrow \textit{Dynamic Pressure @ Deployment} \\ C_x \rightarrow \textit{Shock Load Factor} \\ X_1 \rightarrow \textit{Opening Force Reduction Factor} \\ \left(C_D \cdot S\right)_0 \rightarrow \textit{Nominal Drag Area @ Full Inflation} \end{bmatrix}$$

$$X_1 = f\left(A, \eta\right) \rightarrow \begin{bmatrix} \eta \rightarrow \textit{Inflation Curve Exponent} \\ A \rightarrow \textit{Ballistic Parameter} \end{bmatrix} \quad A = \frac{2 \cdot M_V}{\left(C_D \cdot S\right)_0 \cdot \rho_1 \cdot V_1 \cdot \tau_{infl}}$$

F_{peak} - peak opening load
q_1 - dynamic pressure at start of inflation
C_{D0} - parachute full-open drag coefficient
S_0 - parachute nominal area
C_X - opening load factor (from test data or tables in pages 24 through 26)
X_1 - force reduction factor accounting for deceleration during inflation

A - ballistic parameter
η - inflation curve exponent (dependent on canopy type, see Knacke: Parachute Recovery Systems Design Manual, p. 5-58)
M_V - mass of entry vehicle
ρ_1 - atmospheric density
V_1 - velocity at start of inflation
τ_{infl} - inflation time (See Later Description)

Pflanz's Method (3)

$$A = \frac{2 \cdot M_V}{(C_D \cdot S)_0 \cdot \rho_1 \cdot V_1 \cdot \tau_{infl}}$$

$$\tau_{infl} = n \cdot \frac{D_0}{V_1^k} \rightarrow$$

n – Canopy Fill Constant

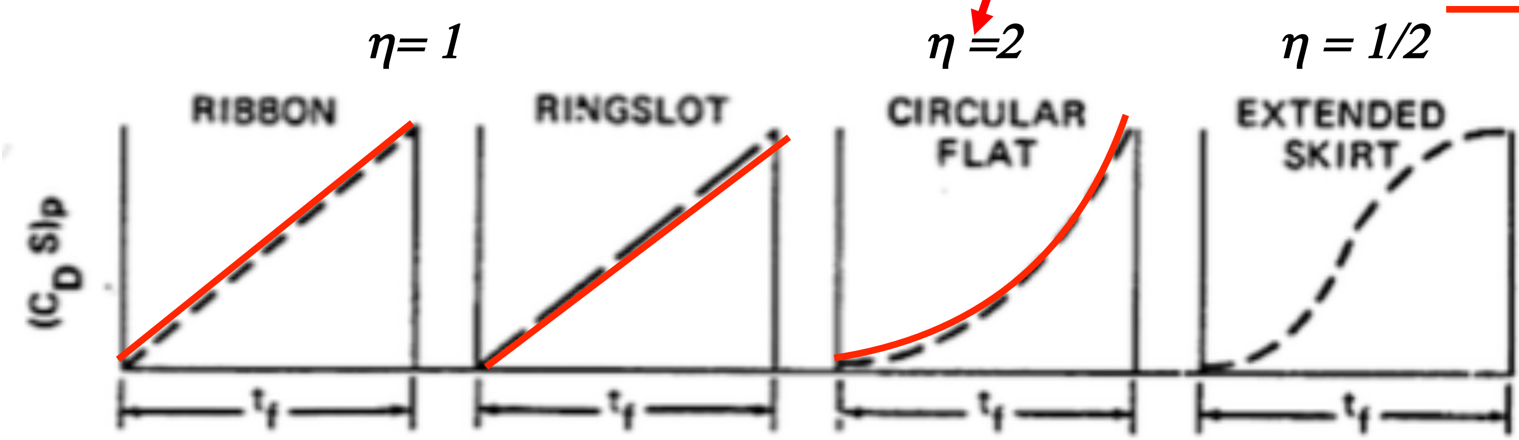
k = Decelleration Exponent

Solid, Elliptical Chute $\rightarrow$ $n \approx 4$, $k \approx 0.85$

Ribbon/Ringslot → $\eta = 1$

Solid, Elliptical, Flat → $\eta = 2$

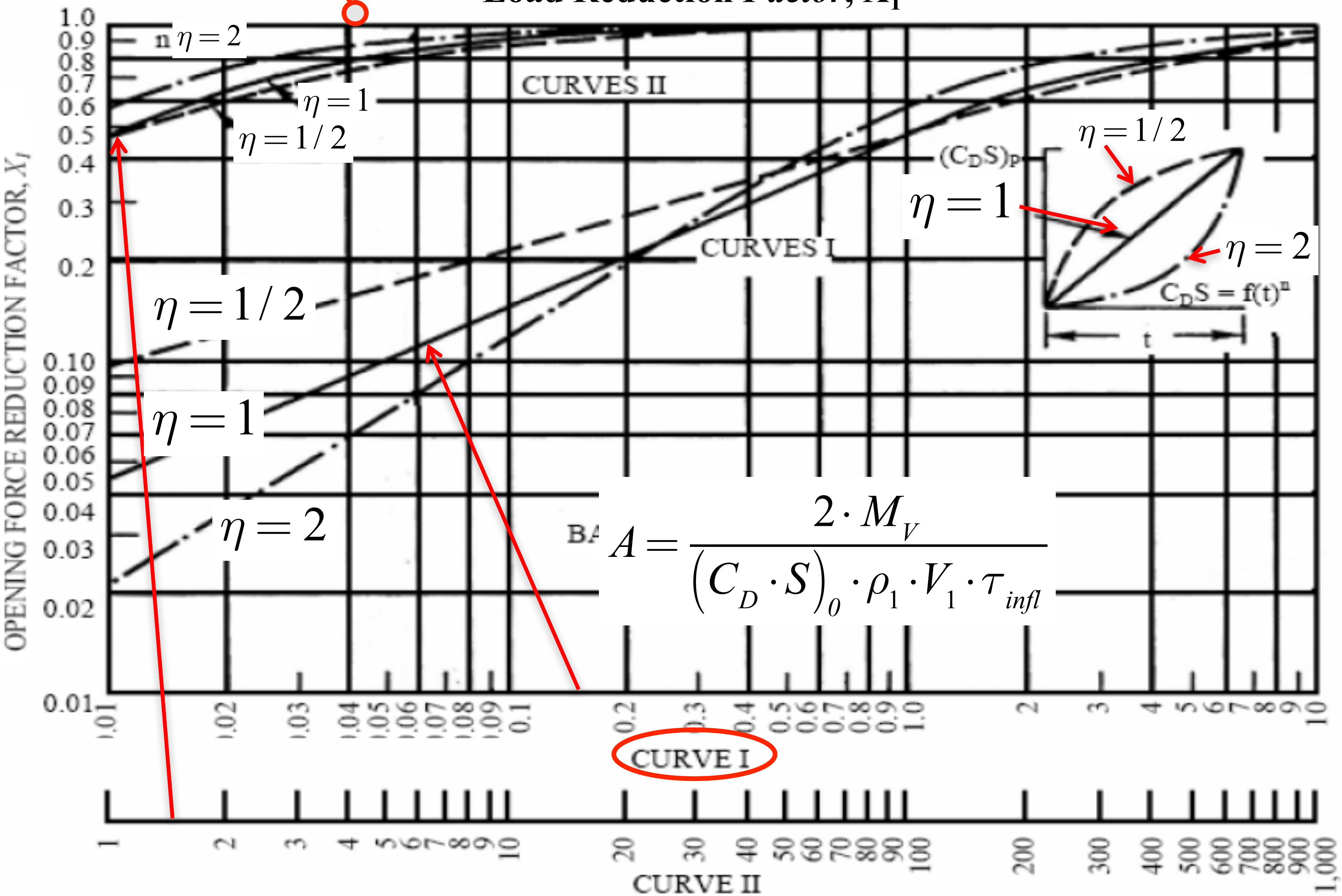
Extended Skirt, Reefed → $\eta = 1/2$



Typical Drag-Area-Versus-Time Increase for Various Parachute Types.

Pflanz's Method (4)

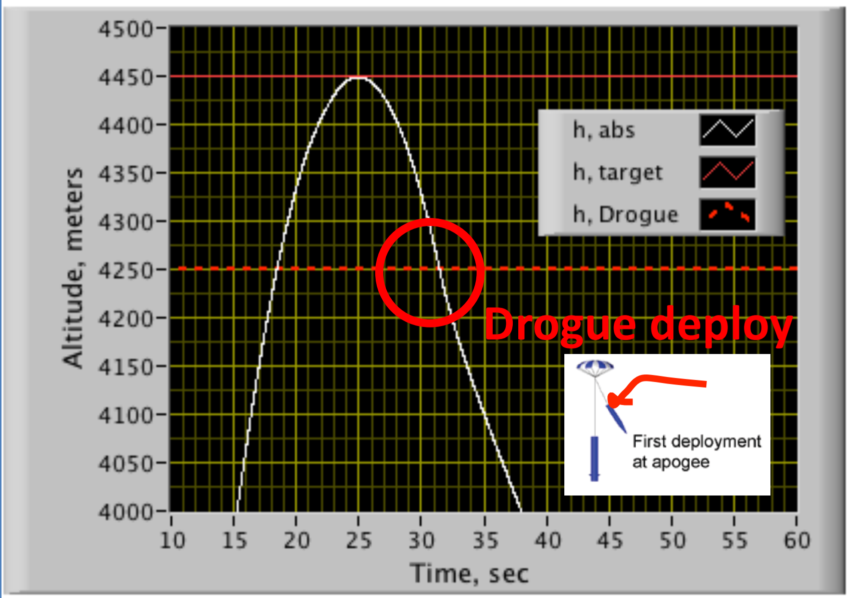
Load Reduction *Factor*, X_1



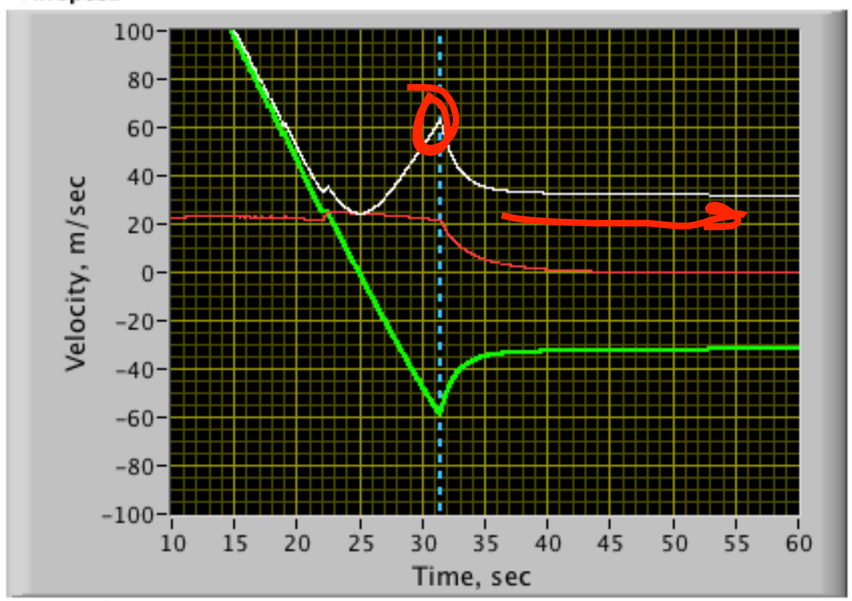
$$A = \frac{2 \cdot M_V}{\left(C_D \cdot S\right)_0 \cdot \rho_1 \cdot V_1 \cdot \tau_{infl}}$$

Pflanz's Method Example

Altitude MSL



Airspeed



Deployment Altitude, MSL Meters: 4251

Deployment Time, sec: 31.45

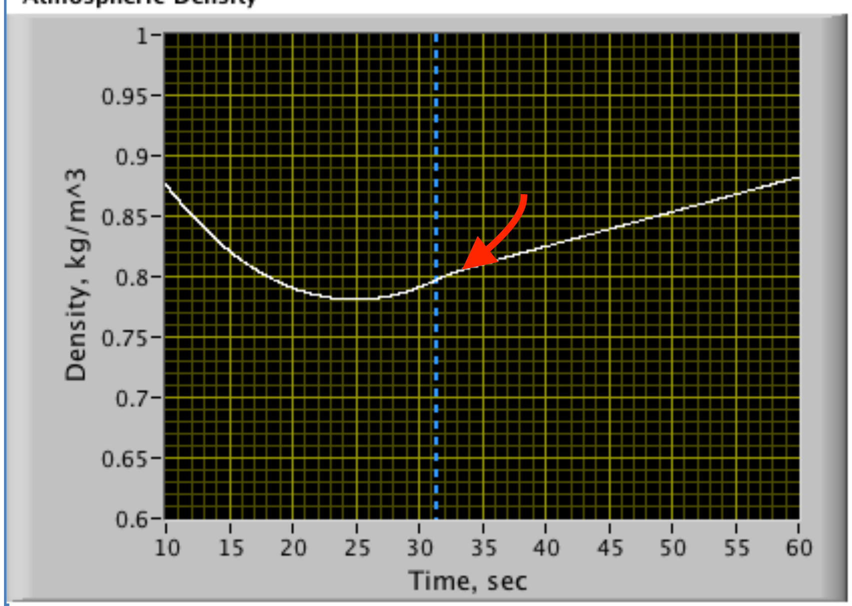
V_1, m/sec: 62.864

ρ_1, kg/m^3: 0.797675

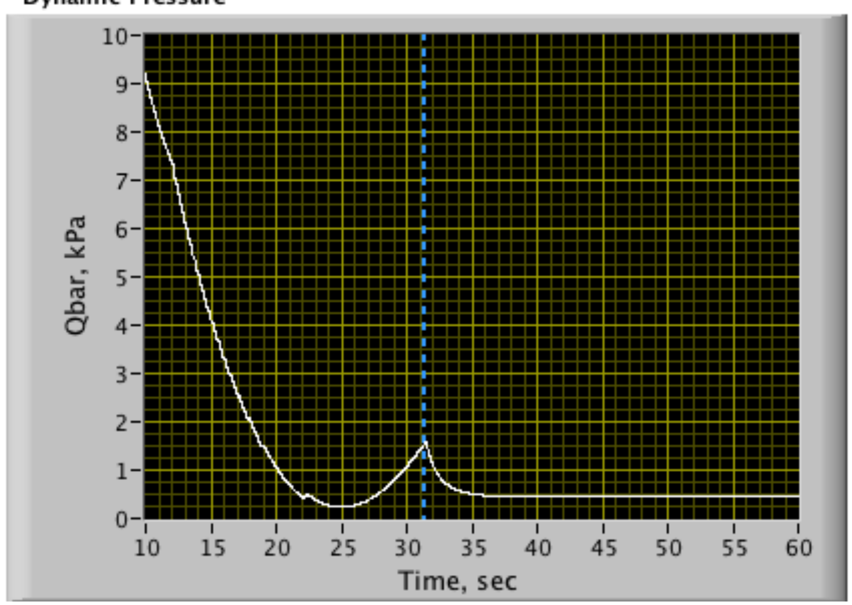
q_1, kPa: 1.57616

$V_{terminal}$, m/sec: 31.5532

Atmospheric Density



Dynamic Pressure



Pflanz's Method Example (2)

Deployment Altitude, MSL Meters: 4251

Deployment Time, sec: 31.45

V1, m/sec: 62.864

ρ1, kg/m^3: 0.797675

q1, kPa: 1.57616

Vterminal, m/sec: 31.5532

Final Vehicle Mass, kg: 10.1151

Parachute CD: 0.85

Vehicle CD0: 0.3415

Vehicle Aref (M^2): 0.036305

Desired Terminal Velocity = 31.55 m/sec … Get Nominal Parachute Size

$$V_{terminal} = \sqrt{\frac{2 M_{vehicle} \cdot g / \rho}{\left(C_D \cdot S\right)_0^{parachute} + \left(C_D \cdot S\right)_{ref}^{vehicle}}}$$

$$S_0^{parachute} = \frac{\dfrac{M_{vehicle} \cdot g}{\left(\frac{1}{2} \rho \cdot V_{\text{terminal}}^2\right)} - \left(C_D \cdot S\right)_{ref}^{vehicle} X\ 2}{C_{D0}^{parachute}} =$$

$$\frac{\dfrac{10.1151 \cdot 9.80716}{0.5 \cdot 0.797675 \cdot 31.5532^2} - 0.3415 \cdot 0.036305 \cdot 2}{0.85} = 0.264745\ \text{M}^2$$

$$\rightarrow D_0^{parachute} = 58.0589_{cm}$$

Pflanz's Method Example (3)

Subsonic Inflation time …

$$\tau_{infl} = n \cdot \frac{D_0}{V_1^k} \rightarrow \begin{matrix} n - Canopy\ Fill\ Constant \\ k = Decelleration\ Exponent \end{matrix}$$

$$Solid,\ Elliptical\ Chute \rightarrow \begin{matrix} n \approx 4 \\ k \approx 0.85 \end{matrix}$$

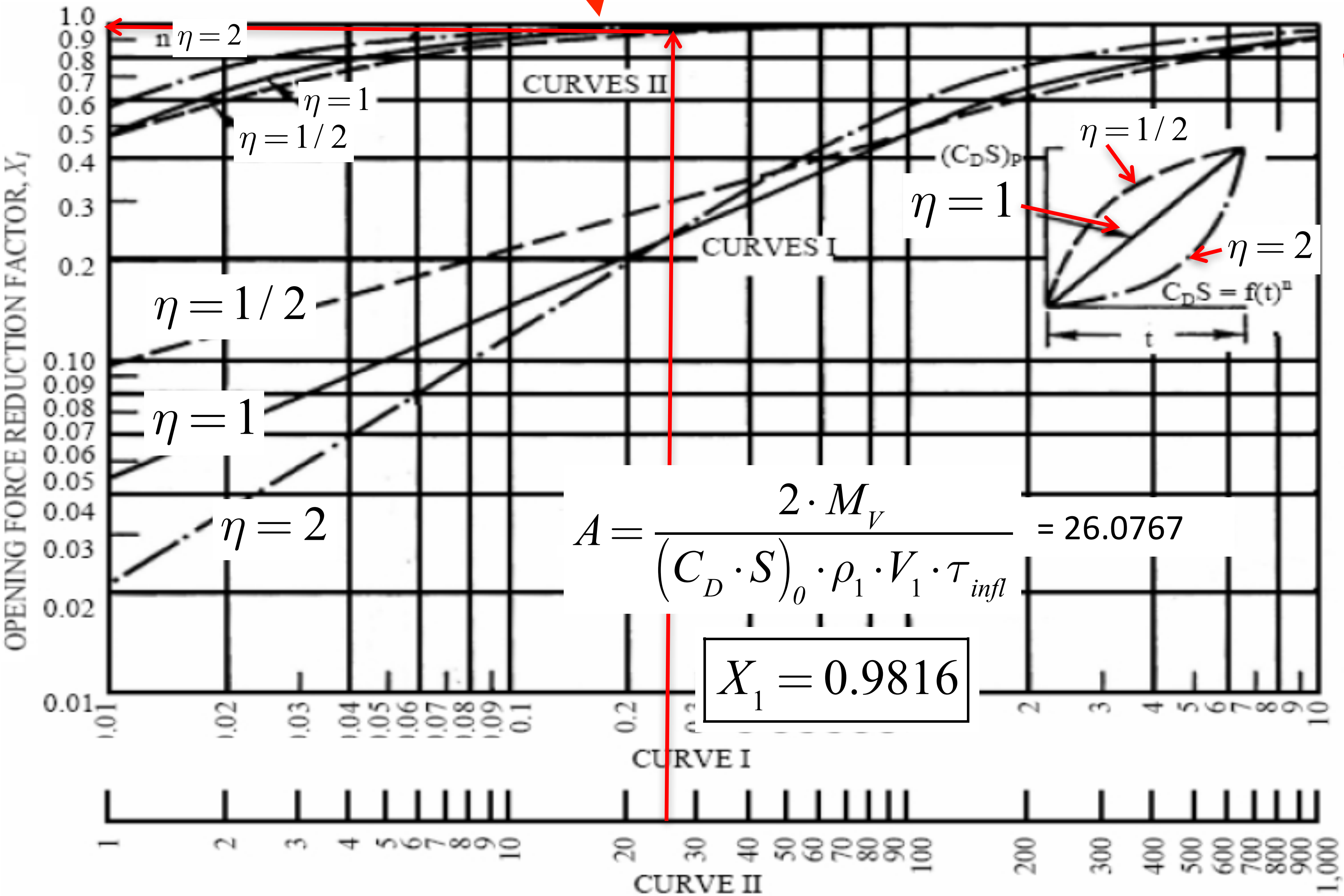
$$\rightarrow D_0^{parachute} = 58.0589_{cm}$$

$$V_1 = 62.864_{m/\sec}$$

$$\tau_{infl} = 4\frac{0.580589}{62.864^{0.85}} = 0.06875 \text{ sec}$$

$$A = \frac{2 \cdot M_V}{(C_D \cdot S)_0 \cdot \rho_1 \cdot V_1 \cdot \tau_{infl}} = \frac{2 \cdot 10.1151}{0.85 \cdot 0.264745 \cdot 0.79765 \cdot 62.864 \cdot 0.0687522} = 26.0767$$

Pflanz's Method Example (5)



$$A = \frac{2 \cdot M_V}{\left(C_D \cdot S\right)_0 \cdot \rho_1 \cdot V_1 \cdot \tau_{infl}} = 26.0767$$

$$X_1 = 0.9816$$

Pflanz's Method Example (5)

$$F_{peak} = \bar{q}_1 \cdot \left(C_D \cdot S\right)_0 \cdot C_x \cdot X_1 \rightarrow \begin{bmatrix} \bar{q}_1 \rightarrow & 1576.2 \text{ Pa} \\ C_x \rightarrow & 1.8 \\ X_1 \rightarrow & 0.9816 \\ \left(C_D \cdot S\right)_0 \rightarrow & 0.22503 \text{ m}^2 \end{bmatrix}$$

Deployment Altitude, MSL Meters: 4251

Deployment Time, sec: 31.45

V_1, m/sec: 62.864

ρ_1, kg/m^3: 0.797675

q_1, kPa: 1.57616

$V_{terminal}$, m/sec: 31.5532

$$= 1576.2 \cdot 0.22503 \cdot 1.8 \cdot 0.9816 = 626.614 \text{ N}$$

$$\boxed{X_1 = 0.9816}$$

Near infinite mass

Pflanz's Method Example 2

Pflanz's Method Example (2)

MER A - Spirit

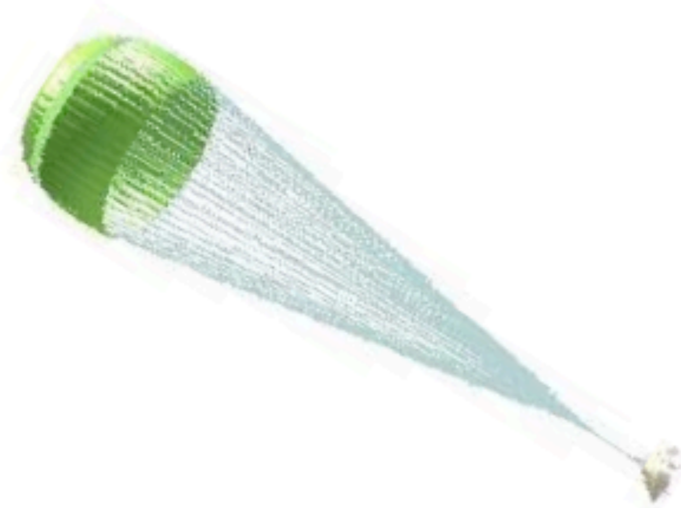
$q_1 = 729$ Pa
$C_{D0} = 0.400$ (at M = 1.75)
$D_0 = 14.1$ m
$S_0 = 156$ m²
$C_X = 1.45$

$m_{EV} = 827$ kg
$\Delta = 0.00863$ kg/m³
$V_1 = 411$ m/s
$t_{inf} = 0.282$ s (from previous discussion on supersonic inflation)

A = 26.5
n = 2 (for DGB parachutes)
$X_1 = 0.98$ (i.e., very close to infinite mass inflation!)

Δ

$F_{peak} = 64{,}641$ N (within 10% of best estimate)



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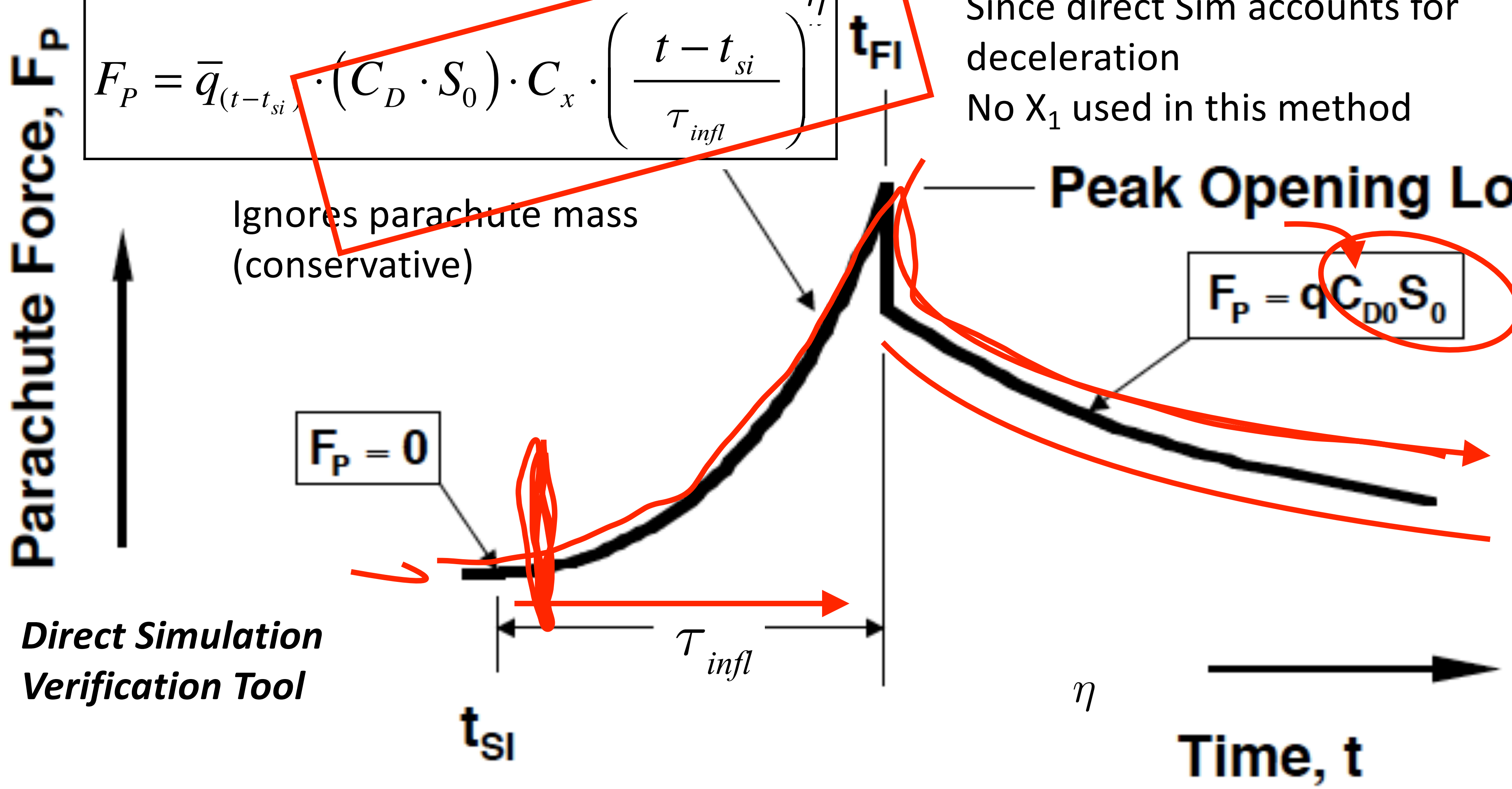
Inflation Curve Methods

$$F_P = \bar{q}_{(t-t_{si})} \cdot (C_D \cdot S_0) \cdot C_x \cdot \left(\frac{t - t_{si}}{\tau_{infl}} \right)^{\eta}$$

Since direct Sim accounts for deceleration

No X_1 used in this method

Ignores parachute mass (conservative)

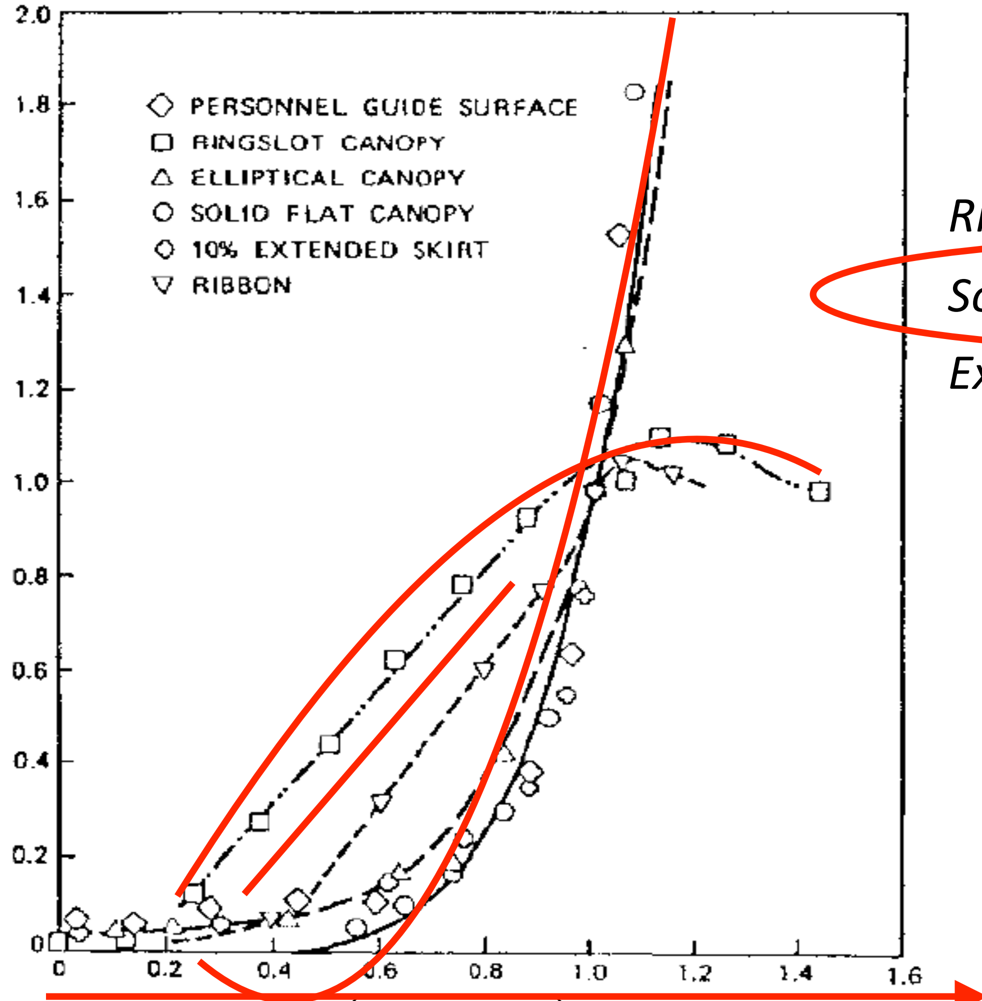


Direct Simulation Verification Tool

$$F_p = \begin{cases} qC_{D,0}S_0C_x\left(\dfrac{t-t_{SI}}{t_{FI}-t_{SI}}\right)^n + M_V\, g\sin\gamma & t_{SI} \le t \le t_{FI} \\ qC_{D,0}S_0 + M_V\, g\sin\gamma & t > t_{FI} \end{cases}$$

Inflation Curve Method (2)

$$\frac{(C_D \cdot S_0)_t}{(C_D \cdot S_0)_{steady}}$$



$$\frac{t - t_{si}}{\tau_{infl}}$$

Inflation Data from Doherr

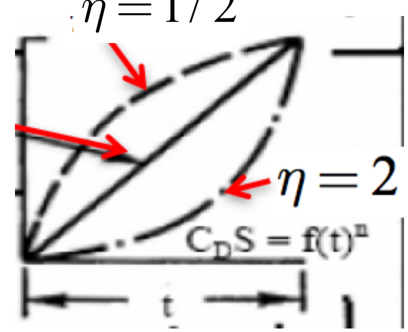
Ribbon/Ringslot → *η=1*

Solid, Elliptical, Flat → *η=2*

Extended Skirt, Reefed → *η=1/2*

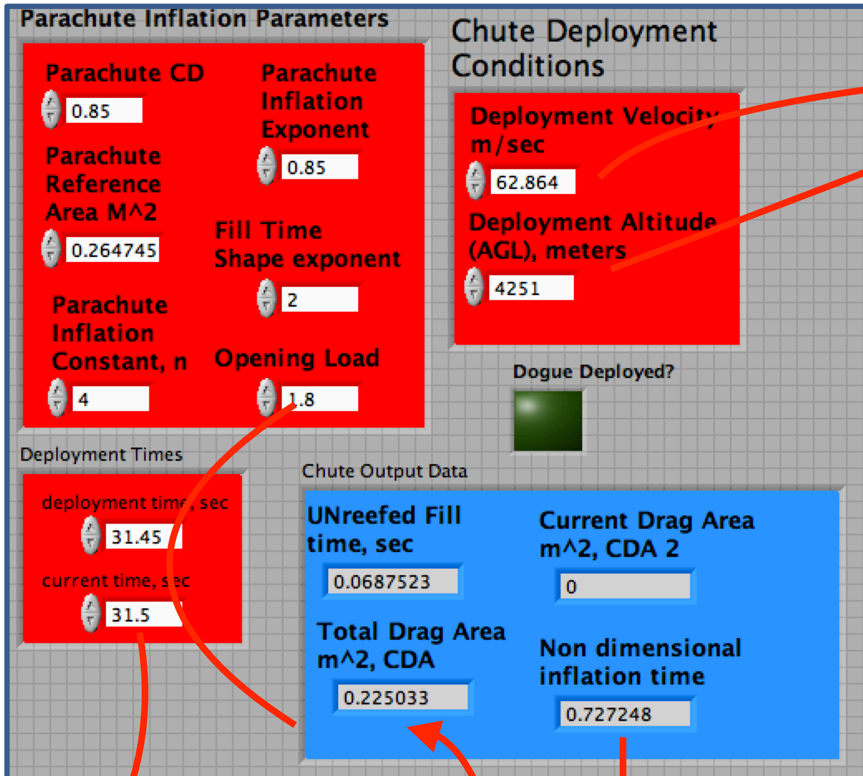
$\eta = 1/2$

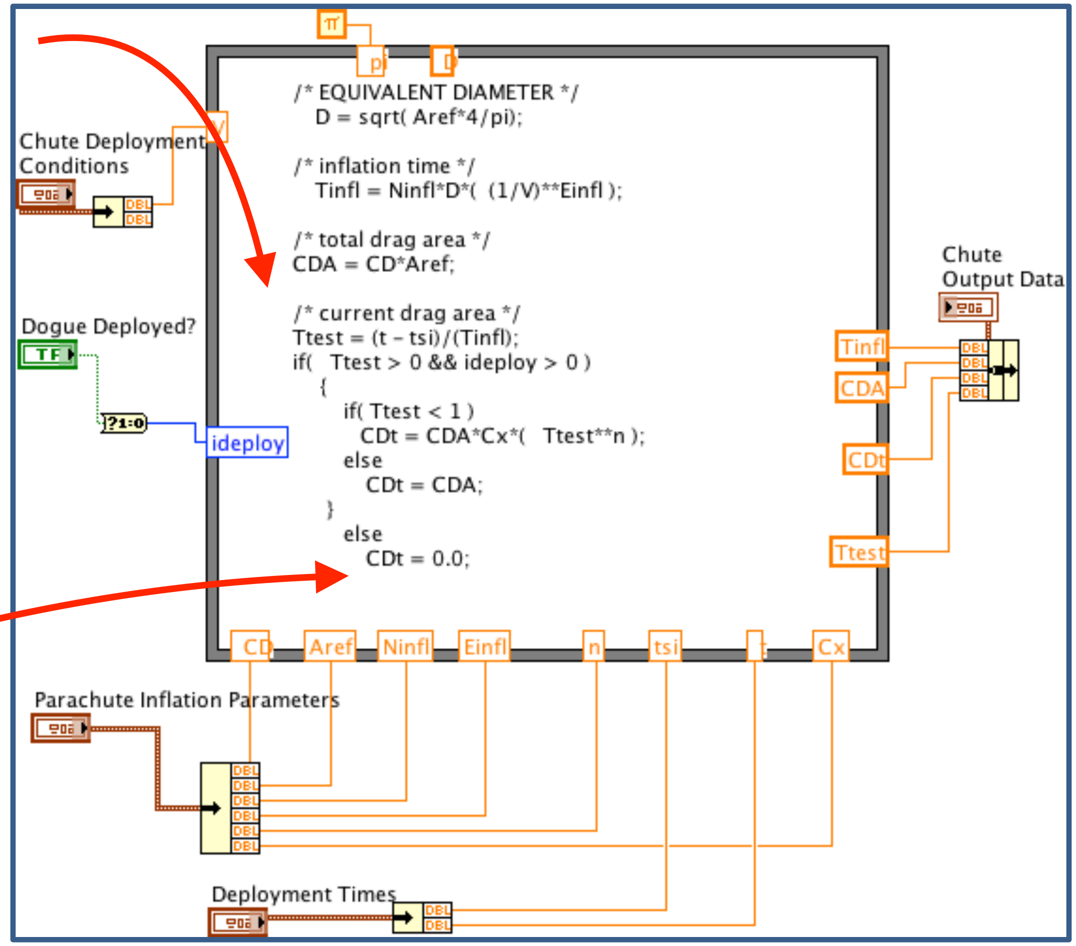
$\eta = 1$



$\eta = 2$

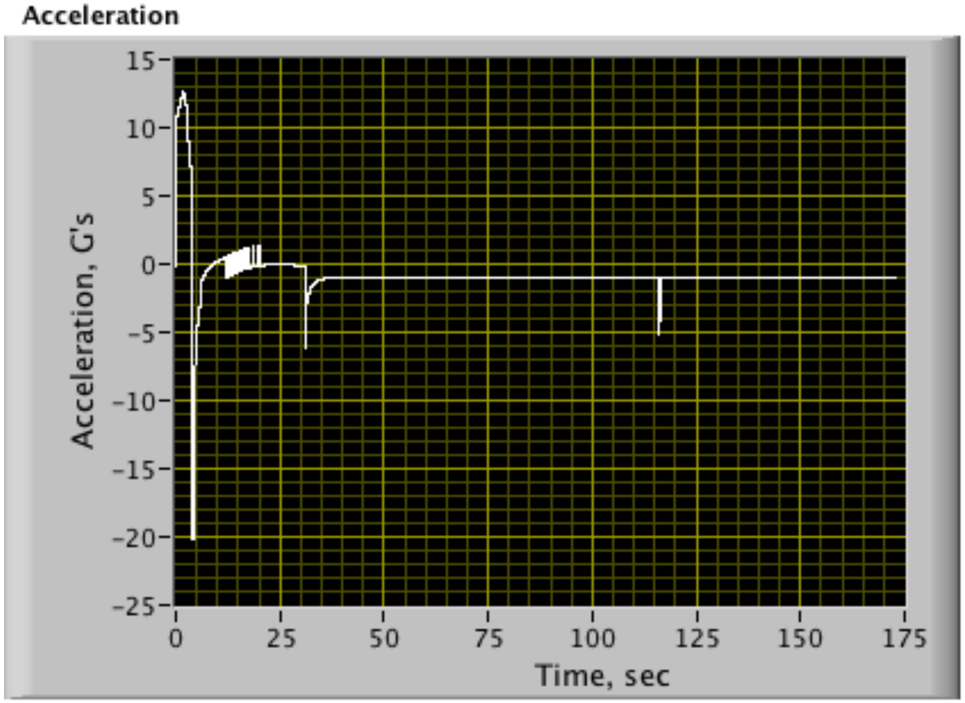
Inflation Curve Method (3)

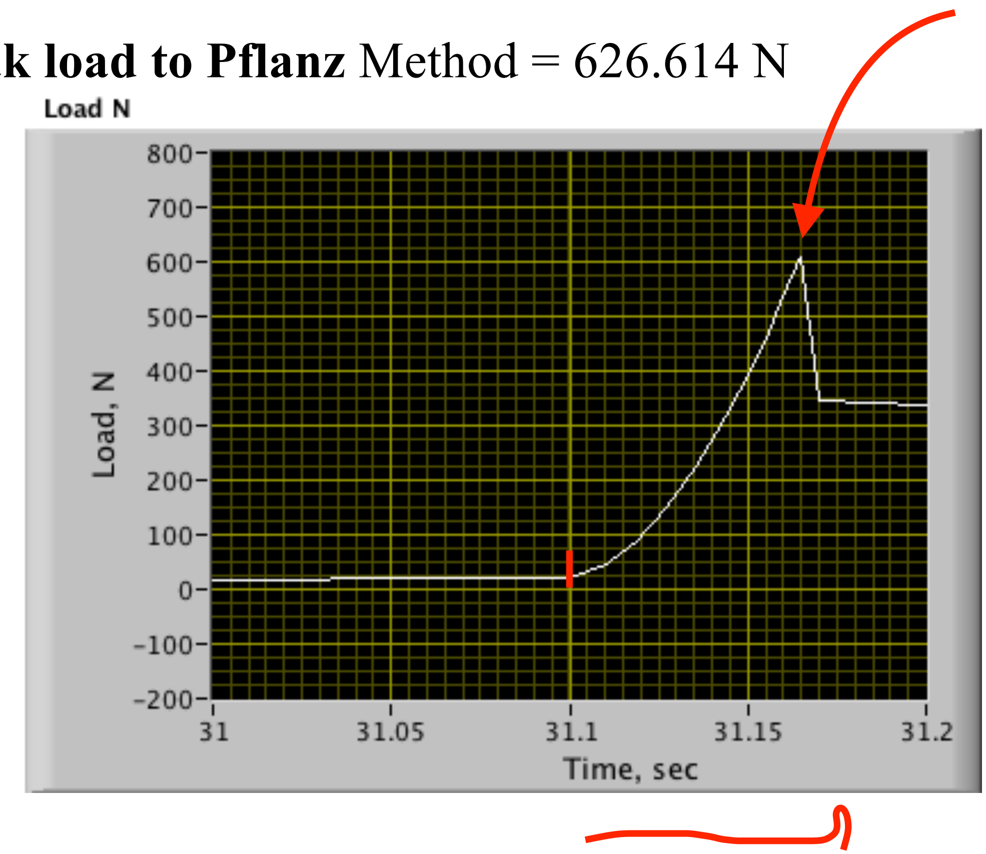




Inflation Curve Method (4)

Direct Simulation Response, compare peak load to Pflanz Method = 626.614 N






Questions??