[12] V. E. Govorov, *On flat modules*, Sibirs. Mat. J. 6 (1965), pp. 300–304.
[13] P. Griffith, *On the decomposition of modules and generalized left uniserial rings*, Math. Ann. 184 (1970), pp. 300–308.
[14] L. Gruson and C. U. Jensen, *Modules algébraiquement compact et foncteurs* $\varprojlim^{(i)}$, C. R. Acad. Sci., Paris, Sér. A 276 (1973), pp. 1651–1653.
[15] C. U. Jensen, *On the vanishing of* $\varprojlim^{(i)}$, J. Algebra 15 (1970), pp. 151–166.
[16] — *Les foncteurs dérivés de* $\varprojlim$ *et leurs applications en théorie des modules*, Lecture Notes in Math. 254 (1972).
[17] R. Kiełpiński, *On Γ-pure injective modules*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. 15 (1967), pp. 127–131.
[18] — and D. Simson, *On pure homological dimension*, ibid. 23 (1975), pp. 1–6.
[19] M. Harada, *Perfect categories*, Osaka J. Math. 10 (1973), pp. 329–368.
[20] O. A. Laudal, *Sur la limite projective et la théorie de la dimension*, Top. et Géom. Dif., Sém. Ehresmann, 1961, 23 p.
[21] B. E. Mares, *Semi-perfect modules*, Math. Z. 83 (1963), pp. 347–360.
[22] B. Mitchell, *Rings with several objects*, Advances in Math. 8 (1972), pp. 1–161.
[23] U. Oberst and H. J. Schneider, *Die Struktur von projektiven Moduln*, Inventiones Math. 13 (1971), pp. 295–304.
[24] B. L. Osofsky, *Upper bound on homological dimension*, Nagoya Math. J. 32 (1968), pp. 315–322.
[25] N. Popescu, *Abelian Categories with Applications to Rings and Modules*, London–New York 1973.
[26] M. Raynaud et L. Gruson, *Critères de platitude et de projectivité. Techniques de „platification" d'un module*, Inventiones Math. 13 (1971), pp. 1–89.
[27] A. V. Roiter, *The unboundedness of the dimension of the indecomposable representations of algebras that have a finite number of indecomposable representations*, Izv. Acad. Nauk SSSR 32 (1968), pp. 1275–1282.
[28] J. E. Roos, *Locally Noetherian categories and generalized strictly linearly compact rings*, Lecture Notes in Math. 92 (1969), pp. 197–277.
[29] C. Schoeller, *Etudé de la catégorie des algébres de Hopf commutatives connexes sur un corps*, Manuskripta Math. 3 (1970), pp. 133–155.
[30] D. Simson, *A remark on projective dimension of flat modules*, Math, Ann. 209 (1974), pp. 181–182.
[31] — *On colimits of injectives in Grothendieck categories*, Arkiv för Matematik 12 (1974), pp. 161–165.
[32] — *Functor categories in which every flat object is projective*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. 22 (1974), pp. 375–380.
[33] — *Serwantność i wymiary homologiczne w lokalnie skończenie przedstawialnych kategoriach Grothendiecka*, Uniwersytet Mikołaja Kopernika, Prace habilitacyjne, Toruń 1974.
[34] B. Stenström, *Purity in functor categories*, J. Algebra 8 (1968), pp. 352–361.
[35] — *Coherent rings and FP-injective modules*, J. London Math. Soc. 2 (1970), pp. 323–329.
[36] — *Pure submodules*, Arkiv för Matematik 7 (1967), pp. 159–171.
[37] M. E. Sweedler, *Hopf Algebras*, New York 1969.
[38] F. Ulmer, *A flatness criterion in Grothendieck categories*, Inventiones Math. 19 (1973), pp. 331–336.
[39] M. Weidenfeld et G. Weidenfeld, *Idéaux d'une catégories préadditive, application aux catégories semi-parfaites*, C. R. Acad. Sci. Paris, Série A, 270 (1970), pp. 1569–1571.

INSTYTUT MATEMATYKI, UNIWERSYTET MIKOŁAJA KOPERNIKA
INSTITUTE OF MATHEMATICS, NICHOLAS COPERNICUS UNIVERSITY

Accepté par la Rédaction le 17. 2. 1975

A locally connected non-movable continuum that fails to separate E^3

by

D. R. McMillan, Jr. * (Madison, Wisc.)

Abstract. A locally connected continuum X (Fig. 2) is constructed by tunneling into a 3-cell in such a way that $E^3 - X$ is connected. The non-movability of X is proven using the three-manifold techniques of Haken and Waldhausen.

1. Introduction. K. Borsuk has introduced and studied the important shape property of movability for compacta. Examples of movable compacta include compact absolute neighborhood retracts and compacta embeddable in 2-manifolds (see [1], [10], and [7]). Some of the more exotic continua, such as solenoids, are not movable ([1]). It seems especially worthwhile to seek convenient characterizations of movable compacta in 3-manifolds. Most examples of locally connected continua in Euclidean 3-space E^3 that come to mind are movable (for example, locally connected one-dimensional continuua are movable), but in general this is not enough to do the job. In particular, Borsuk gave in [2] an example of a locally connected, non-movable continuum in E^3. His example separates E^3 into two pieces. In an effort to focus on what does (and does not) make a *continuum* (i.e., compact, connected Hausdorff space) in E^3 movable, we present an example of a locally connected, non-movable continuum with connected complement in E^3. (This answers the second part of Borsuk's Problem 5.5 in [2].)

A compactum X is *movable* if for some (and hence for every) embedding $X \subset Q$ (= the Hilbert cube), the following holds: Each neighborhood U of X contains a neighborhood V of X such that for each neighborhood W of X, the final stage of some homotopy of V in U throws V into W. Of course, it can be shown that if X lies in a nice space, such as a manifold M, then X is movable by the preceding definition if and only if the corresponding movability statement holds for X with respect to its neighborhoods in M. Our example is constructed from a 3-cell by an infinite sequence of tunneling operations. (See Figure 2: any resemblance to a Christmas tree is coincidental.)

While the example itself is easy to describe, our proof of its non-movability seems rather elaborate. Perhaps simpler proofs and/or examples exist. We rely heavily

* Research supported by N. S. F. grant GP-38877A#1.

on the techniques and concepts of W. Haken ([3], [4]) and F. Waldhausen ([13]). The reader must also be willing to accept omegatators (see our Section 2 and references [6] and [12]) and their elementary properties.

2. Preliminaries. The next few sections contain our most important definitions, notations, and conventions.

All of our mappings, manifolds, submanifolds, etc., are to be piecewise-linear. A "manifold" is connected. A 2-manifold F in a 3-manifold M^3 is usually *properly embedded*, i.e., $\partial F = F \cap \partial M$ (where "∂" denotes boundary of a manifold), and 2-*sided* in the sense of having a collar neighborhood $F\times[-1,1]$ each of whose levels $F\times\{t\}$ is properly embedded. (In such notation, F is always identified with $F\times\{0\}$.) We let Δ^n denote an n-simplex. A piecewise-linear homeomorph of Δ^n is an *n-cell*; a piecewise-linear homeomorph of $S^n = \partial\Delta^{n+1}$ is an *n-sphere*. E^n denotes Euclidean n-space. A *cube-with-handles of genus n* is a 3-manifold piecewise-linearly homeomorphic to the regular neighborhood in E^3 of a finite, connected graph of Euler characteristic $1-n$. The *disk-sum* of two oriented 3-manifolds M_1, M_2 with nonempty boundary is the oriented 3-manifold obtained by pasting M_1, M_2 together via an orientation-reversing homeomorphism between 2-cells D_1, D_2, where $D_i \subset \partial M_i$, $i = 1, 2$.

A disjoint collection ("system") $\{F_i\}$ of 2-manifolds in the 3-manifold M^3 is *compressible in M^3* if for some 2-cell $D \subset M^3$, $D \cap \bigcup F_i = \partial D$ and ∂D bounds no 2-cell in $\bigcup F_i$. Otherwise, the system is *incompressible in M^3*. M^3 is *boundary-irreducible* if ∂M^3 is incompressible in M^3. We say that M^3 is *irreducible* if each 2-sphere in M^3 bounds a 3-cell in M^3. The properly embedded system $\{F_i\}$ is *boundary-compressible in M^3* if some component of some ∂F_i bounds a 2-cell in ∂M^3, or if for some 2-cell $D \subset M^3$, $D \cap \bigcup F_i$ is an arc $A \subset \partial D$ that fails to cut off a 2-cell in $\bigcup F_i$, with $(\partial D) - A \subset \partial M^3$, and with $(\partial D) \cap \bigcup \partial F_i$ consisting of exactly two points. Otherwise, the system is *boundary-incompressible.*

If G is a group and $a, b \in G$, $[a, b]$ denotes the *commutator* $a^{-1}b^{-1}ab$ of a and b. For nonempty subsets A, B of G $[A, B]$ denotes that subgroup of G generated by the set of all commutators $[a, b]$, for $a \in A$, $b \in B$. The smallest normal subgroup of G containing A is denoted $\langle A, G\rangle$. We let G_m denote the mth term in the *lower central series* of G. That is, $G_1 = G$, $G_2 = [G_1, G]$ (the *commutator subgroup*), and in general $G_{m+1} = [G_m, G]$ for $m \geqslant 1$. By G_ω we mean $\bigcap_{m\geqslant 1} G_m$ (the *omegatator subgroup*). Each G_m is normal in G, for $m \leqslant \omega$. A basic fact we use is that $G_\omega = 1$ for each free group G (see pp. 108–109 of [9].)

Of course, our groups are really fundamental groups of arcwise connected spaces M. A *loop in M* is a mapping $f\colon S^1 \to M$. (A "mapping" is always continuous.) A *free homotopy* between loops is a homotopy that may not fix the basepoint. A loop in M determines a conjugate class of elements in the fundamental group $\pi_1(M)$, and it makes sense to consider whether the π_1-class of a loop belongs to a given normal subgroup of $\pi_1(M)$. To say that a loop in M is an *omegatator in M* (where M is an arcwise connected space) means that the π_1-class of the loop belongs to $\pi_1(M)_\omega$.

We sometimes write $\langle J, \pi_1(M^3)\rangle$ when J is a 1-sphere (or *simple closed curve*) in a 3-manifold M^3, to mean the smallest normal subgroup of $\pi_1(M^3)$ containing the π_1-class of a loop corresponding to J. A *2-handle* is a 3-cell with a product structure as $\Delta^2\times[-1, 1]$. To attach a 2-handle *along* a simple closed curve $J \subset \partial M^3$, we paste the 2-handle onto M^3 via a homeomorphism of $(\partial\Delta^2)\times[-1, 1]$ with a regular neighborhood of J in ∂M^3. Finally, Z denotes the infinite cyclic group; $*$ denotes free product; $\approx$ symbolizes homeomorphism of spaces; and $\cong$ symbolizes isomorphism of groups.

3. The continuum X. Our continuum X will be defined as the intersection of a nested sequence $M(1) \supset M(2) \supset \ldots$ in E^3 of compact 3-manifolds with connected boundary. Let $M(1)$ be the compact 3-manifold with connected boundary of genus two shown in Figure 1. This figure shows a 3-cell $T(0)$ and two linked, "spanning"

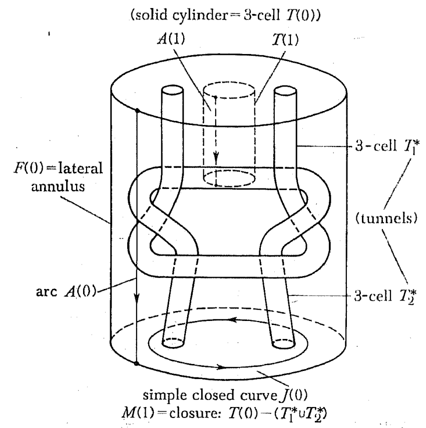


Fig. 1

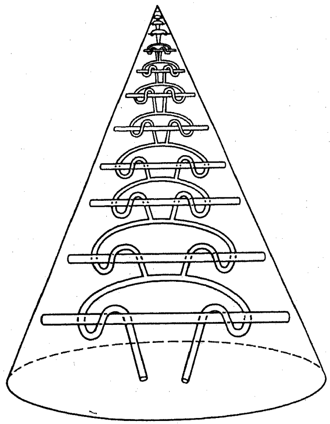
tunnels (3-cells) T_1^* and T_2^*. The closure of $T(0) - (T_1^* \cup T_2^*)$ is $M(1)$. It is shown in [15] that $\pi_1(M(1))$ is not a free group. (In fact, as shown algebraically in [15] and in Section 3 of [6], the simple closed curve $J(0)$ represents a non-trivial omegatator in $M(1)$. The fact that $J(0)$ represents an omegatator can also be seen geometrically from Figure 1, where $J(0)$ bounds a punctured torus in $M(1)$: There is a non-separating simple closed curve in this punctured torus that is freely homotopic in $M(1)$ to $J(0)$. By induction, it can be seen that $J(0)$ represents an element belonging to each term in the lower central series of $\pi_1(M(1))$.)

Some homeomorphism h of E^3 onto E^3 reduces to the identity outside a 3-cell containing $T(0)$, throws the 3-cell $T(0)$ onto the 3-cell $T(1)$, throws the oriented arc $A(0)$ onto the oriented arc $A(1)$, and throws the annulus $F(0)$ onto the closure of $\partial(T(1))-\partial(M(1))$. We can also choose h so that the diameter of $h^n(T(0))$ goes to zero as n goes to infinity. (We denote the n-fold composition of h by h^n.)

If $M(n-1)$ has been defined for some $n\geqslant 2$, we put $M(n)$ equal to the closure of

$$M(n-1)-h^{n-1}(T_1^*\cup T_2^*)\,.$$

Finally, let $X=\bigcap_{n=1}^{\infty} M(n)$ (see Fig. 2). It is clear that X is a locally connected continuum, and that E^3-X is connected. Note that X minus a point is a 3-manifold-with boundary.



X is: the solid right circular cone minus a network of tunnels

Fig. 2

We introduce some notation for later use. $M(n)$ contains some special annuli $F(0),\ldots,F(n-1)$ (defined by Figure 1 and by $F(i)=h^i(F(0))$ for $i>0$), some special arcs $A(0),\ldots,A(n-1)$ (defined by Figure 1 and by $A(i)=h^i(A(0))$ for $i>0$), and some special simple closed curves $J(0),\ldots,J(n-1)$ (defined by Figure 1 and by $J(i)=h^i(J(0))$ for $i>0$). We take $M_i(n)$ to be the closure of: $M(n)$ minus a thin regular neighborhood of $A(i)$ in $M(n)$ for $1\leqslant i\leqslant n-1$.

4. X is not movable. We first collect some facts for our proof.

FACT 1. *$M(n)$ is irreducible.*

Proof. $M(n)$ is irreducible because E^3 is irreducible and $\partial M(n)$ is connected.

FACT 2. *The annuli $F(1),\ldots,F(n-1)$ are disjoint and properly embedded in $M(n)$. Further, $F(i)$ splits $M(n)$ into a cube-with-handles H_i of genus $2i+1$ and a 3-submanifold $B_i(n)$ homeomorphic to $M(n-i)$. Hence, $M_i(n)$ is homeomorphic to the disk-sum of H_i and $B_i(n)$.*

Proof. These statements are easy to verify geometrically by drawing a few diagrams. We leave them to the reader.

FACT 3. *$J(0)$ is a non-trivial omegatator in $M(n)$ for each $n\geqslant 1$. Hence $\pi_1(M(n))$ is not free. Further, there is a homomorphism of $\pi_1(M(n))$ onto a free group of rank $2n$ whose kernel is precisely*

$$\langle J(0),\pi_1(M(n))\rangle=\pi_1(M(n))_\omega\,.$$

Proof. It has been remarked earlier (using [15]) that $J(0)$ is a non-trivial omegatator in $M(1)$. The neatest way to exhibit the desired homomorphism from $\pi_1(M(n))$ onto a free group of rank $2n$ is by means of a geometric construction. Namely, attach a 2-handle to $\partial M(n)$ along $J(0)$ and observe that the resulting 3-manifold has a fundamental group that is free of rank $2n$. We leave this pleasant sketching exercise to the reader. (Figure 3 may help: it shows $M(2)$ from a slightly different perspective.) The homomorphism described has the correct kernel (i. e., the left side of the displayed equation above), by van Kampen's theorem. By [6] Theorem 1, this kernel is also $\pi_1(M(n))_\omega$. Hence, $J(0)$ is an omegatator in $M(n)$.

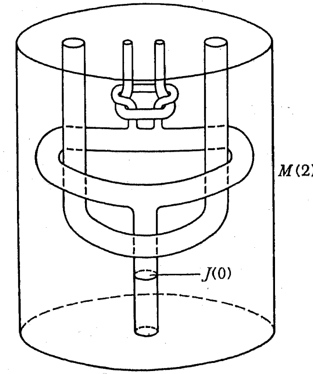


Fig. 3

To prove inductively that $J(0)$ is not contractible in $M(n)$, we need only observe that some retraction

$$M(n)\to B_1(n)\approx M(n-1)$$

takes $J(0)$ homeomorphically to $J(1)$. (In fact H_1, a cube-with-handles of genus 3, retracts onto $F(1)$ in the desired way since $J(1)$ generates a direct summand in the integral first homology of H_1.)

FACT 4. *The system of surfaces $F(1),\ldots,F(n-1)$ is incompressible and boundary-incompressible in $M(n)$.*

Proof. To show incompressibility, it must be proven that there is no 2-cell $D\subset M(n)$ with $D \cap \bigcup F(j) = \partial D$, a non-contractible simple closed curve in some $F(i)$. Any such 2-cell D would lie entirely in H_i or entirely in $B_i(n)$. It would then follow that attaching a 2-handle to ∂H_i (respectively, $\partial B_i(n)$) along ∂D would not affect its fundamental group. But (as can be seen geometrically) the result of attaching a 2-handle in this way to ∂H_i (respectively, $\partial B_i(n)$) is a 3-manifold homeomorphic to $M(i)$ (respectively, a 3-manifold with fundamental group free of rank $2(n-i)$). Since (Fact 3) the fundamental group of $M(j)$ is never free, no such 2-cell D can exist.

To show boundary-incompressibility of this system of annuli, it must first be noted that no component of any $\partial F(i)$ bounds a 2-cell in $\partial M(n)$. This is clear by examining the two pieces into which such a component separates $\partial M(n)$. (Neither is a 2-cell.)

Second, it must be shown that there is no 2-cell $D\subset M(n)$ with $D \cap \bigcup F_j$ equal to an arc $A\subset\partial D$ that is a spanning arc of some $F(i)$, with $(\partial D)-A\subset\partial M(n)$, and with $(\partial D)\cap \bigcup \partial F_j$ consisting of exactly two points. (We assume that A spans $F(i)$, since if ∂A is in one component of $\partial F(i)$, then the closure of one component of $F(i)-A$ is a 2-cell.) Such a 2-cell D would be properly embedded in H_i or properly embedded in $B_i(n)$. Then, the effect of attaching a 2-handle to ∂H_i (respectively, $\partial B_i(N)$) along a component of $\partial F(i)$ would be to replace H_i (respectively, $B_i(n)$) by a 3-manifold R^3 homeomorphic to H_i cut along D (respectively, $B_i(n)$ cut along D.) But (considering the first possibility: $D\subset H_i$) $\pi_1(H_i)$ is free, so $\pi_1(R^3)$ would be free. This is impossible, since it is clear geometrically that a 2-handle can be attached to ∂H_i along a component of $\partial F(i)$ so as to yield $M(i)$. Similarly, if it were true that $D\subset B_i(n)$ we would note that $\pi_1(B_i(n))$ is not free, so $\pi_1(R^3)$ would not be free. But this is absurd, since it is clear geometrically that a 2-handle can be attached to $\partial B_i(n)$ along a component of $\partial F(i)$ so as to yield a 3-manifold with free fundamental group. We conclude that the system of surfaces is boundary-incompressible.

FACT 5. *$M(n)$ is boundary-irreducible.*

Proof. It must be shown that for each properly embedded 2-cell $D\subset M(n)$, ∂D is contractible in $\partial M(n)$. We put D in general position with respect to $\bigcup_{j=1}^{n-1} F(j)$ and induct on k, the number of components of

$$D \cap \bigcup_{j=1}^{n-1} F(j).$$

If $k = 0$, then D is properly embedded either in $B_{n-1}(n)\approx M(1)$, or in some component of $M(n)-\bigcup F(j)$ homeomorphic to H_1 (a cube-with-handles of genus 3). In either case, we could conclude that $M(1)$ is boundary-reducible. (In the second case, attach a 2-handle to the component of $M(n)-\bigcup F(j)$ along a component of $\partial F(i)$ to see this.)

From the boundary-reducibility of $M(1)$, it would follow that $M(1)$ is the disk-sum of two compact 3-manifolds, each bounded by a torus. (Each is a "cube-with-a-possibly-knotted-hole".) Each such 3-manifold is well-known (see [5; Theorem 5]) to admit a mapping onto a cube-with-handles of genus 1 that induces a homeomorphism between their boundaries. (An equivalent concept is that of "boundary-retractability", see [6].) From this it would follow that $M(1)$ admits such a mapping onto a cube-with-handles of genus two. This is known to be false (see [8] and [6]).

If $k>0$, we construct an isotopy of $M(n)$ onto $M(n)$ that moves D off $\bigcup F(j)$. This is a routine argument: Use incompressibility of the $F(j)$'s and irreducibility of $M(n)$ to remove any simple closed curves in $D \cap \bigcup F(j)$; use boundary-incompressibility of the system of $F(j)$'s, plus irreducibility of $M(n)$, plus the "$k = 0$" case above, to remove any arcs in this intersection. We omit the details.

FACT 6. *The loop $J(0)$ is not freely homotopic in $M(1)$ to an omegatator in $M_i(n)$, for $n\geqslant 1$ and $1\leqslant i\leqslant n-1$.*

Proof. By Fact 2, $M_i(n)$ is topologically the disk-sum of a cube-with-handles of genus $(2i+1)$ and a 3-manifold $B_i(n)$. Further, some homeomorphism of $B_i(n)$ onto $M(n-i)$ takes $J(i)$ onto $J(0)$. By Fact 3 and the existence of this homeomorphism, some homomorphism of $\pi_1(B_i(n))$ onto a free group of rank $2(n-i)$ has kernel

$$\langle J(i), \pi_1(B_i(n))\rangle .$$

Hence, some homomorphism of

$$\pi_1(M_i(n)) \cong \pi_1(B_i(n)) * (\text{free group of rank } 2i+1)$$

onto a free group of rank $2n+1$ has kernel

$$\langle J(i), \pi_1(M_i(n))\rangle .$$

By [6] Theorem 1,

$$\langle J(i), \pi_1(M_i(n))\rangle = \pi_1(M_i(n))_\omega .$$

The last displayed equation, plus the fact that $J(i)$ lies in a 3-cell in $M(1)$, imply that the inclusion $M_i(n)\to M(1)$ induces a π_1-homomorphism that maps $\pi_1(M_i(n))_\omega$ trivially. The desired conclusion now follows from Fact 3.

FACT 7. *For $n\geqslant 1$ and $1\leqslant i<j\leqslant n-1$, no non-zero multiple of $J(i)$ homotopes in $M(n)$ into $F(j)$.*

Proof. Attach a 2-handle to $M(n)$ along $J(j)$ and note that some polyhedral, properly embedded 2-cell in the resulting 3-manifold splits it into two pieces. One of these admits a homeomorphism onto $M(j)$ that reduces to the identity on $F(i)$. Hence, by Fact 4 the conjugate class of elements in $\pi_1(M(n))$ determined by a multiple of $J(i)$ meets

$$\langle J(j), \pi_1(M(n))\rangle$$

trivially. This more than proves the above claim.

THEOREM. *The continuum X described above is not movable.*

Proof. Enlarge each $M(n)$ slightly by adding a collar to its boundary, to obtain a compact, polyhedral neighborhood $U(n)$ of X, so that each

$$U(n)\subset \operatorname{Int} U(n-1)\,.$$

and yet $X=\bigcap_{n=1}^{\infty} U(n)$. In particular, $U(n)$ is homeomorphic to $M(n)$. $U(n)$ contains some special annuli, arcs, etc. which correspond to those already described in $M(n)$. We use the same notation for these corresponding objects in $U(n)$, even though they are slightly different.

If X were movable, there would be a fixed integer $n\geqslant 1$ such that for each integer $k\geqslant 0$, some mapping f_k makes the diagram

$$\begin{array}{ccc} U(n) & \xrightarrow{f_k} & U(n+k) \\ & \searrow & \downarrow \\ & & U(1) \end{array}$$

homotopy-commutative. (The unlabeled arrows are inclusions.) We show that each choice of n must fail for sufficiently large k.

Let $n\geqslant 1$ be given. Let N be a finiteness number for $U(n)$. That is, each disjoint collection of N or more compact, properly embedded, incompressible and boundary-incompressible 2-manifolds in $U(n)$ contains a pair of 2-manifolds that are topologically parallel in $U(n)$. (See [3] and [4] for the proof that N exists. Cf. also [14].) We claim that no mapping f makes the diagram

$$\begin{array}{ccc} U(n) & \xrightarrow{f} & U(n+N) \\ & \searrow & \downarrow \\ & & U(1) \end{array}$$

homotopy-commutative.

If there were such an f, it could be assumed (possibly after a homotopy) that f is transverse with respect to the system of surfaces

$$F=\{F(1),\,\ldots,\,F(n+N-1)\}$$

in $U(n+N)$, and that each component of $f^{-1}(F)$ is properly embedded, incompressible and boundary-incompressible in $U(n)$. (We are using Fact 1 and the first part of Fact 4 in $U(n+N)$, plus Fact 5 in $U(n)$, and some standard constructions; see [13] Proposition, p. 60].) We claim that for some j, $1\leqslant j\leqslant n+N-1$, and for each component S (if there are any) of $f^{-1}(F(j))$, the homomorphism of fundamental groups induced by f

$$\pi_1(S)\to\pi_1(F(j))\,,$$

is trivial.

For, if this claim is false there is for each j, $1\leqslant j\leqslant n+N-1$, a component S_j of $f^{-1}(F(j))$ contradicting our assertion. Since $n+N-1\geqslant N$, some S_i is topologically parallel to some S_j in $U(n)$. By the way S_i was chosen, some loop k in S_i is mapped by f to a non-contractible loop in $F(i)$. Since S_i is parallel to S_j in $U(n)$, k freely homotopes in $U(n)$ to a loop in S_j. Applying f to this homotopy in $U(n)$ yields a homotopy in $U(n+N)$ that contradicts Fact 7. The claim follows.

Suppose j is such that for each component S (if any) of $f^{-1}(F(j))$, the induced homomorphism

$$\pi_1(S)\to\pi_1(F(j))$$

is trivial. Then f can be homotoped to a new map (still called f) differing from the old only in tight collar neighborhoods of each component of $f^{-1}(F(j))$, and such that the improved f maps a collar neighborhood of each component of $f^{-1}(F(j))$ to an arc piercing $F(j)$ at a single point in $F(j)-A(j)$. Thus, f has been replaced by a homotopic map of $U(n)$ into the subset

$$U(n+N)-A(j)\approx M_j(n+N)$$

of $U(n+N)$. Hence, by Fact 3, $f(J(0))$ represents an omegatator in $U(n+N)-A(j)$. By Fact 6, $J(0)$ cannot be homotopic in $U(1)$ to $f(J(0))$. Hence, our diagram is not homotopy-commutative. The proof is complete.

References

[1] K. Borsuk, *On movable compacta*, Fund. Math. 66 (1969), pp. 137–146.

[2] — *On a locally connected non-movable continuum*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. 17 (1969), pp. 425–430.

[3] W. Haken, *Some results on surfaces in 3-manifolds*, Studies in Modern Topology, Math. Assoc. Amer. (1968), pp. 39–98.

[4] — *Erratum for "Some results on surfaces in 3-manifolds"* (mimeographed notes).

[5] J. Hempel, *A surface in S^3 is tame if it can be deformed into each complementary domain*, Trans. Amer. Math. Soc. 111 (1964), pp. 273–287.

[6] W. Jaco and D. R. McMillan, Jr., *Retracting three-manifolds onto finite graphs*, Illinois J. Math. 14 (1970), pp. 150–158.

[7] J. Krasinkiewicz, *Continuous images of continua and 1-movability*, Fund. Math. (to appear).

[8] H. W. Lambert, *Mapping cubes with holes onto cubes with handles*, Illinois J. Math. 13 (1969), pp. 606–615.

[9] W. Magnus, A. Karrass and D. Solitar, *Combinatorial group theory: Presentations of groups in terms of generators and relations*, Pure and Appl. Math. 13, New York 1966.

[10] D. R. McMillan, Jr., *Cutting off homotopies on acyclic sets*, Proceedings of Utah Topology Conference (1974), pp. 343–352.

[11] — *One-dimensional shape properties and three-manifolds*, Proc. U. N. C. C. Topology Conference (1974), pp. 367–381.

[12] J. Stallings, *Homology and central series of groups*, J. Algebra 2 (1965), pp. 170–181.

[13] F. Waldhausen, *On irreducible 3-manifolds which are sufficiently large*, Ann. of Math. 87 (1968), pp. 56–88.

[14] A. Wright, *Mappings from 3-manifolds onto 3-manifolds*, Trans. Amer. Math. Soc. 167 (1972), pp. 479–495.

[15] E. C. Zeeman, *Linking spheres*, Abh. Math. Sem. Univ. Hamburg 24 (1960), pp. 149–153.

UNIVERSITY OF WISCONSIN-MADISON

Accepté par la Rédaction 26. 5. 1975