



Jan Mikusiński

Preface to the following paper

This paper is quite unusual. It is not a new paper. It has been published already (in Polish), but the size of the first edition and the circumstances of its publication were out of the ordinary. The edition was limited to seven copies, and the printing was made by the author himself by using X-ray films.

The matter will become clear when we add that the paper appeared in Cracow, in the year 1944, during the Nazi occupation of Poland. It also explains why the author conceals his identity under the initials J. G.-M., and why only initials appear in the acknowledgment at the end of author's introduction.

The time has come to solve this riddle. The letters J. G.-M. stands for Jan G.-Mikusiński (G, being the first letter of Geniusz, the author's patronymic, which he later abandoned). This name is well known nowadays in the world of mathematics. It belongs to an eminent Polish mathematician, the creator of the algebraic approach to operational calculus and the sequential approach to the theory of distributions, the author of numerous books and papers.

The paper "Hypernumbers" represents the first version of the Mikusiński operational calculus (improved afterwards by the use of the Titchmarsh theorem) and contains already the main ideas of this theory.

The date of the edition of the paper is 1944, but the results were already presented by the author in 1943, during one of the secret war-time seminars conducted by Professor Tadeusz Ważewski in Cracow. Its participants were the first persons to come in contact with this theory and to give their opinions to the author. Among them were: A. T. — Andrzej Turowicz, T. W. — Tadeusz Ważewski, J. W. — Jan Weyssenhoff and A. B. — Adam Bielecki, to whom the author pays thanks in the introduction. The first of them is now a professor of mathematics in Cracow and the last is a professor of mathematics in Lublin. Professor Tadeusz Ważewski and Professor Jan Weyssenhoff are no longer among the living.

Now, in 1983, forty years after this discovery, we have an excellent occasion to celebrate a double jubilee: the 70th birthday of Professor Jan Mikusiński and the 40th anniversary of the origin of the Mikusiński operational calculus. On this occasion, we offer the public the first paper on

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this theory. (English translation by Professor Henryk Minc of the University of California).

Presenting "Hypernumbers" in the volume of *Studia Mathematica* dedicated to Professor Jan Mikusiński, we would like to thank him for all the theories and theorems with which he has enriched Polish and world mathematics and for his constant pursuit of beauty, simplicity and elegance in mathematics.

We wish you, Dear Professor, good health and long years of activity to come.

Pupils

Hypernumbers

Part I. Algebra

by

J. G.-M. (Kraków)

Edition of 7 copies; July 1944

Introduction. In this article we introduce an analytic element composed of a complex number and a vector; we call it a "hypernumber". The fundamental difference between a hypernumber and a quaternion lies in differently defined multiplication; moreover, the vector space we consider has somewhat different properties from the one that plays a part in the theory of quaternions.

Hypernumbers find applications in the theory of integral and differential equations reducing certain problems of particular character to purely algebraic problems. By operating with a suitably chosen unit, computations have a very clear form, and some theorems, e.g., in the theory of Fredholm's equation become nearly obvious.

In an application to linear differential equations with constant coefficients computations appear almost identical to Heaviside calculus⁽¹⁾. Hypernumbers thus form a new algebraic basis for this calculus. It is conceptually simpler than theory of Laplace transformation in which Heaviside's calculus found for the first time a rigorous justification⁽²⁾.

As a particular interpretation of hypernumbers one could approximately regard also "commutative functions" (fonctions permutables), the theory of which was developed by Volterra and Péres. However, these authors have not defined accurately the concept of an element composed of a number and a vector; their functions play the part of vectors, and in

⁽¹⁾ Doetsch, p. 344. Heaviside calculus in its original form serves only for finding the integral satisfying initial "zero" conditions; whereas hypernumbers give the general solution.

⁽²⁾ Doetsch.