Electromagnetic multipole moments

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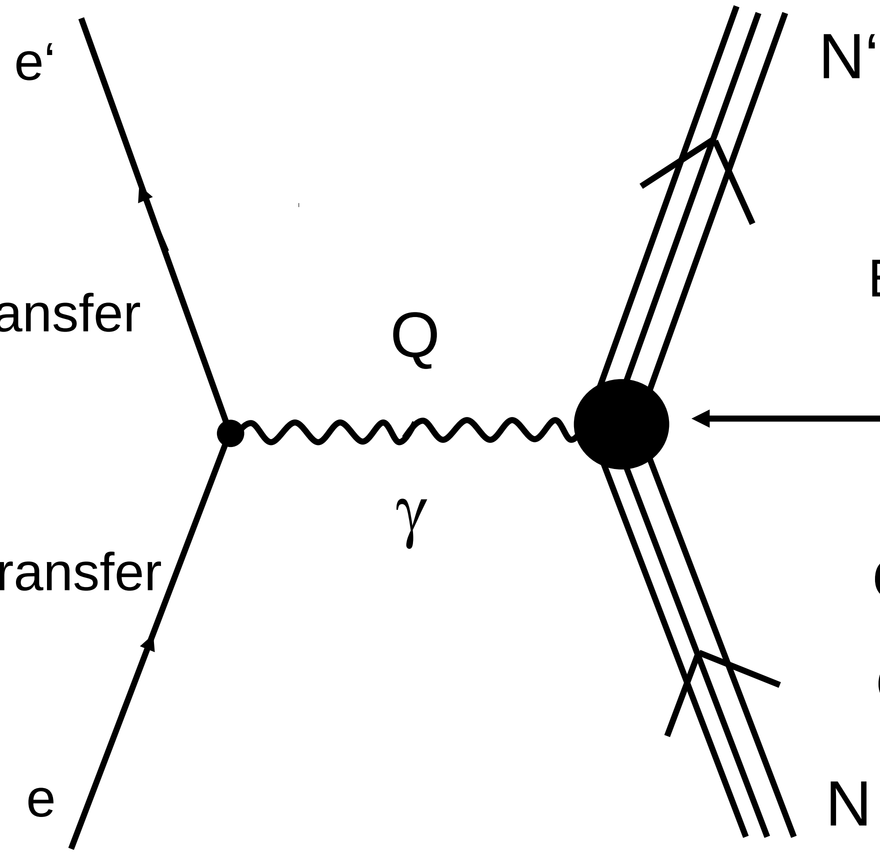
1. Introduction
2. Methods
3. Results
4. Summary

NSTAR 2017

NSTAR 2017, Columbia, 19-24 August 2017

1. Introduction

Elastic electron-nucleon scattering



Q... four-momentum transfer

$Q^2 = -(\omega^2 - \mathbf{q}^2)$

ω...energy transfer

q... three-momentum transfer

Elastic form factors

$$G^{N}_{C,M}(Q^2)$$

$G^{N}_{C}(Q^2)$ charge

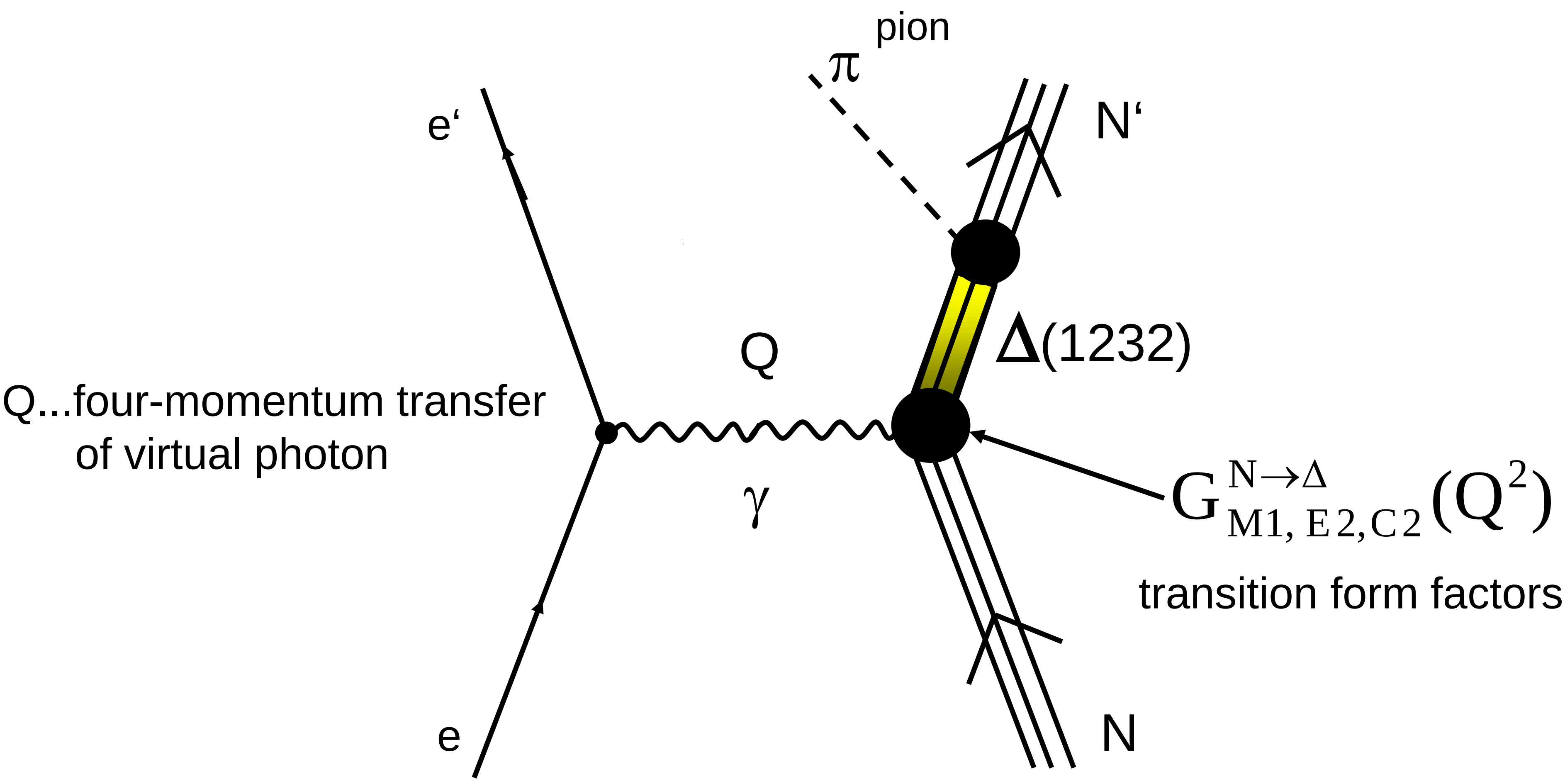
$G^{N}_{M}(Q^2)$ magnetic

e... electron

γ... photon

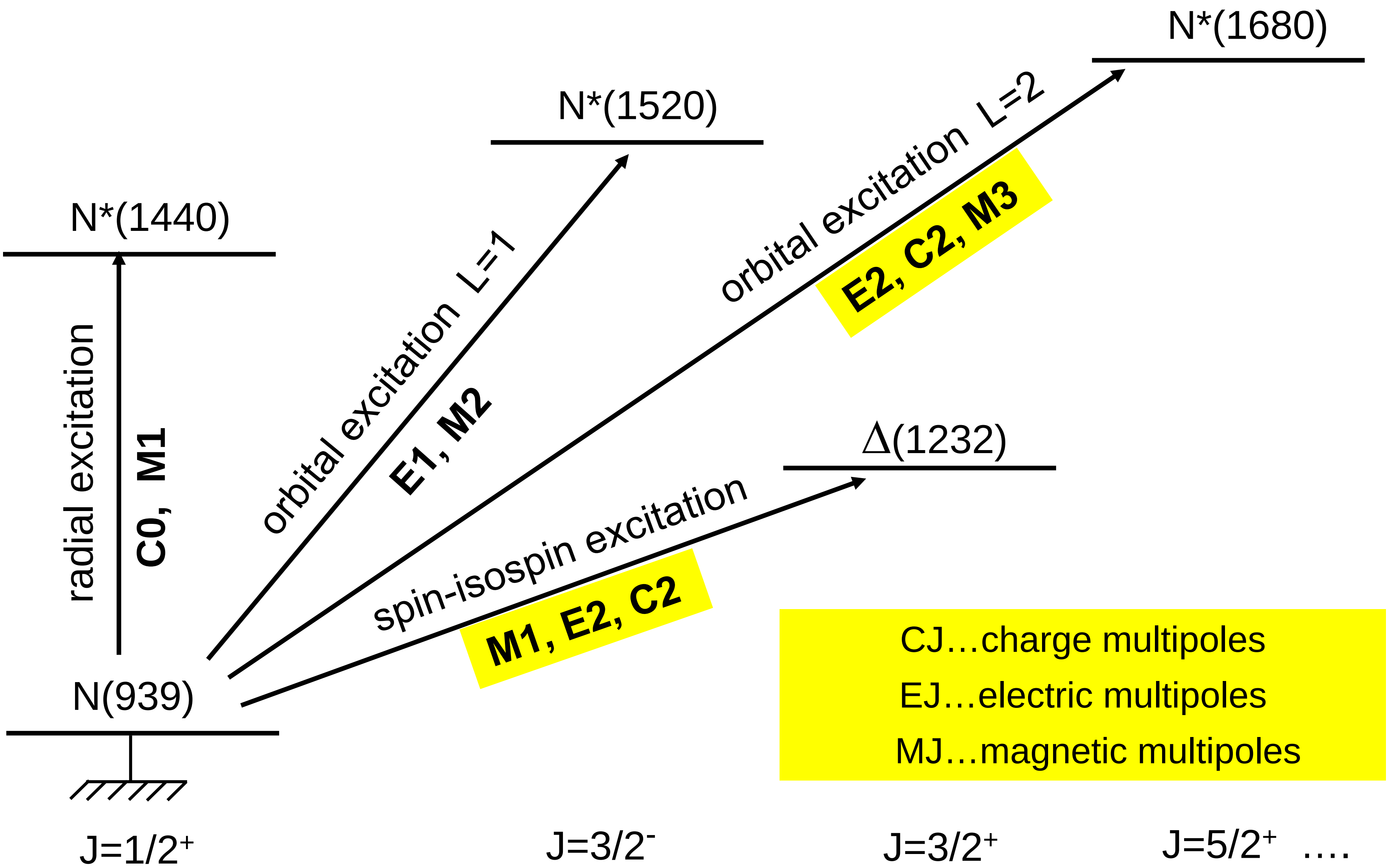
N... nucleon (p,n)

Inelastic electron-nucleon scattering



electro-pionproduction process

Nucleon excitation spectrum



Purpose of this talk

Explore relations

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between

nucleon ground state properties (elastic form factors)

and

transition multipole moments (inelastic form factors).

Electromagnetic multipole moments

What can we learn from

electromagnetic multipole moments?

Electromagnetic multipole moments are directly connected

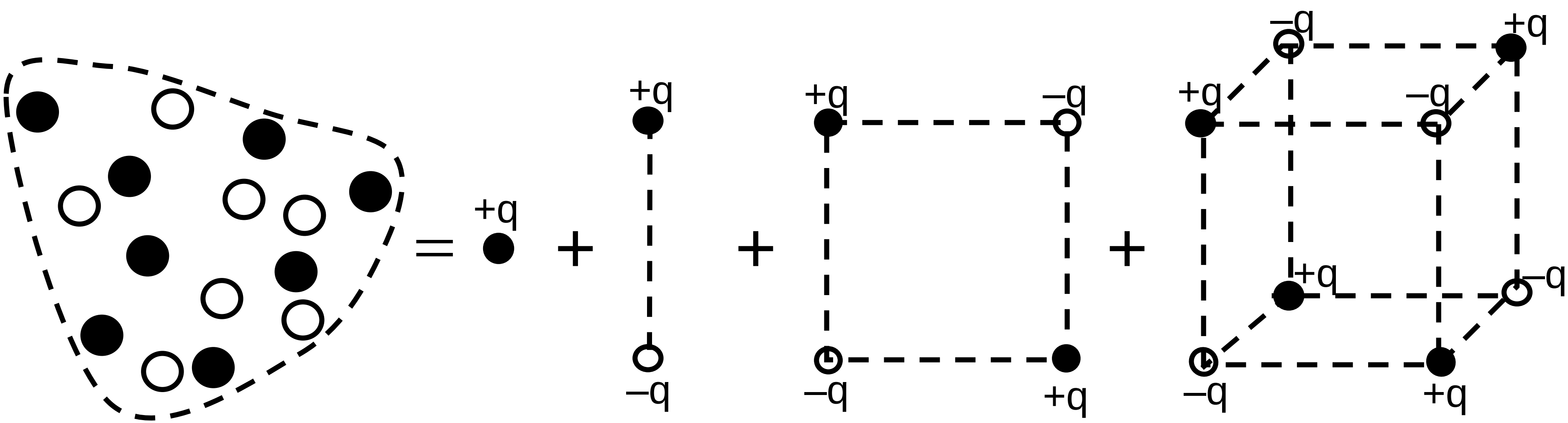
with the **charge and current distributions** in baryons.

Their sign and magnitude provide fundamental information on

- **structure,**
- **size,**
- **shape**

of baryons and their excited states.

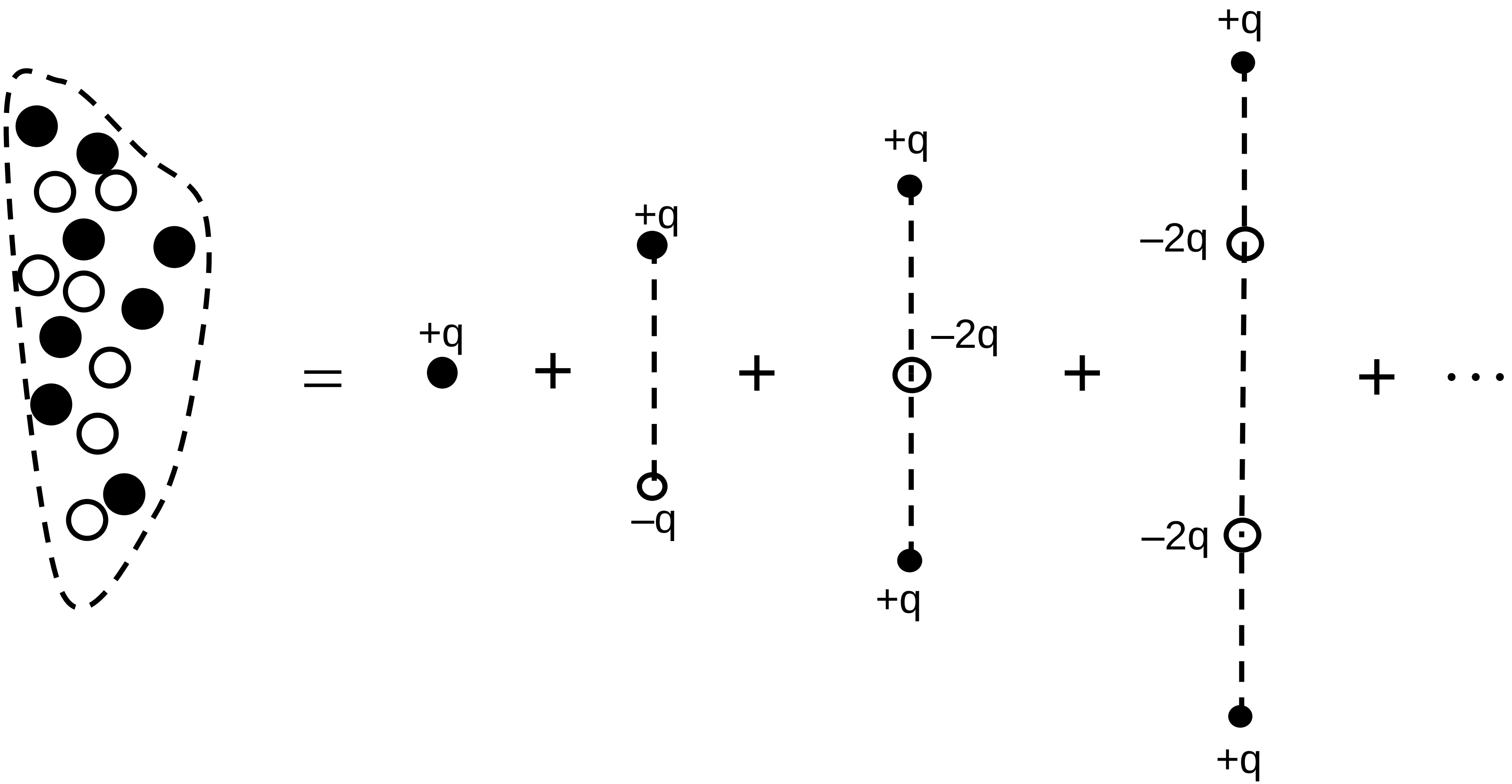
Multipole expansion of charge density ρ



$$\rho \sim q^0 + d^1 + Q^2 + \Omega^3 + \ldots$$

monopole	dipole	quadrupole	octupole
J=0	J=1	J=2	J=3

Another charge configuration



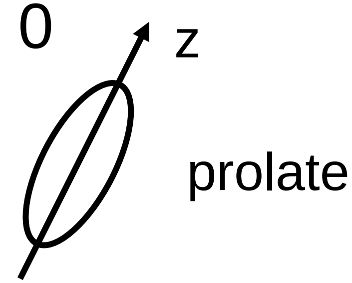
$$\rho \sim \mathbf{q} + \mathbf{d}^1 + \mathbf{Q}^2 + \mathbf{\Omega}^3 + \ldots$$

monopole dipole quadrupole octupole

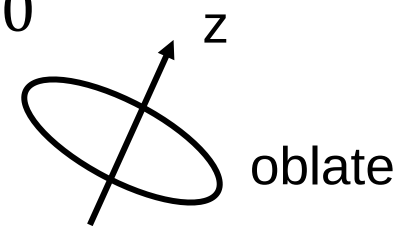
Quadrupole moment Q of baryon B

$$Q = \int \rho_B(\vec{r})\,(3z^2 - r^2)\,d^3\vec{r}$$

If ρ concentrated along z-axis, $3z^2$-term dominates $\rightarrow Q > 0$

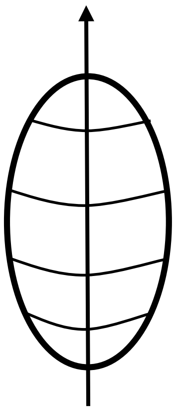


If ρ concentrated in x-y plane, r^2 -term dominates $\rightarrow Q < 0$



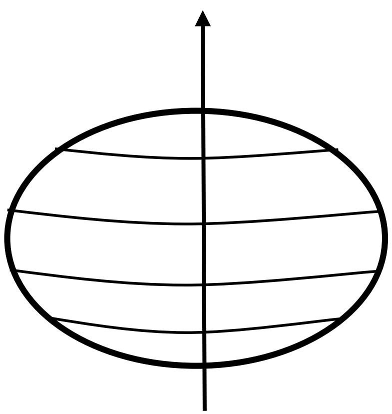
Magnetic octupole moment Ω

The magnetic octupole moment Ω measures the deviation of the magnetic moment distribution from spherical symmetry



$\Omega > 0$ magnetic moment density is prolate

$$\Omega := \frac{3}{8}\int d^3\vec{r}\ \left(\vec{r}\times\vec{J}(\vec{r})\ \right)_z (3z^2 - r^2)$$



$\Omega < 0$ magnetic moment density is oblate

2. Methods

Methods

1. SU(6) spin-flavor operator parameterization
2. $1/N_C$ expansion of operators
3. Current operator algebra

Methods are based on the symmetries of the QCD Lagrangian.

What symmetries are relevant for bound states of u,d,s quarks ?

1. SU(6) spin-flavor symmetry analysis

SU(6) spin-flavor symmetry combines SU(3) multiplets

with

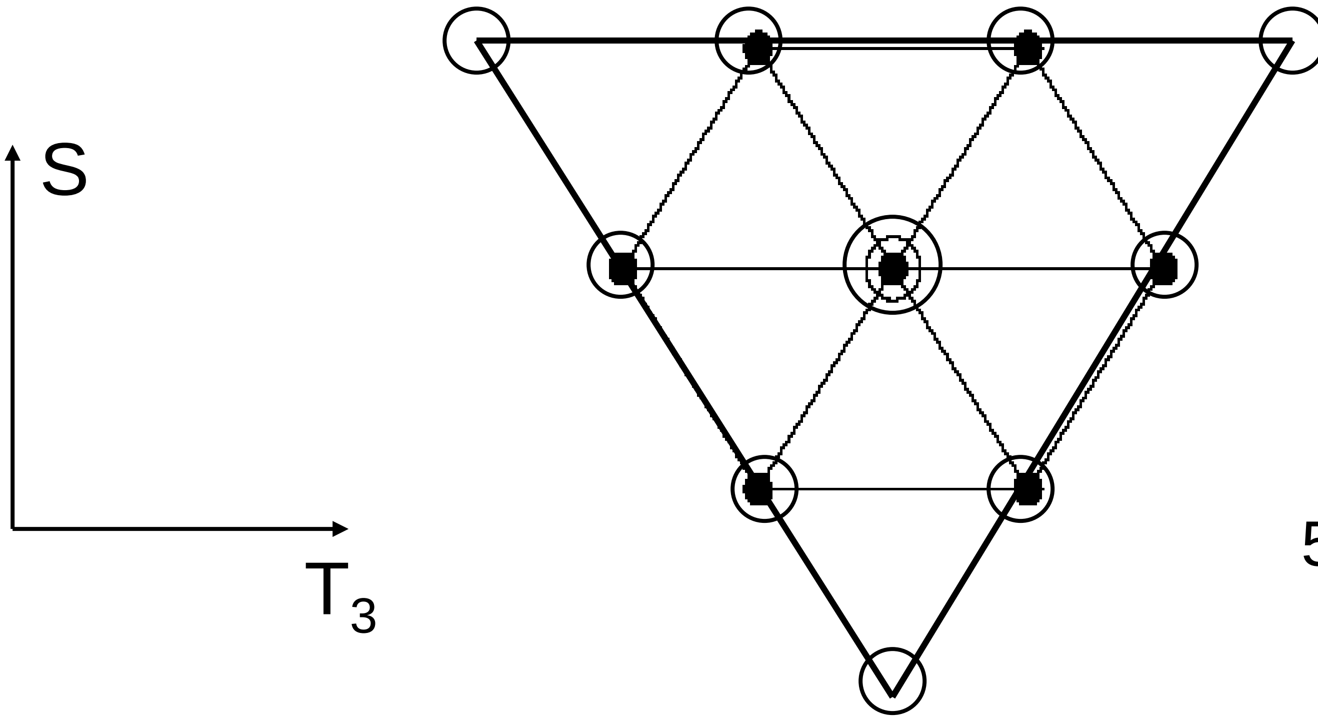
different **spin** and **flavor**

to

SU(6) spin-flavor supermultiplets.

Gürsey, Radicati, Sakita (1964)

SU(6) spin-flavor supermultiplet



baryon
supermultiplet
56 dimensional

$$56 = (8\,,2) + (10\,,4)$$

flavor spin flavor spin

Gürsey-Radicati SU(6) mass formula

$$M = M_0 + M_1\,Y + M_2\left(T(T+1) - \frac{Y^2}{4}\right) + M_3\,J(J+1)$$

$Y \cdots$ hypercharge
$T \cdots$ isospin
$J \cdots$spin

SU(6) symmetry breaking term

$\vec{\sigma}_i \cdot \vec{\sigma}_j$

Relations between octet and decuplet baryon masses

e.g. $M_{\Xi^*} - M_{\Sigma^*} = M_{\Xi} - M_{\Sigma}$

General spin-flavor operator $\hat{O}$

$$\boxed{\hat{O} = \mathrm{A}\,\hat{O}_{[1]} + \mathrm{B}\,\hat{O}_{[2]} + \mathrm{C}\,\hat{O}_{[3]}}$$

one-body two-body three-body

Constants A, B, C are determined from experiment.

$\hat{O}_{[i]}$ … allowed operators in spin-flavor space $i = 1,2,3$

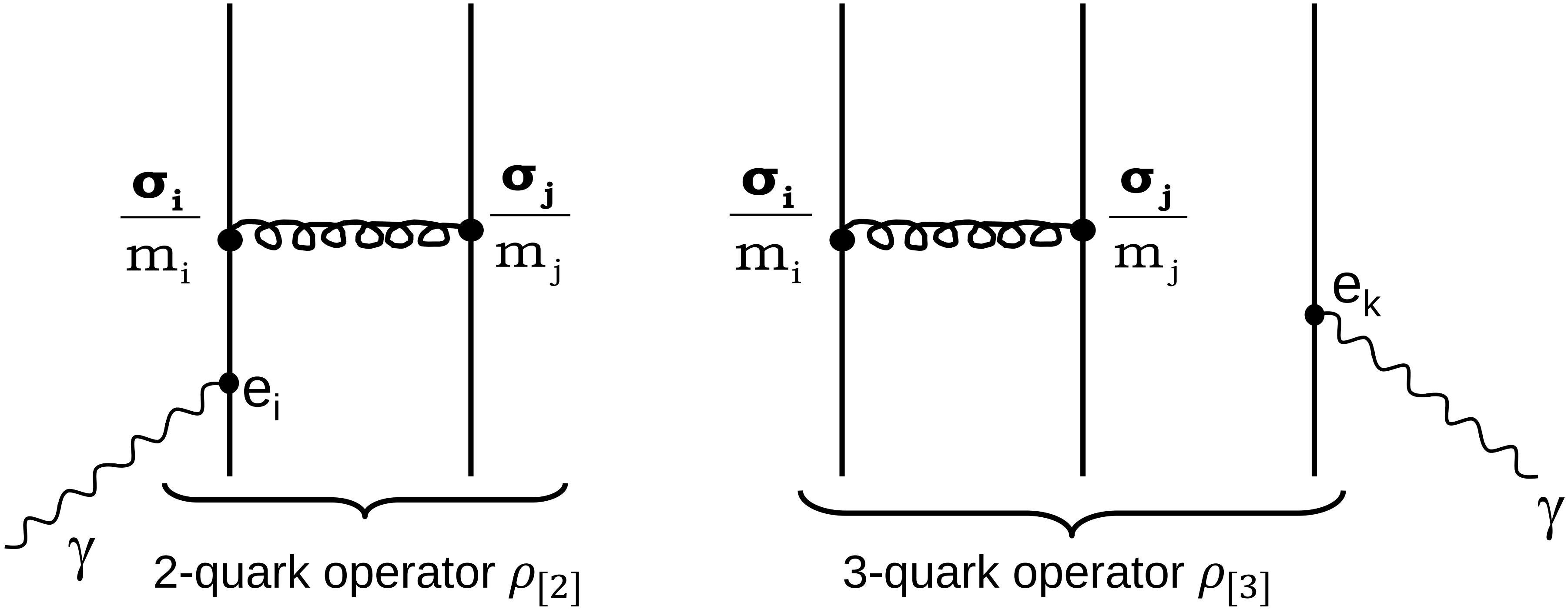
Which spin-flavor operators are allowed?

Operator structures determined from SU(6) group theory.

SU(6) spin-flavor symmetry breaking

e_i ... quark charge
σ_i ... quark spin
m_i ... quark mass

Example: charge operator $\hat{O} = \hat{\rho}$



SU(6) symmetry breaking via **spin** and **flavor dependent** two- and three-quark operators

SU(6) spin-flavor symmetry and its breaking

Connection between observables of different tensor rank J

e.g. charge radii ($J = 0$) and quadrupole moments ($J = 2$).

Example: Multipole expansion of $\rho_{[2]}$ in spin-flavor space

$$\rho_{[2]} = -B \sum_{i \neq j}^{3} e_i \left[2 \underbrace{\vec{\sigma}_i \cdot \vec{\sigma}_j}_{\substack{\text{scalar} \\ (\text{J=0})}} - \underbrace{\left(3\, \sigma_{iz}\, \sigma_{jz} - \vec{\sigma}_i \cdot \vec{\sigma}_j\right)}_{\substack{\text{tensor} \\ (\text{J=2})}} \right]$$

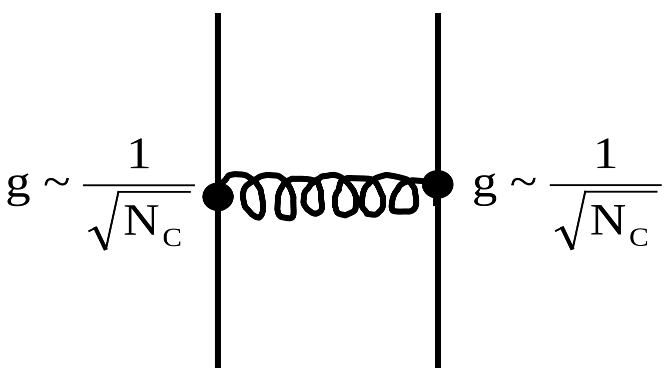
The prefactors of the spin scalar $(+2)$ and spin tensor (-1) terms are determined by the SU(6) group algebra.

2. Large N_C expansion of QCD processes

N_C ... number of colors

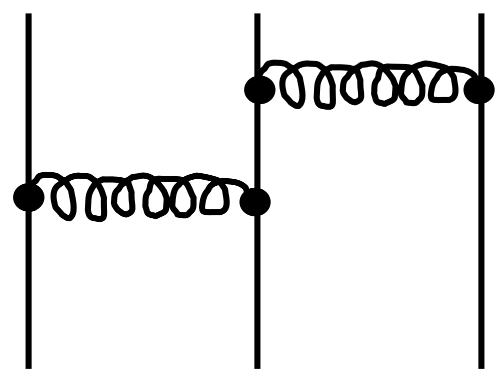
strong coupling $\alpha_S(Q^2) = \frac{g^2(Q^2)}{4\pi} = \frac{12\pi}{(11\,\boldsymbol{N_c} - 2\,N_f)\ln\left(\frac{Q^2}{\Lambda^2}\right)} \sim \frac{\mathbf{1}}{\boldsymbol{N_c}}$

two-quark operator



$O\left(\frac{1}{N_c^1}\right)$

three-quark operator



$O\left(\frac{1}{N_c^2}\right)$

. . .

Example: Quadrupole moment operator

$$\hat{Q}_{[2]} = \frac{B}{N_C} \sum_{i \neq j}^{N_C} e_i \left(3\,\sigma_{iz}\sigma_{jz} - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right)$$

two-body $\quad O\left(\frac{1}{N_C^1}\right)$

$$\hat{Q}_{[3]} = \frac{C}{N_C^2} \sum_{i \neq j \neq k}^{N_C} e_k \left(3\,\sigma_{iz}\sigma_{jz} - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right)$$

three-body $\quad O\left(\frac{1}{N_C^2}\right)$

Note: There is **no one-quark** contribution.

Reason: Cannot build $J = 2$ tensor $\hat{Q}$ from a single $J = 1$ matrix $\boldsymbol{\sigma}_i$.

Advantage of $1/N_C$ analysis

- Large N_C QCD provides a perturbative expansion scheme
- works at **all** energy scales
- **hierarchy** of quark operators due to powers of $1/N_C$

$$\hat{O}_{[1]} \quad > \quad \hat{O}_{[2]} \quad > \quad \hat{O}_{[3]}$$

one-quark	two-quark	three-quark
$O\left(1/N_C^0\right)$	$O(1/N_C^1)$	$O(1/N_C^2)$

$O(1/N_C^1)$ and $O(1/N_C^2)$: supressed

BUT: If $\langle \hat{O}_{[1]} \rangle = 0$ due to selection rules $\hat{O}_{[2]}$ dominant!

3. Current algebra method

- SU(3) generators λ_i satisfy commutation relations (Lie algebra)

$$\boxed{[\lambda_i, \lambda_j] = 2\, i\, f_{ijk}\, \lambda_k}$$

f_{ijk} ··· SU(3) structure constants i, j, k=1,…,8

- Gell-Mann Nishijima relation for electric charge Q

$$\boxed{Q = T_3 + \frac{Y}{2} = \frac{1}{2}\left(\lambda_3 + \frac{1}{\sqrt{3}}\,\lambda_8\right)}$$

- Generalized electromagnetic current densities involve

$T_3 = \frac{1}{2}\lambda_3$, $T_+ = \frac{1}{2}(\lambda_1 + i\lambda_2)$, and $T_- = \frac{1}{2}(\lambda_1 - i\lambda_2)$

Therefore, they also obey SU(3) Lie algebra.

Electromagnetic current commutators

Electromagnetic vector currents obey SU(3) Lie algebra

$$\left[J_i^{\alpha}(\vec{r}),\ J_j^{\beta}(\vec{r}')\right] = i\, f_{\alpha\beta\gamma}\, \delta_{ij}\, \delta(\vec{r}-\vec{r}')\, J_0^{\gamma}(\vec{r}')$$

$J_i^{\alpha}(\vec{r}) \cdots$ four$-$vector current density, i=0, $\cdots$, 3

- i, j $\cdots$ Lorentz 4-vector indices
- α, β, γ $\cdots$ SU(3) flavor indices
- $f_{\alpha\beta\gamma}$ $\cdots$ SU(3) structure constants

$$\mu_p^2 = \frac{1}{6}\, r_p^2$$

Gell-Mann Dashen Lee (1965)

Advantage of current algebra method

Gell-Mann (1964)

No matter how badly SU(3) flavor symmetry is broken,

the SU(3) commutation relations between group generators

are an exact law of nature.

3. Results

N→ Δ transition quadrupole moment

$$\rho_{[2]} = -B \sum_{i \neq j}^{3} e_i \left[\underbrace{2\, \vec{\sigma}_i \cdot \vec{\sigma}_j}_{\substack{\text{scalar} \\ (\text{J=0})}} - \underbrace{\left(3\, \sigma_{iz}\, \sigma_{jz} - \vec{\sigma}_i \cdot \vec{\sigma}_j\right)}_{\substack{\text{tensor} \\ (\text{J=2})}} \right]$$

neutron charge radius

$$r_n^2 = \left\langle 56_n \middle| \rho_{[2]}^{J=0} \middle| 56_n \right\rangle = 4\, B$$

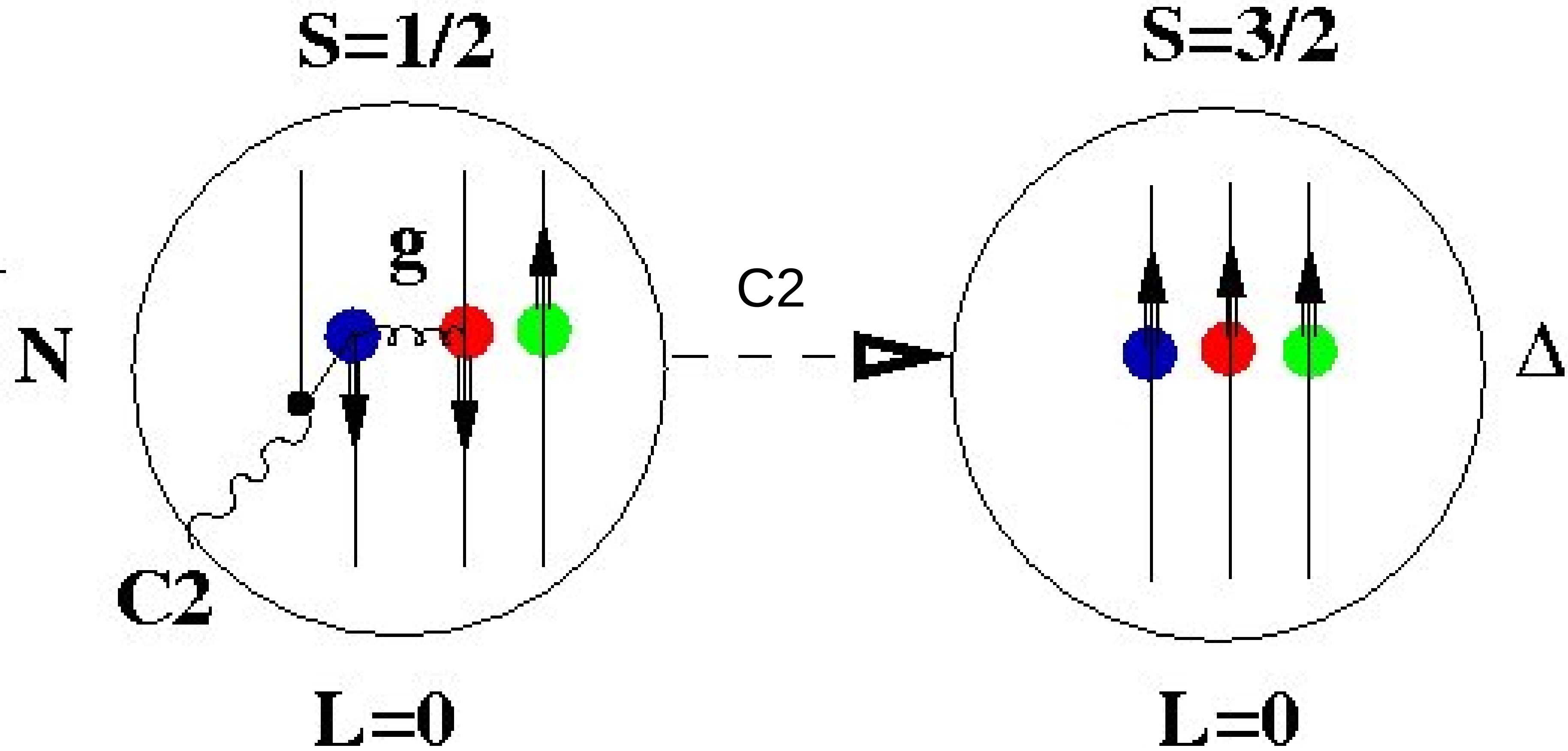
N→Δ transition
quadrupole moment

$$Q_{p\to\Delta^+} = \left\langle 56_{\Delta^+} \middle| \rho_{[2]}^{J=2} \middle| 56_p \right\rangle = 2\sqrt{2}\, B$$

$$Q_{p\to\Delta^+} = \frac{1}{\sqrt{2}} r_n^2$$

Buchmann, Hernandez, Faessler, PRC 55 (1997) 448

Double spin flip mechanism



Simultaneously flipping the spin of two quarks
via two-body exchange current

A. J. Buchmann, E. Hernandez, A. Faessler, Phys. Rev. C55 (1997) 448

$N \rightarrow \Delta$ transition quadrupole moment

Extraction of $p \rightarrow \Delta^+(1232)$ transition quadrupole moment from electron-proton and photon-proton scattering data

data

$$Q_{N\rightarrow\Delta}(exp) = -0.0846(33)\ \mathrm{fm}^2$$ Tiator et al., EPJ ST 198 (2011) 141

$$Q_{N\rightarrow\Delta}(\mathrm{exp}) = -0.108(9)\ \mathrm{fm}^2$$ Blanpied et al., PRC 64 (2001) 025203

theory

$$Q_{p\rightarrow\Delta^+} = \frac{1}{\sqrt{2}} r_n^2 = -0.0821(20)$$ AJB, Hernandez, Faessler, PRC 55, 448

neutron charge radius

N→ Δ quadrupole moment in $1/N_C$

$$Q_{p\to\Delta^+} = \left(\frac{B}{N_C} - 2\frac{C}{N_C^2}\right)\sqrt{\frac{(N_C+5)(N_C-1)}{2}}$$

Including 3-body operators

$$r_n^2 = \left(\frac{B}{N_C} - 2\frac{C}{N_C^2}\right)\frac{(N_C+5)(N_C+3)}{N_C}$$

$$Q_{p\to\Delta^+} = \frac{1}{\sqrt{2}} r_n^2 \underbrace{\frac{N_C}{N_C+3}\sqrt{\frac{N_C+5}{N_c-1}}}_{1}$$

for N_C=3 and N_C=∞

Indicative of a more general validity

AJB, J. Hester, R. Lebed, PRD 66, 056002 (2002)

$N \to \Delta$ form factor relations

$$G_{M1}^{p\to\Delta^+}(Q^2) = -\sqrt{2}\; G_M^n(Q^2)$$

$$\mu_{p\to\Delta^+} = -\sqrt{2}\;\mu_n$$

magnetic form factors
Beg, Lee, Pais, 1964

$$G_{C2}^{p\to\Delta^+}(Q^2) = -\frac{3\sqrt{2}}{Q^2}\, G_C^n(Q^2)$$

$$Q_{p\to\Delta^+} = \frac{1}{\sqrt{2}}\; r_n^2$$

charge form factors
AJB, Phys. Rev. Lett. 93 (2004) 212301

Definition of C2/M1 ratio

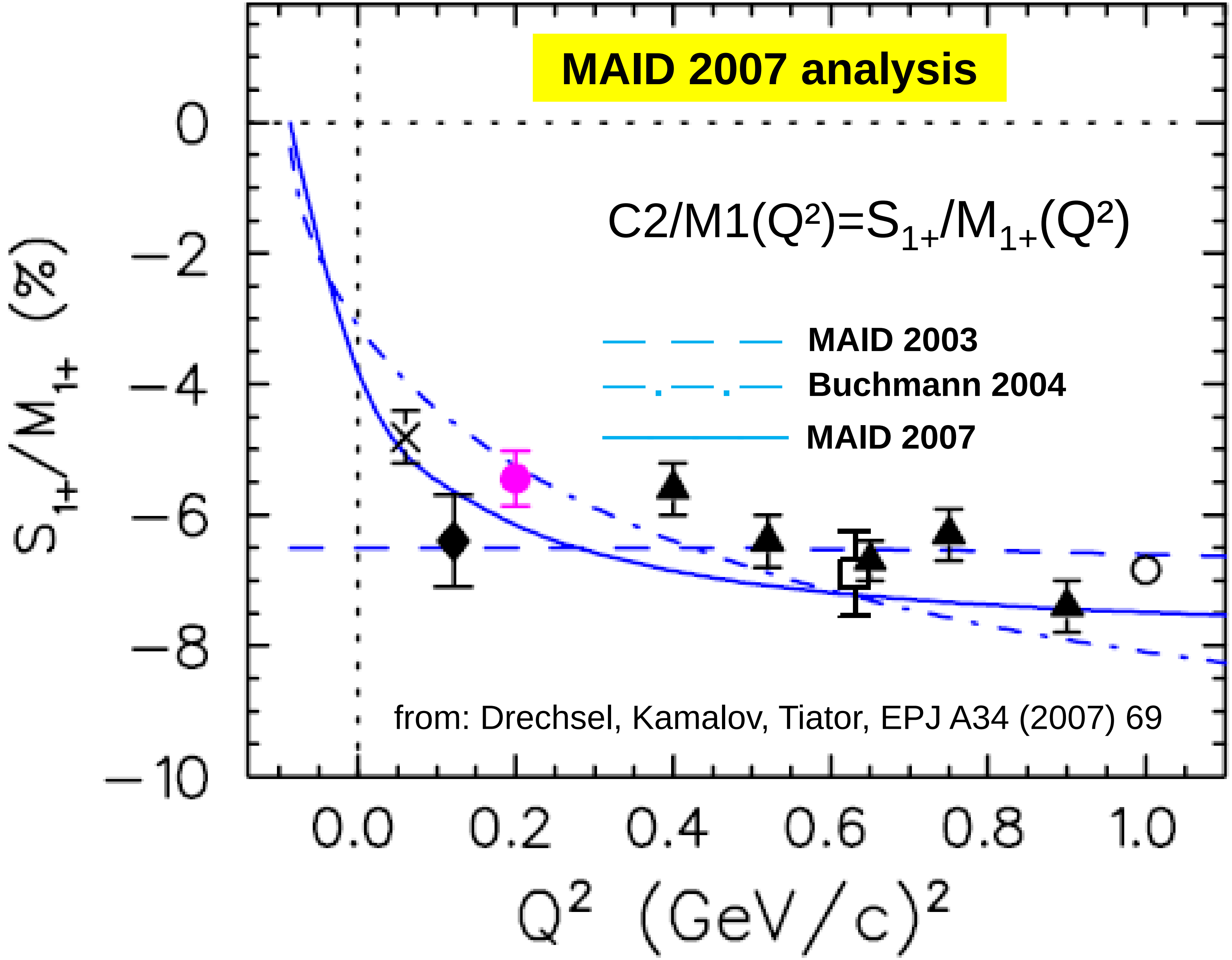
$$\frac{C2}{M1}(Q^2) = \frac{|\vec{q}|\, M_N}{6} \frac{G_{C2}^{p\to\Delta^+}(Q^2)}{G_{M1}^{p\to\Delta^+}(Q^2)}$$

Insert form factor relations

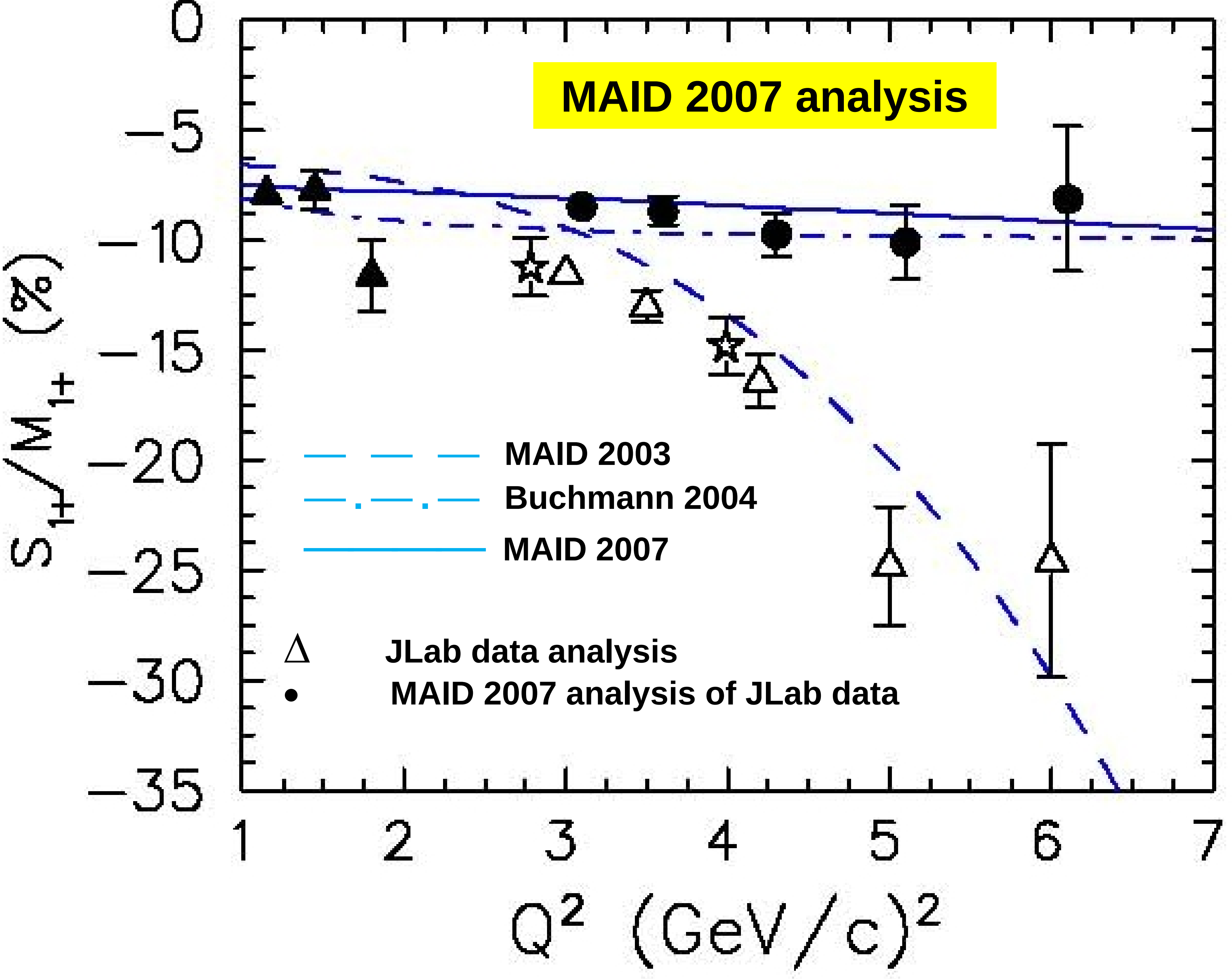
$$\frac{C2}{M1}(Q^2) = \frac{|\vec{q}|}{Q} \frac{M_N}{2Q} \frac{G_C^n(Q^2)}{G_M^n(Q^2)}$$

C2/M1 expressed via neutron elastic form factors

A. J. Buchmann, Phys. Rev. Lett. 93 (2004) 212301

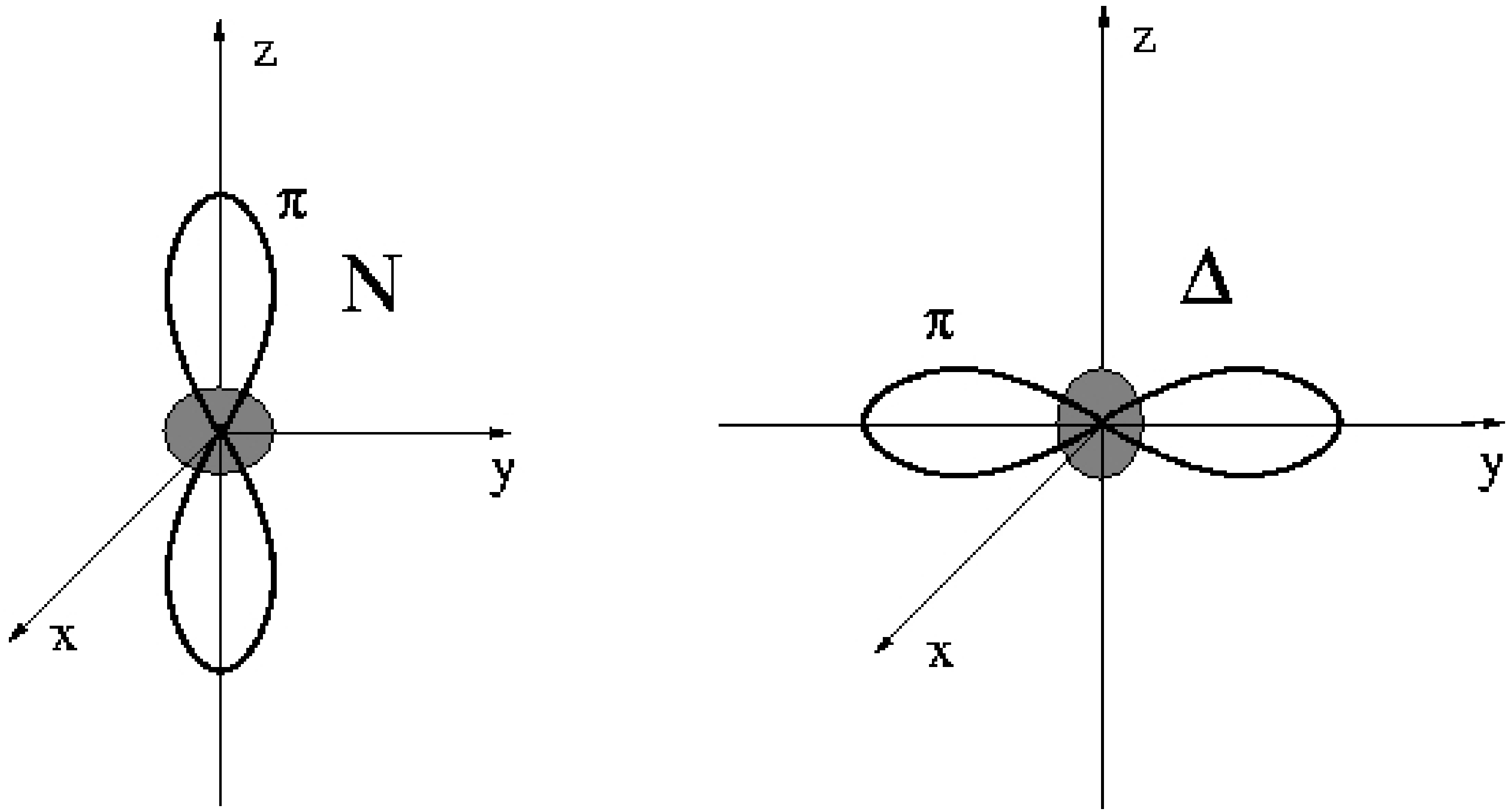


MAID 2007 analysis
C2/M1(Q²)=S1+/M1+(Q²)
MAID 2003
Buchmann 2004
MAID 2007
from: Drechsel, Kamalov, Tiator, EPJ A34 (2007) 69
0
−2
−4
−6
−8
−10
0.0
0.2
0.4
0.6
0.8
1.0
S1+/M1+ (%)
Q2 (GeV/c)2



MAID 2007 analysis
MAID 2003
Buchmann 2004
MAID 2007
Δ JLab data analysis
• MAID 2007 analysis of JLab data
S_{1+}/M_{1+} (%)
Q^2 (GeV/c)^2
0
−5
−10
−15
−20
−25
−30
−35
1
2
3
4
5
6
7

Interpretation in pion cloud model



$Q_0 > 0$ prolate

$Q_0 < 0$ oblate

A. J. Buchmann and E. M. Henley, Phys. Rev. C63, 015202 (2001)

Interpretation in quark model

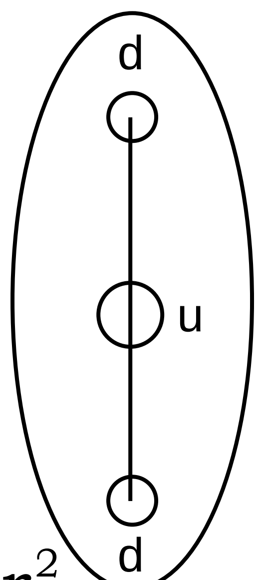
Two-quark spin-spin operators are repulsive
for quark pairs with spin 1.

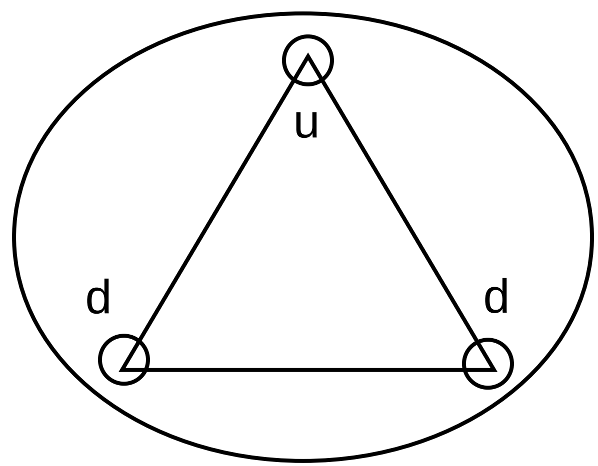
In the neutron, the two down quarks are in a spin 1 state, and are pushed further apart than an up-down pair.
$\rightarrow$ elongated (prolate) charge distribution
$\rightarrow$ negative neutron charge radius.

In the Δ^0, all quark pairs have spin 1. Equal distance between down-down and up-down pairs.
$\rightarrow$ planar (oblate) charge distribution
$\rightarrow$ zero charge radius.

prolate

n





oblate

Δ^0

$$Q_0(n) = -r_n^2$$

A. J. Buchmann, Can. J. Phys. 83 (2005)

$$Q_0(\Delta^0) = r_n^2$$

Quadrupole moment commutator

$$\left[Q_z^\alpha, Q_z^\beta\right] = i\, f_{\alpha\beta\gamma} \int d^3x\, (3\, z^2 - r^2)^2\, J_0^\gamma\, (\vec{x})$$

Evaluate between proton states

$$\left\langle p \middle| \left[Q_z^\alpha, Q_z^\beta\right] \middle| p \right\rangle = \langle p | i\, f_{\alpha\beta\gamma} \int d^3x\, (3\, z^2 - r^2)^2\, J_0^\gamma\, (\vec{x})\, |p\rangle$$

- Insert allowed excited states $\Delta(1232), N^*(1680), \cdots$ on lhs
- Note: The rhs is nonzero even though proton has no spectroscopic quadrupole moment!

$N \rightarrow N^*(1680)$ transition quadrupole moment

$$\underbrace{\langle p|Q_z^+|n(1680)\rangle\langle n\,(1680)|Q_z^-|p\rangle}_{2\,Q^2(p\rightarrow p(1680))_{IV}} = \underbrace{\frac{4}{5}\langle p|\int d^3x\ \ r^4\ J_0^3\ (\vec{x})\ |p\rangle}_{\frac{4}{5}r_{IV}^4(p)}$$

$$Q^2(p\rightarrow p(1680))_{IV} = \frac{2}{5}\,r_{IV}^4(p)$$

$$\boxed{Q(p\rightarrow p(1680))_{IV} = \sqrt{\frac{1}{5}\left(r_p^4 - r_n^4\right)}}$$

r_p^4 ⋯ 4th moment of the proton charge distribution

r_n^4 ⋯ 4th moment of the neutron charge distribution

Comparision with data

Comparision with data (in 10^{-3} GeV$^{-1/2}$) from PDG and MAID

$A_{1/2}(p) = -15 \pm 6,\ A_{3/2}(p) = 133 \pm 12,\ S_{1/2}(p) = -44$ (MAID)

$A_{1/2}(n) = 29 \pm 10,\ A_{3/2}(n) = -33 \pm 9,\ S_{1/2}(n) = 0$ (MAID)

Experiment: $Q(p \to p(1680))_{IV}(exp) = 0.11\ \mathrm{fm}^2$

Theory: $Q(p \to p(1680))_{IV}(theo) = 0.60\ \mathrm{fm}^2$

Using: $r_p^4 = 1.45\ \mathrm{fm}^4$ and $r_n^4 = -0.31\ \mathrm{fm}^4$ **(not well known),**

and only N*(1680) as intermediate state on commutator side

Agreement in sign but not in magnitude.

Measuring baryon magnetic octupole moments

Information on the shape of current distribution in baryons from **sign and magnitude** of Ω.

How can one measure Ω?

Best chance presumably by analyzing the $N \to N^*(1680)$ transition form factors.

$N \rightarrow N^{*}(1680)$ transition octupole moment

$$\Omega(p \rightarrow p(1680)_{IV} = \sqrt{\frac{3}{224}\left(r_p^6 - r_n^6\right)}$$

$r_p^6 \cdots 6$ th moment of the proton charge distribution

$r_n^6 \cdots 6$ th moment of the neutron charge distribution

Use: $r_p^6 = 4.78 \text{ fm}^6$ and $r_n^6 = -1.18 \text{ fm}^6$ **(not well known)**

and only N*(1680) on commutator side

Theory: $\Omega(p \rightarrow p(1680))_{IV}(theo) = 0.28 \text{ fm}^3$

Experiment: $\Omega(p \rightarrow p(1680))_{IV}(exp) = 0.12 \text{ fm}^3$ (PDG)

Agreement in sign but not in magnitude.

Decuplet magnetic octupole moments Ω

Baryon	$\boldsymbol{\Omega(r=1)}$	$\boldsymbol{\Omega(r\neq 1)}$
Δ^-	$-4C$	$-4C$
Δ^0	0	0
Δ^+	$4C$	$4C$
Δ^{++}	$8C$	$8C$
Σ^{*-}	$-4C$	$-4C(1+\mathrm{r}+\mathrm{r}^2)/3$
Σ^{*0}	0	$2C(1+\mathrm{r}-2\mathrm{r}^2)/3$
Σ^{*+}	$4C$	$4C(1+2\mathrm{r}-\mathrm{r}^2)/3$
Ξ^{*-}	$-4C$	$-4C(\mathrm{r}+\mathrm{r}^2+\mathrm{r}^3)/3$
Ξ^{*0}	0	$2C(2\mathrm{r}-\mathrm{r}^2-\mathrm{r}^3)/3$
Ω^-	$-4C$	$-4C\mathrm{r}^3$

$$r = \frac{m_u}{m_s}$$

SU(3) breaking parameter

A.J.B. and E.M. Henley, EPJA 35, 267 (2008)

Decuplet magnetic octupole moments Ω

Determine constant C in pion cloud model $(C = -0.003)$

$$\Omega(\Delta^{+}) = 4C = -0.012\ \mathrm{fm}^3$$

$$\Omega(\Omega^{-}) = -4C/r^3 = 0.003\ \mathrm{fm}^3$$

Comparison with other models:

$\Omega(\Delta^{+}) = -0.0035\ \mathrm{fm}^3$ (Ramalho, Peña, Gross, PLB 678, 355 (2009))

$\Omega(\Omega^{-}) = 0.016\ \mathrm{fm}^3$ (Aliev, Aziz, Savici, PLB 681, 240 (2009))

Models agree in sign but not in magnitude.

4. Summary

Relation between N and Δ form factors

$$G_{C2}^{p\rightarrow\Delta^+}(Q^2) = -\frac{3\sqrt{2}}{Q^2}\, G_C^n(Q^2)$$

N→ Δ charge quadrupole form factor

neutron charge form factor

AJB, Phys. Rev. Lett. 93 (2004) 212301

$N \to N^*(1680)$ transition moments

Relations between electromagnetic $p \to p^*(1680)$ transition moments and ground state properties

$$Q(p \to p(1680))_{IV} = \sqrt{\frac{1}{5}\left(r_p^4 - r_n^4\right)}$$

$$\Omega(p \to p(1680)_{IV} = \sqrt{\frac{3}{224}\left(r_p^6 - r_n^6\right)}$$

Outlook

- Improve current algebra calculation
- Calculate $N \to N^*(1680)$ multipoles using $1/N_C$ expansion
- Calculate quadrupole and octupole moment of $N^*(1680)$