Chapter 5

Kepler's Laws

5.1 Purpose

In this lab, we will investigate the properties of planetary orbits. The motion of the planets had intrigued people throughout history. Johannes Kepler found three empirical laws which were later explained scientifically by Issac Newton's law of gravity and laws of motion.

5.2 Introduction

Almost four centuries ago in 1619, Johannes Kepler published his three laws of planetary motion. These laws were empirical laws; that is, they were derived by examining the shape and speed of the planetary orbits without reference to any underlying physical theory. It wasn't until 1687 that Isaac Newton formulated his theory of gravitation and it was shown that Kepler's Laws are a direct consequence of Newton's Laws. For his analysis, Kepler used the observations of Tycho Brahe. Tycho Brahe was a late 16th Century Danish astronomer, whose measurements of the positions of the planets were made by naked eye, sighting along what was in effect a large sextant.

Kepler's Laws

1. The orbital paths of the planets are elliptical (not circular) with the Sun at one focus of the ellipse.

2. An imaginary line connecting the Sun to any planet sweeps out equal areas of the ellipse in equal time.

3. The square of a planet's orbital period is proportional to the cube of its semi-major axis.

The publication of Kepler's Laws created a revolution in cosmology. Firstly, he had unequivocally dethroned the 'perfect' circle as the basis of planetary orbits. Orbits based upon circles and epicycles had existed in one form or another since the ancient Greeks. And secondly, his second and third laws expressed a direct connection between the motion of a planet and its orbital position, giving birth to the science of orbital dynamics.

5.2.1 Kepler's Constant

Kepler's First Law states that the planets' orbits are described by ellipses with the Sun at one focus (please refer to figure 5.1, below). An ellipse appears as a somewhat flattened circle. If we place the Sun at the focus on the left, then the point in the orbit to the left, which is the planet's closest approach to the Sun, is called the *perihelion*. The farthest point in the orbit is called the *aphelion*.

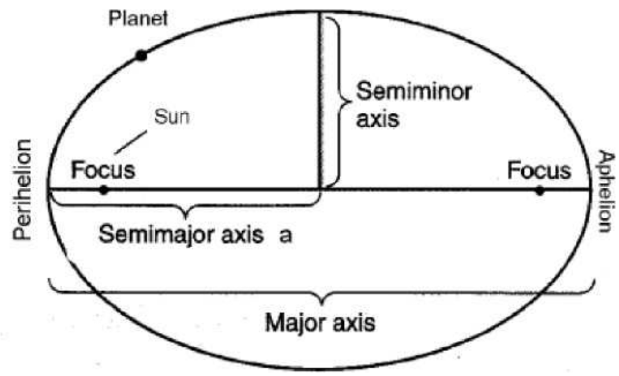

Figure 5.1: Geometry of an Ellipse

The semi-major axis is labeled **a**, and is the distance from the perihelion (or aphelion) to the center of the ellipse. The distance from the Sun's focus to the perihelion is equal to $a \cdot (1 - e)$, where e is the *eccentricity* of the ellipse, which is a measure of how 'flattened' different ellipses are. The distance from the Sun's focus to the aphelion is equal to $a \cdot (1 + e)$. The sum of these two distances is equal to **2a**, and is called the *major axis*.

Kepler's Third Law states that the square of a planet's orbital period, which we will call P, is proportional to the cube of its semi-major axis, a. We can write this in equation form as:

$$\frac{P^2}{a^3} = constant \tag{5.1}$$

Many people get confused about the nature of this constant, which we'll call κ. κ isn't a universal constant, like the speed of light or Newton's G. Rather, κ depends on the particular body that's being orbited (*e.g.*, the Sun). Indeed, we'll discover that the Sun has a unique κ (which we could call $\kappa_{\odot}$) that's different from Jupiter's $\kappa_{♃}$. It's like saying that you'd have a different weight on the Moon than you do on the Earth (in fact, κ is related to your weight!).

Isaac Newton later showed that the constant term is proportional to the inverse of the mass of the sun ($1/\text{Mass}_{\text{Solar Units}}$).

Often, non-standard units (Earth units) are used to calculate the value of this constant. In this system, the orbital period of a planet is measured in years. The distance from the Sun is measured in astronomical units (A.U.). In these units, the value of the constant is 1 $\frac{yr^2}{AU^3}$ since the Earth is one AU from the Sun, the Earth's orbit is nearly circular and the period is one year.

Kepler's grand achievement was in showing that the planetary orbits were best described by ellipses. A circle, however, is also a kind of ellipse with the two foci at the center, an eccentricity of zero, and a semi-major axis equal to the radius of the circle. All of Kepler's laws also hold for a perfectly circular orbit, since it is just a special kind of ellipse.

5.3 Procedure

First open the 'Planetary Orbit Simulator' software in a web browser. The software is on the astronomy lab web page. Go to software and then select 'Kepler'. When you open the software, you should see a window that looks like figure 5.2

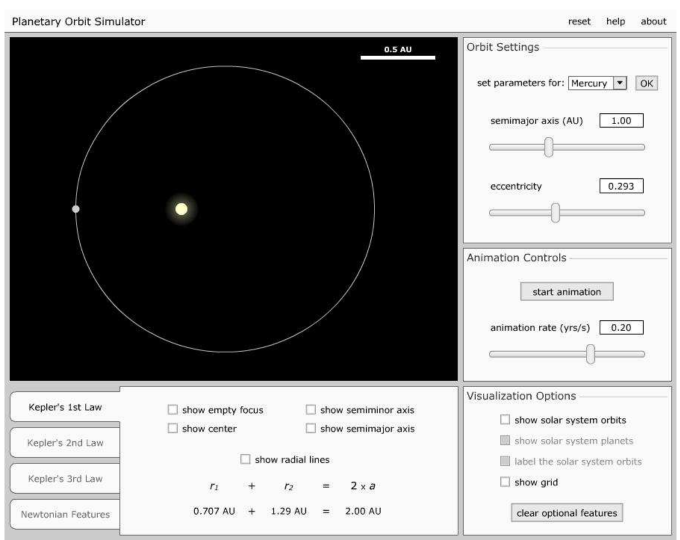

Figure 5.2: Planetary Orbit Simulator software from naap

5.3.1 Orbits around the Sun - Mercury and Mars

First we will examine the orbits of Mercury and Mars. These planets were chosen because their orbits are the most elliptical of the inner planets. The eccentricity of Mars' orbit is 0.093. Mercury's eccentricity is 0.206. By comparison, Venus's nearly-circular orbit has an eccentricity of 0.007.

1. Select the 'First Law' tab if it is not selected. Select 'Mercury' from the upper right pull down menu and click 'ok'. Check all five options (show center, show empty focus, show semimajor axis, show semiminor axis and show radial lines.) in the First law tab. Check the 'show grid' option in the lower right.

2. Place the pointer on the gray circle that represents the planet and hold the left mouse button down. You can drag the planet around to different positions around its orbit. Place the planet at its perihelion all the way to the left of the orbit.

3. In the First law tab, the values for r_1 (the distance to the Sun) and r_2 (the distance to the empty focus) are shown. Record the value for r_1 for Mercury's perihelion below. Move the planet to the its aphelion all the way to the right. Record the value for r_1 for Mercury's aphelion below.

   Mercury's perihelion ____________ Mercury's aphelion ____________

4. Using the relation perihelion $= a \cdot (1 - e) = a - ae$ and aphelion $= a \cdot (1 + e) = a + ae$, calculate the semimajor axis 'a'. (hint: add the two equations and the 'ae' term will drop out). Show your calculation below.

5. Plug the value for the semimajor axis back into either relationship and find the eccentricity. Does your calculated values from the aphelion and perihelion agree with the values given in the upper right of the 'Planetary Orbit Simulator'

6. Using the period of Mercury ($P_{☿}$) of 0.24 Earth years, calculate Kepler's constant below:

7. Select Mars from the pull down menu at the upper right and click 'ok'. Repeat the above procedure for Mars. The period of Mars ($P_{♂}$) is 1.88 Earth years.

   Mars' perihelion ____________ Mars' aphelion ____________

   Mars' Semimajor axis ____________ Mars' eccentricity ____________

   Kepler's constant for Mars____________

5.3.2 The Moons of Jupiter

A short time after hearing of its invention in Holland in the early 1600s, Galileo Galilei constructed a telescope and discovered the four largest satellites of the planet Jupiter. In this part of the exercise we will examine their orbital motions. Since the orbits of these moons of Jupiter have very small eccentricities, the average distance from Jupiter can be used as the semimajor axis.

1. A table of values (Table 5.1)for Galilei's moons of Jupiter is shown below. The data is given in kilometers and earth days so the units for Kepler's constant will be day^2/km^3.

2. Fill in the table using Kepler's 3rd law.

Moon	ave distance (km)	Period (days)	Kepler's Constant (κ)
Io	4.21 x 10^5	1.769	
Europa	6.71 x 10^5	3.551	
Ganymede	1.07 x 10^6	7.155	
Callisto	1.88 x 10^6		4.17 x 10^{-17} day^2/km^3

Table 5.1: Data for the Moons of Jupiter,

5.3.3 Kepler's second law

In this section, we will explore some features of Kepler's 2^{nd} law.

1. Click the 'clear Optional Features' and select the 'Kepler's 2nd law' tab.

2. Set the semimajor axis to 1 AU and the eccentricity to 0.5 by using the slider or typing the values into the appropriate boxes.

3. Click the 'Start Animation' button and check the 'start sweeping'. Click the 'pause animation' button.

4. Place the pointer over the sweep segment, click and hold the left mouse button and drag the sweep segment around the orbit.

5. Where is the sweep segment the thinnest? Where is it the widest? Where is the planet when it is sweeping out each of these segments? What names do astronomers use for these positions?

5.3.4 Questions

1. Convert the value of Kepler's constant for objects around the Sun (≈ 1 yr^2/AU^3) from $\frac{yr^2}{AU^3}$ to SI units of $\frac{s^2}{m^3}$. 1 AU = 1.496 x 10^{11} meters

2. We can combine Newton's Laws with Kepler's Laws to get:

$$\kappa = \frac{P^2}{a^3} = \frac{4\pi^2}{GM},$$

   where G is the Universal Gravitational constant, $G = 6.67 \times 10^{-11} m^3/(s^2 \cdot kg)$, and M is the mass of the Sun (or whatever is being orbited). Using your result from Question 1, solve this equation for M. How does this compare with the accepted value of $M = 1.99 \times 10^{30} kg$? Calculate the % error.

3. The period of Halley's comet is 75.3 years. What is the semimajor axis (a) for Halley's comets highly elliptical orbit?

4. Also from Sir Isaac's Laws, we can derive the speed of an object in orbit around the Sun at any place using

$$v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)},$$

   where r is the distance from the Sun and a is the semi-major axis.

(a) Convert your value of a for **Halley's Comet** from AU to m. The aphelion for Halley's comet is 35.1 AU and the perihelion is 0.586 AU. Convert the aphelion and the perihelion into m.

(b) Calculate the speed of Halley's comet at its perihelion and aphelion.

5.4 Conclusion

Write a conclusion about what you have learned. Include all relevant numbers you have measured with errors. Sources of error should also be included.