

# VII

## Exemplos de

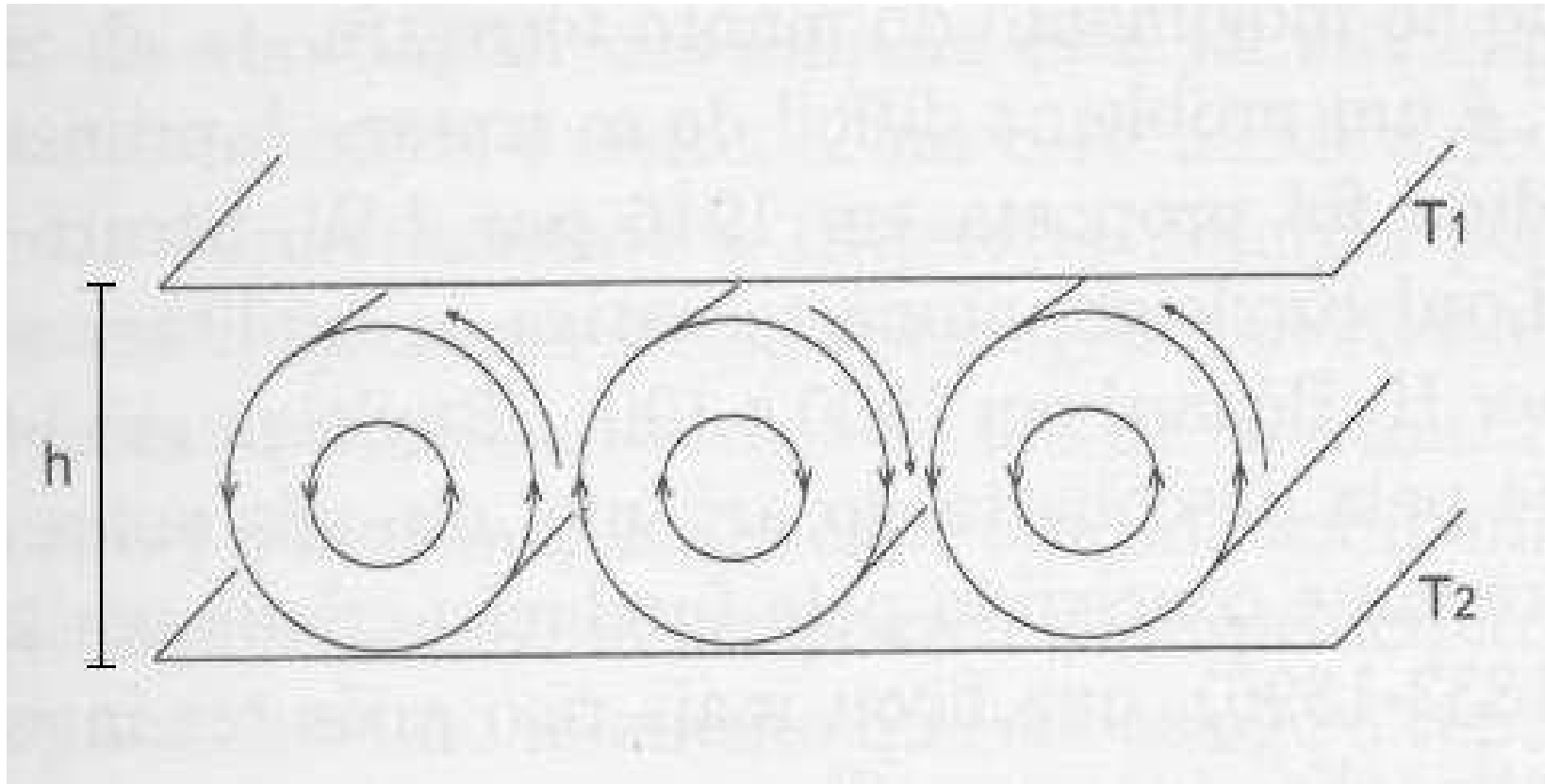
# Atratores Estranhos

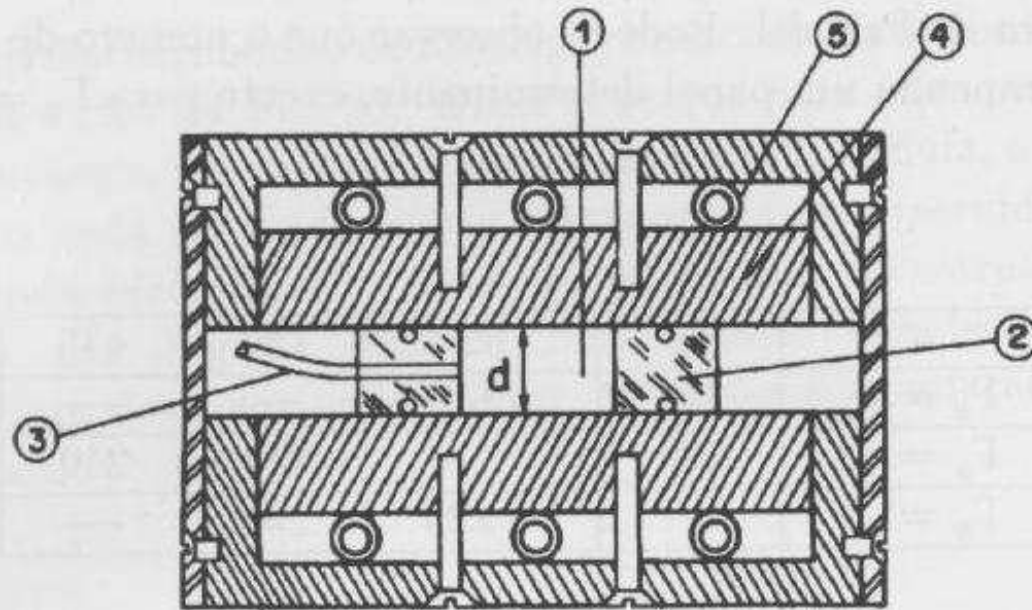
VII – C

Sistema de Lorenz

Primeira Parte

Convecção de  
Rayleigh-Bernard





Esquema do dispositivo para estudo da convecção de Rayleigh-Bénard. 1 - Cavidade do fluido, 2 - peças de "plexiglass" que definem a cavidade, 3 - tubo para introdução do fluido, 4 - placas de cobre, 5 - tubos para água (banho termostático).

Equação de Navier-Stokes

$$\rho \frac{d\vec{v}}{dt} = \vec{F} - \vec{\nabla} p + \mu \nabla^2 \vec{v}$$

$$\frac{dT}{dt} = \kappa \nabla^2 T$$

Equação de Condução do Calor

Equação da continuidade

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

## Equações de Lorenz

$$\frac{dX}{dt} = -\sigma(X - Y)$$

$$\frac{dY}{dt} = rX - Y - XZ$$

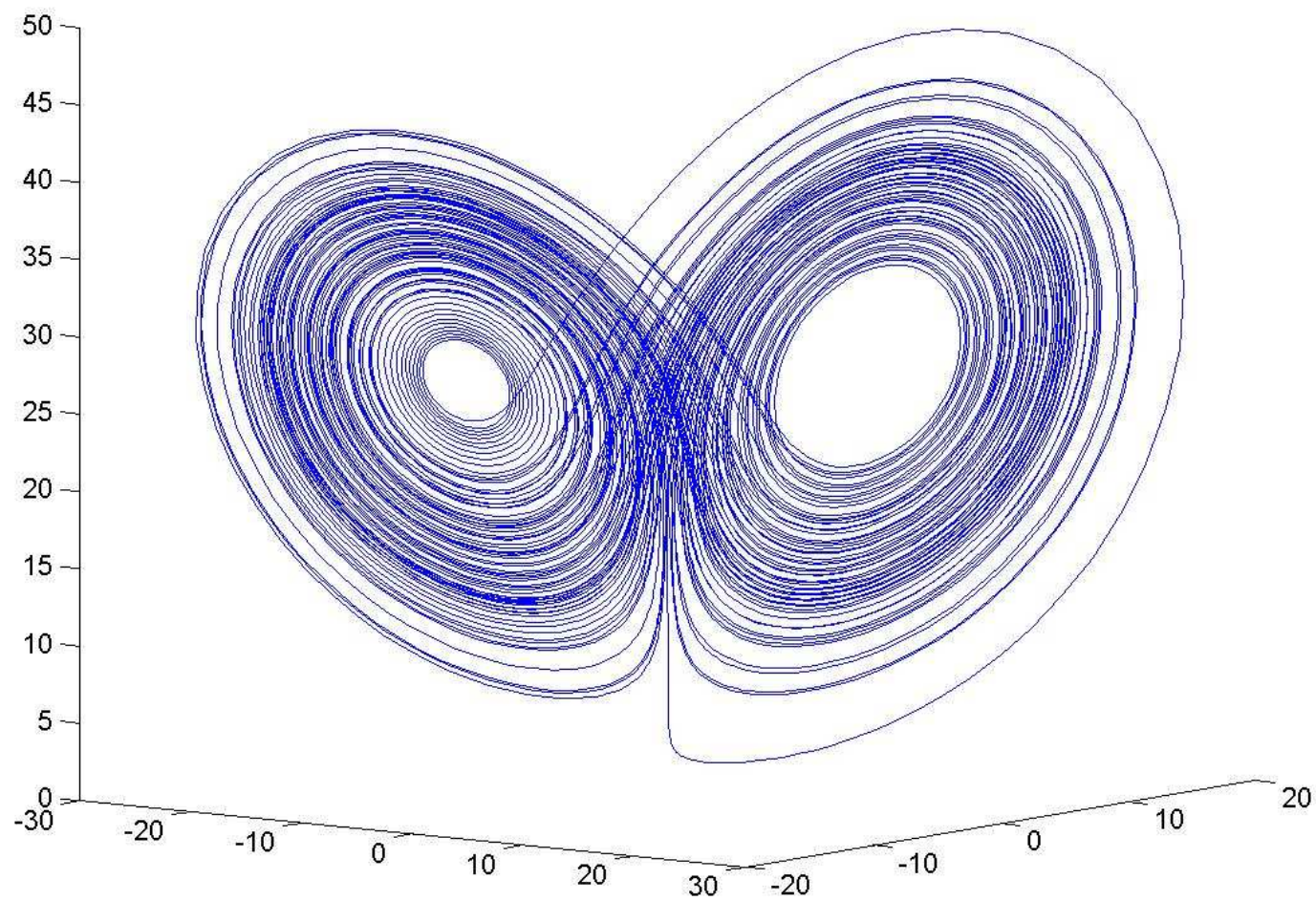
$$\frac{dZ}{dt} = XY - bZ$$

$X$  é proporcional à intensidade da convecção.  $X=0$  implica que não há movimento convectivo, ou seja, o calor é transportado apenas por condução.  $X>0$  implica circulação horária e  $X<0$  circulação anti-horária.

$Y$  é proporcional à diferença de temperatura entre as correntes de fluido ascendente e descendente.

$Z$  é proporcional à distorção do perfil de temperatura vertical, relativamente a um perfil linear. Para  $Z=0$ , a temperatura decresce linearmente.

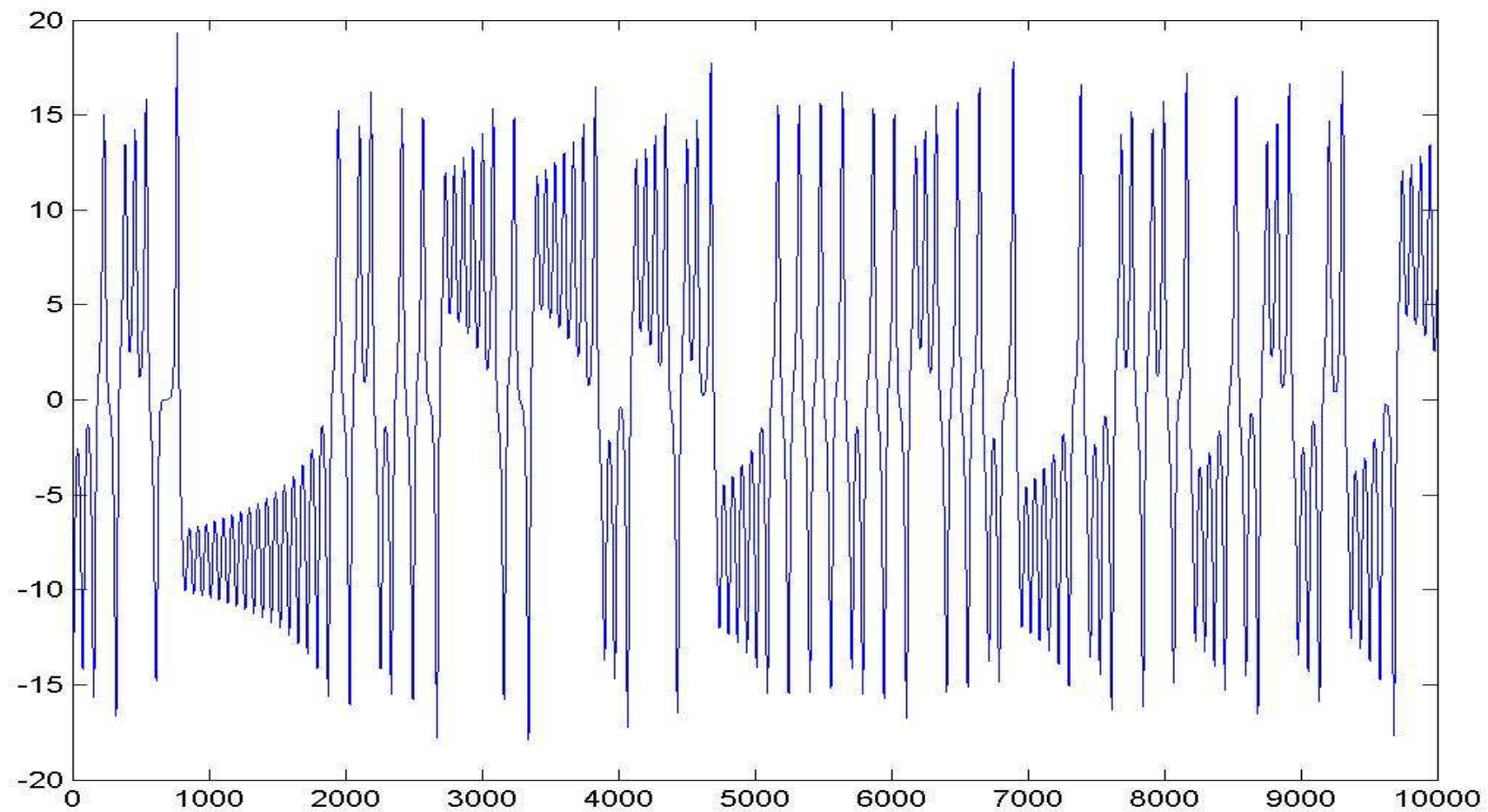


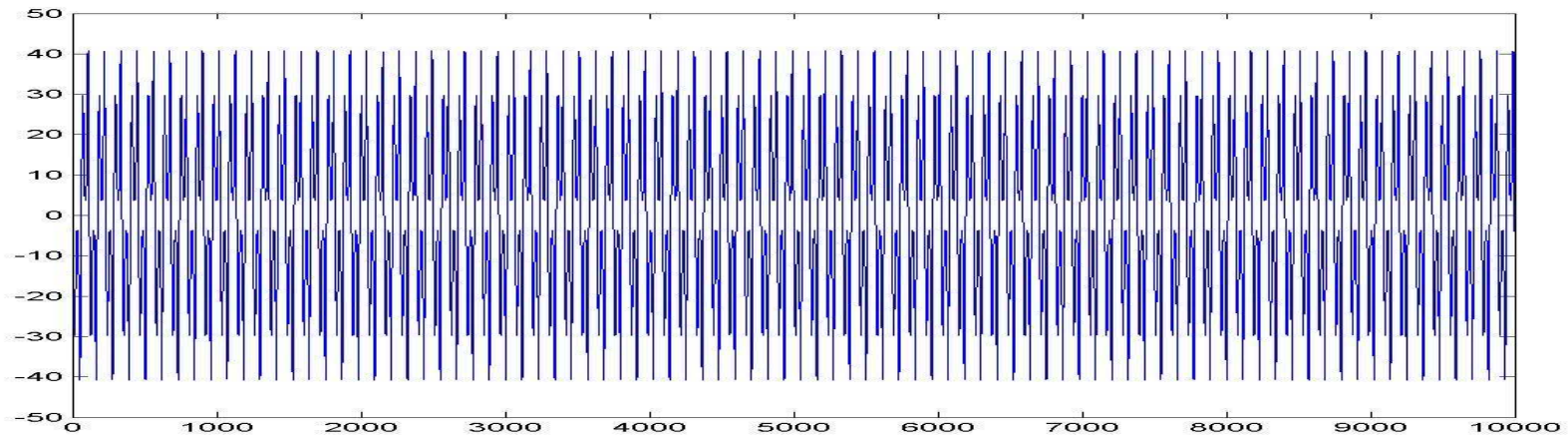


$\sigma = 10$

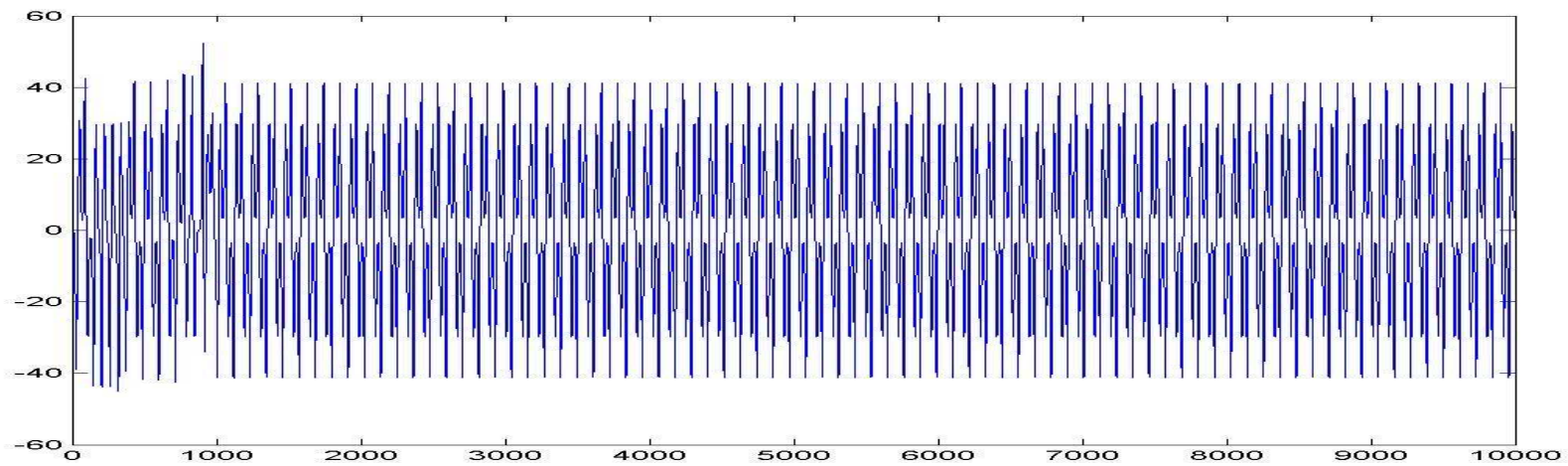
$b = 8/3$

$r = 28$

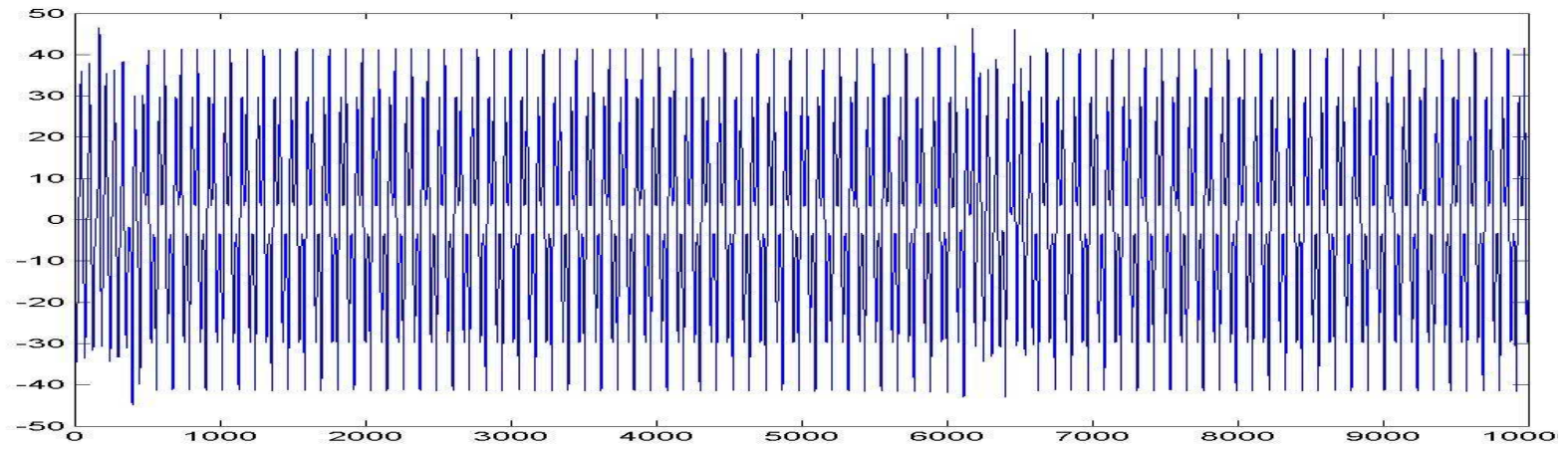




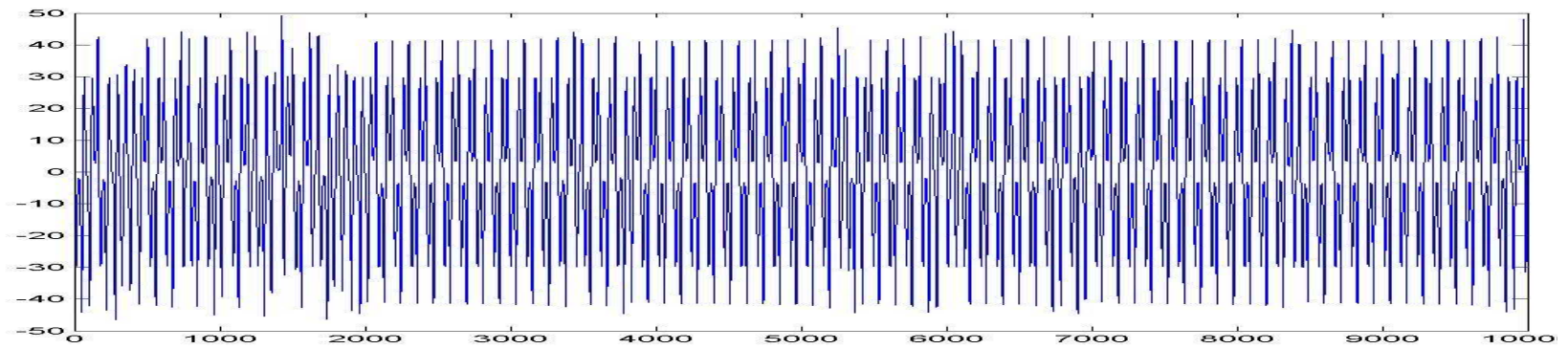
$r=165$



$r=166$

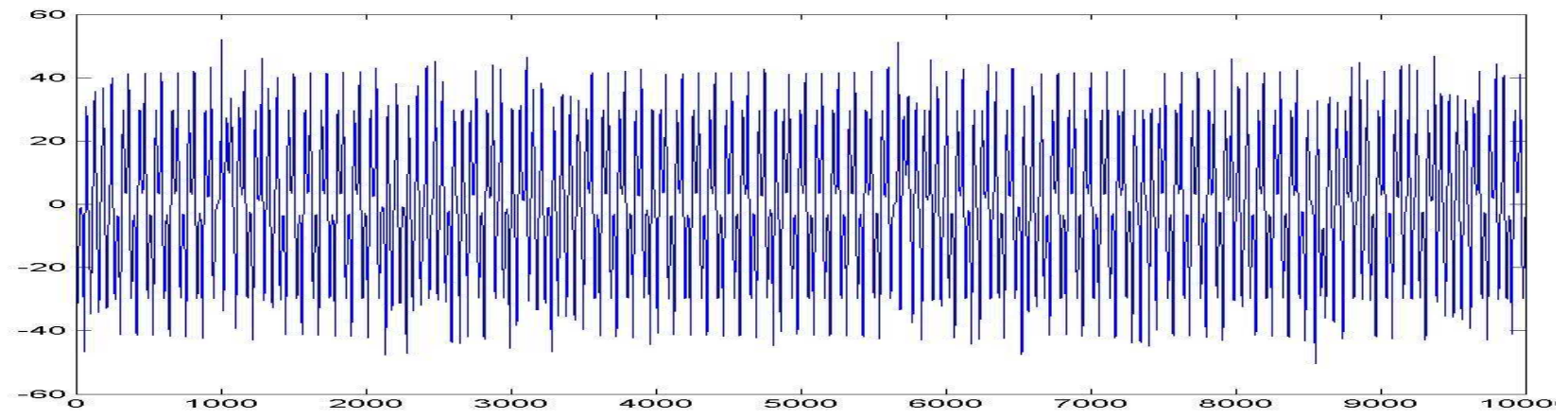


$r=166,1$

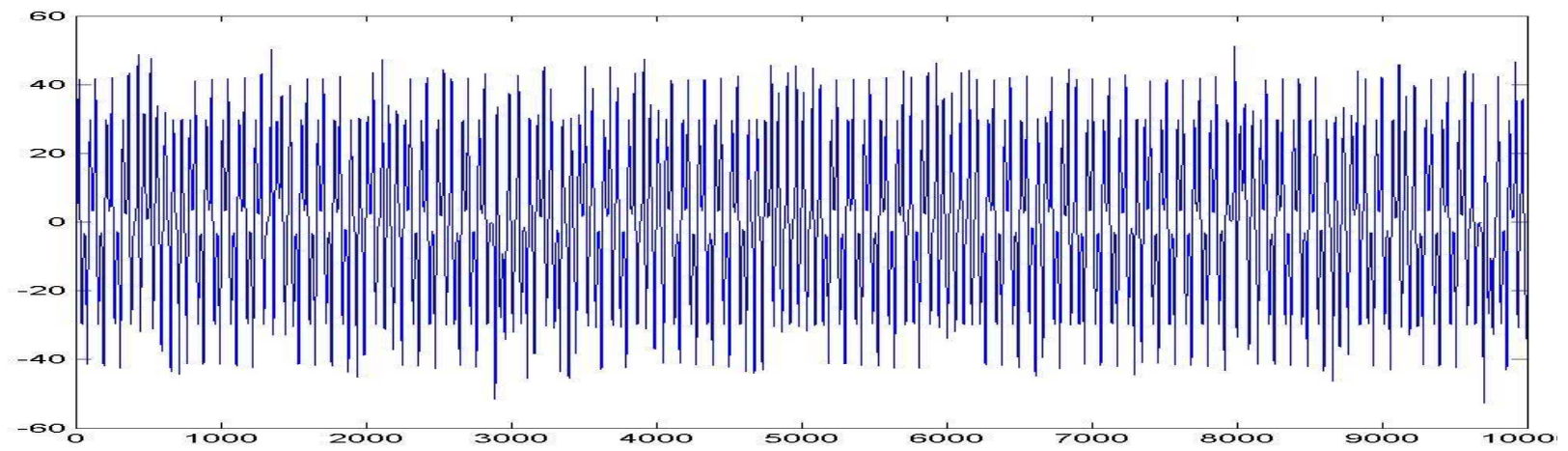


$r=166,2$

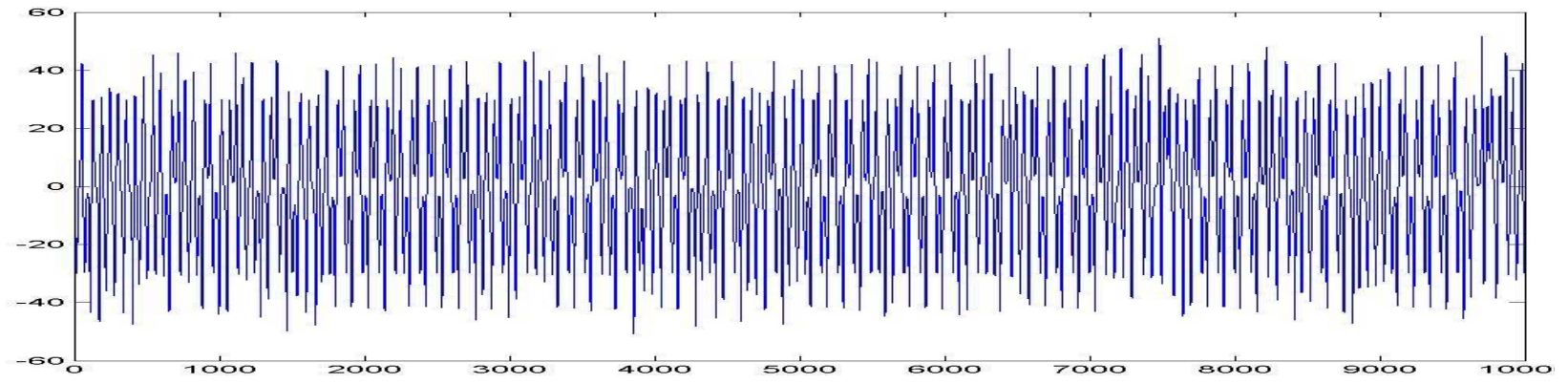




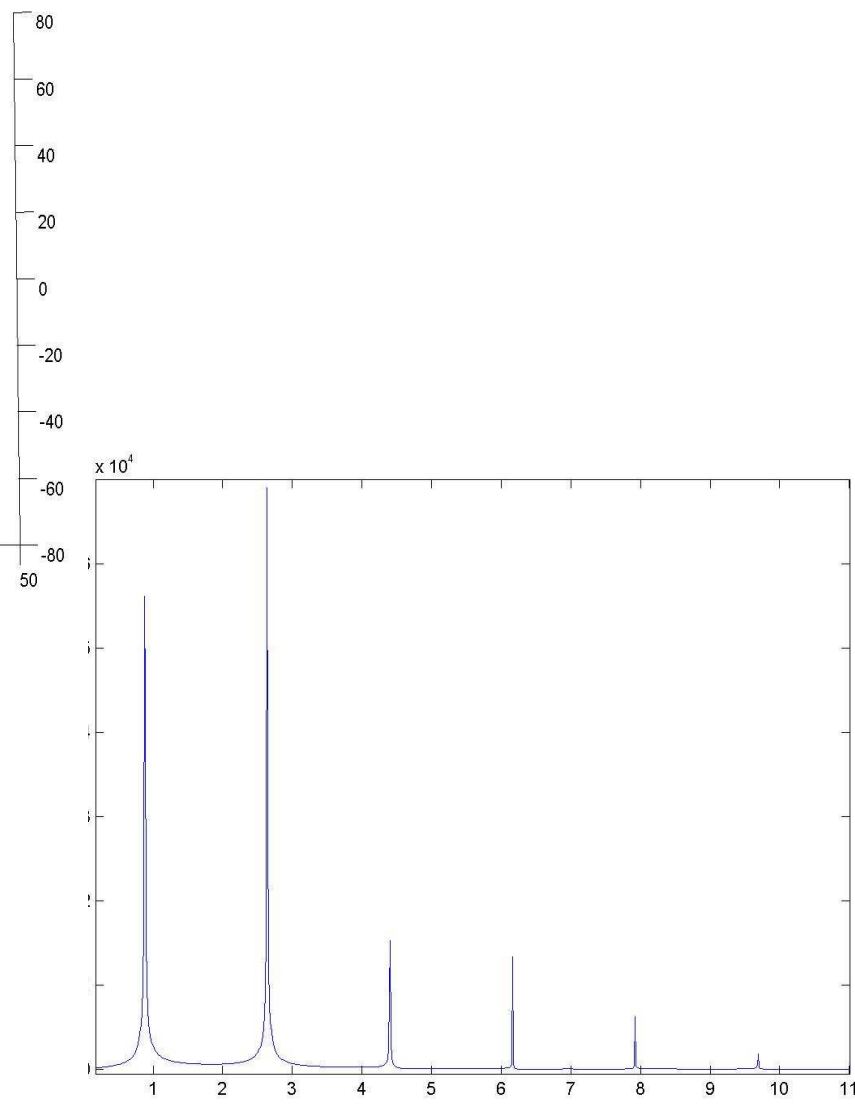
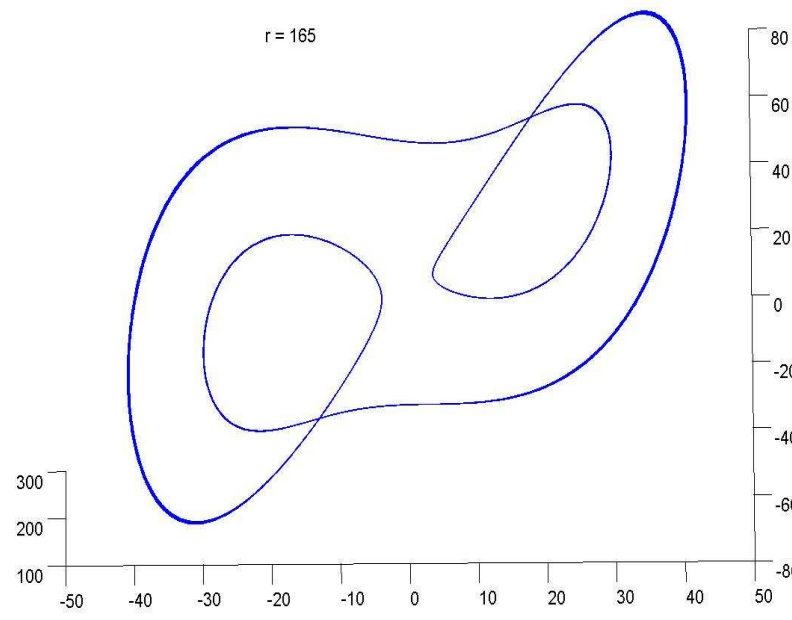
$r=166,4$



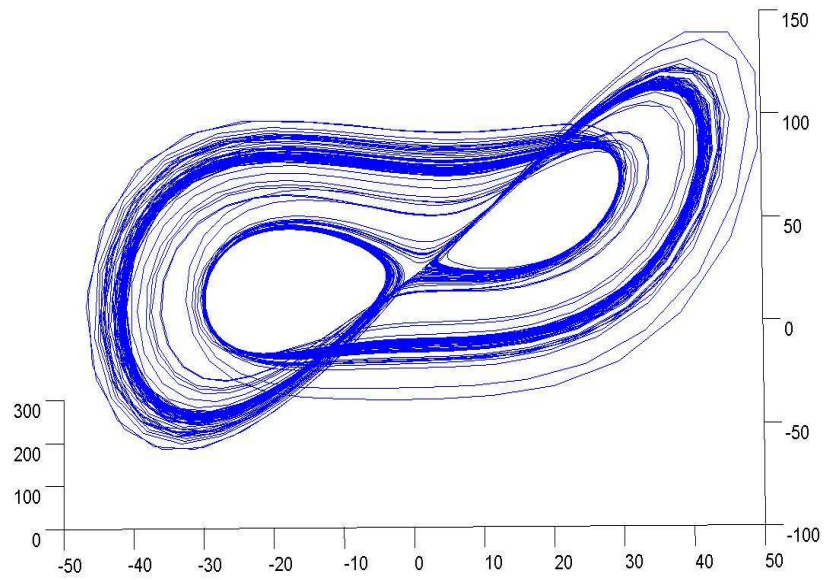
$r=166,6$



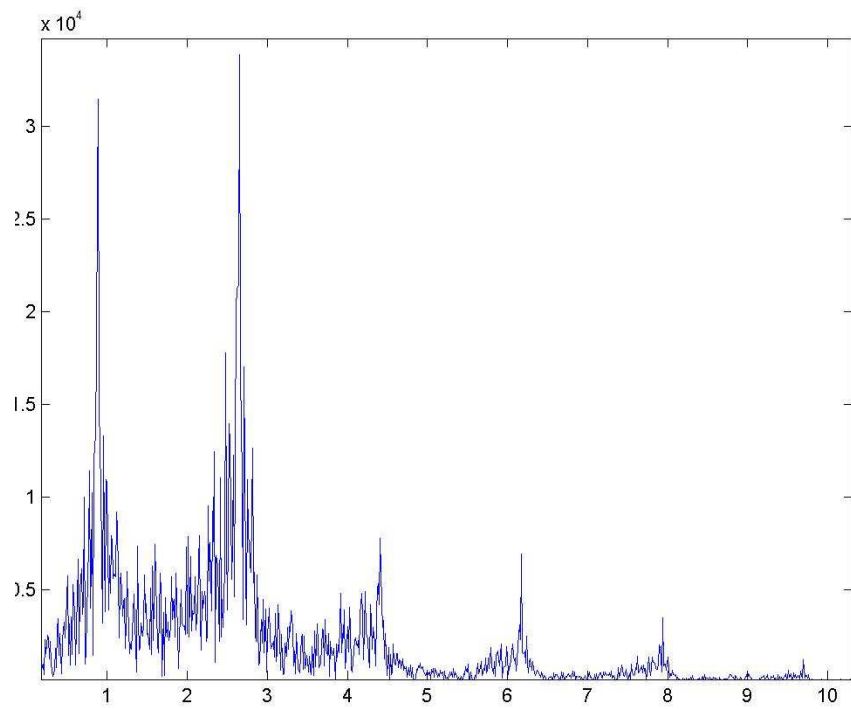
$r=166,8$



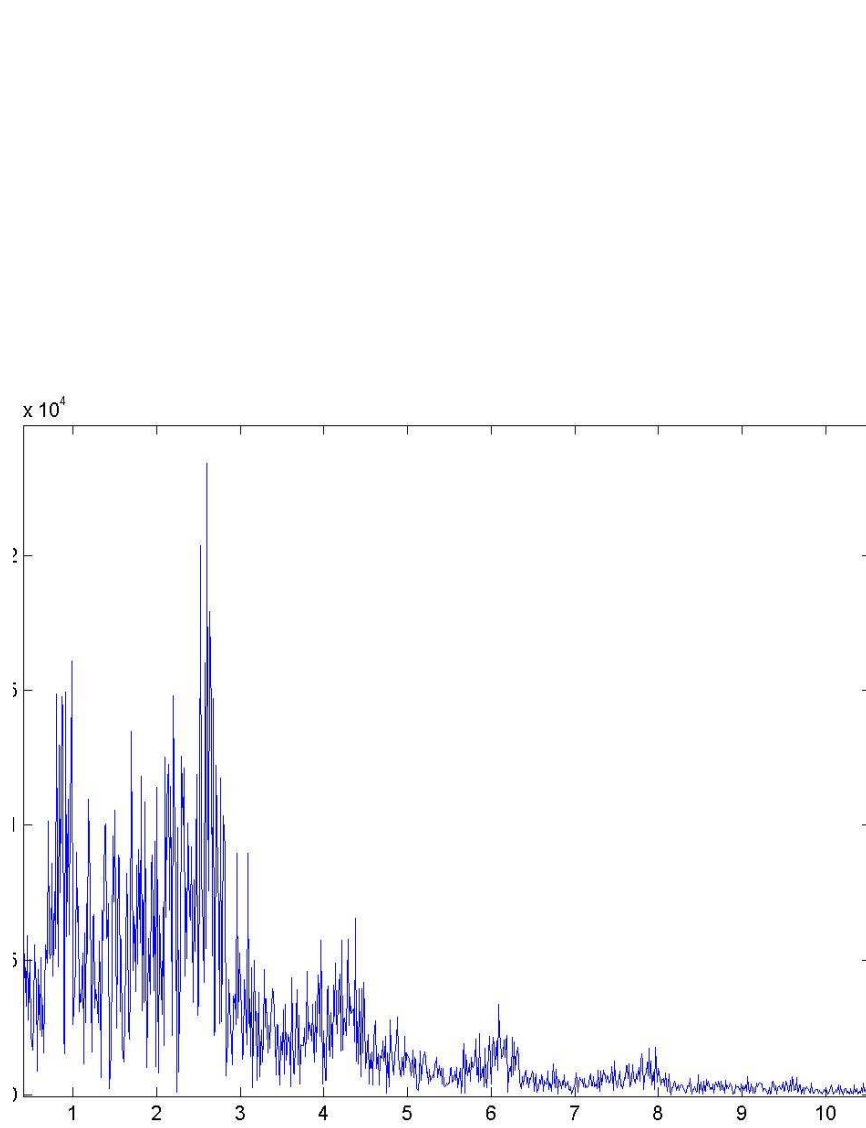
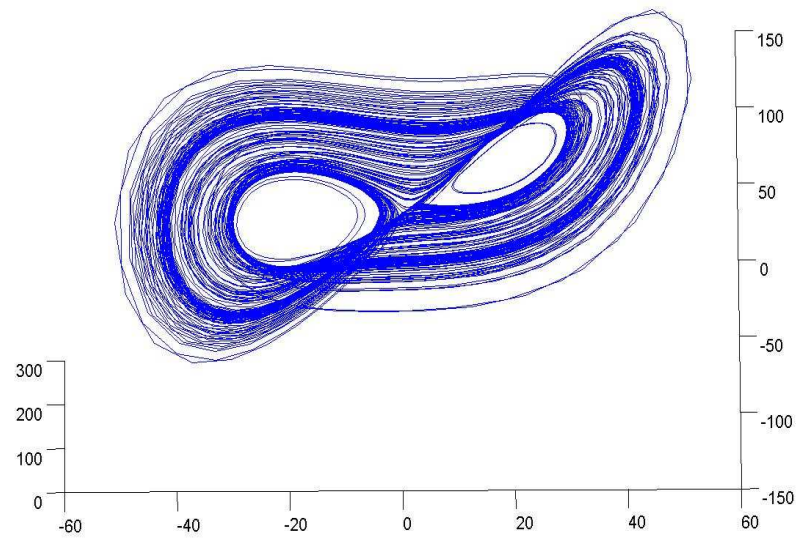
$r=165$



$r=166,2$







$r=166,8$

# Sistema de Lorenz

## Segunda Parte

- E. N. Lorenz, Deterministic Nonperiodic Flow. Journal of Atmospheric Science 20, 130 (1963)
- Primeiro atrator caótico
- Sensibilidade às condições iniciais em um fluido (modelo meteorológico simplificado)

## Sistema de Lorenz

$$\dot{x} = -\sigma x + \sigma y$$

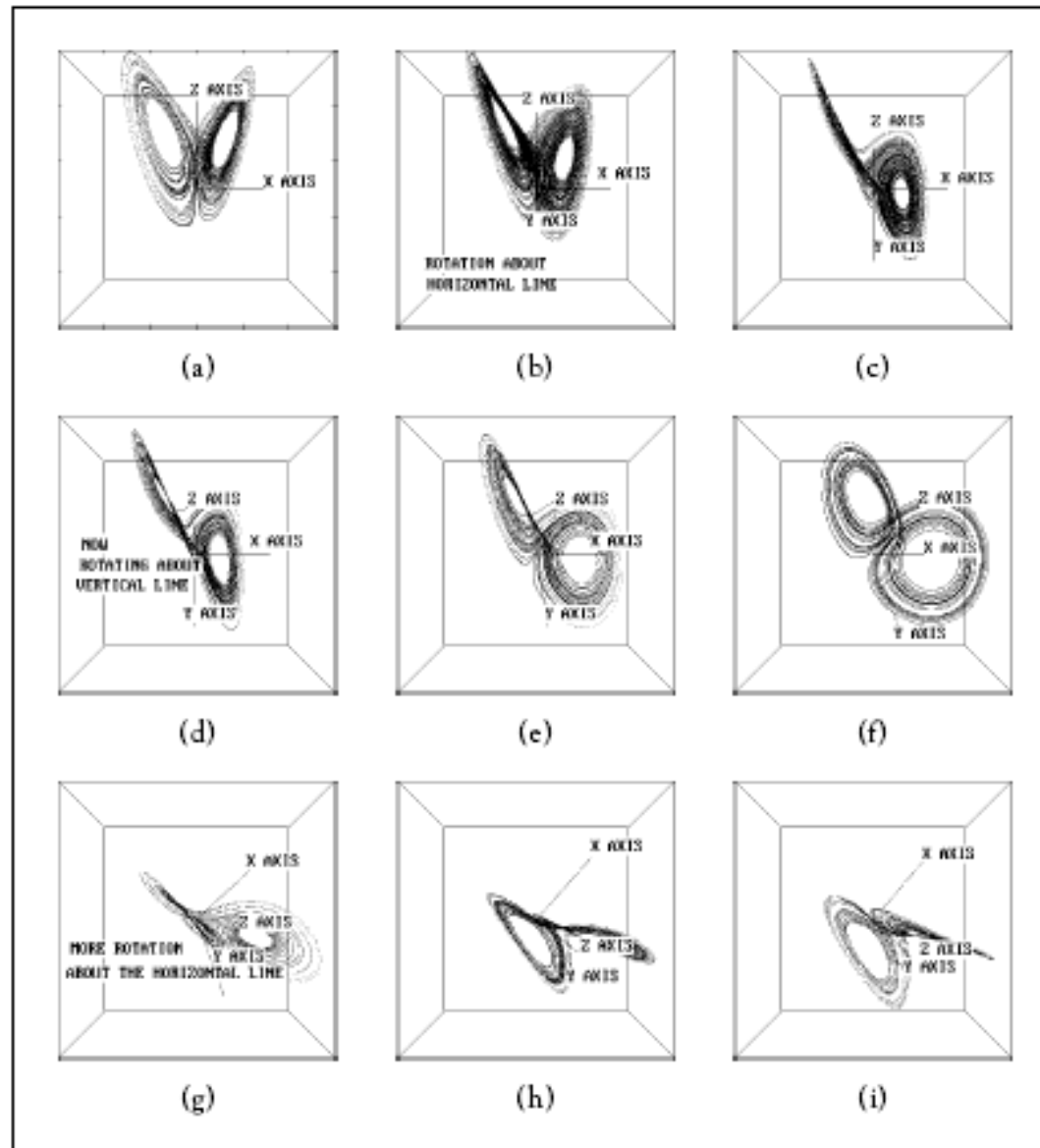
$$\dot{y} = -x y + r x - y$$

$$\dot{z} = x y - b z$$

Variáveis:  $x, y, z \rightarrow$  espaço de fase tridimensional

Parâmetros de controle:  $\sigma, r, b$

# Atrator Caótico Sistema de Lorenz

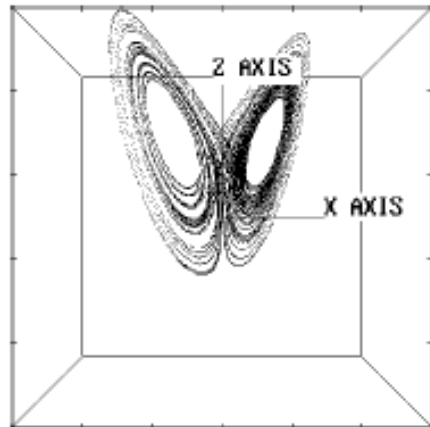


**Figure 9.2** Several rotated views of the Lorenz attractor with  $r = 28$ .

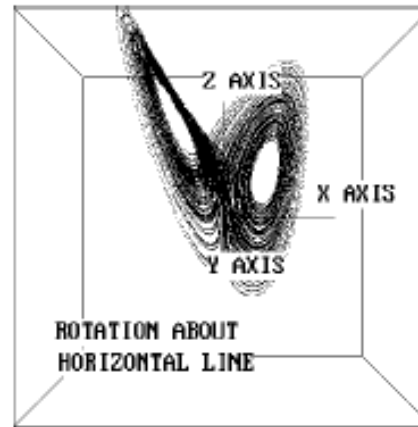
In frames (a)–(c), the attractor is tipped up (rotated about the horizontal  $x$ -axis) until the left lobe is edge-on. In frames (d)–(f), the attractor is rotated to the left, around a vertical line. In frames (g)–(i), more rotation about a horizontal line.

Chaos  
Alligood et al.

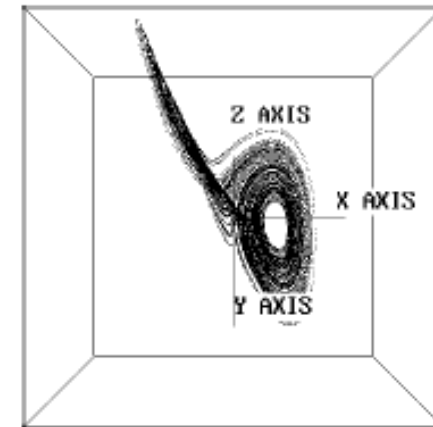
# Ampliação do Atrator de Lorenz



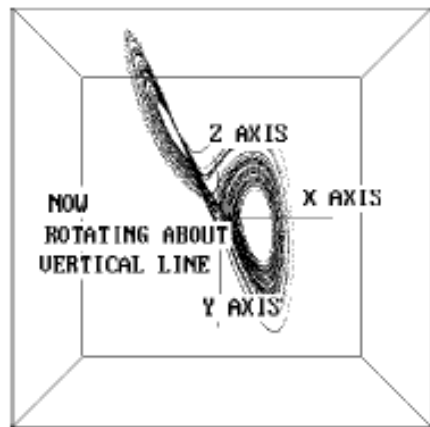
(a)



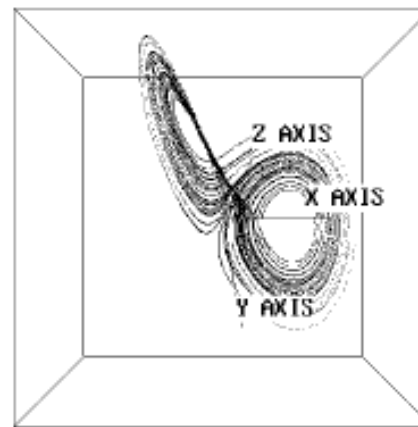
(b)



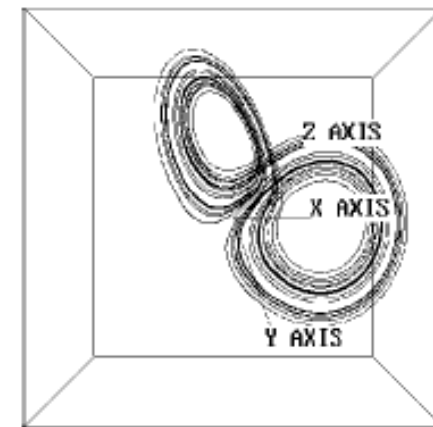
(c)



(d)

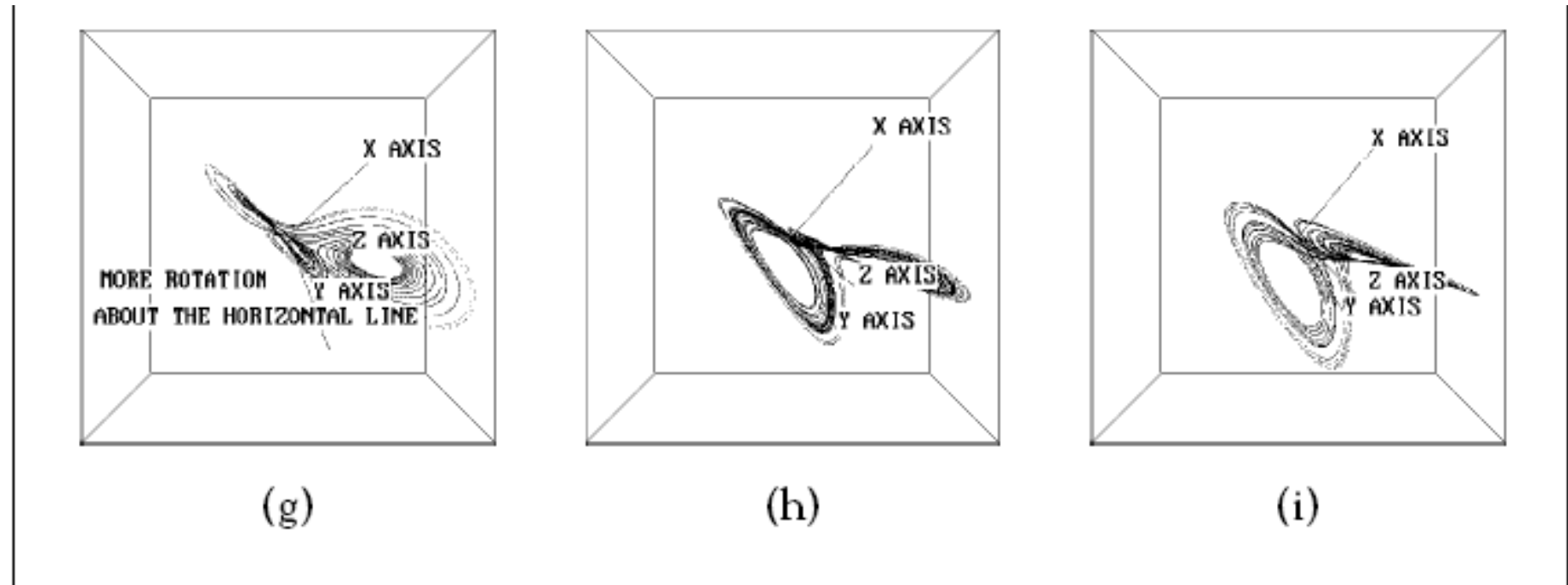


(e)



(f)

## Ampliação do Atrator de Lorenz



**Figure 9.2** Several rotated views of the Lorenz attractor with  $r = 28$ .

In frames (a)–(c), the attractor is tipped up (rotated about the horizontal  $x$ -axis) until the left lobe is edge-on. In frames (d)–(f), the attractor is rotated to the left, around a vertical line. In frames (g)–(i), more rotation about a horizontal line.

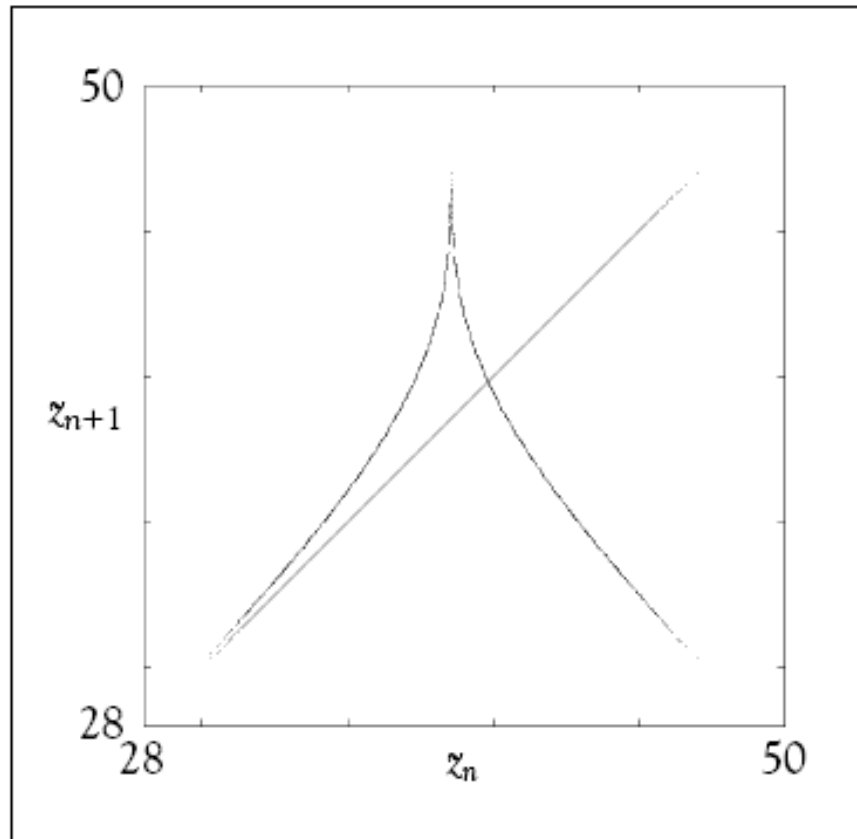
## Atratores do Sistema de Lorenz

$r$	Attractor
$[-\infty, 1.00]$	$(0, 0, 0)$ is an attracting equilibrium
$[1.00, 13.93]$	$C_+$ and $C_-$ are attracting equilibria; the origin is unstable
$[13.93, 24.06]$	Transient chaos: There are chaotic orbits but no chaotic attractors
$[24.06, 24.74]$	A chaotic attractor coexists with attracting equilibria $C_+$ and $C_-$
$[24.74, ?]$	Chaos: Chaotic attractor exists but $C_+$ and $C_-$ are no longer attracting

**Table 9.1** Attractors for the Lorenz system (9.1).

For  $\sigma = 10$ ,  $b = 8/3$ , a wide range of phenomena occur as  $r$  is varied.

## Mapa de Retorno do Atrator de Lorenz

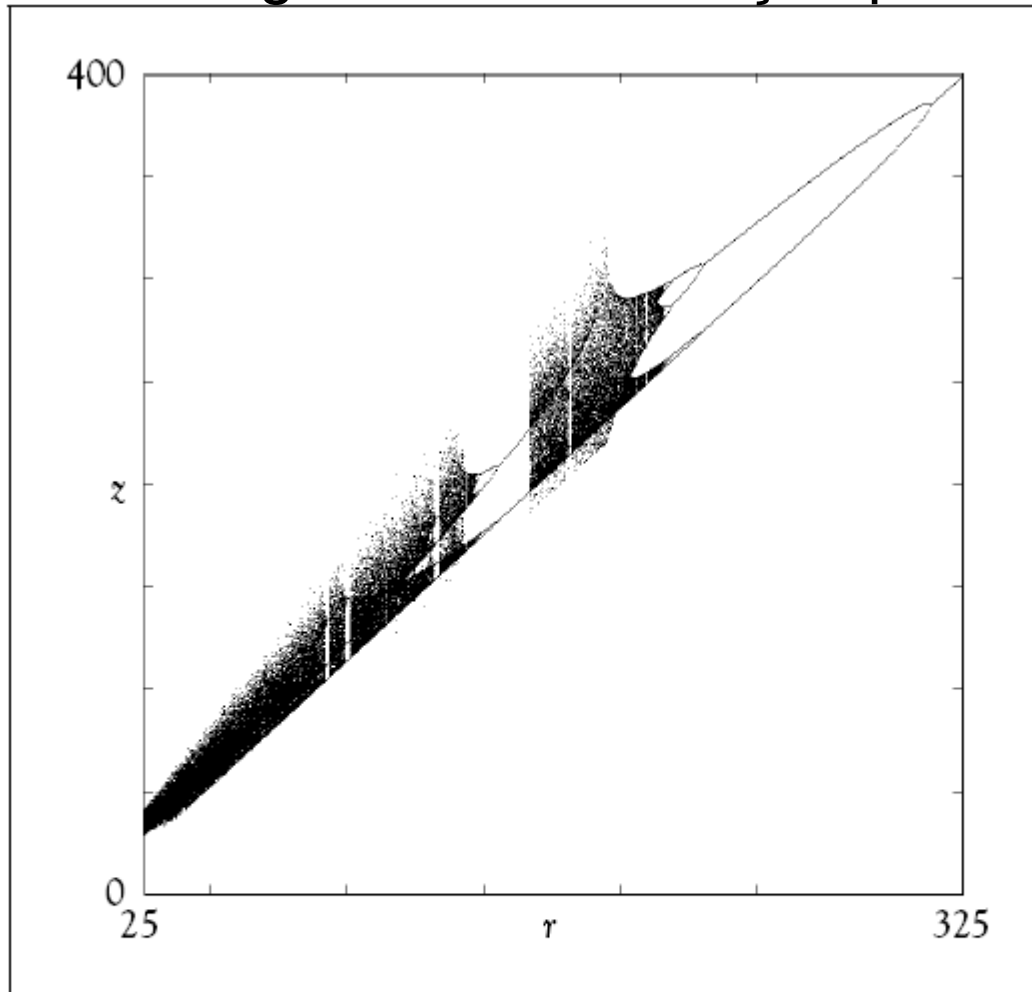


**Figure 9.3** Successive maxima of  $z$ -coordinate of Lorenz attractor.

Each plotted dot on the tent-like map is a pair  $(z_n, z_{n+1})$  of maximum  $z$ -coordinates of loops of the trajectory, one following the other. The nearly one-dimensional nature of the map arises from the very strong volume contraction.



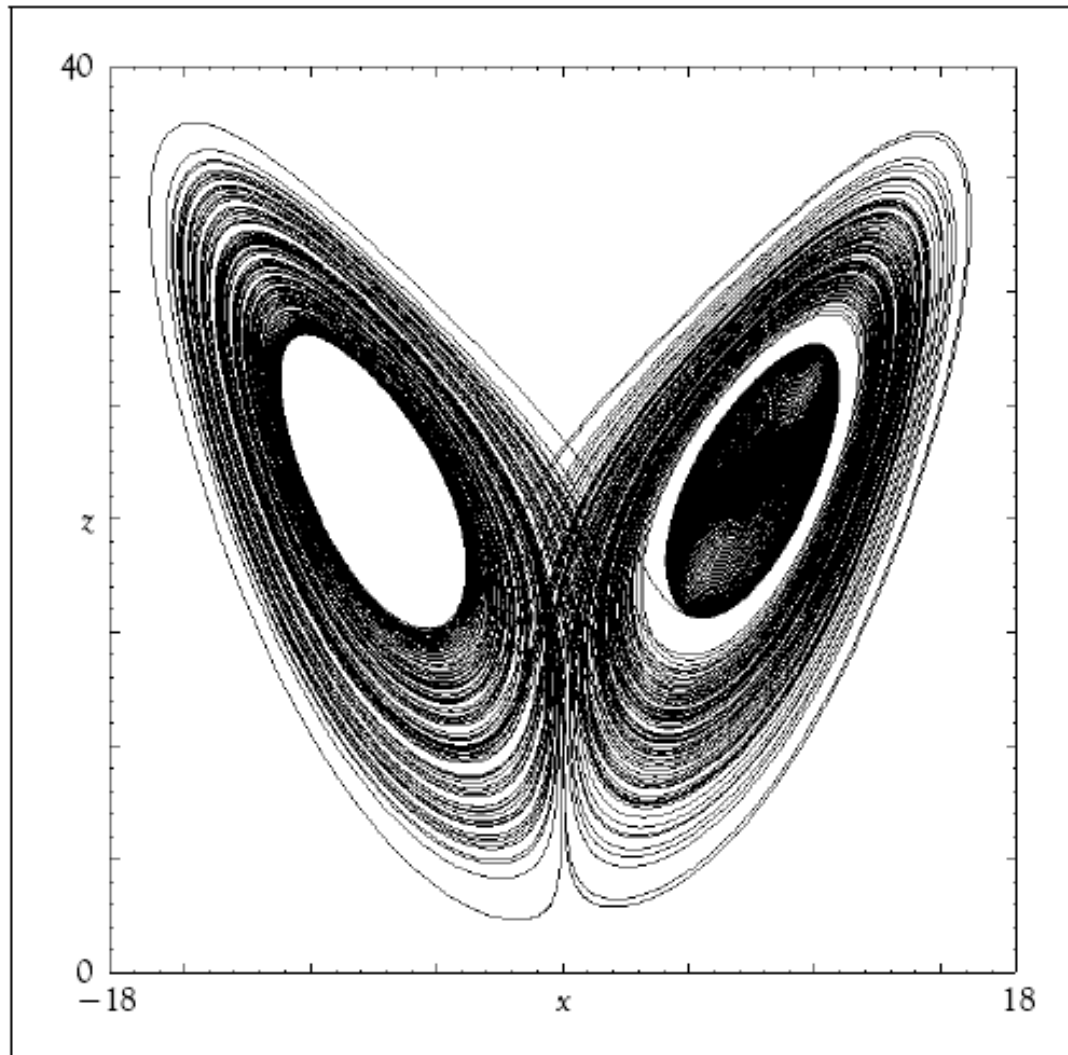
## Diagrama de Bifurcação para o Sistema de Lorenz



**Figure 9.4 Bifurcation diagram of the Lorenz tent map.**

The asymptotic behavior of the tent map of Figure 9.3 is plotted as a function of the bifurcation parameter  $r$ . The points plotted above each  $r$  correspond to the  $z$ -maxima of the orbit, so that 1 point means a period- $T$  orbit, 2 points correspond to a period- $2T$  orbit, and so on.

## Transiente Caótico no Sistema de Lorenz



**Figure 9.5** Transient chaos in the Lorenz equations.

A trajectory of the Lorenz system has been plotted using  $b = 8/3$  and  $\sigma = 10$ , the same values Lorenz used, but here  $r = 23$ . When  $r < r_1 \approx 24.06$  there is

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## Sistema de Roessler

$$\dot{x} = -y - z$$

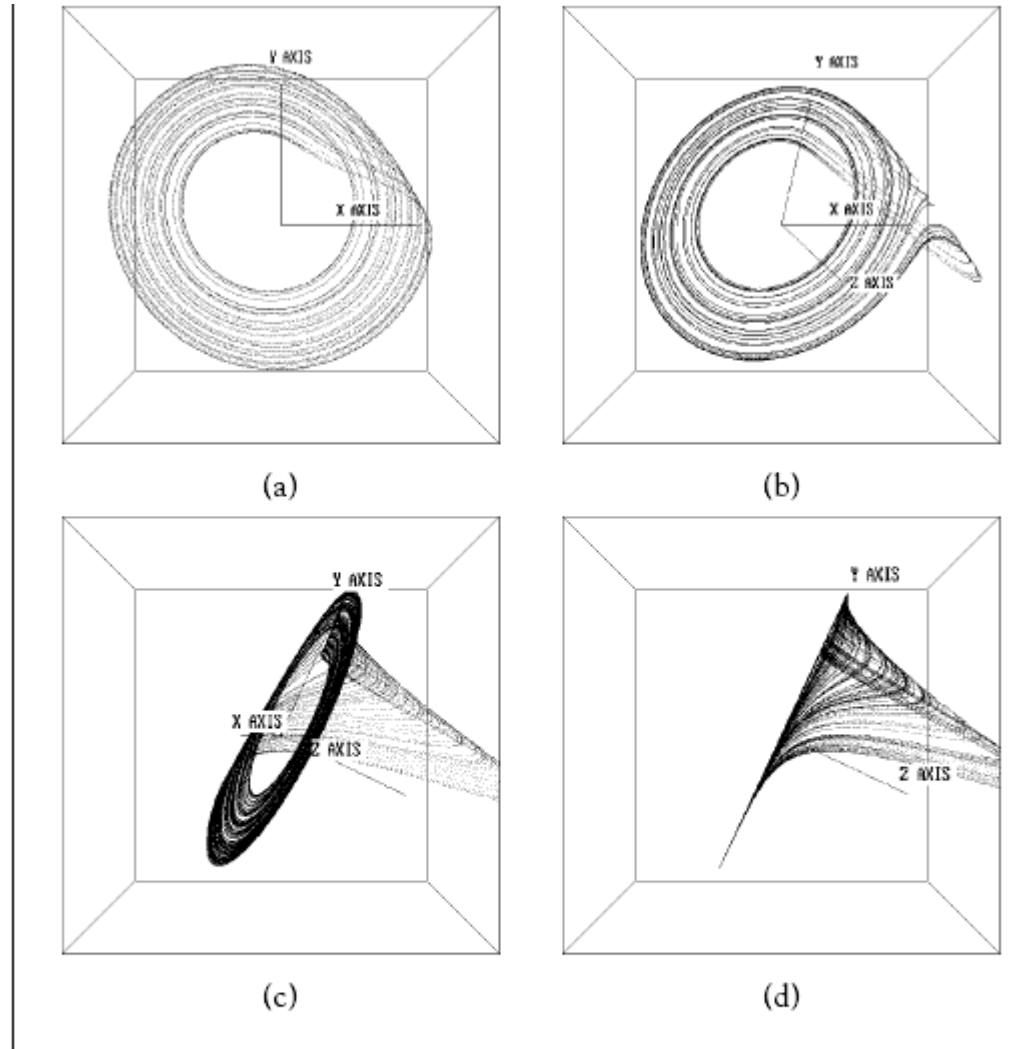
$$\dot{y} = x + a y$$

$$\dot{z} = b + (x - c) z$$

Variáveis:  $x, y, z \rightarrow$  espaço de fase tridimensional

Parâmetros de controle:  $a, b, c$

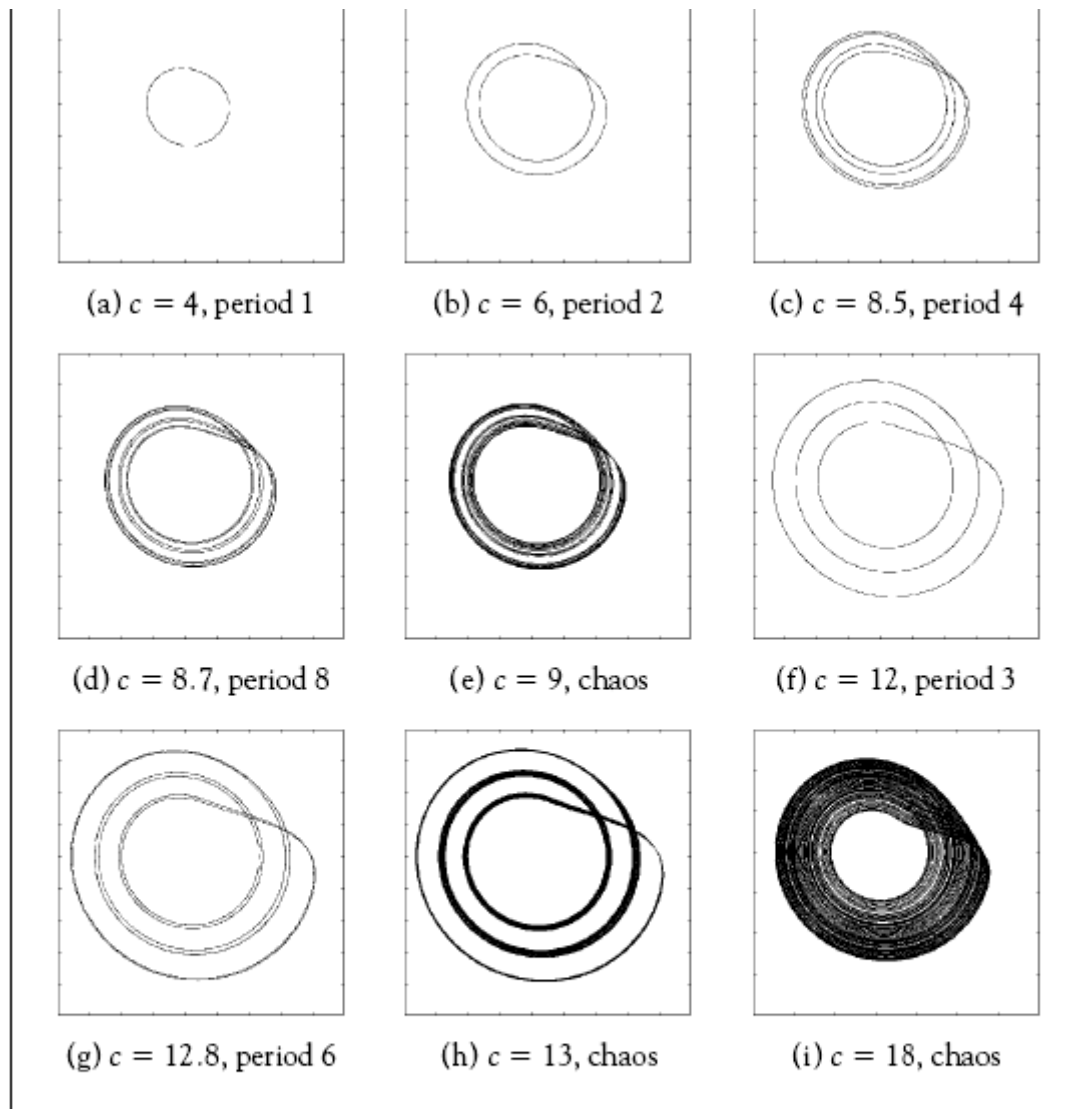
# Atrator Caótico de Roessler



**Figure 9.6 The Rössler attractor.**

Parameters are set at  $a = 0.1$ ,  $b = 0.1$ , and  $c = 14$ . Four different views are shown. The dynamics consists of a spiraling out from the inside along the  $xy$ -plane followed by a large excursion in the  $z$ -direction, followed by re-insertion to the vicinity of the  $xy$ -plane. Part (d) shows a side view. The Lyapunov dimension is 2.005—indeed it looks like a surface.

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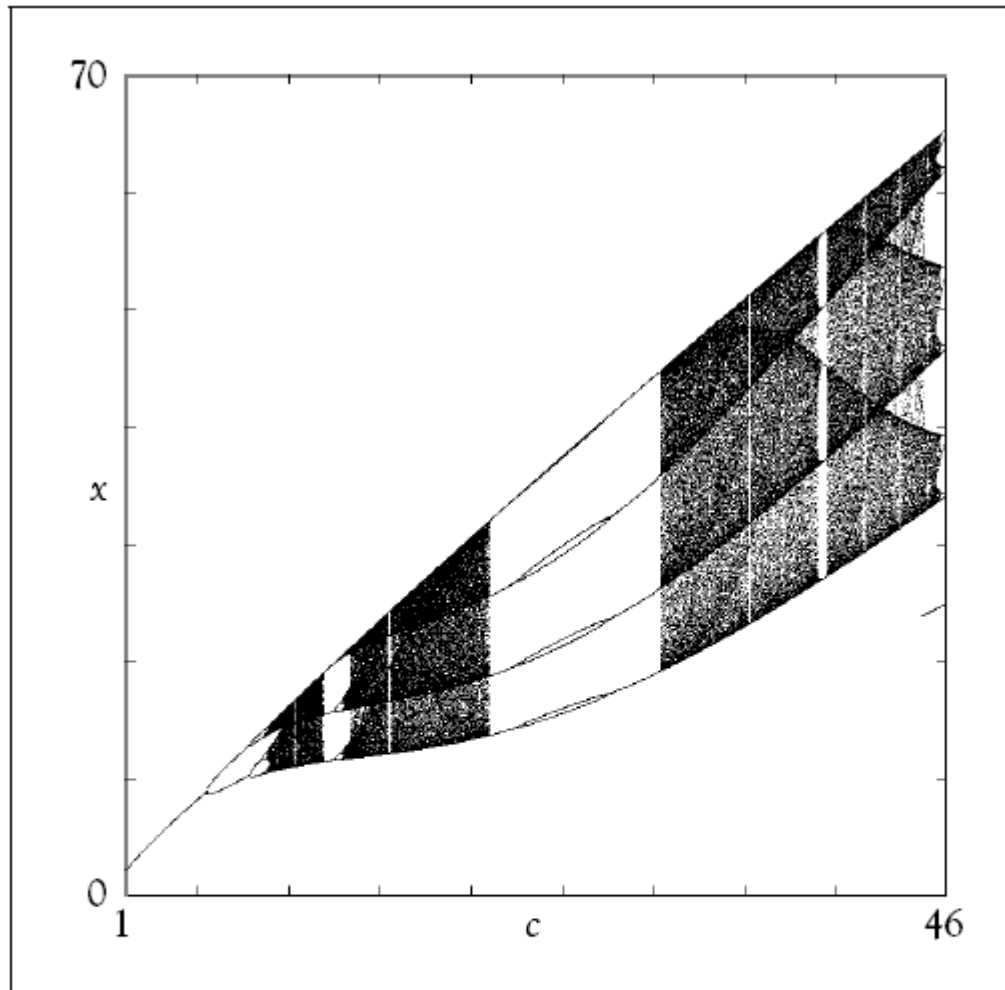
Atratores do  
Sistema de Rössler  
para diferentes valores  
do parâmetro  $c$

$$a = b = 0.1$$

**Figure 9.7** Attractors of the Rössler system as  $c$  is varied.

Fixed parameters are  $a = b = 0.1$ . (a)  $c = 4$ , periodic orbit. (b)  $c = 6$ , period-doubled orbit. (c)  $c = 8.5$ , period four. (d)  $c = 8.7$ , period 8. (e)  $c = 9$ , thin chaotic attractor. (f)  $c = 12$ , period three. (g)  $c = 12.8$ , period six. (h)  $c = 13$ , chaotic attractor. (i)  $c = 18$ , filled-out chaotic attractor

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## Diagrama de Bifurcação Sistema de Roessler

Máximos locais  
da variável x

$$a = b = 0.1$$

**Figure 9.8 Bifurcation diagram for the Rössler equations.**

The parameters  $a = b = 0.1$  are fixed. The horizontal axis is the bifurcation parameter  $c$ . Each vertical slice shows a plot of the local maxima of the  $x$ -variable of an attractor for a fixed value of the parameter  $c$ . A single point implies there is a periodic orbit; two points mean a periodic orbit with “two loops”, the result of a period doubling, and so on. Near  $c = 46$  the attractor disappears abruptly.

Chaos  
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# Caos no Circuito Elétrico de Chua

Parâmetros de Controle  
Controle das Oscilações  
Atratores

- M. S. Baptista e I. L. Caldas - Physica D (1999).
- R. O. Medrano-T., M. S. Batista e I. L. Caldas, Physica D (2003).

# Circuito de Chua

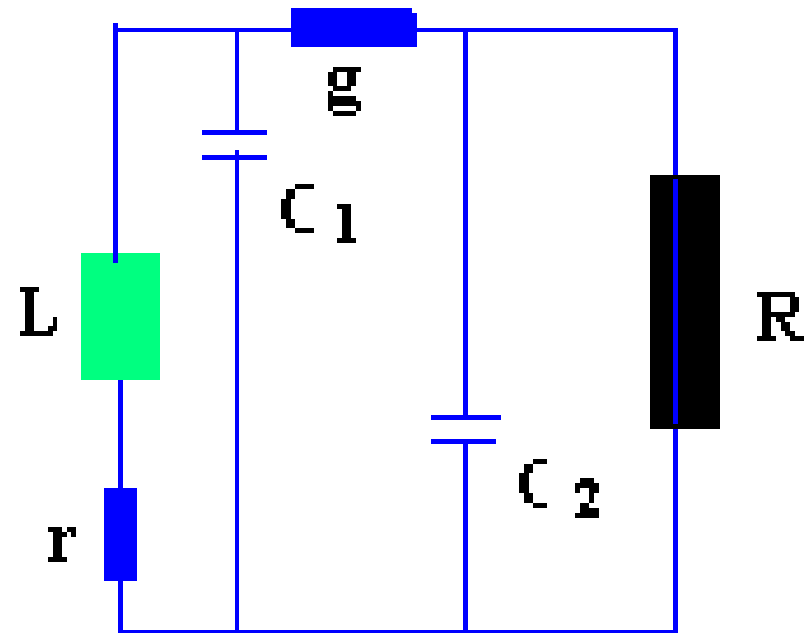
R elemento linear por partes

- Variáveis dinâmicas:

$V_{c1}$  tensão

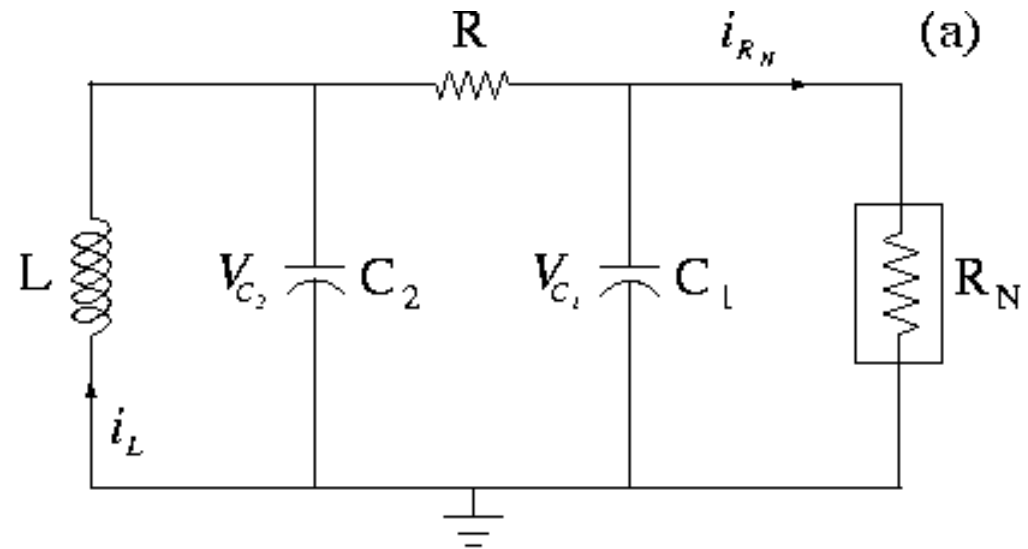
$V_{c2}$  tensão

$i_L$  corrente



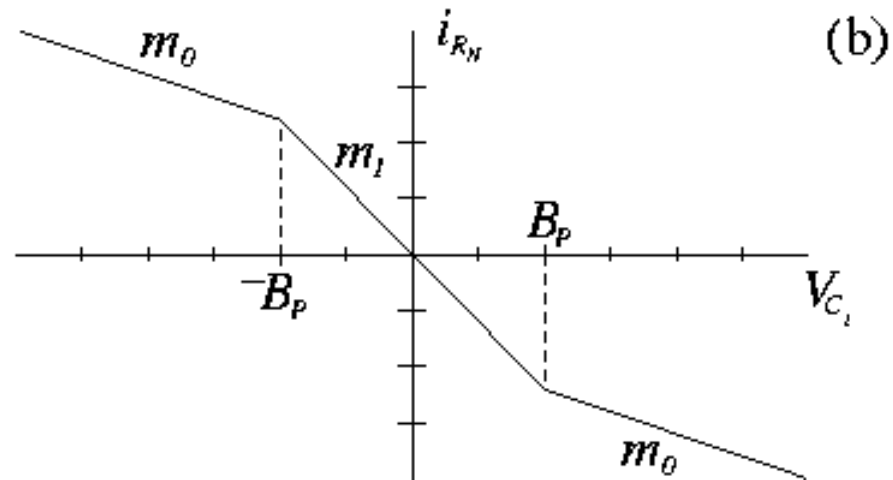


# Circuito de Chua



Curva Característica

Linear por partes



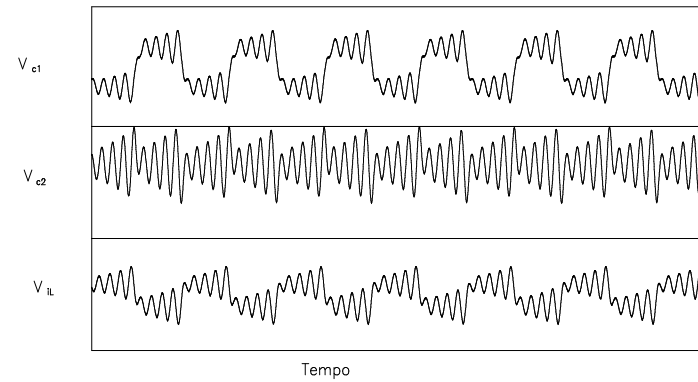
# Experiment

## Periodic Attractor

$V_{c1}$  voltage across  $C_1$

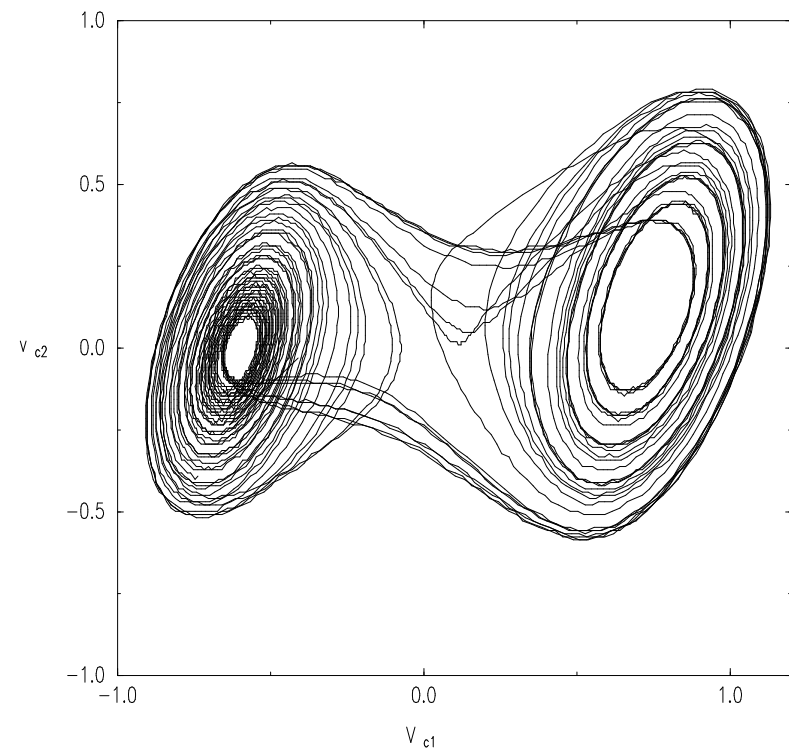
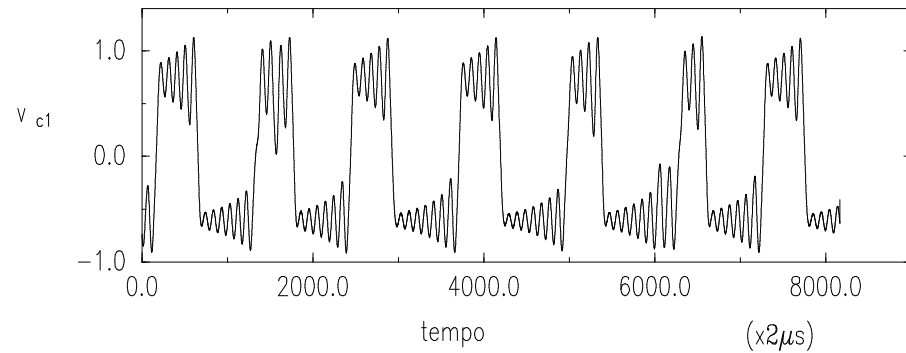
$V_{c2}$  voltage across  $C_2$

$i_L$  current through  $i_L$



# Experiment

- Double Scroll Atrator



## O Circuito de Chua

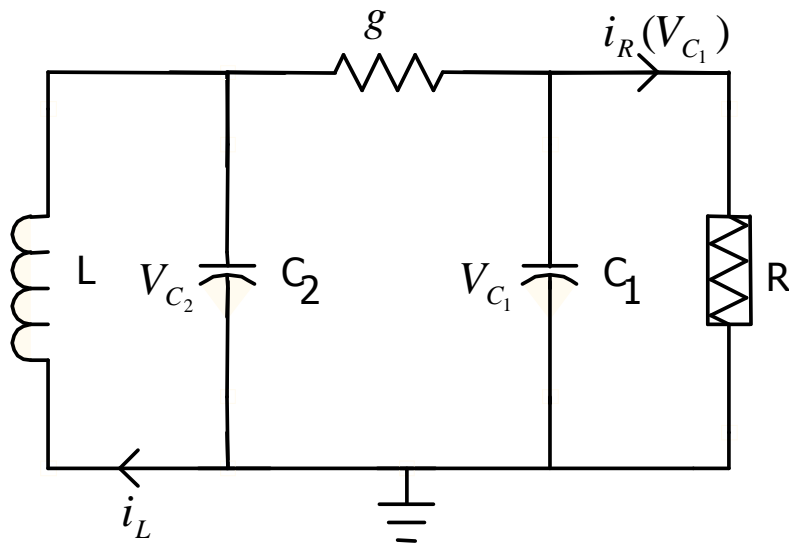


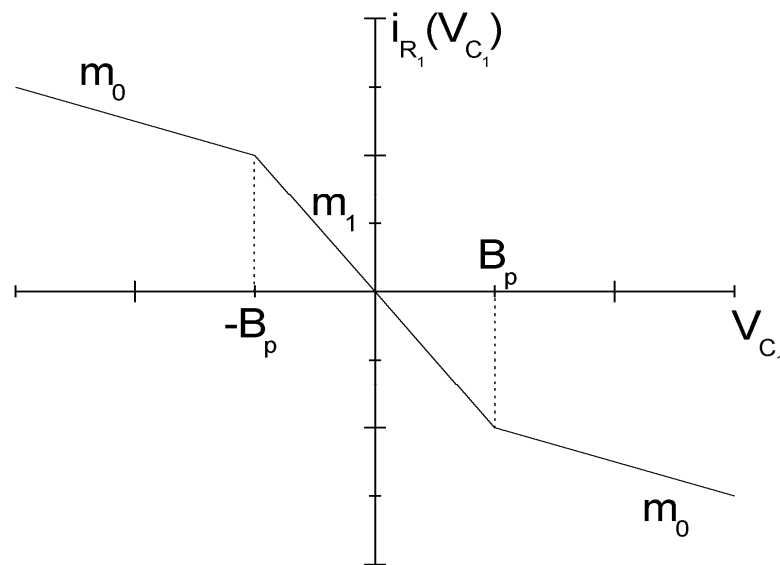
Fig.1. Circuito de Chua. R é a resistência não linear.

Aplicando a lei de Kirchoff ao circuito:

$$\begin{aligned}C_1 \dot{V}_{C_1} &= g(V_{C_2} - V_{C_1}) - i_R(V_{C_1}) \\C_2 \dot{V}_{C_2} &= g(V_{C_1} - V_{C_2}) + i_L \\Li_L &= -V_{C_2}\end{aligned}$$

Simetria ímpar:  $f(x) = -f(-x)$

## Resistência Linear por Partes



Função da curva característica da resistência linear por partes:

$$i_R(V_{C_1}) = \begin{cases} m_0 V_{C_1} + (m_1 - m_0) B_p, & V_{C_1} \geq B_p \\ m_1 V_{C_1}, & |V_{C_1}| \leq B_p \\ m_0 V_{C_1} - (m_1 - m_0) B_p, & V_{C_1} \leq -B_p \end{cases}$$

Fig. 2. Curva característica da resistência linear por partes.

# Sistema Adimensional

Mudança de variáveis:

$$x = \frac{V_{C_1}}{B_p}, \quad y = \frac{V_{C_2}}{B_p} \quad e \quad z = \frac{i_L}{gB_p}$$

$$\alpha = \frac{C_2}{C_1}, \quad \beta = \frac{C_2}{g^2 L}, \quad \tau = \frac{g}{C_2} t,$$

$$a = \frac{m_1}{g} \quad e \quad b = \frac{m_0}{g}$$

$$C_1 \dot{V}_{C_1} = g(V_{C_2} - V_{C_1}) - i_R(V_{C_1})$$

$$C_2 \dot{V}_{C_2} = g(V_{C_1} - V_{C_2}) + i_L$$

$$L \dot{i}_L = -V_{C_2}$$



$$\dot{x} = \alpha[y - x - k(x)]$$

$$\dot{y} = x - y + z$$

$$\dot{z} = -\beta y$$

$$i_R(V_{C_1}) = \begin{cases} m_0 V_{C_1} + (m_1 - m_0) B_p, & V_{C_1} \geq B_p \\ m_1 V_{C_1}, & |V_{C_1}| \leq B_p \\ m_0 V_{C_1} - (m_1 - m_0) B_p, & V_{C_1} \leq -B_p \end{cases}$$



$$k(x) = \begin{cases} bx + (a - b), & x \geq 1 \\ ax, & |x| \leq 1 \\ bx - (a - b), & x \leq -1 \end{cases}$$

# Atratores do Sistema

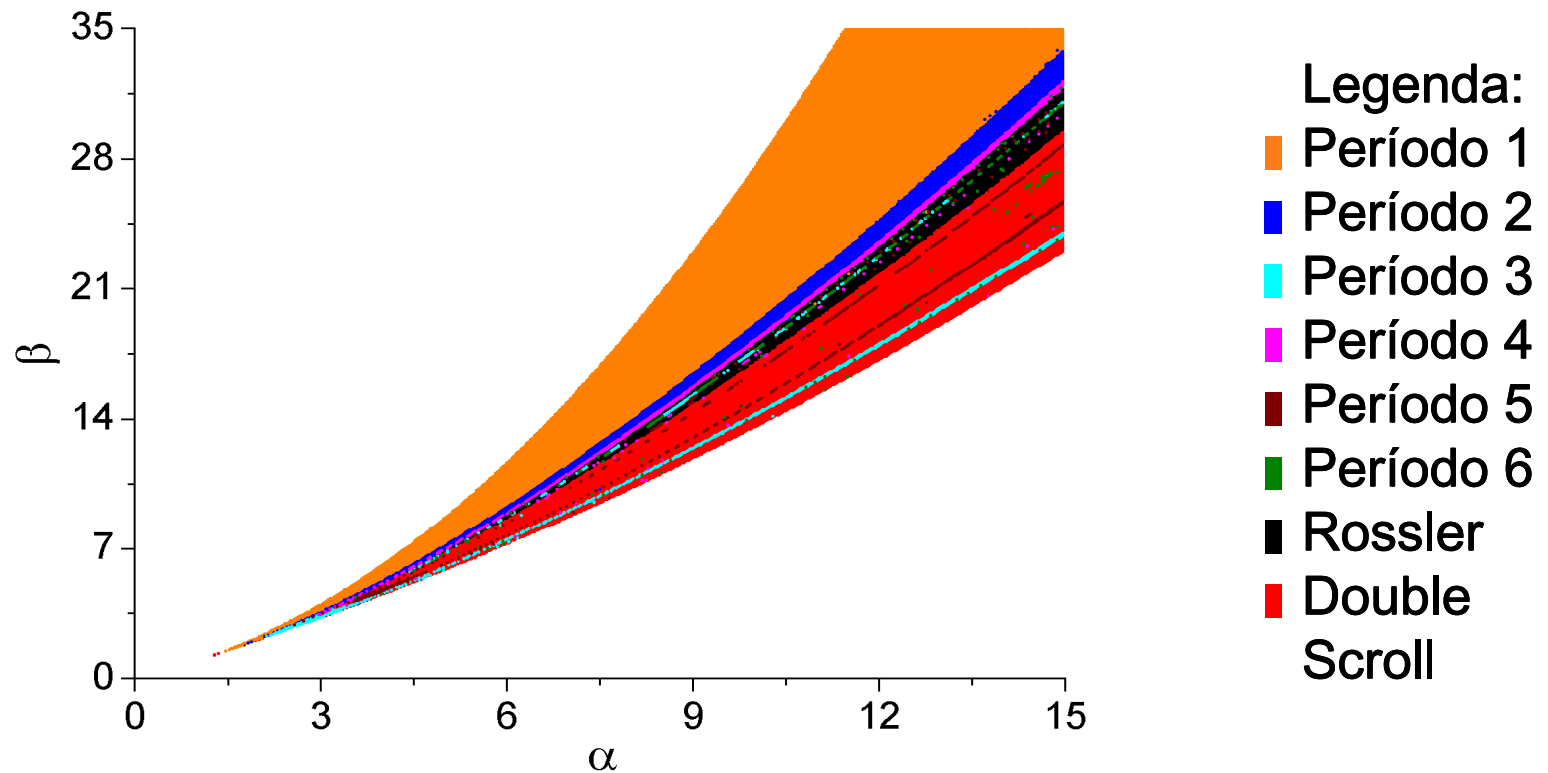


Fig. 3. Atratores no espaço dos parâmetros.

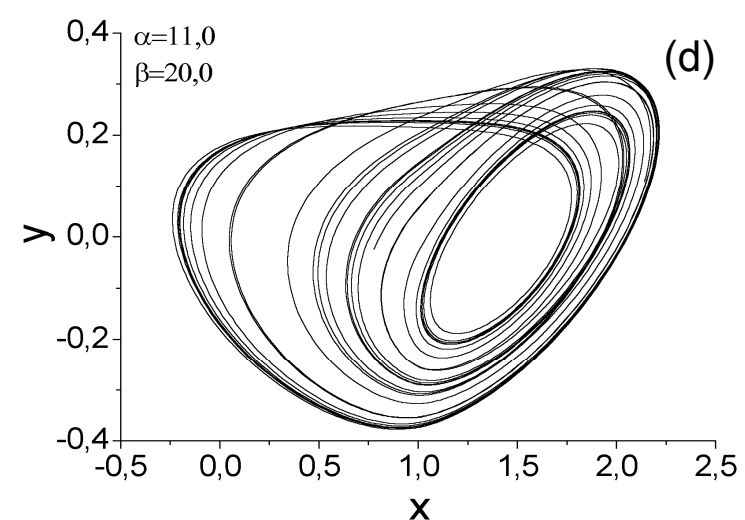
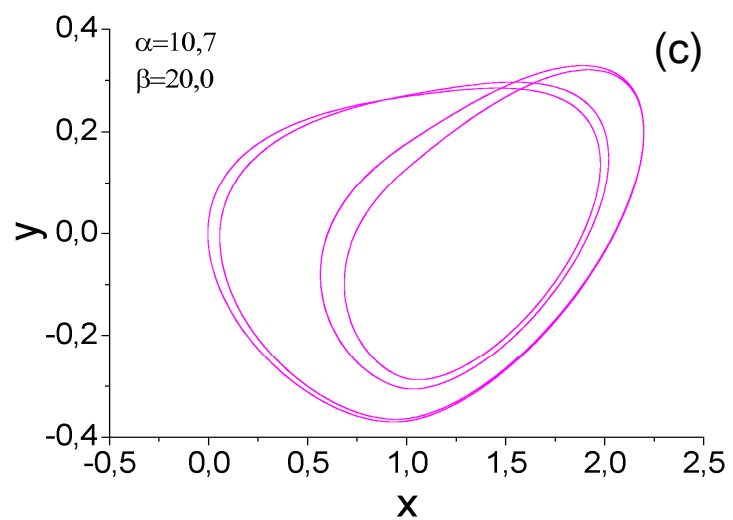
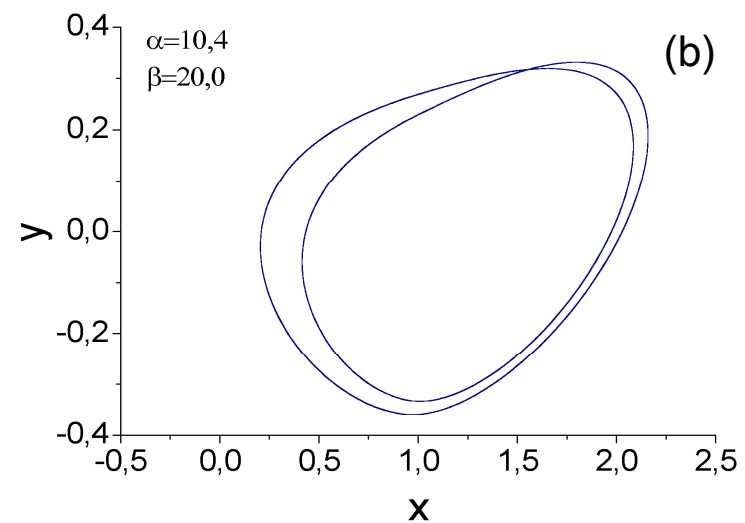
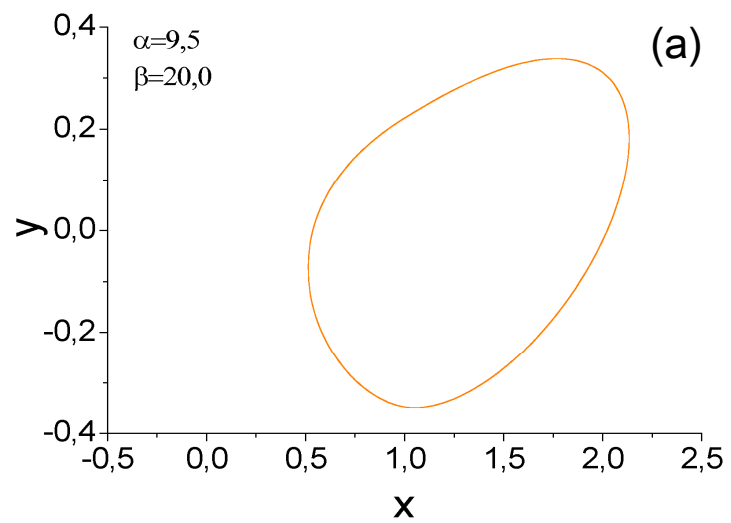
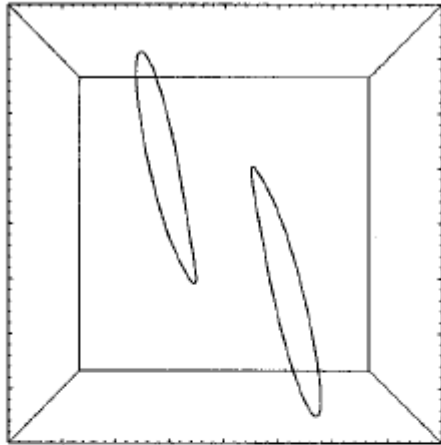


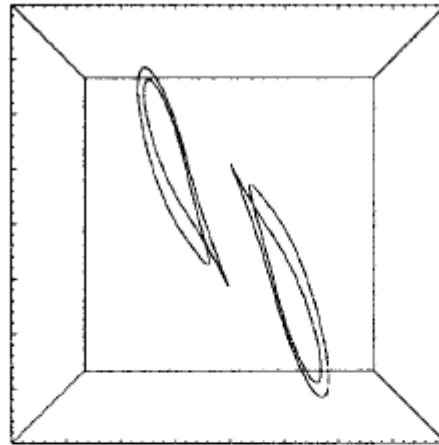
Fig. 4. Atratores: (a) Período 1, (b) Período 2, (c) Período 3, (d) Tipo Rössler.



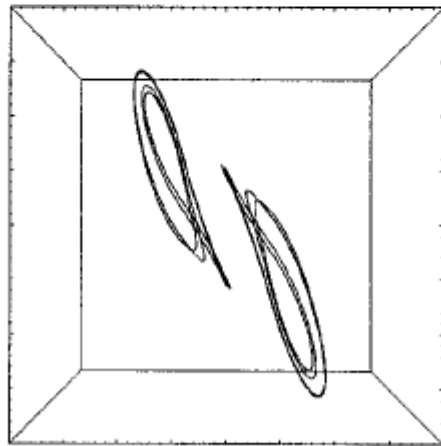
# Atratores do Circuito de Chua



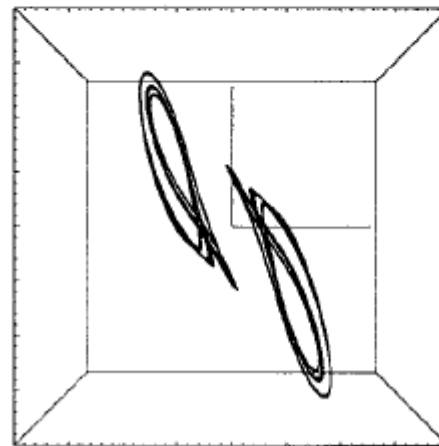
(a)  $c_3 = 50$ , period 1



(b)  $c_3 = 35$ , period 2

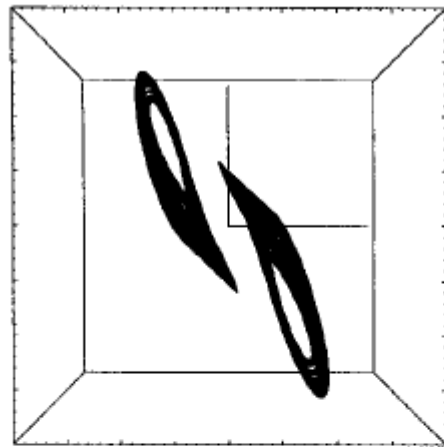


(c)  $c_3 = 33.8$ , period 4

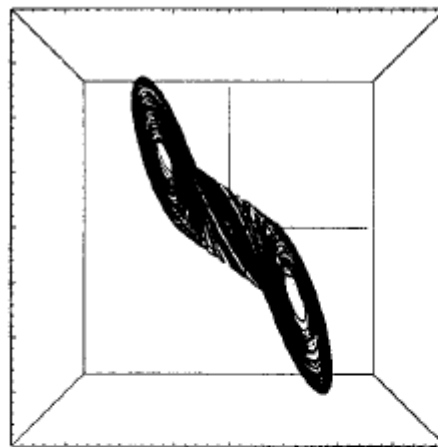


(d)  $c_3 = 33.6$ , chaos $\times 2$

## Atratores do Circuito de Chua



(e)  $c_3 = 33$ , chaos $\times 2$



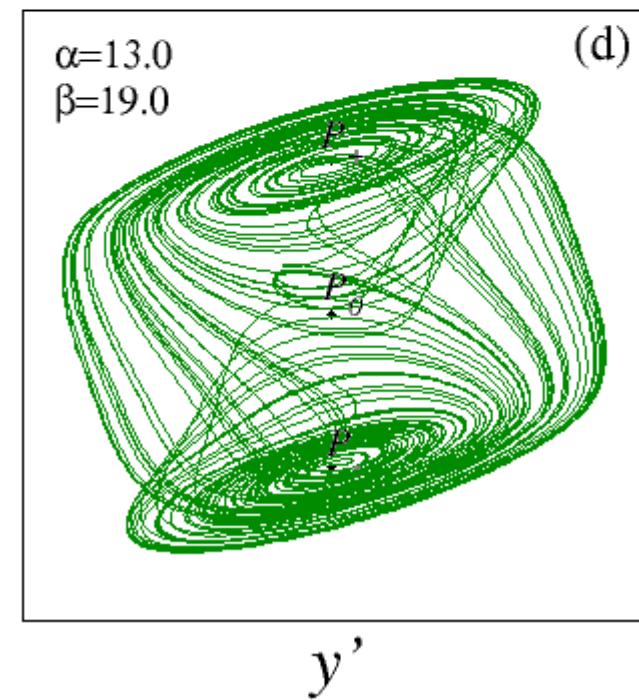
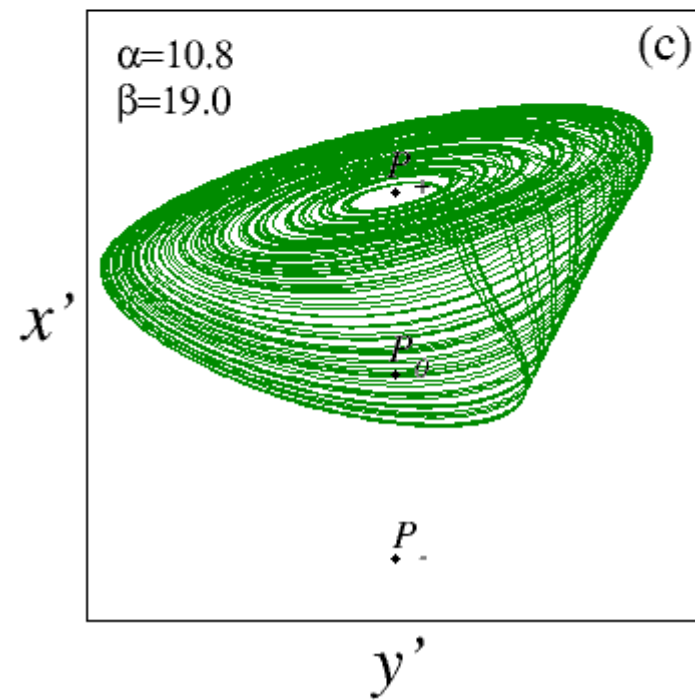
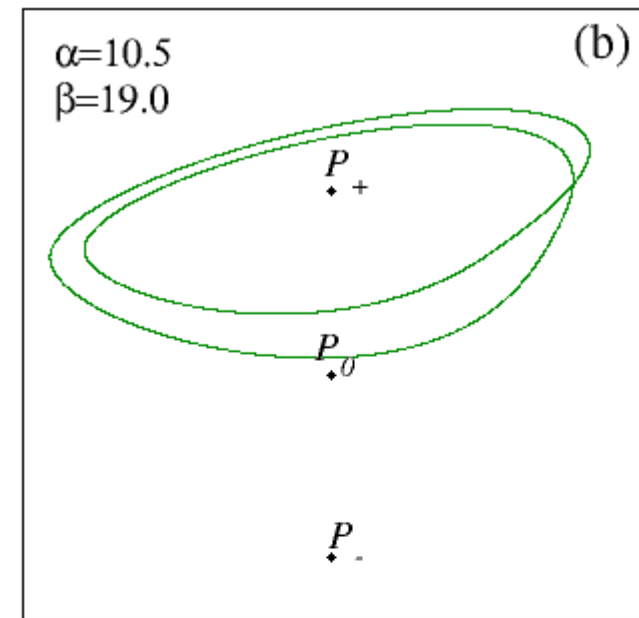
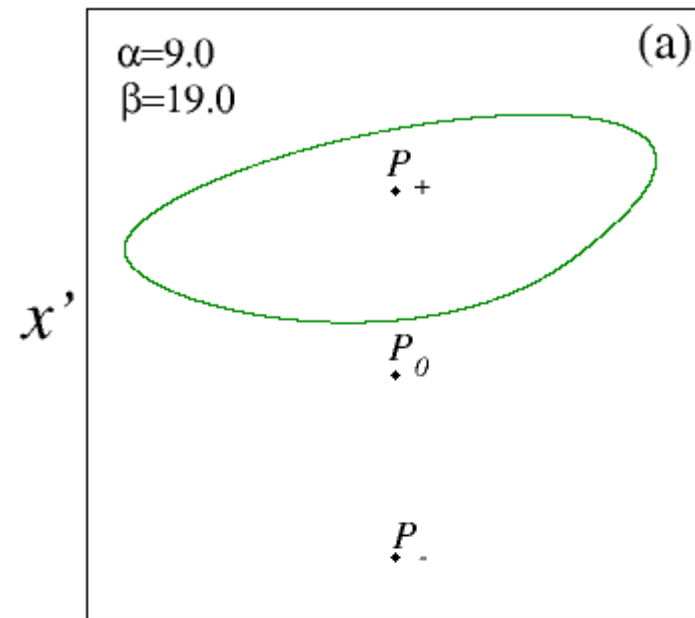
(f)  $c_3 = 25.58$ , double scroll chaos

### Figure 9.10 Chua circuit attracting sets.

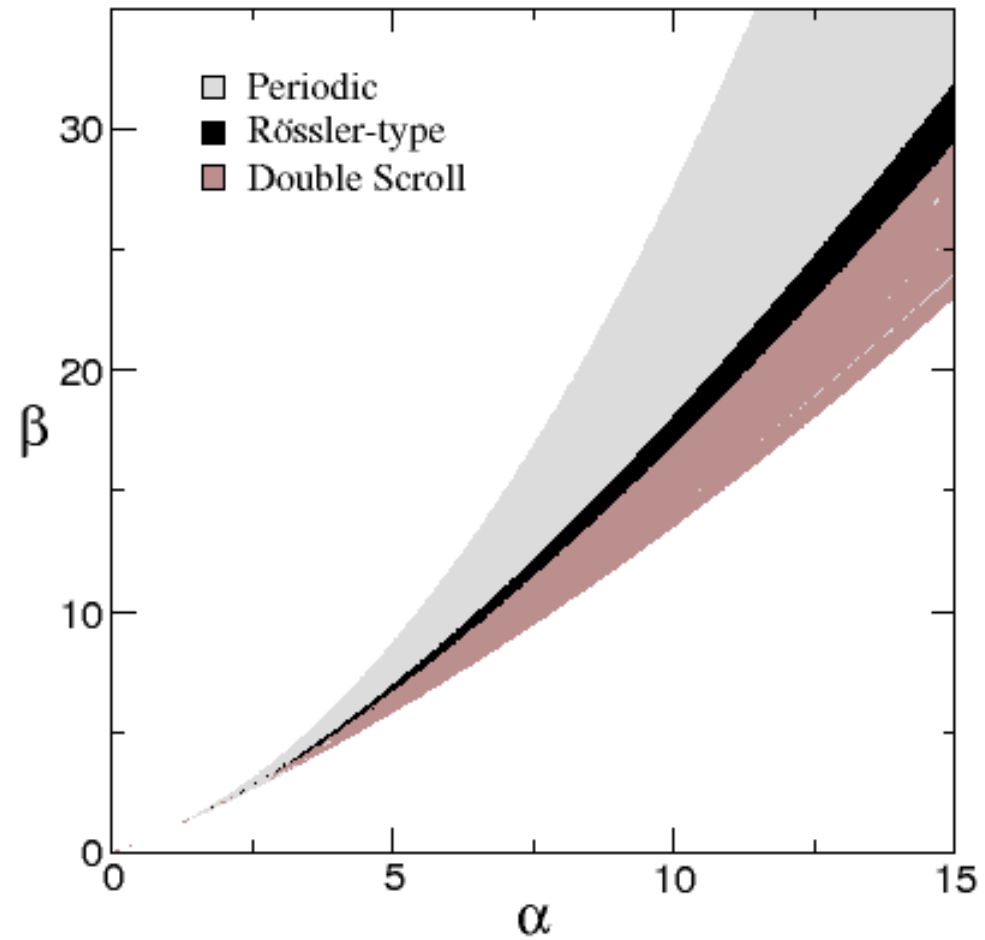
Fixed parameters are  $c_1 = 15.6$ ,  $c_2 = 1$ ,  $m_0 = -8/7$ ,  $m_1 = -5/7$ . The attracting set changes as parameter  $c_3$  changes. (a)  $c_3 = 50$ , two periodic orbits. (b)  $c_3 = 35$ , the orbits have “period-doubled”. (c)  $c_3 = 33.8$ , another doubling of the period. (d)  $c_3 = 33.6$ , a pair of chaotic attracting orbits. (e)  $c_3 = 33$ , the chaotic attractors fatten and move toward one another. (f)  $c_3 = 25.58$ , a “double scroll” chaotic attractor. This attractor is shown in color in Color Plate 18.

Chua  
Alligood et al.

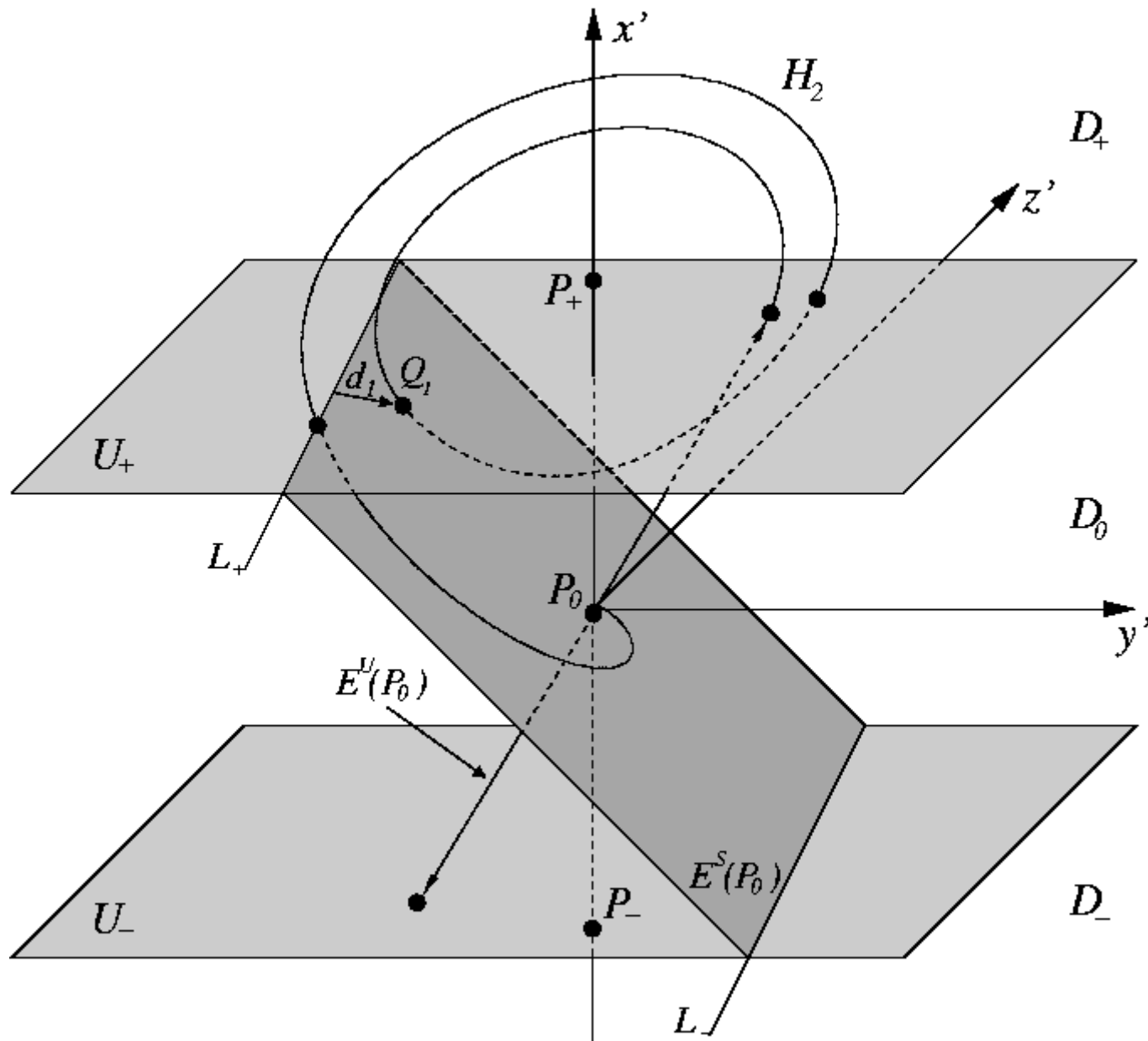
Atratores  
e  
pontos fixos  
instáveis



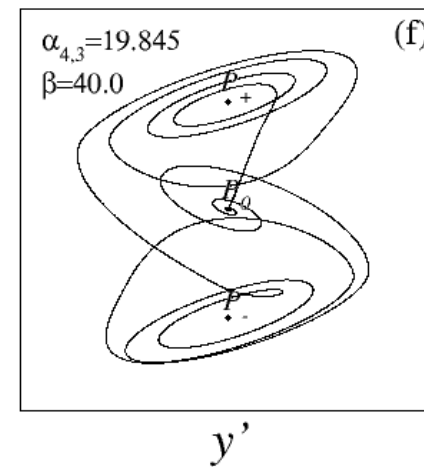
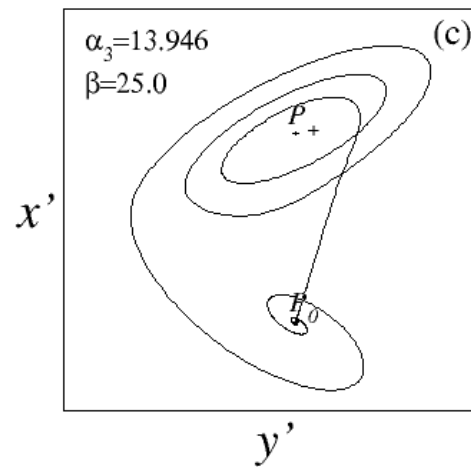
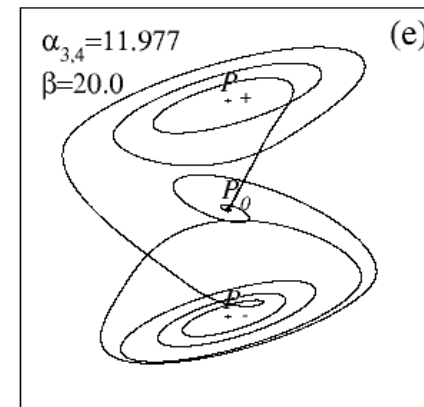
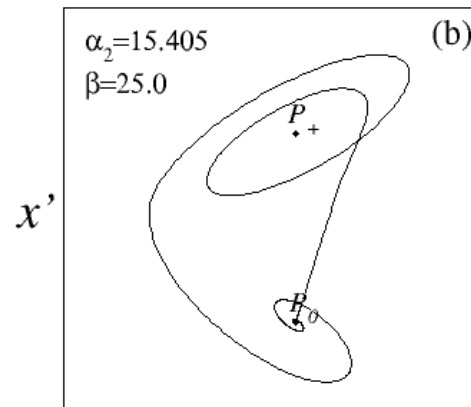
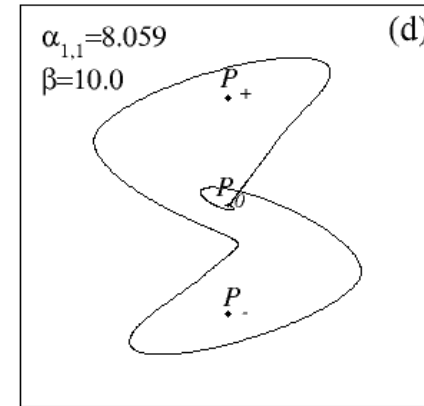
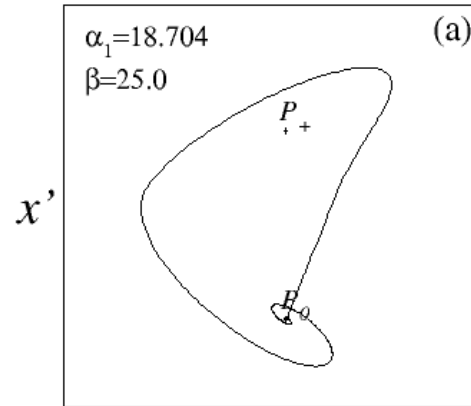
# Atratores no Espaço dos Parâmetros



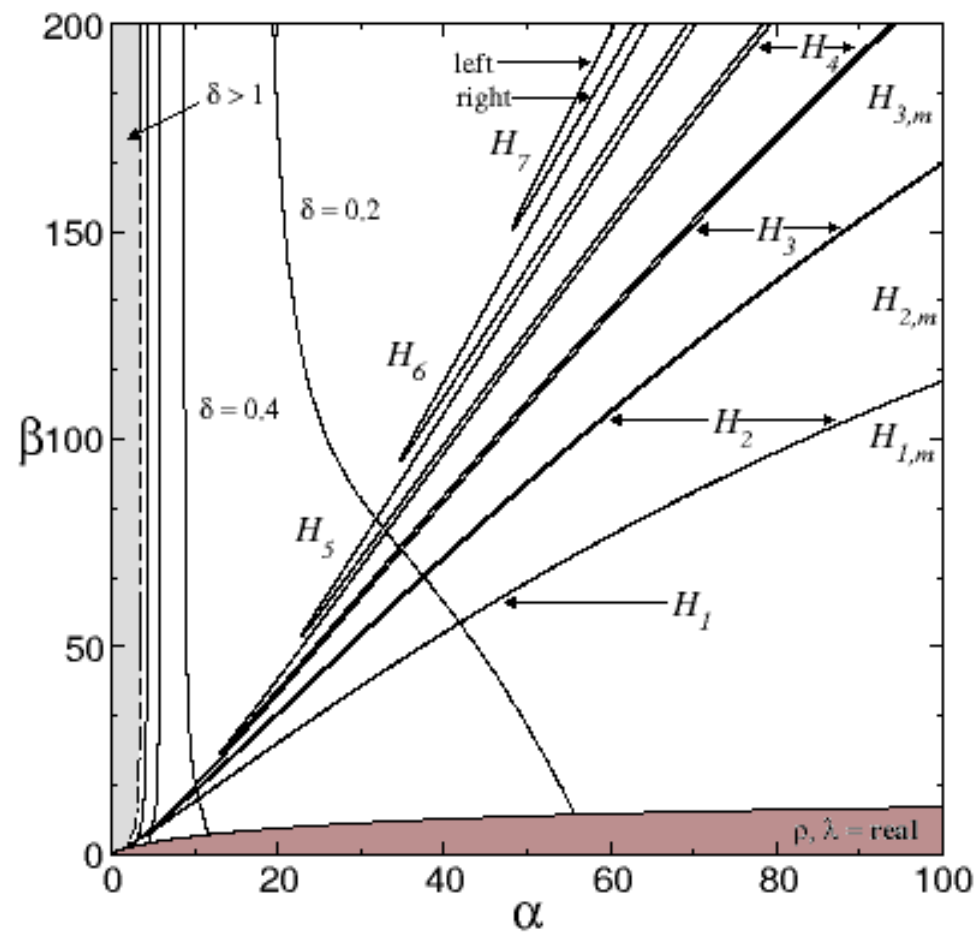
# Variedades do Circuito de Chua



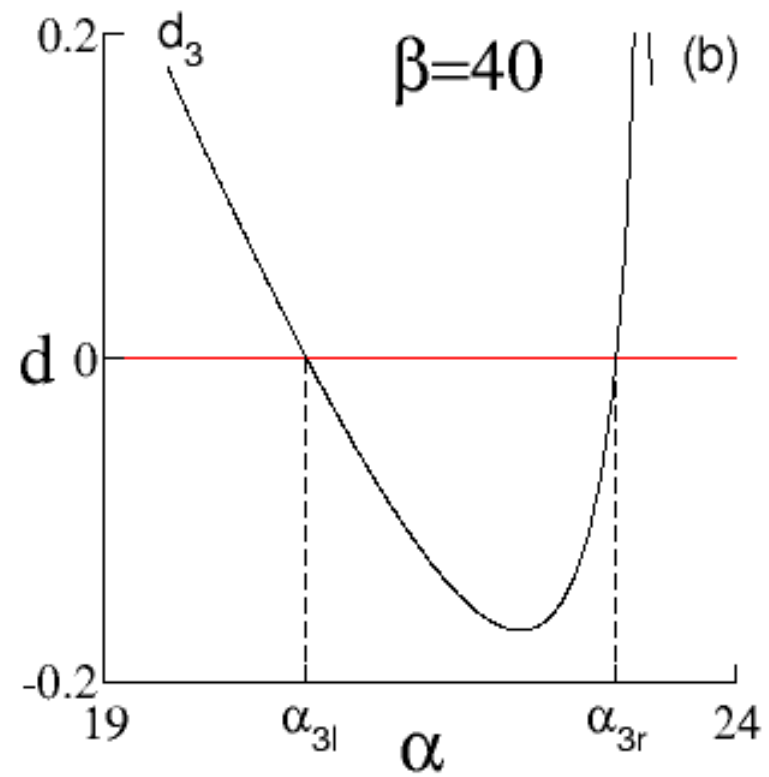
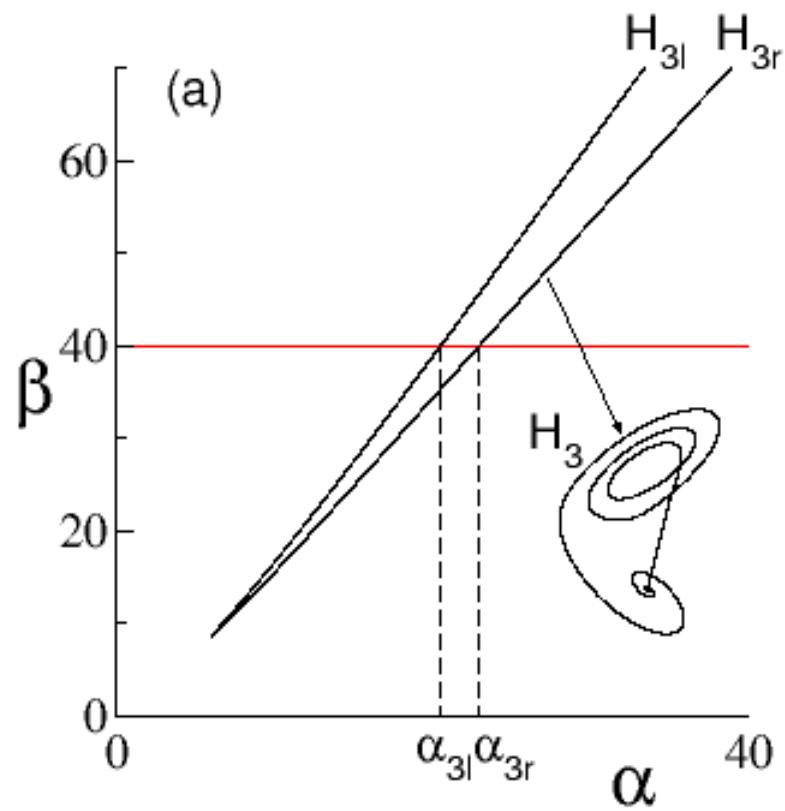
# Órbitas Homoclínicas



# Órbitas Homoclínicas Espaço dos Parâmetros



# Família de Órbitas Hoclínicas Espaço dos Parâmetros





# Circuito de Chua Perturbado

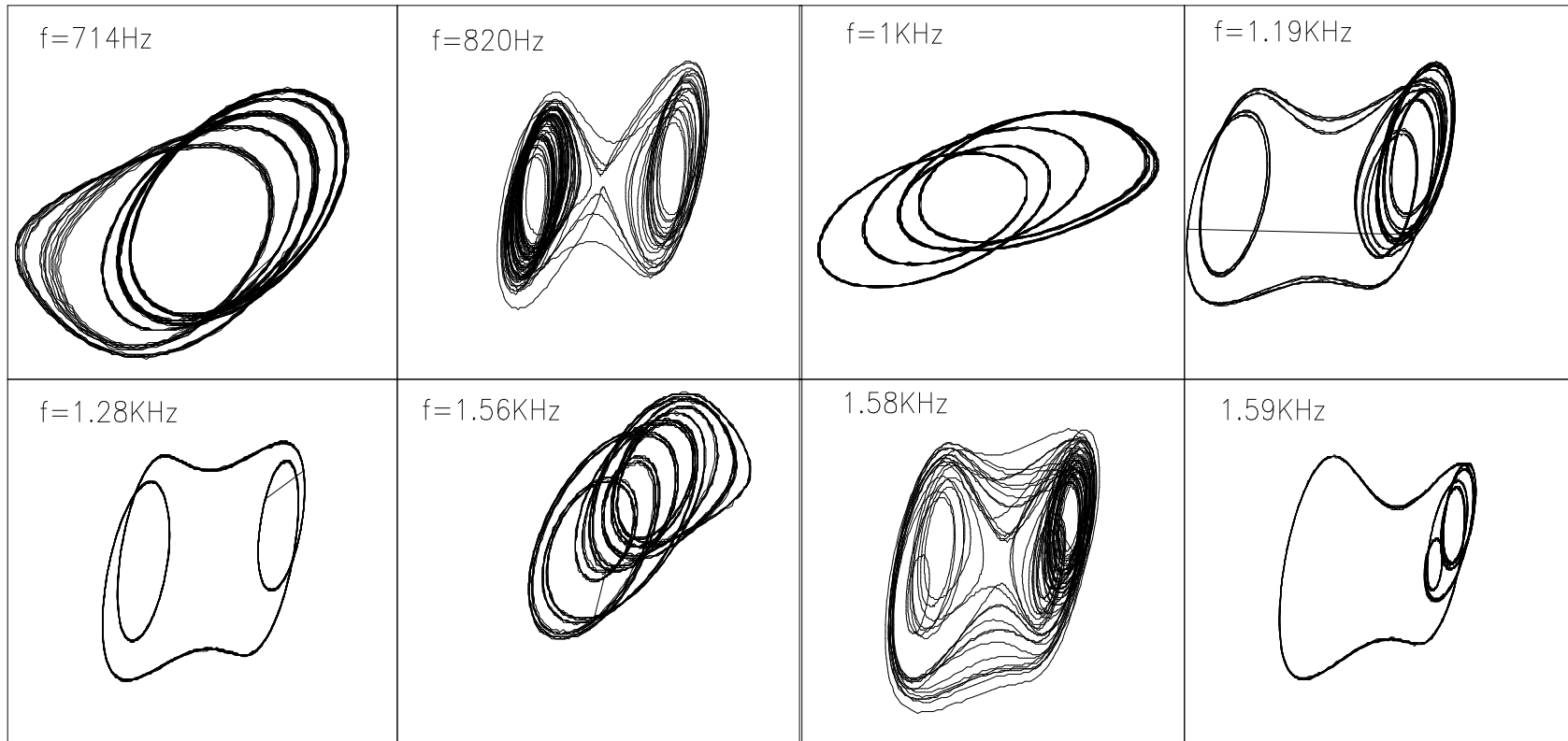
Oscilação forçada

Sincronização de dois circuitos

# Perturbação Senoidal

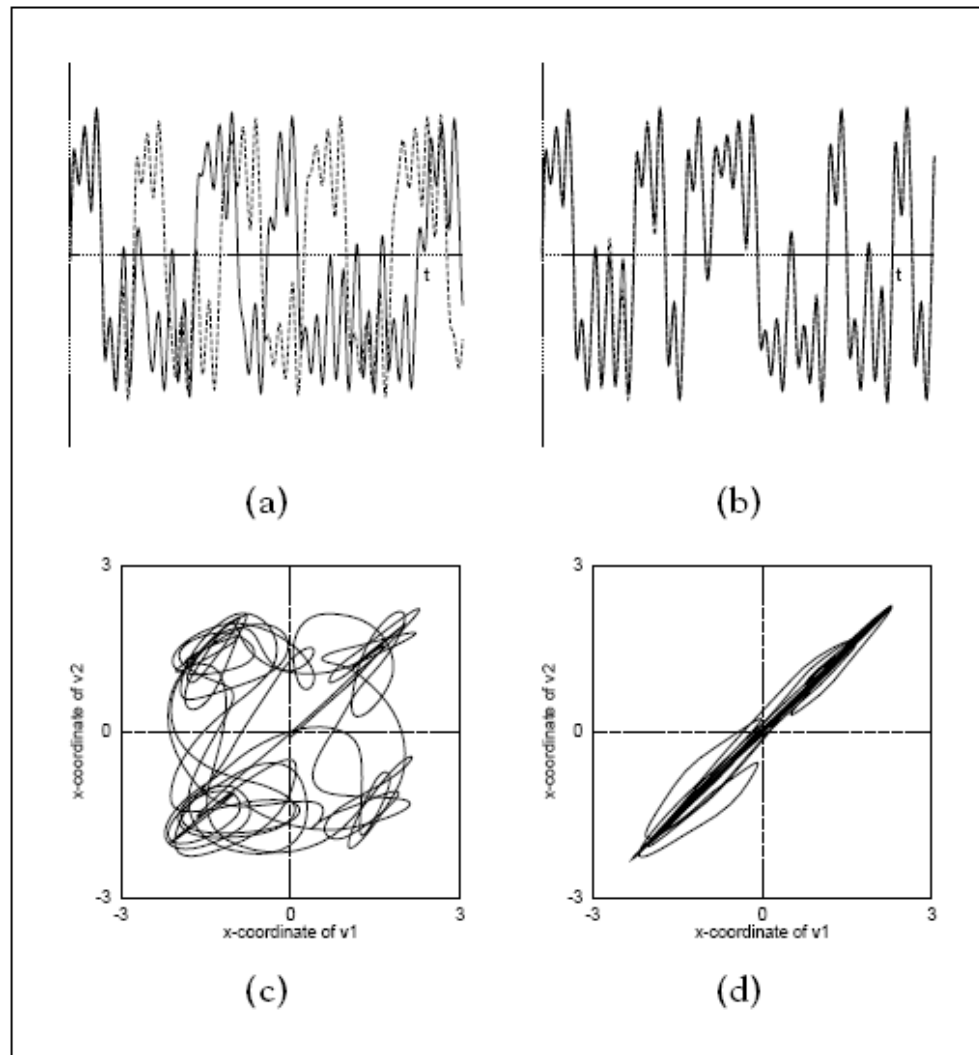
(Tese de doutoramento, Murilo Baptista, IF-USP, 1996)

Amplitude = 14Volts



## Sincronização de Dois circuitos de Chua

(tese de doutoramento  
Elinei dos Santos  
IF-USP, 2001)



**Figure 9.14 Synchronization of the Chua attractor.**

(a) Time traces of the  $x$ -coordinates of  $v_1$  (solid) and  $v_2$  (dashed) for coupling strength  $c = 0.15$ . (b) Same as (a), but for  $c = 0.30$ . (c) A simultaneous plot of one curve from (a) versus the other shows a lack of synchronization. (d) Same as (c), but using the two curves from (b). The plot lines up along the diagonal since the trajectories are synchronized.

Chaos  
Alligood et al.