Capitalization of Commuting Cost Decrements in Urban Land Rent

April 4, 2005

Abstract

We observe that for an equilibrium monocentric, open city, the aggregate commuting cost savings resulting from a unit distance cost decrement are precisely capitalized in aggregate land rent increase. Large unit distance cost decrements, given Cobb-Douglas utility functions, are undercapitalized in land rent increase. (file: trancap.tex)

- key words: land rent change, capitalization of commuting cost reduction
- JEL classification: R11, R42, O47

1. Introduction

An idea of long standing is that an improvement in transportation infrastructure should show up in higher land rents and in some cases one could contemplate paying for the improvement by taxing the new rent. Variable costs are, more or less, covered by users while fixed costs get capitalized, more or less, in increased land rents.[1] Here we consider the textbook monocentric city and observe that a commuting cost decrement which benefits all residents gets precisely capitalized in an increase in total land rent. Of interest here is that the expression for commuting cost savings takes the form of total miles of commuting before the cost change multiplied by the cost-change; that is, $-Q(N)dt$ for $Q(N)$ total miles of commuting for N residents, and dt for the unit-distance, cost-change. We parameterize utility with the Cobb-Douglas form and we are able to compute expressions for the case of discrete or "large" changes in commuting cost per unit distance. For these large changes, we observe that rent increase under-capitalizes the change in total commuting cost. We then explain how the results "go through" directly with quite general utility functions. We also comment on how the analysis for an exogenous wage increment goes through in a similar fashion. Our inference is:

[1] Lewis and Robinson [1984] related the cost of the construction of the Canadian Pacific Railway across western Canada in the 1880's to the revenues gleaned by the company from selling off land along the railway, land deeded to the company by the government of Canada. Mohring [1961] considered the impact of transportation improvements in Seattle. For distinct subway installations in Toronto, Dewees [1976] and Bajic [1983] related house price increases along the lines to the installation of the new infrastructure. Robson [1976] and Kanemoto [1980; Chapters 4 and 5] took up a different matter, namely the allocation of land to roads in an abstract urban area. Marshall [1916; p. 674] suggested that more than three quarters of the benefit of technical progress in manufactures in Britain in the nineteenth century ended up in lowering the transport costs "of men and goods, of water and light, of electricity and news". And on page 679 he notes "The rental of urban land in England is now much greater than the rent of agricultural land: and in order to estimate the full gain of the landlords from the expansion of population and general progress, we must reckon in the values of the land on which there are now railroads, mines, docks, etc."

when we compare two cities, large and small, the large one is the way it is because of (a) past commuting cost improvements and (b) past technical changes resulting in currently higher wages. Both of these "effects" are capitalized, somewhat imperfectly, in the higher rent observed for the larger city. We make use of "the open city" assumption which involves identical workers in all places achieving the same utility level. Costless migration between cities yields this "arbitrage" condition.[2]

2. The Analysis

Production in a city occurs in the center of disk and involves no space. A wage w is offered to workers who commute from the surrounding, radially-symmetric residential area. A worker's round-trip commuting costs are linear in distance, with dollar cost tx for radial distance x.[3] At x the worker spends her residual income, $w - tx$ on composite good $c(x)$, costing p dollars per unit and on housing $h(x)$, costing $R(x)$ per unit. For simplicity, $h(x)$ is assumed to be simply land, leaving $R(x)$ as rent per unit of land at x. As the residential area expands, it encroaches on land available at the edge at rent ρ. We make use of "the open city assumption" which fixes the utility level of a representative worker at the level in a hypothetical system of cities. Utilities are equalized by free migration of like workers between cities. Wages are however not equalized and difference in wages in different cities are attributed to a productivity advantage associated with a larger city (eg. Ciccione and Hall [1996]).

For the Cobb-Douglas case, the open city assumption implies $\overline{U} = c(x)^a h(x)^{1-a}$ or $h(x) = \left[\frac{\overline{U}}{c(x)^\alpha}\right]^{\frac{1}{1-\alpha}}$. We also have $pc(x) = \alpha[w - tx]$ and $R(x)h(x) = (1-\alpha)[w -$

[2]Equalization of one period utility levels by costless migration is obviously a simplification. The more valid and more complicated case is that of equalization of lifetime, present value utility levels. See for example Eaton and Eckstein [1997] and Lucas [2004].

[3]Non-linear commuting costs create problems for our analysis and are discussed below.

$tx]$. Hence $\overline{x} = \frac{w}{t} - \frac{\rho\left(\frac{1-\alpha}{\alpha}\right)^{\alpha}\left(\frac{p}{\rho}\right)^{\alpha}\overline{U}}{(1-\alpha)t}$ and $R(x) = \frac{(1-\alpha)[w-tx]c(x)^{\frac{\alpha}{1-\alpha}}}{\overline{U}^{\frac{1}{1-\alpha}}}$. And since $c(x) = \alpha[w - tx]\ /p$, we have

$$R(x) = \frac{(1-\alpha)}{\overline{U}^{\frac{1}{1-\alpha}}}\left(\frac{\alpha}{p}\right)^{\frac{\alpha}{1-\alpha}}[w - tx]^{\frac{1}{1-\alpha}}.$$

Since the number of residents in the annulus at x is $\frac{2\pi x dx}{h(x)}$ $(= N(x)dx)$, substitutions along the same lines as above for $R(x)$, yield

$$N(x)dx = K[w - tx]^{\frac{\alpha}{1-\alpha}} x dx \text{ for } K = \left[2\pi\left(\frac{\alpha}{p}\right)^{\frac{\alpha}{1-\alpha}}\right]/\overline{U}^{\frac{1}{1-\alpha}}.$$

We need the value of total round-trip commuting travel in the city, namely $\int_0^{\overline{x}} xN(x)dx$. We make use of $d\overline{x}/dw = 1/t$. Substitution from above yields

$$\begin{aligned}\widehat{xN} &= \int_0^{\overline{x}} xN(x)dx \\ &= K\int_0^{\overline{x}}[w - tx]^{\frac{\alpha}{1-\alpha}} x^2 dx.\end{aligned}$$

The integral for total rent is

$$R = (1-\alpha)K\int_0^{\overline{x}}[w - tx]^{\frac{1}{1-\alpha}} x dx.$$

We now draw on Leibnitz's Rule for differentiating "under the integral sign" and obtain

$$\begin{aligned}\frac{dR}{dt} &= K\int_0^{\overline{x}}[w - tx]^{\frac{\alpha}{1-\alpha}} x^2 dx + (1-\alpha)K[w - t\overline{x}]^{\frac{1}{1-\alpha}}\overline{x}d\overline{x}/dt \\ &= \widehat{xN} + \rho 2\pi\overline{x}d\overline{x}/dt\end{aligned}$$

and hence our central result:

$$dR - \rho 2\pi\overline{x}d\overline{x} = \widehat{xN}\ dt,$$

the increment in net land rent in the residential area equals transportation cost reduction, where the latter is the pre-change total commuting miles multiplied by the incremental reduction in cost per mile.[4] Though we do have a pure result: transportation cost saving is exactly capitalized in land rent increment, the expression for transportation cost saving is quite special. When we let the change in t be discrete or "large", we computed that net land rent change under-capitalizes the corresponding transportation cost saving.

3. General Functional Forms

Our above result turns on the linearity of commuting costs but not on the Cobb-Douglas form for the utility function. To see this, we need to check that the comparative statics result $\frac{dR(x)}{dt}$ at each fixed x is $-\frac{x}{h(x)}$, regardless of whether the utility function is Cobb-Douglas or not. With a general utility function, at each x, a consumer-household satisfies

$$\begin{aligned}\frac{U_h}{U_c} &= R(x), \text{ for } p \text{ set at unity,}\\ w - tx &= c + hR(x)\\ \overline{U} &= U(c,h).\end{aligned}$$

These are three equations in c, h, and $R(x)$. Consider now the comparative statics result, $\frac{dR(x)}{dt}$. From the third equation, we get $U_c dc + U_h dh = 0$. This combines with the first equation to yield

$$\{dc + R(x)dh\}U_c = 0.$$

From the second equation, we get

$$-xdt = \{dc + R(x)dh\} + hdR(x) = 0.$$

[4]This result can be expressed as $\frac{\partial R}{\partial t} = \widehat{xN}$, where the partial derivative means differentiation with respect only t directly.

From the last two equations, we infer that $\frac{dR(x)}{dt} = -\frac{x}{h}$ at each x. This is the key relationship that allowed us to make use of Liebnitz's Rule above with the Cobb-Douglas utility function, and to get $dR - \rho 2\pi\overline{x}d\overline{x} = \widehat{xN}\ dt$, the capitalization result. Hence our capitalization result does not turn on our choice of the Cobb-Douglas functional form.

However, if commuting costs were non-linear we could not work with $-xdt$ above and could not obtain the precise capitalization result which we get.[5] This is interesting because it indicates that capitalization is not trivially valid. Nonlinear commuting costs cause precise capitalization of transportation infrastructure improvements to not go through.

4. Wage Changes

The same procedure we have just appealed to yields the important capitalization result for an exogenous wage increment: namely $dR - \rho 2\pi\overline{x}d\overline{x} = Ndw$. This indicates that a wage increment for all workers residing in the residential area is precisely capitalized in a net rent increase.[6] This is of interest because a wage increment can be thought of as deriving from some technical progress in goods production for the urban area.[7] Land rent can only be relatively high if demanders of sites can pay the going prices. High urban land rents derive from relatively high urban incomes. When the incomes derive from producers in the city, we can link relatively high land rent to relatively high wages paid in the city. What drives up wages is the tricky issue. The city may have some special location advantage over

[5] The linear commuting costs allows us to do our analysis at each x independently of nearby x values. Nonlinear commuting costs destroys this independence.

[6] This can be expressed as $\frac{\partial R}{\partial w} = N$. This result "goes through" with general utility functions and non-linear commuting costs.

[7] Ciccione and Hall [1996] estimate the productivity premia associated with higher density of economic activity in the United States. These premia are remarkably large, particularly for the county comprising New York City.

other cities such as a good port[8] or the city may have been the repository of considerable technical progress in its goods production sector. Larger cities probably have large groups of workers with abundant human capital, which amounts to a roundabout way of saying that a technical advantage is being capitalized in the land rent of larger cities.[9] This leads to the view that relatively high land rents in large cities should be viewed as residues of past technical progress. Diluting the capitalization of wage increases in land rent could be the presence of large city disamenities such as congestion, crime, noise etc. Some of the wage increase would "leak" into expenditure for a larger "primary" commodity bundle to compensate the worker for the disamenities. In any case, when we see a large city with its large "rent tent" we should ponder how well the extra rent, relative to a smaller city, is measuring past technical change for the city and past commuting cost improvements.

Our calculations above were for a single utility-taking city in a large system of cities. In the long run, technical change in production will show up in various increases in the utility level for some super successful cities in the system. Above we obtain, $\frac{dR}{d\overline{U}} < 0$ or an exogenous increase in the parametric utility level of a city, shrinks the city. Hence the first order effect of a rise in $\overline{U}$ for a large successful city is to shrink the neighboring cities which are not experiencing rapid technical progress. The residents of the quiescent cities experience an increase in $\overline{U}$ but the these quiescent cities themselves shrink. In shrinking, land rent declines and this permits a resident to experience the welfare increase.

[8]Davis and Weinstein [2002] reflect on the question of whether a particular city is large because it exploited a positive geographic efficiency factor (eg. being a good port) or because it has benefitted particularly from scale economies.

[9]Black and Henderson [1997] have a model of two types of cities growing at a constant rate, with human capital accumulation and exogenous population growth driving the system.

5. Concluding Remarks

We have seen how the textbook monocentric residential area yields up capitalization results: those for a transportation improvement and for wage-augmenting technical change. These are interesting properties of the textbook model and are useful for shedding light on city-wide development and change. Many transportation specialists are interested in capitalization of a neighborhood transportation improvement, not city-wide changes. Thus our analysis becomes I think a parable for such specialists rather than new guidelines for planning. Our analysis could however be reworked for an examination of the improvement of one large artery in a city and such a study of capitalization might be useful in practical transportation planning.

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