Credit risk and rating assignments with parent-subsidiary links

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Abstract

This paper explores the relationship between leverage, credit risk and rating when a parent issues a guarantee for its subsidiary's debt. It analyzes the consequences of the guarantee on rating assignments, when the subsidiary's leverage is either exogenous or at its optimal level. It computes rating assignments on the basis of a structural model of credit risk for the two firms that explicitly incorporates the guarantee in favour of the subsidiary.

Default probability is lower and overall credit quality - rating included - is higher in the subsidiary than in a stand alone company, thanks to the guarantee, if its debt is fixed. However, the opposite result can hold at the optimal level of debt, since the guarantee allows to raise much higher external financing from the subsidiary - thus increasing bankruptcy costs together with tax avoidance. Starting from a BBB situation, we show that, under optimal leverage, the credit standing and rating of the subsidiary can either improve or worsen, depending on the correlation between the holding and the subsidiary operating profit. We perform a symmetric analysis for the holding.

Our model reproduces the positive relationship between rating gaps in groups and selective default observed in practice. It also differentiates spreads, for given rating, according to the ownership status (parent, subsidiary), while reproducing on average per-rating observed spreads.

Keywords: credit risk, default risk, structural models, rating assignment, parent-subsidiary.

JEL classification numbers: G32, G33, G34.

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1 Introduction

Academic models of credit risk, including the ones used for structured products and credit derivative pricing, usually consider a company as a stand-alone unit. However, companies often own - at least partially - a subsidiary unit, which issues debt in its own name. These parent-subsidiary links characterize private equity arrangements, as well as joint ventures, project financing LBOs and MBOs, and business groups. This paper describes the main features of a model that is able to account for the effect of the parent or subsidiary status on credit risk, providing as a by-product a suitable rating assignment.

The parent-subsidiary (or group, for short) link may imply very different relationships depending on legal covenants, informal support agreements, ownership levels and shared names (Samson, 2001). The model captures the case - very frequently observed - in which the parent provides a credible guarantee, but still enjoys limited liability. Thus the parent may choose the selective default of its subsidiary,i.e. to let the subsidiary default, only when its own cash flow is insufficient for supporting it. We then determine the optimal leverage policy. This maximizes the joint value of the parent and its subsidiary, by trading off bankruptcy costs and the tax advantage of debt. We analyze how this affects default probabilities, recovery rates, the associated spreads and ratings. Thus, our model advances on current rating predictions by explicitly accounting for parent-subsidiary links.

We contrast the credit quality of group and stand alone firms using a model with default costs and taxes (Luciano and Nicodano, 2008), which nests Leland (2007) model of a stand alone when the guarantee issued by the parent is not credible. It turns out that the optimal parent debt is zero: this ensures that the holding never incurs into default costs, being moreover able to transfer all its cash flow to its subsidiary in case of need. The subsidiary debt, and hence total group debt, exceeds that of two comparable stand alone firm. The guarantee by the holding lowers the expected default costs, relative to the stand alone case. This makes it optimal for the group to increase leverage, so as to reduce the tax burden.

Leland's model for stand alone firms gives optimal leverage, default probabilities, recovery and spreads close to the average observed ones, when calibrated to BBB companies. Our contribution consists in computing default probabilities and map them into ratings for group affiliated firms, using as benchmark the ones that lead to a BBB stand alone one. By so doing, we provide a rating assignment which, while simplified, captures the effect of the ownership links and the associated guarantees. For instance, the subsidiary has a B3 assignment when the correlation coefficient between parent and subsidiary is 0.2, that improves to Aa1 when the correlation falls to -0.8. This is because its higher leverage induces a higher default probability (hence a lower rating assignment than a stand alone unit) unless the parent is able to frequently support its subsidiary, as in the negative correlation case.

We also compute the optimal group leverage and rating assignment when the subsidiary cannot lever up more than a stand alone. This, beside being realistic when thin capitalization rules are enforced, allows the reader to disentangle the effect of the guarantee from that of the extreme leverage choice described above. In this case, the credit evaluations for the subsidiary differ from those of a stand alone, despite their common face value of debt. In particular, its marginal default probability is much smaller (3.62% instead of 11.2%), since the holding can support it. Its implied rating is equal to A3 (Aaa) when cash flow correlation is equal to 0.2 (-0.8), hence always higher than in the BBB stand alone case.

We also remark the consistency of our prediction with existing evidence on the occurrence of selective versus joint default (Emery and Cantor, 2005), in particular with the positive relationship between rating gaps in groups and the occurrence of selective default. We finally examine the consistency of the model-implied spreads with the average observed spreads per-rating.

The main driver of our results is the possibility, given by separate incorporation, of rescue conditional on non endangering the parent. Such a conditional rescue has been highlighted by a number of empirical studies of parent-subsidiary internal capital markets. It is consistent with the evidence in Dewaelheyns and Van Hulle (2006), who report that "private business groups support struggling subsidiaries [..]. However, once groups profitability turns negative, groups tend to terminate support to weak subsidiaries".

The rest of the paper is organized as follows. Section 2 sketches the model. Section 3 presents the numerical evaluation of optimal group leverage, when the subsidiary is either debt-constrained or not. It also provides details on default probabilities and recovery upon default. Section 4 maps these credit characteristics into an implied rating, and discusses how they differ with respect to comparable stand alone activities when the correlation between cash flow varies. It also compares model- implied spreads to observed ones.

2 The model

In this section we briefly sketch the theoretical set up, which is thoroughly explained in Luciano and Nicodano (2008).

We consider a no arbitrage environment with two dates $t = \{0, T\}$. There are two activities, and each activity i generates a random future operational cash flow X_i at time $t = T$. X_i is a continuous random variable. The risk free interest rate over the time period T is r_T. With a tax rate equal to τ_i, the aftertax value of the operational cash flow at $t = 0$ is its discounted expected value:

$$(1 - \tau_i)(1 + r_T)^{-1}\mathrm{E}X_i \tag{1}$$

where $\mathrm{E}X_i$ is evaluated under the risk neutral measure. The owners can "walk away" from negative cash flows thanks to limited liability. Thus the value of each activity with limited liability is

$$V_{0i} = (1 - \tau_i)(1 + r_T)^{-1}\mathrm{E}X_i^+ \tag{2}$$

where $X_i^+ = \max(X_i, 0)$. Firms can issue zero-coupon bonds at time $t = 0$, due, with absolute priority, at $t = T$, with principal value P_i. They have an incentive to do so as interest on debt is a deductible expense. However, debt will also increase the probability of default, which is assumed to cost a fraction α_i of (positive) cash flows and to cause a loss proportional to the firm value[1].

Let $D_{0i}(P_i)$ denote the value, at $t = 0$, of debt. Taxable income is the operational one net of interest payments:

$$X_i - (P_i - D_{0i}(P_i)) \tag{3}$$

The zero-tax level of cash flows or tax shield, X_i^Z, is then

$$X_i^Z(P_i) = P_i - D_{0i}(P_i) \tag{4}$$

while operational cash flows, net of tax payments, are

$$X_i^n = X_i^+ - \tau_i(X_i - X_i^Z)^+ = \begin{cases} 0 & X_i < 0 \\ X_i & 0 < X_i < X_i^Z \\ X_i(1-\tau) + \tau X_i^Z & X_i > X_i^Z \end{cases} \tag{5}$$

Similarly to Merton (1974), default occurs when net operational cash flow at T is smaller than the face value of the debt:

$$X_i^n < P_i \tag{6}$$

[1] In our model indeed firm value and cash flow X_i coincide at maturity T

namely, when gross cash flows are smaller than X_i^d, the default threshold, defined as

$$X_i^d(P_i) = P_i + \frac{\tau_i}{1-\tau_i} D_{0i}(P_i) = \frac{P_i - \tau_i X_i^Z}{1-\tau_i} \tag{7}$$

The level of debt determines both the probability of default, PRD_i, the recovery rate R_i and the spread y_i

$$y_i = (P_i/D_{0i})^{1/T} - 1 - r_T \tag{8}$$

We assume that the leverage policy of the firm aims at maximizing the sum of equity and debt, which in turn pairs the after-tax asset value of the firm. The value of equity and debt is the expected present value of cash flows accruing to shareholders and lenders respectively. Clearly, such cash flows vary with parent subsidiary links.

If the two activities, $i = 1, 2$, are separately incorporated and independently managed, as in Leland (2007), the face value of debt issued by firm i maximizes the value of firm i:

$$\nu_{0i}(P_i) = E_{0i} + D_{0i} \tag{9}$$

By no arbitrage the value of equity is simply

$$E_{0i}(P_i) = (1 + r_T)^{-1} \mathrm{E}(X_i^n - P_i)^+ \tag{10}$$

The payoff to lenders at time $t = T$ will equal P_i when $X_i > X_i^d$ and the firm is solvent. Lenders will absorb a tax liability $\tau_i(X_i - X_i^Z)$ in default when $X_i^Z < X_i < X_i^d$ and will suffer default costs, when the firm is insolvent. Debt present value $D_{0i}(P_i)$ -where $\mathbf{1}_{\{\bullet\}}$ is the usual indicator function -follows:

$$D_{0i}(P_i) = \\ (1+r_T)^{-1} \mathrm{E} \left[\begin{array}{c} (1-\alpha_i) X_i \; \mathbf{1}_{\{0 < X_i < X_i^Z\}} + \\ \left[(1-\alpha_i) X_i - \tau_i (X_i - X_i^Z)\right] \mathbf{1}_{\{X_i^Z < X_i < X_i^d\}} + \\ + P_i \; \mathbf{1}_{\{X_i > X_i^d\}} \end{array} \right] \tag{11}$$

When the two activities are still separately incorporated, but one of the two - the parent company - can transfer cash flows to the subsidiary in order to honour debt, both equity of the parent and debt of the subsidiary change[2]. Let us denote with X_h and X_s the pretax operational cash flows of the parent or holding ($i = h$) and the subsidiary ($i = s$), with $X_i^d, X_i^Z, i = h, s$ their thresholds. Please notice that, since the holding and subsidiary optimal debts will differ from the stand alone ones, also their thresholds will.

The transfer takes place if the subsidiary is in default while the parent is not, and if the parent is not drag into default by rescue:

$$\begin{cases} 0 < X_s < X_s^d, \\ X_h > X_h^d \\ X_h^n - P_h > P_s - X_s^n \end{cases} \tag{12}$$

The amount of the transfer is $(P_s - X_s^n)\mathbf{1}_{\{A\}}$, where A is the event described by (12).

[2] Cash or asset transfers are the way in which our stylized model can incorporate those forms of support from the parent to the subsidiary which in reality take more complex forms, such as transfer pricing at off-market prices, support in restructuring or renegotiating the terms of debt, collateral provision.

The initial owner or shareholder is assumed to choose the face value of debt in the parent and in the subsidiary so as to maximize levered group value. If we include in the holding equity (E_{0h}) dividends from the subsidiary, the group value is:

$$\nu_{0g} = \nu_0(P_h, P_s; \omega) = E_{0h} + D_{0h} + (1-\omega)E_{0s} + D_{0s} \tag{13}$$

where ω is the ownership share of the parent in the subsidiary: $\omega \in [0,1]$. For the sake of simplicity, we consider here the case in which control is exerted, but dividends are null: $\omega = 0$. We also posit $\alpha_i = \alpha$ and $\tau_i = \tau$. As a consequence of the transfer, the equity value of the holding is:

$$E_{0h}(P_h, P_s) = (1+r_T)^{-1}\mathrm{E}\left[(X_h^n - P_h)^+ - (P_s - X_s^n)\mathbf{1}_{\{A\}}\right] \tag{14}$$

The payoff to subsidiary lenders is the same as in the stand alone case, in the states where no transfer takes place and $X_s < X_s^Z$ (event B) or $X_s^Z < X_s < X_s^d$ (event C) . It changes in the transfer area (event A). The subsidiary debt becomes:

$$D_{0s}(P_s, P_h) =$$

$$(1+r_T)^{-1}\,\mathrm{E}\left[\begin{array}{c} X_s(1-\alpha)\mathbf{1}_{\{B\}}+ \\ +\left[X_s(1-\alpha) - \tau(X_s - X_s^Z)\right]\mathbf{1}_{\{C\}}+ \\ +P_s\left[\mathbf{1}_{\{A\}} + \mathbf{1}_{\{X_s > X_s^d\}}\right] \end{array}\right] \tag{15}$$

3 Credit risk: a base case

We study the credit risk implications of the organizational structures outlined above assuming that - for each company - annual cash flows are Normal i.i.d.[3]

We start from a base case, whose parameters are borrowed from Leland (2007), in which companies have identically - although not independently - distributed cash flows. The parameters which characterize the symmetric case are calibrated to firms that - as stand alone - would issue BBB-rated unsecured debt. In particular, the debt maturity is assumed to be five years, consistent with investment grade evidence. Given an annual riskless interest rate of 5%, expected operational cash flow for each activity, $Mu = 127.6$, is chosen such that its present value is normalized to $X_0 = 100$. Operational cash flow at the end of 5 years has a standard deviation (Std) of 49.2, i.e. it has an annual standard deviation $\sigma = 22\%$. The tax rate $\tau = 20\%$ and the default cost parameter $\alpha = 23\%$ are chosen so as to generate optimal leverage and recovery rates consistent with the BBB choice (see Leland, 2007).

We consider both the case in which there is limited borrowing capacity and the case in which leverage is optimized, as in the theoretical model described in the previous section.

3.1 Limited borrowing capacity

We now optimize leverage in the holding, imposing a subsidiary debt level equal to the optimal stand alone one (57.2 as in Leland).This situation models - in a simplified way - caps on the subsidiary debt due to external covenants, agency costs or regulatory constraints.[4] We expect an improvement

[3] A full study, together with a detailed presentation of the numerical results, is performed in Luciano and Nicodano (2008).

[4] There are at least two types of regulatory interventions which can cap the subsidiary debt. First, several jurisdictions impose to subsidiary managers to act in the interest of the subsidiary shareholders, rather than implement what is optimal for the group (Hadden, 1996). Higher leverage and a lower value of equity in the subsidiary could be considered as a violation of the rule, especially when subsidiaries' shareholders do not have any stake in the holding. This occurs for instance when holdings are not listed on public exchanges. Second, regulation against thin capitalization can prevent high leverage in the subsidiary.

in the credit standing of the subsidiary, all others equal, with respect to a stand alone situation. The subsidiary can indeed be rescued, while a stand alone cannot. Without debt optimization, we expect the credit standing - and the rating - of the subsidiary to improve, since it is now supported by the holding. The support is not unconditional: the improvement in the subsidiary rating cannot therefore be too strong, at least for high correlation between the subsidiary and holding cash flows. Since rescue is conditional on joint survival, there cannot be a negative impact on the credit standing of the holding. At most, it affects the payoff to its shareholders, which dismiss part of their dividends in favour of the subsidiary's debtholders. This is evident from formulas (14) and (15) above, which describe the value of the holding equity and of subsidiary debt, respectively.

Let us focus for the moment on the case $\rho = 0.2$, which is the one adopted by Leland. We obtain the following.

- The optimal debt in the parent company has a face value which is close to the subsidiary one. The relative leverage of parent and subsidiaries (50% and 53%) is close to the one observed in Belgian and Italian groups (Bianco and Nicodano, 2006; Dewaelheyns and Van Hulle, 2007). Similarly, the implied optimal group leverage (51%) is close and slightly smaller than the stand alone leverage, consistently with the empirical evidence in Deloof and Verschueren (2006), Dewaelheyns and Van Hulle (2007).

- All the endogenous credit evaluations for the subsidiary differ from those of a stand alone, despite their common face value of debt. In particular, its marginal default probability is much smaller (3.62% instead of 11.2%), since the holding can support it. Overall, the spread it deserves is also smaller, even if its recovery unfavourably compares to that of the stand alone.

- The evaluations for the holding are better than those of a stand alone, too, because the face value of its debt and its marginal default probability are slightly smaller than the stand alone ones (respectively 54 and 9.63% instead of 57.2 and 11.2%). Since the holding provides support only when this does not endanger her lenders, and has a lower face value of debt than a stand alone, it deserves a lower spread than a stand alone.

- The probability of selective default of the parent, namely default with survival of the subsidiary, by far exceeds that of the subsidiary (7.8% instead of 1.8%), since the latter has similar leverage but receives support from its parent[5].

- Thanks to the support mechanism, the joint default probability of the group (1.81%) is smaller than that of two equally correlated, stand alone companies.

Moving out of the $\rho = 0.2$ correlation case, we observe that the holding - and therefore the group - debt slightly decreases as diversification opportunities vanish. The results obtained above for mild positive correlation remain qualitatively unaffected. They are simply numerically affected by the reduction in leverage when correlation is higher than 0.2.

3.2 Optimal leverage

Let us consider now the implications for the assessment of credit risk of the theoretical model of section 2, without constraints on the subsidiary debt. We expect debt to be put a hundred per cent on the subsidiary's shoulders, since there it can be re-paid, possibly with the help of the holding,

[5] Please notice that marginal and selective default probability of a firm in a group differ, in that the first includes also the cases of joint default, while the second accounts only for default of the firm under scrutiny and survival of the other.

even when the subsidiary alone would be unable to face it. At the same time, it does not seem to be worth putting debt at the parent's level, since the subsidiary cannot intervene to help her. This mechanism should work as long s the subsidiary can save bankruptcy costs - thanks to her parent's help - and increase its tax shield. It is likely that, at a given point, increasing debt in the subsidiary would not be beneficial any more, since the holding will become unable to help. As a consequence, we expect an interior debt for the subsidiary to be optimum. In correspondence to maximum debt, we expect the subsidiary to be more levered than a stand alone, and therefore to deserve an higher default probability, higher spread, and, overall, a lower credit rating than a stand alone. we expect it to compare "unfavourably" to a stand alone: the increase in overall group-value is paid by the subsidiary. Nonetheless, we expect it to compare "favourably" to an equally levered stand alone, in the sense of having lower default probability, spread, and, overall, a better credit rating. When the correlation coefficient between the units' cash flows is equal to 0.2, we obtain the following results.

- The overall group debt is raised by the subsidiary, with the holding being unlevered, as this is the best way to solve the bankruptcy-costs versus tax-saving trade-off. [6]This ensures that the holding never incurs into default, being moreover able to transfer all of its cash flow to its subsidiary in case of need. The subsidiary is saved from bankruptcy, so as to avoid the proportional loss of value inherent in default, 52.35% of the times - with our parametrization. It is not rescued only when this would deplete the overall group value without avoiding bankruptcy costs, as the holding company uses its limited liability in these contingencies. [7]
- The optimal face value of debt for the subsidiary is higher than that of two stand alone companies. The guarantee by the holding lowers the expected default costs, relative to the stand alone case. This makes it optimal for the group to increase leverage, so as to reduce the tax burden.This is consistent with the empirical evidence in Dewaelheyns and Van Hulle (2006), who notice that the "decreased potential costs of financial distress allow group members to ex ante take on more debt, thus realizing more tax gains". It is a fortiori consistent with the very high leverage observed in project financing, LBOs and private equity, which are closer to our assumption of no agency costs.
- Leverage impacts on the marginal and selective default probabilities. The latter goes to 46.2 % for the subsidiary, a number much higher than the selective default probability of both the parent (7.8%) and the subsidiary itself (1.8%) in the constrained case. Selective default is hence much more likely under optimal than under constrained leverage.
- The recovery rate falls from 48.1% for the stand alone to 31.2% for the subsidiary. Indeed, the parent is more likely to be unable to provide support when the subsidiary's losses are larger - leaving these low recovery cases to lenders. The decline in recovery when the default probability increases - or, equivalently, the increase in loss given default - is an important feature to capture, since empirical evidence supports it (see for instance Altman and Fanjul, 2004).
- As a consequence of high default probability and low recovery, the credit spread dramatically increases for the subsidiary with respect to the stand alone (8.4% versus 1.26% over five years).
- Selective default of the subsidiary - i.e. default with survival of the parent - occurs in 46.2% of the possible cases, while rescue occurs in 52.35% of the occurrences. These are the two

[6] There are less extreme leverage situations when the holding has on average higher cash flows than its subsidiary. Similarly, optimal leverage converges to the stand alone one as the credibility of the guarantee tends to zero. See Luciano and Nicodano (2008) for details.

[7] This outcome is also a potential explanation for the coexistence of highly leveraged transactions with a relatively low occurrence of defaults in private equity and LBOs (Jensen, 2007).

most likely scenarios. Joint default, given that the parent is unlevered, is very rare (0.34%) compared to joint default of two stand alone firms (2%).

What happens when we change correlation from the base case 0.2? One may expect that the optimal face values of debt in groups will converge to the stand alone level as correlation among cash flow increases, since the transfers from the parent to the subsidiary will become less likely. This intuition is incorrect: debt in the parent continues to be zero, because this still allows to eliminate the parent bankruptcy costs and to maximize the possibility of rescue in the subsidiary. In the base case explored so far, the tax shelter differential between raising debt in the parent and raising it in the subsidiary is evidently not strong enough to move debt from the subsidiary to the parent - i.e. from the company which can be rescued to the other one. Not only the parent remains unlevered, but the debt of the subsidiary increases: support from the holding decreases, recovery tends to decrease and this allows for further debt to be issued by the subsidiary. Moreover, the optimal level of debt is higher. In fact that expected default costs are increasing in subsidiary's debt, because not only its default probability but also because proportional default costs conditional on default are larger. But, the larger is ρ, the likelier it is that the cash flow in the parent suffices to rescue the subsidiary when conditional default costs - that are proportional to the subsidiary cash flow - are also large.

Overall, starting from BBB firms, the existence of guarantees together with separate incorporation rationalizes the extreme leverage levels observed in practice for organizations such as private equity funds, LBOs and MBOs. Extreme leverage turns out to be accompanied by a relatively low default probability in our model: even in this respect, our model rationalizes empirical evidence, since the occurrence of default is quite rare in the going-private world, despite high leverage. On the contrary, our model with limits to subsidiaty's debt seems to reproduce the credit features of listed traditional business groups. These are more likely than private groups to be constrained in borrowing by agency costs, covenants or regulatory interventions. In a companion paper (Luciano and Nicodano 2008) we show that these results are robust with respect to the ownership share.

4 Rating assignment for group members

The activities studied in the previous section deserve a BBB rating in Standard & Poor's scale when they are incorporated as stand alone (Leland, 2007), i.e. when they do not support each other. This section assigns a rating to those activities once they are affiliated to the group. It will become apparent how the guarantee may lead to ratings which differ markedly for group affiliates and stand alone firms. We determine ratings with the following procedure:

- we compute their model-implied default probabilities, over the fixed calibration horizon (5 years)[8]. Such probabilities will differ depending on the role of the firm in the group (parent, subsidiary) and on whether we consider groups with limited leverage capacity or optimally levered groups (constrained and unconstrained ones);
- we compare such default probabilities with the historically observed probabilities (frequencies) of firms in different ratings;
- we assign to each firm the rating which would have produced the same default frequency as its model default probability.

[8] Please notice that the probabilities we have presented so far are risk neutral ones, as needed for recovery and spread computations. Given the structural nature of the underlying model it is however possible to compute the corresponding historical probabilities, which are called here model default probabilities. In order to do that, we assumed a risk premium equal to 4%, motivated as in Huang and Huang (2003).

To be specific, we take as a reference for default frequencies the five-year average default occurrences over the period 83-99, as resulting from Moody's statistics, unadjusted for withdrawals (see Keenan et al., 2001). We first map a Standard & Poor BBB firm - our base case - into a Baa3 Moody's rating: indeed our stand alone firms have a default probability of 4%, which is (included in the range of) the default frequency of Baa3 firms. We then repeat the mapping for different group members, in different (constrained and unconstrained) group configurations.

Table 1 below presents our rating assignments, for different correlation levels. Such assignments are obviously oversimplified and must be understood as a first approximation.[9] Nonetheless, they have at least two advantages:

- they are a direct implication of a model that explicitly accounts for guarantees in holding-subsidiary structures;
- as such, it is easy to study their behavior with respect to asset correlation, ownership share (and eventually individual firm's features, such as size and volatility). In Table 1 we indeed present the rating assignment as a function of asset correlation, for the cases of limited (upper part) and unlimited borrowing capacity (lower part). We also report the spread implied by the model as well as the spread observed in some empirical studies, namely Huang and Huang (2003) and Elton et al. (2001)

Table 1

Default probability, spread, rating assignments and leverage for different correlation levels.

Leverage is computed as the ratio between market debt value and total firm value $\left(\frac{D_0^*}{V_0^*}\right)$.

HH and EG = observed spreads in Huang and Huang (2003), Elton et al. (2001).

	cash-flow correlation				
	-0.8	-0.2	0	0.2	0.8
unconstrained subsidiary					
hist def prob	0.30%	13.80%	19.35%	22.13%	32.02%
closest implicit rating	Aa1	Ba3	B2	B3	Caa-C
model spread (bp)	174	683	805	842	1040
observed spread (bp)	Aa: 65 HH	Ba:320 HH	B:470 HH	B:470 HH	B: 470 HH
leverage	80.53%	72.24%	70.95%	70.32%	68.82%
constrained subsidiary					
hist def prob	0.00%	0.14%	0.36%	0.74%	2.30%
closest implicit rating	Aaa	Aaa	Aa1	A3	Baa2
model spread (bp)	16	36	47	58	98
observed spread (bp)	Aaa: 55 HH	Aaa: 55 HH	Aa: 65 HH	A: 96 HH, 74 EG	Baa: 158 HH, 121 EG
leverage	53.47%	53.20%	53.04%	52.89%	52.35%
constrained holding					
hist def prob	3.96%	3.76%	3.76%	3.39%	2.89%
closest implicit rating	Baa3	Baa3	Baa3	Baa3	Baa3
model spread (bp)	125	121	121	112	100
observed spread (bp)	Baa:158 HH, 121 EG	Baa:158 HH, 121 EG	Baa:158 HH, 121 EG	Baa:158 HH, 121 EG	Baa:158 HH, 121 EG
leverage	52.75%	51.77%	51.65%	49.92%	47.07%

[9] They could be adjusted, based on more specific considerations (ring fencing, common names, common country and the like).

The main features of the previous table are the following:

- When debt is at its optimal level, the subsidiary can either improve or worsen its ranking depending on correlation: it would deserve from Aa1 to C, instead of the Baa3 of a stand alone. Leverage in the subsidiary is so high that its rating worsens, unless the correlation between the cash flows is so low that rescue is effective in most cases. Indeed, the corresponding stand alone leverage is 52%, while the subsidiary one in this case goes from 68.82% to 80.53%. These results are reported in the "unconstrained subsidiary" section of Table 1.
- When the subsidiary has limited borrowing capacity, its leverage (which stays always above 52%) increases with respect to the stand alone case (52%). Nonetheless, its credit rating improves with respect to the stand alone situation, because of the parent guarantee. It moves from Baa3 as a stand alone into a range including Aaa (when correlation is low and negative) to Baa2 (when correlation is high and positive). The holding is not too much affected: its rating stays at the Baa3 level, with a slight reduction in default occurrence. The results are reported in the "constrained subsidiary" and "constrained holding" sections of Table 1.

In Table 1 we omit the rating of the unconstrained holding, because it is unlevered: its default probability would correspond to the probability of negative cash flows. With our modelling parameters, the unconstrained holding qualifies as a Aaa company. As a consequence, the rating gap between the parent and its affiliate is lower in the constrained case (Baa3 for the holding versus Aaa to Baa2 for the subsidiary) than in the unconstrained one (Aaa for the holding versus Aaa1 to C for the subsidiary). Empirical evidence - reported by Emery and Cantor (2005) - shows that the likelihood of selective defaults increases with the rating gap between group members before default. In our model, selective default is more likely in the unconstrained than in the constrained case (see the previous section): the model is therefore able to reproduce the positive relationship between rating gap and selective default observed in practice.

Table 1 presents also the theoretical spreads. Such spreads differ according to the company status in the group, holding rating fixed. Consider the Baa3 case. The holding is Baa3 in the constrained case, when it has a spread ranging from to 125 to 100 basis points (bp from now on). On average, this is lower than the model spread of the stand alone with the same rating, 126 bp (not reported in the current table) because of its lower leverage. At the opposite, an unconstrained subsidiary, when Baa3, has a spread higher than the stand alone with the same rating. It pays already 174 bp when it is Aa1, due to its lower recovery upon default, which in turn originates from the way in which the parent supports it. Finally, a constrained subsidiary pays even a lower spread than a constrained holding, for given rating, given that it has a similar leverage but receives support. It is well known in practice that a single rating encapsulates many different realities: our model reproduces this feature.

Below we proceed to compare our model-implied spread with observed ones. The observed one are measured out of samples that aggregate parent, subsidiaries and stand alone companies. We therefore expect observed "average" spreads to be lower than those for unconstrained subsidiaries, but slightly higher than the ones of both parent companies and constrained subsidiaries.

In Table 1 we compare our theoretical spreads with the observed per-rating ones reported in Huang and Huang (HH,2003) and Elton et al. (2001). Their statistics are given per alphabetical rating, while we have an alphanumerical scale in Table 1: they are therefore coarser than ours. In addition, they do not include the C category.

Observe the constrained cases first. When asset correlation moves from -0.8 to 0.8, the holding and the subsidiary display model spreads ranging from 125 to 100 bp, and from 16 to 98 respectively. The observed spreads for the corresponding ratings are as follows: 158 bp according to Huang and Huang (121 bp according to Elton et al. (2001) for an "average company" with the same rating as the holding, and a range from 55 to 158 for an "average company" with the same rating as the constrained subsidiary. As expected, and consistent with the fact that observations cover different company types, the model produces spreads lower than the observed per-rating ones.

Consider now the unconstrained case. Our subsidiary has a model spread which ranges from 174 to 1040 bp, with a rating moving from Aa1 to C. Consistent with the above remarks, for each rating the model spread remains higher than the observed one.

5 Summary and conclusions

This paper has explored the relationship between leverage, credit risk and rating when there is a guarantee between firms. It has described a structural model of a parent and a subsidiary, which issues debt in its own name under a guarantee by the parent. It has analyzed consequences on rating assignment, both when leverage is not optimized and when it is. The model provides an implied rating assignment to each group member, based on whether it guarantees the other members (our holding) or receives support by them (our subsidiary); and on whether it can - or cannot - optimally lever up. Furthermore, the model can also be calibrated to other company characteristics, such as size, cash-flow volatility and cash flow correlation.

The paper presents ratings associated to cash flow correlations, and argues that these model-implied rating assignments are reliable on two different grounds. First, the model is able to reproduce the positive relationship between rating gaps between group members and selective default observed in practice. It also differentiates spreads, for given rating, according to the parent-subsidiary status, while reproducing on average per-rating observed spreads.

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