Index Numbers

Index numbers are used to compare the current unit price for an item or a specified collection of items to the price in a *base period*. They vary in the methods used for combining the current and base prices of the items in the collection [the *weighting* of the prices for different items]. One important use of indexes is *deflating* a series of spending or price values to reflect the change in prices (and of the value of money) to allow comparisons across time.

1. **Relatives** Price for *one item* relative to a given year (no question of weighting)
   $I = \left(\frac{P_n}{P_0}\right) \times 100$ with P_n = price now, P_0 = price at time 0 (base time)

2. **Aggregated price indices**

   (a) **Unweighted** (same weight for all items — ignores frequency of purchase/use)
   $I = \left(\frac{\sum P_n}{\sum P_0}\right) \times 100$ (sum is over all items in the aggregation – the collection)

   (b) **Weighted** [attempts to account for quantity purchased difference is: quantity in what year?]

      i. Laspeyres (weighted by relative amounts in *base year*)
      $I = \left(\frac{\sum P_n Q_0}{\sum P_0 Q_0}\right) \times 100$ [change in price at weight (value) of base year]

      ii. Paasche (weighted by relative amounts in *current* year)
      $I = \left(\frac{\sum P_n Q_n}{\sum P_0 Q_n}\right) \times 100$ [change in price at weight (value) of current year]

      iii. Fixed weight
      $I = \left(\frac{\sum P_n Q_a}{\sum P_0 Q_a}\right) \times 100$ [change is price at weight (value) determined by some fixed consideration (generalization of Paasche and Laspeyes)]

      iv. Fisher ideal price index
      $I = \sqrt{Laspeyres \times Paasche}$

3. **Aggregate price relative indexes**

   (a) Unweighted
   $$I = \frac{\sum \frac{P_n}{P_0} \times 100}{N}$$

   (b) Weighted
   $$I = \frac{\sum \frac{P_n}{P_0} \times 100 \times v}{\sum v} = \frac{\sum P_n Q}{\sum P_0 Q} \times 100$$

      i. Laspeyres if $v = P_0 Q_0$ (if $Q = Q_0$)

      ii. Paasche if $v = P_0 Q_n$ (if $Q = Q_n$)

      iii. Fixed if $v = P_0 Q_a$ (if $Q = Q_a$)

4. **Quantity indexes**

   (a) Quantity relative
   $I = \frac{Q_n}{Q_0} \times 100$ with Q_n = quantity now, Q_0 = quantity at time 0 (base time)

   (b) Weighted aggregate quantity index
   $I = \frac{\sum Q_n w}{\sum Q_0 w} \times 100$
   where wW is the weight for each item which could be a value added or a price for some specific year.
   If $w = P_0$ this is Laspeyres with roles of P and Q reversed.

5. **Deflating a Series by Price Indexes**
   To compare figures from different years for some quantity represented in dollars (or pesos or any other currency) we want to remove the effects of inflation. Otherwise, increases in dollar amounts which are due only to a rise in the overall level of prices would exaggerate (or understate) the changes in real value. This is referred to a s*deflating the series*. This is typically done for year-to-year comparisons of Gross Domestic Product, cost of housing, mean and median household income, etc.

Using an index to deflate and compare prices between years:

To compare prices at any time t to prices in a base year:

(a) Using an index that is also based in year 0: adjusted price $P_a = \dfrac{P(t)}{I(t)} \times 100$
($P(t)$ = current price or price in year of interest) $I(t)$ = Index value for current year (year of interest), P_a = price in base year dollars.

(b)) Using an index that is based in a year y different from the base year : adjusted price $P_a = \dfrac{P(t))}{I(t)} \times I(0)$
($P(t)$ = current price or price in year of interest) $I(t)$ = Index value for current year (year of interest), $I(0)$ = index for the year to use as base year (both $I(t)$ and $I(y)$ based on year $y)P_a$ = price in base year dollars.

The percentage increase in *overall prices* from year t_1 to year t_2 is given by $\dfrac{I(t_2) - I(t_1)}{I(t_1)} \times 100$.
The percentage increase in *the price of one good* from year t_1 to year t_2 is given by $\dfrac{P_a(t_2) - P_a(t_1)}{P_a(t_1)} \times 100$.

Example for class:

ITEM	UNIT	PRICE 1996 dollars	QUANTITY 1996	PRICE 2006 dollars	QUANTITY 2006
Gasoline	gallon	\$.95	1000	\$2.28	900
Oil	quart	\$.90	15	\$2.89	13
Tires	each	\$72	3	\$97	2
Insurance	policy	\$980	1	\$1240	1

INDEX NUMBERS - EXERCISES

1. The only data concerning the prices in Iran is from two different sources, but the two sources have different base years.

Year	Source A	Source B
1998	100	
1999	106	
2000	113	
2001	121	
2002	125	100
2003		107
2004		111
2005		118

(a) Splice the two indexes to form a single index with 1998 as the base year.

(b) Splice the two indexes so that 2002 is the base year.

2. The Consumer Price Index for January 2006 was 305.4 and for February it was 307.4.
(a) Does this mean that prices increased by 2% during the month?

(b) By what percent did prices increase during the month?

(c) What is the annual inflation rate based on the change in prices during this month?

3. The Consumer Price Index (base 1982-84) is as given below:

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
CPI	130.7	136.3	140.3	144.5	148.2	152.4	156.9	160.5	163.0	166.6

(a) What was the inflation rate for 1999?

(b) If you earned $15,000 in 1993, how much would you need in 1998 to have the same purchasing power?

(c) You earned $200 per week in 1994. In 1995 you got a 10% raise and in 1996 you got a 7% raise. How has your purchasing power changed?

4. In arranging for short-term credit with the bank Top Notch Corp. needs to project its cash needs on a monthly basis. To help in doing this the following monthly indexes for cash requirements have been developed:

Month	Index	Month	Index
January	40	July	90
February	100	August	120
March	140	September	180
April	60	October	140
May	110	November	80
June	90	December	50

(a) The cash needs for Top Notch are expected to average $1,100,000 per month for 2005. Translate the needs into a need for each month, based on the indexes.

(b) If Top Notch spent $600,000 in January, how well does this expenditure fit the 1.1 million dollar per month forecast?

(c) If Top Notch needs $630,000 for January, what would be their projected need for the whole year?

(d) If Top Notch had planned to need $13,985,000 for 2005, how much should they need for March?

5. The average weekly earnings for persons employed in manufacturing is given below:

Year:	1990	1991	1992	1993	1994	1995
Earnings:	$133.75	142.44	154.69	166.06	176.40	189.51

(a) Using the CPI values from exercise 3, deflate this time series.

(b) From part (a), comment on what was happening to the real earnings of people employed in manufacturing.

6. The following table shows the price of a share of stock for the first nine months of 2003.

Month	1	2	3	4	5	6	7	8	9
PRICE	33.1	33.9	34.7	35.0	34.9	35.4	34.7	38.8	37.1

a) Form a price index with January as the base.

b) Form a price index with April as the base.

7. The following data show lumber production in billions of board feet in the United States for various years (from the Statistical Abstracts of the United States, 1988 p.642):

Year	1976	1977	1978	1979	1980	1981	1982	1983
Production	37.0	39.4	40.5	40.6	35.4	31.7	30.0	34.6

Construct an index of lumber production with 1980 as the base year.

8. The following data show prices and quantities of food purchased by a family in 2000 and 2005.

Commodity	Price 2000	Quantity 2000	Price 2005	Quantity 2005
Meat-pound	$2.60	9	$3.20	11
Vegetables-pound	1.18	8	1.28	7
Milk-gallon	1.79	5	2.09	5
Bread-loaf	0.45	3	0.65	4
Fruit-pound	1.15	3	1.50	6

Use 2000 as the base year and assume that prices are unit prices to calculate the following 2005 price indexes.

a) Laspeyres

b) Paasche

c) Fisher Ideal

9. Explain a possible deficiency of the Laspeyres price index.

10. Explain a possible deficiency of the Paasche price index.

11. Explain what the Laspeyres index found in 8(a) tells us about the price of the market basket.

12. An employee notices that the Consumer Price Index increased from 326.4 in 2005 to 340.4 in 2006. The employee observes that 340.4 - 326.4 = 14.0 and requests a 14% raise in order to maintain her standard of living. What should management say to the employee?