

## Estudio Crítico

### CIRCLING GOTTLÖB FREGE\*

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*Frege: Importance and Legacy* is based on papers on logic, philosophy of mathematics, epistemology, and philosophy of language presented by some of the foremost American and European Frege scholars at a conference on foundational problems in Frege's works and modern logic held in Munich in July 1991. The stated ambition of its editor, Matthias Schirn, is to display both the breadth and the significance of current Frege research as well as to make good a second claim of the title that Frege left a legacy, a set of questions to be answered (something about which there can, of course, be no doubt).

The discussion of Frege's work over the last fifteen years, Schirn considers, has been motivated by the desire to "locate his work more accurately in the history of logic, mathematics and philosophy," "bring into sharp focus and reassess both his logicism and his arithmetical platonism," "examine more thoroughly particular aspects of his logical theory," "analyze his mathematical work in *Grundgesetze*," "investigate the various facets of his epistemology," "provide a systematic account of his semantics and to develop further certain central ideas of it" (p. 28). The book itself emphasizes the importance of Frege's philosophy of mathematics, to which 2/3 of the work is devoted; Frege's work on the philosophy of language or epistemology plays a subsidiary role.

Schirn's conviction that "to reveal errors or shortcomings in Frege's work may well go hand in hand with admiration for its major achievements, the power and depth of his argument and the lucidity of both his exposition and his style" (p. 1) sets the tone of the collection and also reveals something of the maturity of the editor's approach. Several of the

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articles anthologized here raise really pertinent questions of more fundamental significance than one usually finds in the literature on Frege and advance theses that warrant further thorough, competent investigation, thus lending the work a spirit of lively, intelligent inquiry. So the work genuinely represents a step forward in providing a truer likeness of Frege's ideas as based on what he actually wrote and not just on a likable, even fanciful, interpretation of his work which more mirrors what many analytic philosophers have believed or wished he had written.

Looking beyond the titles of the papers, or even the stated intentions of the editor, one finds most of the book directly or indirectly circling in on three important themes: the underlying reason for the inconsistency of the *Grundgesetze* system; Frege's Platonism and the role of logical objects in his thought; and Frege's suggestive, but incomplete, remarks on epistemology.

George Boolos, Michael Dummett and Christian Thiel try to locate the mistake in Frege's reasoning that led to the inconsistency in his system, to what Dummett terms the "colossal blunder" that caused the *Grundgesetze* to go "so disastrously wrong" (p. 253).

Rightly noting that there is as yet no unanimity as to the "real" reasons for the failure, in "On the Structure of Frege's System of Logic," Thiel points the finger at the horizontal which, he argues, invites "inappropriate liberality" in admitting *Wertverläufe* like the class that does not belong to itself, hand in hand with the function names out of which they were formed. He calls these "Trojan expressions" and suggests that excluding them might be a solution to Frege's inconsistency problem (p. 275).

Peter Simons's paper, "The Horizontal," complements Thiel's. Despite the inconsistency, Simons says, Frege's logic "has its own oddities and beauties, and can be studied for its own sake both from an historical and from a logico-aesthetic point of view" for what this reveals about how Frege thought (pp. 281–82). Avoiding doing so in a way in which the inconsistency of the system might impinge on his efforts, Simons pokes into "interstices" of Frege's system which he "had neither the time nor, later, the inclination to investigate" (p. 282). Studying the "massive reinterpretation" Frege's notation underwent between the *Begriffsschrift* and the *Grundgesetze*, Simons comes to thirteen conclusions regarding Frege's reintroduction and reinterpretation of the horizontal (known by him to be redundant) in the *Grundgesetze*, a change which Simons con-



siders to have had "the greatest interpretative repercussions" and to have brought about a "thorough change of sense" (pp. 285–86).

In "Whence the Contradiction," George Boolos disputes Dummett's claim in *Frege: Philosophy of Mathematics* that "the serpent of inconsistency" entered Frege's paradise via the second-order quantifier, a diagnosis which Boolos calls subtle, powerful and unified, but too recherché. According to Boolos "the culprit is the obvious one, Basic Law V" (pp. 235–36), which he brands as "simply a (higher-order) logical falsehood" (p. 249). It was "not so much Frege's insouciance concerning second-order quantifiers that was responsible for his downfall," Boolos maintains, "as his adoption of a theory about a function from second- to first-order objects that could not possibly be true, facilitated by a lingering attachment to the idea that 'contextual definitions' like Hume's principle and Basic Law V, are, if not logically true, then near enough as could make no difference" (p. 245). What Dummett has taken to be the cause of all the trouble, Boolos thinks, should be considered a 'background condition' (p. 239).

Dummett responds to these criticisms in his usual sweeping, categorical style. Frege, Dummett holds, wanted to know what justifies us in assuming the existence of abstract objects, those of a fundamental mathematical theory, in particular. The basis of his answer was the context principle, by which reference to a range of abstract objects is justified if the senses of sentences involving it can be stipulated without presupposing the existence of those objects. The reference of the terms of the theory had to be stipulated by laying down the values of functions, including concepts, that take their referents as arguments and this procedure was validated by a consistency proof that a unique reference had been stipulated for every well-formed expression. This proof was integral to his entire conception of the manner in which one was to justify introducing a range of abstract objects, but it breaks down in the presence of second-order quantification.

Unfortunately, Dummett, Boolos and Thiel have not inquired very far back in their search for the origins of the inconsistency. They shed light on the workings of the inconsistency as it appeared in the *Grundgesetze*, where it first reared its ugly head. By the time the *Grundgesetze* was published, though, the beast of inconsistency had already been slouching around in Frege's "paradise" for some time. The stage for its disturbing appearance was set in the *Grundlagen* and it was just waiting to be found, the way Columbus discovered America, once Frege finalized and



formalized his theories in a way that one could see upon what the whole construction rested.

Yet, only Boolos peeks back into the *Grundlagen*, offering a two paragraph guess as to how Frege might have been "hoodwinked" into putting forth Basic Law V in connection with 'the Julius Caesar problem' and merely concluding that, though we may guess at Frege's trains of thought, we cannot really explain how the serpent entered Eden (pp. 249–50), a skirting of key issues which would do little to disabuse a cynic of the suspicion that, for fear of unearthing ugly and embarrassing problems deeply embedded in the very foundations of analytic philosophy, those most competent to track down the origins of the inconsistency prefer to wring their hands in consternation rather than to set out in earnest to track down the source of the trouble. Whence Basic Law V? Why did Frege feel forced, as he more than once said, almost against his will, to mandate it in the first place? Instead of circling around the issues why not lasso the critter and find out what tempted Frege to put it in the *Grundgesetze* in the first place?

Asking how a serpent of inconsistency might have entered a logical system implies that some nefarious force has been allowed to worm its way into a logic to commit its villainy there. But logical inconsistency does not mysteriously enter into logical systems from the outside. So the only answer to Dummett's question is that the creator somehow planted it there to begin with. The use of the second-order quantifier or the horizontal could not have unearthed the contradiction until Frege had committed himself to a particular systematization of his ideas that he felt obliged to espouse because he saw no way of avoiding the absurd inferences or sterility of the theory of number he embraced in the *Grundlagen*.

The first purpose of his formal language, Frege said in the *Begriffsschrift* was to provide the most reliable test of the validity of a chain of inferences and to point out every presupposition that tries to sneak in unnoticed so that its origin might be investigated. Now, if that is so, and if one of the great merits of Frege's work is, as so many claim, that he created a system so clear that, to borrow Schirn's phrase, "all expressions wear their logical form on their sleeves" (p. 122), then one should be able to start with Frege at square one and reason with him to find the source of the "colossal blunder."

When Russell pointed out the inconsistency, Frege himself immediately pointed to Basic Law V. Both men wrote quite a bit on the source



of the contradiction and came to some very specific, and similar, conclusions. Yet, much of what they wrote on the specific causes of the contradiction has been ignored (or, in Frege's case, lost). For example, the scholar intent upon examining the connections between the theory of classes and the various uses of the definite article which both Frege and Russell ultimately concluded had given rise to the paradoxes still enters practically virgin territory.

As it happens, insight into the origins of the inconsistency is most nearly provided by other papers in this collection. The really probing, thought-provoking remarks Schirn makes about Frege's philosophy of arithmetic in his introduction and in his "On Frege's Introduction of Cardinal Numbers as Logical Objects" are based on a close, observant and lucid reading of the relevant texts that really comes to grips with the issues involved in Frege's introduction of courses-of-values. So, while intentionally refraining from making direct pronouncements about the inconsistency of Frege's theory, Schirn actually provides much needed insight into what Boolos calls "the genesis of Frege's error in putting forth Basic Law V" (p. 249).

In his paper, Schirn thoroughly characterizes the essential features of Frege's foundational program, conscientiously ferreting out problems inherent in his analysis of numerical statements and the introduction of cardinal numbers as logical objects in the *Grundlagen*. Frege's principal scientific concern, Schirn maintains, was to lay the logical foundations of number theory and analysis. To accomplish this, he reasons, Frege probably believed it mandatory to establish not only the purely logical nature of the natural and real numbers, but also their objectual status. Schirn concludes the first half of his paper "by claiming that unless someone has succeeded in refuting Paul Benacerraf's ontological argument against number-theoretic platonism, the conception of numbers as objects remains a dogma bequeathed to us by Frege" (p. 32). In the second half of the paper, Schirn argues that Frege's attempted definitions of number in the *Grundlagen* prove inadequate in "resolving the pervasive indeterminacy of reference affecting the cardinality operator" (p. 32).

Such concern with Frege's Platonism and the role of logical objects in his thought is in fact a major theme of this collection. This is as evident in Bob Hale's and Crispin Wright's critical discussion of Hartry Field's ideas in "Nominalism and the Contingency of Abstract Objects" as it is in Dummett's argument about the crucial role that the contradiction producing second-order quantifiers may have played in fulfilling Frege's aim



of justifying us in assuming the existence of abstract objects. Bob Hale's paper "Singular Terms" is "concerned exclusively with the problem of formulating acceptable criteria for singular termhood, of the general kind required ... to subserve the Fregean argument for numbers as objects" (p. 439). In "Frege's Treatment of Indirect Reference," Richard Mendelsohn addresses yet another facet of objecthood as he studies problems of sense and reference in oblique contexts. Even Terence Parson's attempt "to produce theories of truth and meaning, to see what assumptions are needed for what results, and to explore some of the options that are left open" and his conclusion "that theories of truth and meaning turn out to be independent of Frege's doctrine that sense determines reference" (p. 372) can be no stranger to such considerations. Platonism of course takes center stage in the book's epistemological discussions of Frege's Third Realm.

In "On Positing Mathematical Objects," Michael Resnik studies the philosophical and methodological issues involved in the theories that Frege, Cantor, Dedekind and Hilbert espoused relative to the introduction (through postulation, creation, definition, discovery) of mathematical objects. Resnik concludes that it is wise to review the lessons we can draw from Frege's realism, Dedekind's structuralism and Hilbert's postulationism. The second half of his paper is devoted to depicting how "we can combine a realist, mathematical structuralism with a postulational epistemology to obtain a coherent philosophical view" (p. 56).

In "Frege versus Cantor: On the Concept of Number," W.W. Tait also situates Frege's achievements in relation to those of Dedekind and Cantor. Frege's discussions of others in his field, Tait maintains, "are often characterized less by clarity than by misinterpretation and lack of charity, and on many matters, both of criticism of other scholars and of substance, his analysis is defective" (p. 72). Tait deems it unfortunate that Frege's evaluation of the efforts of his contemporaries "lives on in much of the philosophical literature, where respected mathematicians ... are regarded as utterly muddled about the concept of number and great philosophers, such as Cantor and Dedekind, are treated as philosophical naifs ..." (p. 73). "Not only have we inherited from Frege a poor regard for his contemporaries," laments Tait, "but, taking the critical parts of his *Grundlagen* as a model, we in the Anglo-American tradition of analytic philosophy have inherited a poor vision of what philosophy is" (p. 73).

Tait particularly takes aim at Frege's criticisms of Cantor and Dedekind as reflected in Dummett's starry-eyed vision of Frege as "the



greatest philosopher of mathematics yet to have written" (pp. 70–71). Focusing on the numerous important and interrelated issues surrounding abstractionism and psychologism, equinumerosity in terms of one-to-one correspondence, extensionality, Tait thoroughly examines the wide-ranging implications of criticisms leveled by Frege and Dummett.

For example, in response to Dummett's contention that Frege undoubtedly gave Hume's principle of equinumerosity "its most exact formulation and its most acute philosophical defense," Tait argues that Frege actually misunderstood Hume and failed to give Cantor his due (pp. 104–05). In defense of Dedekind and Cantor, Tait writes that in neither case is the abstractionism employed "subject to the criticism that it is psychologistic: For neither of them are numbers psychological objects nor are the laws of number to be understood in any way as subjective." (p. 82). Tait's paper ends with a reminder that Frege's "assumption in the *Grundgesetze* that every concept has an extension was an act of recklessness, forewarned against by Cantor already in 1883 and again ... in 1885" (p. 112).

Frege's pronouncements in the area of epistemology have drawn as much attention on the part of scholars for what he said as for what he did not say. In this collection, Eva Picardi, Gottfried Gabriel and Tyler Burge take up philosophical questions raised by Frege's scant, but tantalizing remarks on the subject.

Picardi opens her discussion of Frege's anti-psychologism by noting that there is hardly a piece of writing by Frege "where he misses the opportunity to stigmatize the evil of psychologism" (p. 307). Judging appeals to Frege's conception of epistemology, whose bare outlines can at best be surmised, to be not only an unpromising but a positively misleading way of approaching Frege's anti-psychologism, she argues that Frege's complaints against psychologism were essentially semantic in nature, that he aimed to defeat psychologism through theory of meaning.

The main fault of psychologism for Frege, she explains, lies in "a mistaken picture of language which turns the objectivity of sense and the communication of thoughts into a mystery" (p. 308). One facet of this is a confusion of logical and psychological laws that results from an extreme form of naturalism whose "chief defect is not just that it disregards the claims of *a priori* knowledge without offering any alternative account, but that, by embodying a relativistic notion of truth, it issues in a form of extreme subjectivism as regards meaning" (p. 309).



Countering those who would contend that Frege himself fell into a new form of psychologism or Kantian transcendentalism, she maintains that it is precisely Frege's realistic conception of truth, in Dummett's sense (p. 320), which "provides the link between his anti-psychologism in logic and his anti-psychologism in the account of meaning," that for Frege "nothing short of the classical notion of truth can give us a correct account of the meaning we attach to our utterances" (p. 309).

Lastly, to dramatize difficulties that Frege's theory of sense encounters on its own ground, she explores what she calls a feeble suggestion on her part that in his later years Frege may have sought to ward off psychologism by espousing a path for securing objectivity of content which did not go through language, but made a straightforwardly metaphysical appeal to a third non-actual, non-sensible realm of independent entities outside of space and time because he ultimately could not see how else adequately to guarantee the objectivity of sense and thoughts. In an appendix, she hypothesizes that Ernst Mach may have been an unnamed target of attacks in Frege's late writings.

In "Frege's 'Epistemology in Disguise'," Gabriel takes quite the opposite approach. For him, "Frege had an immediate interest in logical and epistemological questions, but only a mediate interest in questions in the philosophy of language." His actual goal, Gabriel considers, was "the construction of a logical language to serve a particular purpose: carrying out the epistemologically motivated logicist program" (p. 332). Frege's main works, Gabriel declares, "can be seen as an attempt to clarify the 'epistemological nature' of arithmetic" and are "dedicated to the epistemological aim of obtaining a new understanding of analytical judgments which differs from Kant's views" (p. 331). Gabriel shows how Frege "provides proof-theoretical criteria for decisions about the epistemic nature of truth and entire sciences," how he uses the gap-free chain of deductions in the hope of finding the exact conditions under which a proposition is true (p. 339). Frege, Gabriel contends, "recognizes non-logical reasons as reasons, and thereby acknowledges epistemology as an argumentative basic discipline which is to be distinguished from the psychology of knowledge" and seems "content with a blurred distinction between logic and epistemology" (p. 345).

Burge too considers Frege's main project to be "to explain the foundations of arithmetic in a such a way as to enable us to understand the nature of our knowledge of arithmetic" (p. 347). In "Frege on Knowing the Third Realm," he discusses an "intensification" of the puzzle about



the dearth of facts about Frege's theory of knowledge of the foundations in the light of what Burge calls his own short, but incomplete, explanation that Frege believed he had little to add to the traditional rationalist account of knowledge, that he assumed "that we can know arithmetic and its foundations purely through reason, and that individuals are reasonable and justified in believing basic foundational truths" (p. 348).

Burge then discusses Frege's theory that both the thought contents constituting the proof-structure of mathematics and the subject matter of those thought contents (extension, functions) exist in a third realm, different from the realm of physical objects and that of mental entities, to which all logic, and thus all sciences, are committed and are about. In this case, the problem then becomes the very traditional one "of understanding how reason alone could justify one in believing that a thought is true, when the thought has a subject matter that is as independent of anyone's thinking as Frege indicates it is" (p. 349).

All agree that Frege was not effusive on the topic of epistemology *per se*. He was, however, unsparing when it came to defeating and belittling those holding competing views, mainly psychologizing logicians and formalists, and much of that involved confronting them on epistemological ground. Moreover, as embarrassing as it may be to the heirs of the British empiricist tradition who have embraced Frege, he was perfectly explicit about his hostility to empiricism which in his mind, as no reader of his could fail to notice, was closely linked with psychologism. This is perfectly clear in his reviews of Cantor and Husserl, but Frege leaves no doubt about this in "On Sense and Meaning" and other important writings. Now this plain expression of animosity towards empiricism on Frege's part could be used to dispel mysteries surrounding his epistemological views, and his rationalism and belief in a third realm in particular. Yet it is totally ignored in the papers anthologized here, which again circle important, and potentially embarrassing, issues.

Such circumspection, however, has been an abiding characteristic of Frege research, which got off to a regrettably late start following a long "don't confuse me with the facts" era during which analytic philosophy thrived in English-speaking countries. This paved the way for the introduction of a fictional Frege who was pleasing to numbers of philosophers and fired their imaginations, leaving behind a goldmine of theses to refute; ever widening the distance between themselves and the facts, philosophers had lost sight of the origins of analytic philosophy, so that



the Frege that so many philosophers finally came to know and love was rather a free creation of Michael Dummett's mind.

In spite of the undeniable efforts in recent years to locate Frege's work more accurately in the history of logic, mathematics and philosophy, we still do not have an adequate and thorough account of the context in which Frege thought his thoughts. And many of the thoughts found in this book would grow in sophistication if submitted to a really conscientious examination of Frege's writings (as Schirn or Simons have done) and if thoroughly studied in relation to the intellectual context of the times (as Tait has done). For loath as Frege was to give credit where credit was due, he was not, as he has so often been portrayed, laying the foundations of number theory and analysis in a vacuum. He shared the goals of the most insightful and prophetic of his peers, among them Cantor, Husserl, Dedekind, Weierstrass and Peano.

Frege scholarship is only now coming of age and philosophers really should do some soul searching and ask why it has taken roughly a hundred years for it to reach the level of sophistication reflected in this book. For example, it is perfectly astonishing that, as Thiel notes, Frege's pessimistic conclusion regarding the formulation of an intuitively acceptable notion of set (or class or *Begriffsumfang*) has stimulated so little investigation (p. 268).

This slow maturation of Frege scholarship overall is reflected in the fact that many articles in this collection fill in gaps in the scholarship, or make up for other shortcomings, of other papers anthologized here. For example, much of Boolos's and Dummett's exchange turns on questions involving quantifying over infinitely extensible concepts and would grow in sophistication by examining Frege's and Cantor's exchange of ideas, in the way Tait does in his paper. Likewise, Hale begins his paper on singular terms alluding to the argument "which is at least implicit in *Grundlagen* for the existence of numbers as objects" (p. 438), while, as Schirn's paper makes perfectly clear, the argument for the existence of numbers as objects in the *Grundlagen* is as explicit as could be. And it is impossible to think that, as Boolos and Dummett both maintain, Frege hadn't "the glimmering of a suspicion of the existence of indefinitely extensible concepts" (pp. 234, 235), or that, as Thiel writes, "in Frege's time classes were clearly identified with the extensions of concepts" (p. 269). For in a text, which Tait cites and discusses (p. 109), Cantor, the father of set theory and one of the great specialists on indefinitely extensible sets and inconsistency, warned Frege that it was an error to take the exten-



sion of a concept as the foundation of the number concept because only in certain cases is the extension of a concept quantitatively determined.

While it is interesting and informative to find the papers complementing each other in this way, it is nevertheless not completely satisfying to find scholars of this caliber using knowledge which should by now be the common property of all Frege scholars to fill in lapses in the scholarship of their peers. So in mirroring the state of current Frege research this collection also reflects certain shortcomings still present in Frege research itself.

One last remark. In all fairness, it must be noted that Edmund Husserl (whom only Tait mentions briefly) is the invisible presence in practically every paper anthologized here. This is as much the case when Tait disparages Frege's damaging attacks on Husserl's teacher Karl Weierstrass and Husserl's close friend and colleague Georg Cantor, as it is in Tait's and Picardi's discussions of Frege's anti-psychologism. Likewise, the interest Thiel manifests in Boolos's suggestion that Frege's extensions might be replaced by abstract objects with respect to a particular equivalence relation (p. 269) brings to mind Husserl's late nineteenth century campaign to show that the entire formal basis upon which the calculus of classes rests is valid for the relationships between conceptual objects and that one could solve logical problems without making the detour through classes.

In the 1890s, Husserl was already hard at work laying bare the follies of extensional logic, by which he meant a calculus of classes. In the *Philosophy of Arithmetic* he had insightfully criticized Frege's recourse to extensions. He also had much of interest to say about epistemology, Platonism, realism and postulationism in mathematics, and about problems encountered when trying rigorously to derive all of mathematics from the concept of cardinal number. He grappled with a good number of the theses advanced in this work, and it is a shame that his philosophy of logic and mathematics has not found a place in it, where it could still shed light on the issues under discussion.