HW 6.5.1: Parametric Equations – Projectile Motion

$$x(t) = (v_0 \cos\theta)t$$
$$y(t) = h_0 + (v_0 \sin\theta)t - 16t^2$$

1. Partnering up with Tiger Weeds is Jordan Spittoon out of the Dallas area. He hits a shot that goes 280 feet and skims the top of a 120-foot tree at the peak of the ball's path.
   a. Use regression and/or matrices to create an equation for the path of the ball, *h(x)*.

   b. Write the equation for the golf ball's path using the vertex form of the equation for a parabola.

   c. What is the angle at which the golf ball takes off?

   d. In order to introduce the parameter of time, *t*, into this situation, find the time it takes the ball to fall to the ground from its maximum height. ($h(t) = -16t^2$, gravity using feet)

   e. Find the ball's speed when it hits the ground.

   f. Write a parametric relation for the ball's path that incorporates time and relates this parameter to the distance and height that the ball travels.

2. A baseball is hit when it is 3 feet above the ground and leaves the bat with initial velocity of 150 feet per second and at an angle of elevation of 20°.
   a. Write a parametric equation that relates time to the horizontal distance and height of the ball.

   b. There is a fence that is 400 feet from home plate. How long does it take the ball to reach the fence?

c. The fence is 20 feet tall; does the ball clear the fence earning the batter a homerun?

3. A golfer hits a ball with an initial velocity of 133 feet per second and an angle of 36° for the horizontal.
   a. How long is the ball in the air (hang time)?

   b. How far does the ball travel horizontally?

   c. What is the maximum height of the ball?

   d. If there were a 10 ft/sec headwind, would it affect the horizontal or vertical distance? How? Write a new parametric equation that incorporates the headwind in the equation.

Answers:

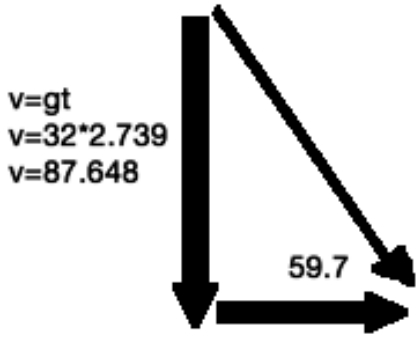
1.

a. $h(x) = -0.006122x^2 + 1.7143x$

b. $h(x) = -0.00612(x-140)^2 + 120$

c. 59.7°

d. 2.739 seconds



e.

$\sin(59.7^\circ) = \dfrac{v_y = 87.648}{v_{landing}}$, $v_{landing} = 101.5\ ft/\sec$

f.

$x(t) = (101.5\cos(59.7))t$

$y(t) = (101.5\sin(59.7))t - 16t^2$

2.

a.

$x(t) = (150\cos(20))t$

$y(t) = 3 + (150\sin(20))t - 16t^2$

b. $400 = (150\cos(20))t$, $t = 2.838$ seconds

c. $y(2.838) = 3 + (150\sin(20))t - 16t^2 = 19.73$ ft. No, it is not a homerun.

3.

a. 4.886 seconds

b. 525.73 ft

c. 95.49 ft

d. It will only affect the horizontal component. Since wind is a velocity, it needs to be multiplied by t, and the negative is due to the headwind (a tailwind would have been positive).

$x(t) = 133\cos(36)t - 10t$