Philosophy 231
More Paraphrase; Schematization; Interpretations; Truth-Tables

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Wesleyan University

Fall 2014

Class Outline

1. More Logical Paraphrase
2. Schematization
3. Interpretation of Schemata
4. Truth-Tables

A Message from our Sponsors

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NEVER, NEVER, NEVER

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read '$\supset$' as 'implies'

The Biconditional

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The biconditional sign, $\equiv$ is just an abbreviation.

That is to say,

$$p \equiv q$$

means exactly the same thing as

$$(p \supset q).(q \supset p)$$

Paraphrasing Conditionals

Paraphrasing into the symbol '$\supset$' poses more difficulties than the other logical symbols. So I want to go over three expressions that can be paraphrased using '$\supset$'.

‘only if’

- Consider the following statement

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    A car will start only if there’s gas in the tank.

‘only if’

- Consider the following statement

  A car will start only if there’s gas in the tank.

- The way to think about paraphrasing this is to consider whether we will take the statement to be $\top$ or $\perp$, under various conditions.

‘only if’

- For example, suppose some car starts, but there’s no gas in the tank (because it’s a combination gas electricity car) is the statement $\top$ or $\bot$?

‘only if’

- For example, suppose some car starts, but there’s no gas in the tank (because it’s a combination gas electricity car) is the statement $\top$ or $\bot$?
- Now, suppose a car doesn’t start, but there is gas in the tank, is the statement $\top$ or $\bot$?

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- For example, suppose some car starts, but there's no gas in the tank (because it's a combination gas electricity car) is the statement $\top$ or $\bot$?
- Now, suppose a car doesn't start, but there is gas in the tank, is the statement $\top$ or $\bot$?
- So, our paraphrase is:

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- Now, suppose a car doesn't start, but there is gas in the tank, is the statement $\top$ or $\bot$?
- So, our paraphrase is:

$$\text{(That car will start)} \supset \text{(there's gas in its tank)}$$

Necessary and Sufficient Conditions

If a conditional statement is $\top$, then

- Its antecedent is a sufficient condition for the consequent

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- Its consequent is a necessary condition for the antecedent

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> You will pass the course provided that you attend all the lectures.

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- Suppose you attend all the lectures, but I didn’t pass you, would you say that I’m a liar?

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  > You will pass the course provided that you attend all the lectures.

- Suppose you attend all the lectures, but I didn’t pass you, would you say that I’m a liar?
- What about if you passed the course, despite not having attended all the lectures?

‘provided that’

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- Suppose you had an enemy, who comes around and reminds me of what I said, and says that I ought to flunk you; would I be right to reply that what I did is consistent with my previous statement?
- So, here’s the paraphrase:

$$\text{(You attend all the lectures)} \supset \text{(you will pass the course)}$$

‘unless’

- Consider the following sentence:

'unless'

- Consider the following sentence:

  I won't go to the party unless I finish the logic problem set.

'unless'

- Consider the following sentence:

  > I won't go to the party unless I finish the logic problem set.

- Suppose you go to the party without finishing the problem set. Would you have kept your word?

‘unless’

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'unless'

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- Suppose you go to the party without finishing the problem set. Would you have kept your word?
- So the statement seems to be $\perp$ in the same condition as

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- Is this the right paraphrase?

‘unless’

- You might say no, when you think about possible situations in which you do finish the problem set.

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is $\top$ no matter whether its antecedent is $\top$ or $\bot$.

'unless'

- You might say no, when you think about possible situations in which you do finish the problem set.
- Given our reading of $\supset$,

$$\text{(I will go to the party)} \supset \text{(I finish the logic problem set).}$$

  is $\top$ no matter whether its antecedent is $\top$ or $\bot$.
- But, if you in fact finish the problem set, then surely you will go to the party, right?

‘unless’

- Really?

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- Even if you do finish the problem set, something else might happen to prevent you from going to the party.

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- Maybe you get a call from a friend who is depressed and you miss the party because you were cheering her up.

‘unless’

- Really?
- Even if you do finish the problem set, something else might happen to prevent you from going to the party.
- Maybe you get a call from a friend who is depressed and you miss the party because you were cheering her up.
- So the possibility in which you finish the problem set is compatible with your not going to the party.

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  ‘unless’ to be the same as ‘$\vee$’.

- Can you see why?

‘unless’ is the same as ‘$\vee$’

- If

$$\text{not } p \text{ unless } q$$

is paraphrased as

$$p \supset q$$

'unless' is the same as '$\vee$'

- If

$$\text{not } p \text{ unless } q$$

  is paraphrased as

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- Then

$$p \text{ unless } q$$

  is paraphrased as

$$-p \supset q$$

‘unless’ is the same as ‘$\vee$’

- Now,

$$-p \supset q$$

is $\perp$ only when $-p$ is $\top$ and q is $\perp$,

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- Now,

$$-p \supset q$$

is $\perp$ only when $-p$ is $\top$ and q is $\perp$,

- I.e., only when p is $\perp$ and q is $\perp$.

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- Now,

$$-p \supset q$$

  is $\bot$ only when $-p$ is $\top$ and q is $\bot$,
- I.e., only when p is $\bot$ and q is $\bot$.
- But these are the truth-conditions of $p \vee q$

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- How would you paraphrase

  The Wesleyan faculty is productive even though the Wesleyan administration overburdens it?

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- You will never go wrong if you follow them

Standard Paraphrases

'p only if q'	'$p \supset q$'
'p provided that q'	'$q \supset p$'
'not p unless q'	'$p \supset q$'
'p unless q'	'$-p \supset q$' or '$p \vee q$'
'p if and only if q'	'$p \equiv q$'
'p just in case q'	'$p \equiv q$'
'p even though q'	'$p.q$'

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General Approach to Paraphrase

As Goldfarb puts it on p. 18, there are generally speaking, three steps involved in logical paraphrase

- Identify the English expressions that are used like our truth functional connectives.
- Demarcate the constituent sentences of the sentence, and make the appropriate changes to turn these sentences into statements.
- Determine the grouping of the constituent statements.

Example of Paraphrase: Truth-functional Connective Phrases

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Example of Paraphrase: Grouping and Assembling the Paraphrase

If the tree rings have been correctly identified and the mace is indigenous, then the Ajo culture antedated the Tula unless the latter was contemporary with or derivative from that of the present excavation.

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- What about the consequent?

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$$p \supset r$$
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$$p \vee q$$
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$$(p.q) \supset \ (r \vee \ (s \vee \ t))$$

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Example of Convention for Bracketing

This isn't that difficult to understand. All it amounts to is that if we write

$$- \; p \, . \, q$$

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$$- \, p \, . \, q$$

We mean

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- Assigning a truth value to each distinct letter of a schema. This is called interpretation by assignment.

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- Replacing each letter appearing in a schema with a statement. This is called interpretation by replacement.
- The result is to convert a schema back to a statement of logical English.
- Assigning a truth value to each distinct letter of a schema. This is called interpretation by assignment.
- Notation: '$p := \perp$' means that the sentence letter p is assigned the value $\perp$.

Interpretation of Schemata

There are two notions of interpretation of schemata

- Replacing each letter appearing in a schema with a statement. This is called interpretation by replacement.
- The result is to convert a schema back to a statement of logical English.
- Assigning a truth value to each distinct letter of a schema. This is called interpretation by assignment.
- Notation: '$p := \perp$' means that the sentence letter p is assigned the value $\perp$.
- The result of such an assignment is that the entire schema is determined as $\top$ or $\perp$.

Example of Interpretation by Assignment

- Let's consider the schema:

$$p \vee q \supset r$$

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$$p \vee q \supset r$$

- An interpretation by assignment is:

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- The truth value of this schema under this interpretation is computed by using the rules for the truth functional connectives. Can you tell me what it is?

Example of Interpretation by Assignment

- Let's consider the schema:

$$p \vee q \supset r$$

- An interpretation by assignment is:

$$p := \top, q := \bot, r := \top$$

- The truth value of this schema under this interpretation is computed by using the rules for the truth functional connectives. Can you tell me what it is?
- It's $\top$, because the schema is a conditional, and its consequent is assigned $\top$. The assignments to p and to q, in this case, don't matter, because a conditional is $\top$ if it has a $\top$ consequent, no matter what the truth-value of its antecedent.

Standard Procedure to Write a Truth-Table

There is a standard procedure for constructing truth tables: the rows of the table are written down in a fixed order. Let's look at how it's done with 3 sentence letters, p, q, and r. For 3 letters there are $2^3 = 8$ possible combinations of truth-values. Each letter is $\top$ in half of those and $\bot$ in half; i.e., $\top$ in 4 interpretations and $\bot$ in the other 4.

Standard Procedure to Write a Truth-Table

We begin with p, and first fill the first 4 rows with $\top$.

p	q	r

Standard Procedure to Write a Truth-Table

We begin with p, and first fill the first 4 rows with $\top$.

p	q	r
$\top$		
$\top$		
$\top$		
$\top$		

Standard Procedure to Write a Truth-Table

We begin with p, and first fill the first 4 rows with $\top$.
Then we fill the remaining 4 with $\bot$

p	q	r
$\top$		
$\top$		
$\top$		
$\top$		

Standard Procedure to Write a Truth-Table

We begin with p, and first fill the first 4 rows with $\top$.
Then we fill the remaining 4 with $\bot$

p	q	r
$\top$		
$\top$		
$\top$		
$\top$		
$\bot$		
$\bot$		
$\bot$		
$\bot$		

Standard Procedure to Write a Truth-Table

Next we work on q, in the 4 rows in which p is $\top$.

p	q	r
$\top$		
$\top$		
$\top$		
$\top$		
$\bot$		
$\bot$		
$\bot$		
$\bot$		

Standard Procedure to Write a Truth-Table

Next we work on q, in the 4 rows in which p is $\top$.
We fill half of those with $\top$

p	q	r
$\top$		
$\top$		
$\top$		
$\top$		
$\bot$		
$\bot$		
$\bot$		
$\bot$		

Standard Procedure to Write a Truth-Table

Next we work on q, in the 4 rows in which p is $\top$.
We fill half of those with $\top$

p	q	r
$\top$	$\top$	
$\top$	$\top$	
$\top$		
$\top$		
$\bot$		
$\bot$		
$\bot$		
$\bot$		

Standard Procedure to Write a Truth-Table

Next we work on q, in the 4 rows in which p is $\top$.
We fill half of those with $\top$
Then the rest with $\bot$

p	q	r
$\top$	$\top$	
$\top$	$\top$	
$\top$		
$\top$		
$\bot$		
$\bot$		
$\bot$		
$\bot$		

Standard Procedure to Write a Truth-Table

Next we work on q, in the 4 rows in which p is $\top$.
We fill half of those with $\top$
Then the rest with $\bot$

p	q	r
$\top$	$\top$	
$\top$	$\top$	
$\top$	$\bot$	
$\top$	$\bot$	
$\bot$		
$\bot$		
$\bot$		
$\bot$		

Standard Procedure to Write a Truth-Table

Next we work on q, in the 4 rows in which p is $\top$.
We fill half of those with $\top$
Then the rest with $\bot$
Then we do the same for the 4 rows in which p is $\bot$

p	q	r
$\top$	$\top$	
$\top$	$\top$	
$\top$	$\bot$	
$\top$	$\bot$	
$\bot$		
$\bot$		
$\bot$		
$\bot$		

Standard Procedure to Write a Truth-Table

Next we work on q, in the 4 rows in which p is $\top$.
We fill half of those with $\top$
Then the rest with $\perp$
Then we do the same for the 4 rows in which p is $\perp$

p	q	r
$\top$	$\top$	
$\top$	$\top$	
$\top$	$\perp$	
$\top$	$\perp$	
$\perp$	$\top$	
$\perp$	$\top$	
$\perp$		
$\perp$		

Standard Procedure to Write a Truth-Table

Next we work on q, in the 4 rows in which p is $\top$.
We fill half of those with $\top$
Then the rest with $\bot$
Then we do the same for the 4 rows in which p is $\bot$

p	q	r
$\top$	$\top$	
$\top$	$\top$	
$\top$	$\bot$	
$\top$	$\bot$	
$\bot$	$\top$	
$\bot$	$\top$	
$\bot$	$\bot$	
$\bot$	$\bot$	

Standard Procedure to Write a Truth-Table

Now we work on r. It's just alternating $\top$ and $\bot$ down the third column:

p	q	r
$\top$	$\top$	
$\top$	$\top$	
$\top$	$\bot$	
$\top$	$\bot$	
$\bot$	$\top$	
$\bot$	$\top$	
$\bot$	$\bot$	
$\bot$	$\bot$	

Standard Procedure to Write a Truth-Table

Now we work on r. It's just alternating $\top$ and $\bot$ down the third column:

p	q	r
$\top$	$\top$	$\top$
$\top$	$\top$	
$\top$	$\bot$	
$\top$	$\bot$	
$\bot$	$\top$	
$\bot$	$\top$	
$\bot$	$\bot$	
$\bot$	$\bot$	

Standard Procedure to Write a Truth-Table

Now we work on r. It's just alternating $\top$ and $\bot$ down the third column:

p	q	r
$\top$	$\top$	$\top$
$\top$	$\top$	$\bot$
$\top$	$\bot$	
$\top$	$\bot$	
$\bot$	$\top$	
$\bot$	$\top$	
$\bot$	$\bot$	
$\bot$	$\bot$	

Standard Procedure to Write a Truth-Table

Now we work on r. It's just alternating $\top$ and $\bot$ down the third column:

p	q	r
$\top$	$\top$	$\top$
$\top$	$\top$	$\bot$
$\top$	$\bot$	$\top$
$\top$	$\bot$	
$\bot$	$\top$	
$\bot$	$\top$	
$\bot$	$\bot$	
$\bot$	$\bot$	

Standard Procedure to Write a Truth-Table

Now we work on r. It's just alternating $\top$ and $\bot$ down the third column:

p	q	r
$\top$	$\top$	$\top$
$\top$	$\top$	$\bot$
$\top$	$\bot$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	
$\bot$	$\top$	
$\bot$	$\bot$	
$\bot$	$\bot$	

Standard Procedure to Write a Truth-Table

Now we work on r. It's just alternating $\top$ and $\bot$ down the third column:

p	q	r
$\top$	$\top$	$\top$
$\top$	$\top$	$\bot$
$\top$	$\bot$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\top$
$\bot$	$\top$	
$\bot$	$\bot$	
$\bot$	$\bot$	

Standard Procedure to Write a Truth-Table

Now we work on r. It's just alternating $\top$ and $\perp$ down the third column:

p	q	r
$\top$	$\top$	$\top$
$\top$	$\top$	$\perp$
$\top$	$\perp$	$\top$
$\top$	$\perp$	$\perp$
$\perp$	$\top$	$\top$
$\perp$	$\top$	$\perp$
$\perp$	$\perp$	
$\perp$	$\perp$	

Standard Procedure to Write a Truth-Table

Now we work on r. It's just alternating $\top$ and $\bot$ down the third column:

p	q	r
$\top$	$\top$	$\top$
$\top$	$\top$	$\bot$
$\top$	$\bot$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\top$
$\bot$	$\top$	$\bot$
$\bot$	$\bot$	$\top$
$\bot$	$\bot$	

Standard Procedure to Write a Truth-Table

Now we work on r. It's just alternating $\top$ and $\bot$ down the third column:

p	q	r
$\top$	$\top$	$\top$
$\top$	$\top$	$\bot$
$\top$	$\bot$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\top$
$\bot$	$\top$	$\bot$
$\bot$	$\bot$	$\top$
$\bot$	$\bot$	$\bot$

A Full Truth-Table

Let's now calculate the truth-table for a fairly simple schema:

$$(p \supset q) \equiv -r$$

First we write across the top the parts of the schemata, from less complex to more complex.

p	q	r			
$\top$	$\top$	$\top$			
$\top$	$\top$	$\bot$			
$\top$	$\bot$	$\top$			
$\top$	$\bot$	$\bot$			
$\bot$	$\top$	$\top$			
$\bot$	$\top$	$\bot$			
$\bot$	$\bot$	$\top$			
$\bot$	$\bot$	$\bot$			

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p	q	r	$p \supset q$		
$\top$	$\top$	$\top$			
$\top$	$\top$	$\bot$			
$\top$	$\bot$	$\top$			
$\top$	$\bot$	$\bot$			
$\bot$	$\top$	$\top$			
$\bot$	$\top$	$\bot$			
$\bot$	$\bot$	$\top$			
$\bot$	$\bot$	$\bot$			

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p	q	r	$p \supset q$	$-r$	
$\top$	$\top$	$\top$			
$\top$	$\top$	$\bot$			
$\top$	$\bot$	$\top$			
$\top$	$\bot$	$\bot$			
$\bot$	$\top$	$\top$			
$\bot$	$\top$	$\bot$			
$\bot$	$\bot$	$\top$			
$\bot$	$\bot$	$\bot$			

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First we write across the top the parts of the schemata, from less complex to more complex. Then the whole schema.

p	q	r	$p \supset q$	$-r$	
$\top$	$\top$	$\top$			
$\top$	$\top$	$\bot$			
$\top$	$\bot$	$\top$			
$\top$	$\bot$	$\bot$			
$\bot$	$\top$	$\top$			
$\bot$	$\top$	$\bot$			
$\bot$	$\bot$	$\top$			
$\bot$	$\bot$	$\bot$			

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p	q	r	$p \supset q$	$-r$	$(p \supset q) \equiv -r$
$\top$	$\top$	$\top$			
$\top$	$\top$	$\bot$			
$\top$	$\bot$	$\top$			
$\top$	$\bot$	$\bot$			
$\bot$	$\top$	$\top$			
$\bot$	$\top$	$\bot$			
$\bot$	$\bot$	$\top$			
$\bot$	$\bot$	$\bot$			

The Truth-Values of $p \supset q$

- Since a conditional is $\top$ if its antecedent is $\bot$, we can immediately write a $\top$ in all the rows in which p is $\bot$.

p	q	r	$p \supset q$	$-r$	$(p \supset q) \equiv -r$
$\top$	$\top$	$\top$			
$\top$	$\top$	$\bot$			
$\top$	$\bot$	$\top$			
$\top$	$\bot$	$\bot$			
$\bot$	$\top$	$\top$			
$\bot$	$\top$	$\bot$			
$\bot$	$\bot$	$\top$			
$\bot$	$\bot$	$\bot$			

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- Since a conditional is $\top$ if its antecedent is $\bot$, we can immediately write a $\top$ in all the rows in which p is $\bot$.

p	q	r	$p \supset q$	$-r$	$(p \supset q) \equiv -r$
$\top$	$\top$	$\top$			
$\top$	$\top$	$\bot$			
$\top$	$\bot$	$\top$			
$\top$	$\bot$	$\bot$			
$\bot$	$\top$	$\top$	$\top$		
$\bot$	$\top$	$\bot$	$\top$		
$\bot$	$\bot$	$\top$	$\top$		
$\bot$	$\bot$	$\bot$	$\top$		

The Truth-Values of $p \supset q$

- Since a conditional is $\top$ if its antecedent is $\bot$, we can immediately write a $\top$ in all the rows in which p is $\bot$.
- A conditional is also $\top$ if its consequent is $\top$ so we can write $\top$ in the rows in which q is $\top$.

p	q	r	$p \supset q$	$-r$	$(p \supset q) \equiv -r$
$\top$	$\top$	$\top$			
$\top$	$\top$	$\bot$			
$\top$	$\bot$	$\top$			
$\top$	$\bot$	$\bot$			
$\bot$	$\top$	$\top$	$\top$		
$\bot$	$\top$	$\bot$	$\top$		
$\bot$	$\bot$	$\top$	$\top$		
$\bot$	$\bot$	$\bot$	$\top$		

The Truth-Values of $p \supset q$

- Since a conditional is $\top$ if its antecedent is $\perp$, we can immediately write a $\top$ in all the rows in which p is $\perp$.
- A conditional is also $\top$ if its consequent is $\top$ so we can write $\top$ in the rows in which q is $\top$.

p	q	r	$p \supset q$	$-r$	$(p \supset q) \equiv -r$
$\top$	$\top$	$\top$	$\top$		
$\top$	$\top$	$\perp$	$\top$		
$\top$	$\perp$	$\top$			
$\top$	$\perp$	$\perp$			
$\perp$	$\top$	$\top$	$\top$		
$\perp$	$\top$	$\perp$	$\top$		
$\perp$	$\perp$	$\top$	$\top$		
$\perp$	$\perp$	$\perp$	$\top$		

The Truth-Values of $p \supset q$

- Since a conditional is $\top$ if its antecedent is $\bot$, we can immediately write a $\top$ in all the rows in which p is $\bot$.
- A conditional is also $\top$ if its consequent is $\top$ so we can write $\top$ in the rows in which q is $\top$.
- In the remaining rows $p \supset q$ is $\bot$.

p	q	r	$p \supset q$	$-r$	$(p \supset q) \equiv -r$
$\top$	$\top$	$\top$	$\top$		
$\top$	$\top$	$\bot$	$\top$		
$\top$	$\bot$	$\top$			
$\top$	$\bot$	$\bot$			
$\bot$	$\top$	$\top$	$\top$		
$\bot$	$\top$	$\bot$	$\top$		
$\bot$	$\bot$	$\top$	$\top$		
$\bot$	$\bot$	$\bot$	$\top$		

The Truth-Values of $p \supset q$

- Since a conditional is $\top$ if its antecedent is $\bot$, we can immediately write a $\top$ in all the rows in which p is $\bot$.
- A conditional is also $\top$ if its consequent is $\top$ so we can write $\top$ in the rows in which q is $\top$.
- In the remaining rows $p \supset q$ is $\bot$.

p	q	r	$p \supset q$	$-r$	$(p \supset q) \equiv -r$
$\top$	$\top$	$\top$	$\top$		
$\top$	$\top$	$\bot$	$\top$		
$\top$	$\bot$	$\top$	$\bot$		
$\top$	$\bot$	$\bot$	$\bot$		
$\bot$	$\top$	$\top$	$\top$		
$\bot$	$\top$	$\bot$	$\top$		
$\bot$	$\bot$	$\top$	$\top$		
$\bot$	$\bot$	$\bot$	$\top$		

The Truth-Values of $-r$

This is easy, we just reverse the truth-values of the third column

p	q	r	$p \supset q$	$-r$	$(p \supset q) \equiv -r$
$\top$	$\top$	$\top$	$\top$		
$\top$	$\top$	$\bot$	$\top$		
$\top$	$\bot$	$\top$	$\bot$		
$\top$	$\bot$	$\bot$	$\bot$		
$\bot$	$\top$	$\top$	$\top$		
$\bot$	$\top$	$\bot$	$\top$		
$\bot$	$\bot$	$\top$	$\top$		
$\bot$	$\bot$	$\bot$	$\top$		

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p	q	r	$p \supset q$	$-r$	$(p \supset q) \equiv -r$
$\top$	$\top$	$\top$	$\top$	$\bot$	
$\top$	$\top$	$\bot$	$\top$	$\top$	
$\top$	$\bot$	$\top$	$\bot$	$\bot$	
$\top$	$\bot$	$\bot$	$\bot$	$\top$	
$\bot$	$\top$	$\top$	$\top$	$\bot$	
$\bot$	$\top$	$\bot$	$\top$	$\top$	
$\bot$	$\bot$	$\top$	$\top$	$\bot$	
$\bot$	$\bot$	$\bot$	$\top$	$\top$	

Truth-Values of the Entire Schema

Finally, we calculate the final column from the truth-values of the 4th and 5th columns:

p	q	r	$p \supset q$	$-r$	$(p \supset q) \equiv -r$
$\top$	$\top$	$\top$	$\top$	$\bot$	
$\top$	$\top$	$\bot$	$\top$	$\top$	
$\top$	$\bot$	$\top$	$\bot$	$\bot$	
$\top$	$\bot$	$\bot$	$\bot$	$\top$	
$\bot$	$\top$	$\top$	$\top$	$\bot$	
$\bot$	$\top$	$\bot$	$\top$	$\top$	
$\bot$	$\bot$	$\top$	$\top$	$\bot$	
$\bot$	$\bot$	$\bot$	$\top$	$\top$	

Truth-Values of the Entire Schema

Finally, we calculate the final column from the truth-values of the 4^{th} and 5^{th} columns:

p	q	r	$p \supset q$	$-r$	$(p \supset q) \equiv -r$
$\top$	$\top$	$\top$	$\top$	$\bot$	$\bot$
$\top$	$\top$	$\bot$	$\top$	$\top$	
$\top$	$\bot$	$\top$	$\bot$	$\bot$	
$\top$	$\bot$	$\bot$	$\bot$	$\top$	
$\bot$	$\top$	$\top$	$\top$	$\bot$	
$\bot$	$\top$	$\bot$	$\top$	$\top$	
$\bot$	$\bot$	$\top$	$\top$	$\bot$	
$\bot$	$\bot$	$\bot$	$\top$	$\top$	

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p	q	r	$p \supset q$	$-r$	$(p \supset q) \equiv -r$
$\top$	$\top$	$\top$	$\top$	$\bot$	$\bot$
$\top$	$\top$	$\bot$	$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$	$\bot$	$\bot$	
$\top$	$\bot$	$\bot$	$\bot$	$\top$	
$\bot$	$\top$	$\top$	$\top$	$\bot$	
$\bot$	$\top$	$\bot$	$\top$	$\top$	
$\bot$	$\bot$	$\top$	$\top$	$\bot$	
$\bot$	$\bot$	$\bot$	$\top$	$\top$	

Truth-Values of the Entire Schema

Finally, we calculate the final column from the truth-values of the 4^{th} and 5^{th} columns:

p	q	r	$p \supset q$	$-r$	$(p \supset q) \equiv -r$
$\top$	$\top$	$\top$	$\top$	$\bot$	$\bot$
$\top$	$\top$	$\bot$	$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\top$
$\top$	$\bot$	$\bot$	$\bot$	$\top$	
$\bot$	$\top$	$\top$	$\top$	$\bot$	
$\bot$	$\top$	$\bot$	$\top$	$\top$	
$\bot$	$\bot$	$\top$	$\top$	$\bot$	
$\bot$	$\bot$	$\bot$	$\top$	$\top$	

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p	q	r	$p \supset q$	$-r$	$(p \supset q) \equiv -r$
$\top$	$\top$	$\top$	$\top$	$\bot$	$\bot$
$\top$	$\top$	$\bot$	$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\top$
$\top$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$
$\bot$	$\top$	$\top$	$\top$	$\bot$	
$\bot$	$\top$	$\bot$	$\top$	$\top$	
$\bot$	$\bot$	$\top$	$\top$	$\bot$	
$\bot$	$\bot$	$\bot$	$\top$	$\top$	

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p	q	r	$p \supset q$	$-r$	$(p \supset q) \equiv -r$
$\top$	$\top$	$\top$	$\top$	$\bot$	$\bot$
$\top$	$\top$	$\bot$	$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\top$
$\top$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$
$\bot$	$\top$	$\top$	$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\bot$	$\top$	$\top$	
$\bot$	$\bot$	$\top$	$\top$	$\bot$	
$\bot$	$\bot$	$\bot$	$\top$	$\top$	

Truth-Values of the Entire Schema

Finally, we calculate the final column from the truth-values of the 4$^{\text{th}}$ and 5$^{\text{th}}$ columns:

p	q	r	$p \supset q$	$-r$	$(p \supset q) \equiv -r$
$\top$	$\top$	$\top$	$\top$	$\bot$	$\bot$
$\top$	$\top$	$\bot$	$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\top$
$\top$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$
$\bot$	$\top$	$\top$	$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\bot$	$\top$	$\top$	$\top$
$\bot$	$\bot$	$\top$	$\top$	$\bot$	
$\bot$	$\bot$	$\bot$	$\top$	$\top$	

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p	q	r	$p \supset q$	$-r$	$(p \supset q) \equiv -r$
$\top$	$\top$	$\top$	$\top$	$\bot$	$\bot$
$\top$	$\top$	$\bot$	$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\top$
$\top$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$
$\bot$	$\top$	$\top$	$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\bot$	$\top$	$\top$	$\top$
$\bot$	$\bot$	$\top$	$\top$	$\bot$	$\bot$
$\bot$	$\bot$	$\bot$	$\top$	$\top$	

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Finally, we calculate the final column from the truth-values of the 4th and 5th columns:

p	q	r	$p \supset q$	$-r$	$(p \supset q) \equiv -r$
$\top$	$\top$	$\top$	$\top$	$\bot$	$\bot$
$\top$	$\top$	$\bot$	$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\top$
$\top$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$
$\bot$	$\top$	$\top$	$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\bot$	$\top$	$\top$	$\top$
$\bot$	$\bot$	$\top$	$\top$	$\bot$	$\bot$
$\bot$	$\bot$	$\bot$	$\top$	$\top$	$\top$

Now it's Homework Time

DL p. 254, Problem 4 (a)-(c); paraphrase and schematize:

(a) The curse will be effective and neither Fasolt nor Fafner will retain the Ring.

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(c) Wotan and Alberic will not both be satisfied.