1-D AEROTHERMODYNAMIC CYCLE ANALYSIS AND DESIGN OF A TURBOJET ENGINE WITH A TURBINE BURNER

TURBOMACHINES MINI PROJECT

By :

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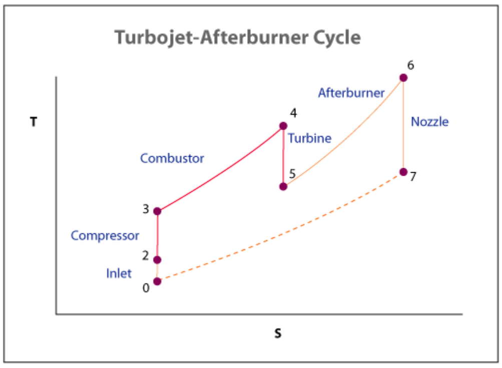
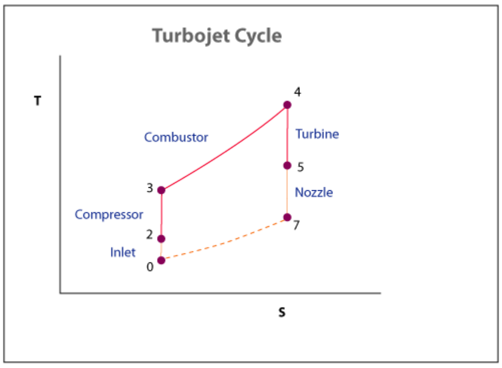
Abstract

In conventional turbojet engines, fuel is burned in separate combustors before the high pressure gas expands in the turbine. But Sirignano and Liu have found that when combustion is performed continuously in the turbine (generating turbine power at the same time), the specific thrust /power and efficiency of the turbojet engine is increased. This is termed as the turbine burner. Hence this work involves performing a thermodynamic analysis of the turbojet engines of various configurations with and without a turbine burner (CTB and ITB). Furthermore, it was found that the Turbine burner (of Continuous Turbine burner configuration) could not be achieved practically. Instead, an Inter-stage turbine burner (ITB), an alternative to CTB could be achieved practically. The thermodynamic analysis of ITB too has been performed and compared with CTB. Thermodynamic analysis has been performed by varying the major parameters (turbine inlet temperature, Compression ratio and Flight Mach number). A numerical code has also been developed in MATLAB to design the turbojet engine(theoretical) of any configuration and any number of stages (multi-stage turbine burner) whose results have been verified with the existing results obtained in benchmark journal papers. The code enables one to find parameters like ST,TSFC, various kinds of efficiencies etc.

This work is briefly divided into a number of sections. The introduction stage introduces the concept of turbojet engine (ITB and CTB) and explains the thermodynamic cycle on which it operates. Literature review section reviews the works which have been done on analysis of Turbojet engines operating on various cycles and works on ITB . Method of approach section briefly discusses the equations, assumptions and processes comprised in the thermodynamic cycle . In the section Results and Discussions, the plots of major parameters when varied with flight mach number, compression ratio and turbine inlet temperature are recorded . The findings of this thermodynamic analysis have been explained in the conclusions section. The numerical code in MATLAB is available in the Appendix section.

Introduction





A turbojet engine consists of a compressor coupled to a turbine, and a burner(or a combustor) in between. It also consists of an inlet and exhaust nozzles. The exhaust nozzle converts the internal energy of the hot gas into thrust. The work extracted by the turbine is to drive the compressor. A conventional turbojet engine runs on a Brayton cycle. In the Brayton cycle, the air is compressed isentropically(in a compressor), burned at constant pressure (in a combustor) and finally expanded to the starting pressure (in a turbine).

The thermal efficiency of a Brayton cycle can be improved by increasing the pressure ratio. But on increasing the pressure ratio, the total temperature of the flow entering the combustor increases which will limit the heat to be added to the combustor due to a limitation on the maximum turbine inlet temperature. Though addition of an afterburner after turbine expansion increases the power level of the engine, but the overall cycle efficiency reduces. To prevent this reduction in the efficiency due to the addition of an afterburner, the concept of turbine- burner was proposed. In a turbine-burner, one adds heat in the turbine where the pressure level is higher than that of the afterburner. In an ideal turbine-burner (continuous-turbine burner), heat is added to the flow while the rotor work is obtained such that the stagnation temperature in the turbine remains constant. Though it is impractical to perform combustion in the rotor and obtain work in the same time, inter-stage turbine-burners can be introduced as an alternative. As the number of inter-stage turbine-burners tends to infinity, the continuous turbine-burner configuration can be achieved.

The reheat cycle is the most widely used method to increase the thrust. The ITB which has been introduced in this work is an application of reheat cycle.

Literature Review

The concept of turbine burner was proposed by Sirignano et al in 1997. Since in the turbine burner, constant stagnation temperature is maintained in the turbine expansion process, this isothermal heat addition process is a Carnot cycle, the most efficient cycle. They carried out a thermal analysis for a single spool turbojet engine. They used simplified assumptions like perfect gas, constant gas properties, ideal component efficiencies and no turbine cooling. They found that the TB gave much better performance improvements with increasing compression ratios over increasing flight mach number. At high compression ratios, the TB produced equal specific thrust as those produced by the conventional turbojet engines.

Though the turbine burner engines seemed to be favourable, there were some challenges which had to be overcome.

High speed combustion results in mixing and reaction in turbine passages which results in:

- Unsteady flows
- Large acceleration (of the order of 10^5g)to produce hydrodynamic instability and large straining of flows

  A mixture of burned and unburned gases enters the turbine from the main combustor, which results in variations in temperature, species composition and density. Since the turbine blades and stator turn the flow, Rayleigh Taylor instabilities occur. These instabilities may result in the formation of recirculation zones and vertical cells, which affects the aerodynamics of the flow. Hence the high acceleration flows poses problems in the field of ignition and flameholding. Flameholding and flame stabilization in high acceleration flows and high centrifugal forces is itself a different field in Combustion.

In 2001, Liu and Sirignano introduced the concept of ITB(Inter-Stage Turbine burner). The TB configuration which they had analysed earlier was a CTB (Continuous Turbine burner). In ITB, the turbine stators and nozzles were converted into combustors. Hence an infinite number of ITBs were required to approach a Continuous Turbine burner cycle (CTB).

But these problems and challenges of CTB were overcome with the ITB configuration as in these combustion occurred in the secondary combustors.

Similar to ITB, Sequential Combustion cycle (SCC) has also been discovered. SCC means that there is a second combustion after the HPT to reheat the gas before expanding in LPT. Vogeler applied SCC to high bypass ratio turbofan engines. He found that the SCC increased the thrust of the engine without needing much advancement in the material and turbine cooling methods. He considered variable specific heats due to compression, heat addition and expansion processes. He analysed two configurations of turbofan engine - single spool and twin spool. SCC analysis revealed that the single spoon engine was better than the twin-spool engine. The reason was that the expansion of the high pressure gas already took place in HPT. The low pressure ratio across LPT did not allow the effective usage of this energy.

Andriana et al investigated the performance of a turbojet engine with a CTT (Constant temperature cycle). Their cycle was similar to the cycle proposed by Sirignano et al (TB which helped in maintaining a constant stagnation temperature). They performed only an off-design analysis of the turbojet engine. Their model assumed that the flows at the inlet and exhaust nozzles were choked.

Several researchers have also performed numerical analysis in ITB to determine if combustion is feasible. Siow and Yang presented the computational result of the combusting flow field inside a simplified ITB. ITB, located between LPT and HPT, was subject to different inflow velocities, flame-holder sizes and shapes. Their results demonstrated that the flow and combustion stability depended strongly on Reynolds number.

Mawid carried out the three-dimensional computational fluid dynamics (CFD) computations to guide the ITB experiments. They claimed that it was very important to identify the key design parameters for the best performance and to optimize the ITB design configurations. The results presented that the intense combustion occurs inside the circumferential cavity.

CTB,CTT are similar to TB cycles. They maintain an isothermal combustion inside the turbine passages. On the contrary, SCC and ITB cycles employ a secondary combustor after the expansion process in a turbine. But the ITB is a constant pressure secondary combustor in between the turbine stages, unlike the CTB which is a constant temperature combustor. Unlike the CTB, work cannot be extracted from the ITB as the combustion does not occur in the turbine rotors.

Method of Approach

The performance of a turbojet engine can be assessed by two parameters:

- Specific Thrust (ST): Specific thrust is defined as the thrust per unit mass flux of air.

  $$ST = \frac{T}{\dot{m}_a}$$

  $\dot{m}_a$ is the mass flow rate of the whole engine. A higher ST means a higher thrust level for the same engine cross section and thus smaller engine size and lighter weight.
- Thrust specific Fuel consumption rate (TSFC): TSFC is defined as the fuel flow rate per unit thrust.

  $$TSFC = \frac{\dot{m}_f}{T}$$

  m_f is the total fuel-mass flow rate for the complete engine. A low TSFC means less fuel consumption for the same thrust level.

TSFC is determined through Propulsion efficiency and thermal efficiency.

$$\text{Thermal efficiency} = \eta_t = \frac{KE\ of\ exhaust\ gas - KE\ of\ inlet\ air}{m_f Q_R}$$

where Q_R is the calorific value of the fuel.

$$\text{Propulsion efficiency} = \eta_p = \frac{Tu}{KE\ of\ exhaust\ gas - KE\ of\ inlet\ air}$$

where u is the flight velocity.

$$\text{Overall efficiency} = \eta_o = \eta_t \times \eta_p$$

Thermal efficiency indicates how efficient the engine converts heat to kinetic energy.The propulsion efficiency indicates how efficiently the engine uses the kinetic energy for propulsion purposes.

Fig1 shows the variation of TSFC vs ST. From the variation of TSFC and ST, both the parameters trade off with each other. Hence the desirable trend is to prefer the right hand side region.

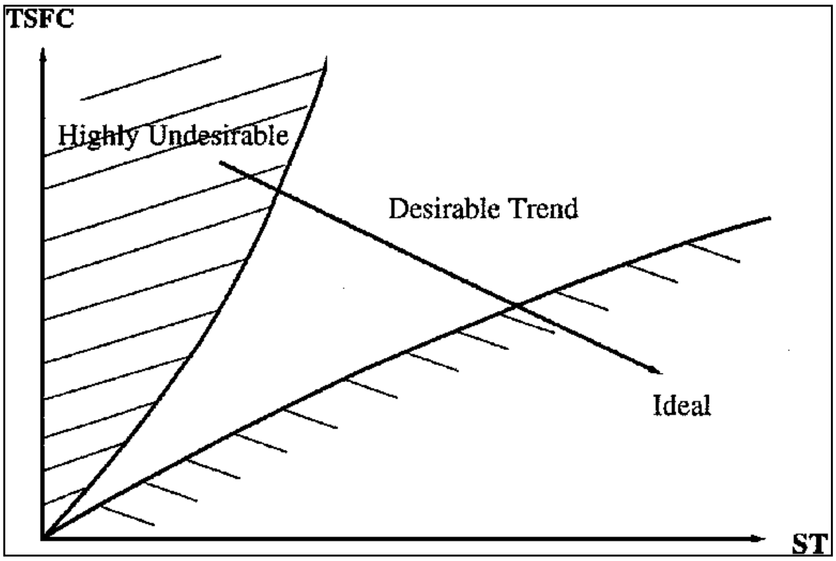


Figure 1 : Desired TSFC and ST range

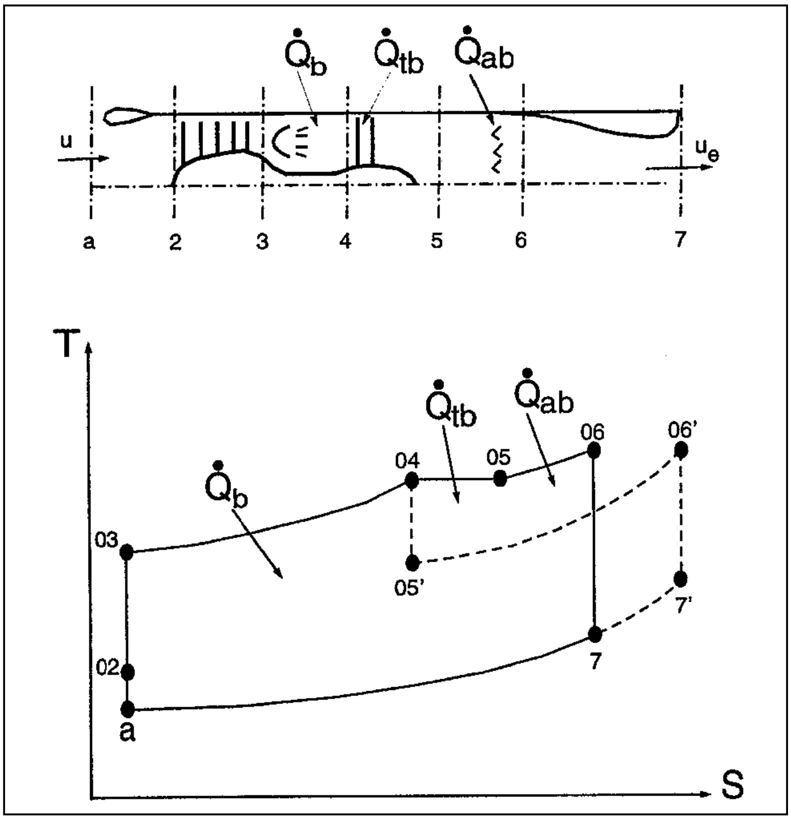


Figure 2 : Comparison of thermodynamic cycles with and without turbine-burner

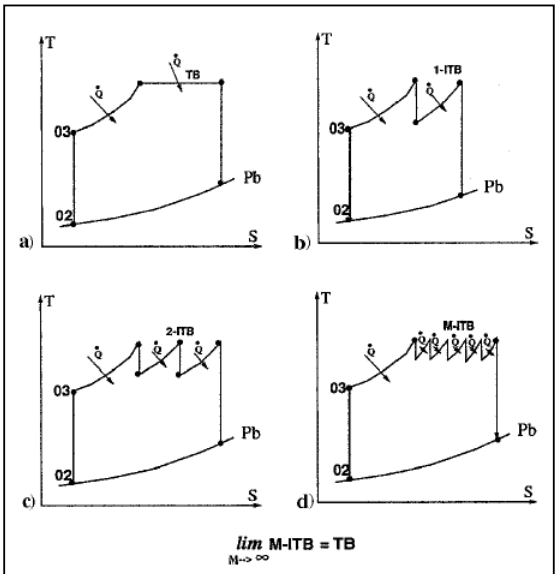


Fig. 3 TB and the *M*-ITB cycles.

Table 1 List of efficiencies used

Component	Avg. Specific heat ratio	Symbol	Value
Inlet/diffuser adiabatic efficiency	1.40	η_d	0.95
Compressor adiabatic efficiency	1.37	η_c	0.85
Main burner combustion efficiency	1.35	η_b	0.95
Main burner total pressure recovery coefficient		π_b	0.99
Main turbine adiabatic efficiency	1.33	η_t	0.95
Turbine-burner combustion efficiency		η_{tb}	0.96
Additional turbine-burner total pressure recovery coefficient		π_{tb}	0.98
Afterburner combustion efficiency		η_{ab}	0.98
Afterburner total pressure recovery coefficient		π_{ab}	0.99
Nozzle adiabatic efficiency	1.36	η_n	0.97

Table 2 List of Processes

Deceleration of fluid flow in diffuser section	a - 02
Compression in a compressor	02 - 03
Heat addition in a main burner	03 - 04
Expansion in a turbine of conventional configuration	04 – 05’
Afterburner section of conventional configuration	05’ – 06’
Expansion in a nozzle of conventional configuration	06’ – 7’
Turbine burner	04 – 05
Afterburner section of turbine burner	05 - 06
Expansion in a nozzle	06 - 7

Assumptions used while using the equations:

- No air bleeding
- Complete expansion
- Perfect gas and constant properties
- Turbine power exactly balanced by the compressor work

Four types of configurations of turbojet engines were used:

- Conventional turbojet engine without afterburner
- Conventional turbojet engine with afterburner
- Turbojet engine with a turbine-burner(CTB), without an afterburner
- Turbojet engine with a turbine-burner(CTB) and an afterburner
- Turbojet engine with 1-stage Turbine burner(1-ITB)
- Turbojet engine with 2-stage Turbine burner(2-ITB)

Equations used are (1) (2) :

I. No turbine burner and no after burner

1. Process a-02 in a diffuser and inlet section

$$\boldsymbol{T_{02}} = \boldsymbol{T_a}\left\{\mathbf{1} + \left[\frac{\boldsymbol{\gamma} - \mathbf{1}}{\mathbf{2}}\right]\mathbf{M^2}\right\}$$

$$P_{02} = P_a\left\{1 + \eta_d\left[\frac{T_{02}}{T_a} - 1\right]\right\}^{\gamma/(\gamma-1)}$$

Where γ is the ratio of specific heats of air and M is the flight Mach number.

2. Compression in a compressor 02 – 03

$\frac{p_{03}}{p_{02}} = \pi_c$

$$\frac{T_{03}}{T_{02}} = 1 + \frac{1}{\eta_c}\left[{\pi_c}^{(\gamma-1)/\gamma} - 1\right]$$

3. Heat addition in a main burner 03 – 04

Through the energy equation, it can be shown that the ratio of mass flow rate of air to the mass flow rate in the burner is

$$\frac{\dot{m_{fb}}}{m_a} = \frac{C_p(T_{04} - T_{03})}{Q_R\eta_b - C_pT_{04}}$$

where Q_R is the heating value (calorific value) of the fuel.

4. Expansion in a turbine of conventional configuration 04 – 05'

During this expansion, the turbine generates power to balance the power consumed by the compressor.

$P_t = P_c$

$$\dot{m}_tC_{pt}(T_{04} - T_{05'}) = \dot{m}_cC_{pc}(T_{03} - T_{02})$$

By assuming that $\dot{m}_t C_{pt} = \dot{m}_c C_{pc}$, the stagnation temperature and pressure ratios are:

$$\frac{T_{05'}}{T_{04}} = 1 - \frac{T_{03} - T_{02}}{T_{04}}$$

$$\frac{p_{05'}}{p_{04}} = \left[1 - \frac{1}{\eta_t}\left(1 - \frac{T_{05'}}{T_{04}}\right)\right]^{\gamma/(\gamma-1)}$$

5. Expansion in a nozzle 06 – 7

The stagnation temperatures at 06 and 07 will remain the same for complete expansion of the nozzle.

$$T_{07} = T_{06}$$
$$p_7 = p_a$$

The exit velocity at the end of the nozzle is

$$u_e = \sqrt{2\eta_n T_{06} C_p [1 - (p_a/p_{06})^{(\gamma-1)/\gamma}]}$$

II. Afterburner, no turbine burner

The processes from a-05’ remains the same as that of the “No turbine burner and no afterburner” configuration.

1. Afterburner section of conventional configuration 05’ – 06’

$$\frac{p_{06'}}{p_{05'}} = \pi_{ab}$$

$$\frac{\dot{m_{fab}}}{m_a + \dot{m_{fb}}} = \frac{C_p(T_{06'} - T_{05'})}{Q_R\eta_{ab} - C_p T_{06'}}$$

2. Expansion in a nozzle 06 – 7

The exit velocity at the end of the nozzle is:

$$u_e = \sqrt{2\eta_n T_{06'} C_p [1 - (p_a/p_{06'})^{(\gamma-1)/\gamma}]}$$

III. Turbine burner, no afterburner

The processes from a-04 remains the same as that of the “No turbine burner and no afterburner” configuration.

1. Process 04 – 05 in a Turbine burner

   In a turbine burner, controlled burning occurs in such a way that the temperature in the turbine remains constant. But the energy released in the combustion process is converted to turbine work.

$$T_{05} = T_{04}$$

The turbine burner fuel mass flow rate is:

$$\dot{m}_{fb} = \frac{P_t}{Q_R\eta_{tb}}$$
$$T_{av} = 0.91T_{04}$$
$$s_5 - s_4 = \frac{P_t}{T_{av}(m_a + \dot{m_{fb}})}$$

$$\frac{p_{05}}{p_{04}} = \pi_{tb} exp\lfloor -(s_5 - s_4)/R \rfloor$$

2. Expansion in a nozzle 06 – 7

The exit velocity at the end of the nozzle is:

$$u_e = \sqrt{2\eta_n T_{06} C_p [1 - (p_a/p_{06})^{(\gamma-1)/\gamma}]}$$

IV. Turbine burner and afterburner

The processes from a-05 remains the same as that of the “Turbine burner and no afterburner” configuration.

1. Afterburner section of turbine burner 05 – 06

$$\frac{p_{06}}{p_{05}} = \pi_{ab}$$

$$\frac{\dot{m_{fab}}}{m_a + \dot{m_{ftb}} + \dot{m_{fb}}} = \frac{C_p(T_{06} - T_{05})}{Q_R \eta_{ab} - C_p T_{06}}$$

2. Expansion in a nozzle 06 – 7

The exit velocity at the end of the nozzle is:

$$u_e = \sqrt{2\eta_n T_{06} C_p [1 - (p_a/p_{06})^{(\gamma-1)/\gamma}]}$$

To evaluate the turbojet performance,

$$ST = \left[\left(1 + \frac{m_{fb+}\dot{m_{ftb}} + \dot{m_{fab}}}{\dot{m_a}}\right) u_e - u\right]$$

$$TSFC = \left(\frac{m_{fb+}\dot{m_{ftb}} + \dot{m_{fab}}}{\dot{m_a} ST}\right)$$

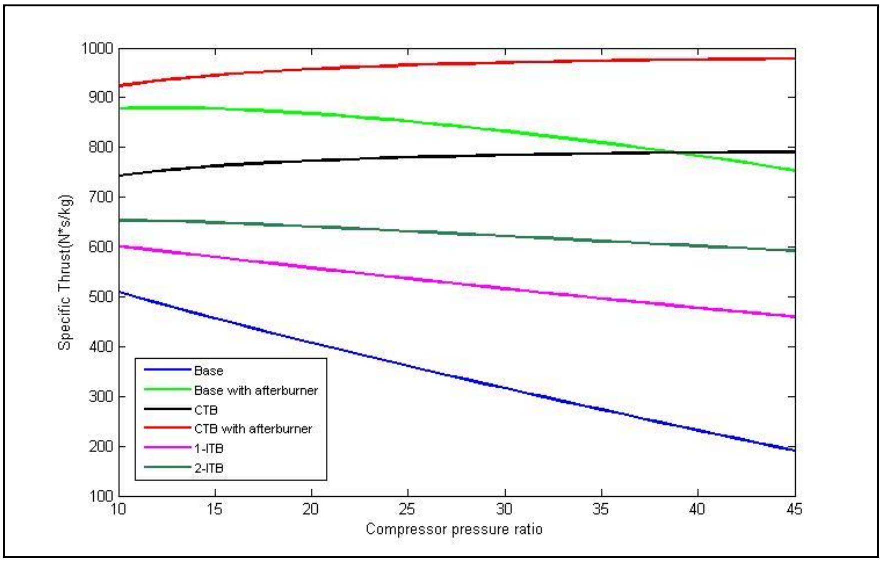
The above equations were solved and graphs were plotted in Engineering Equation Solver and MATLAB.

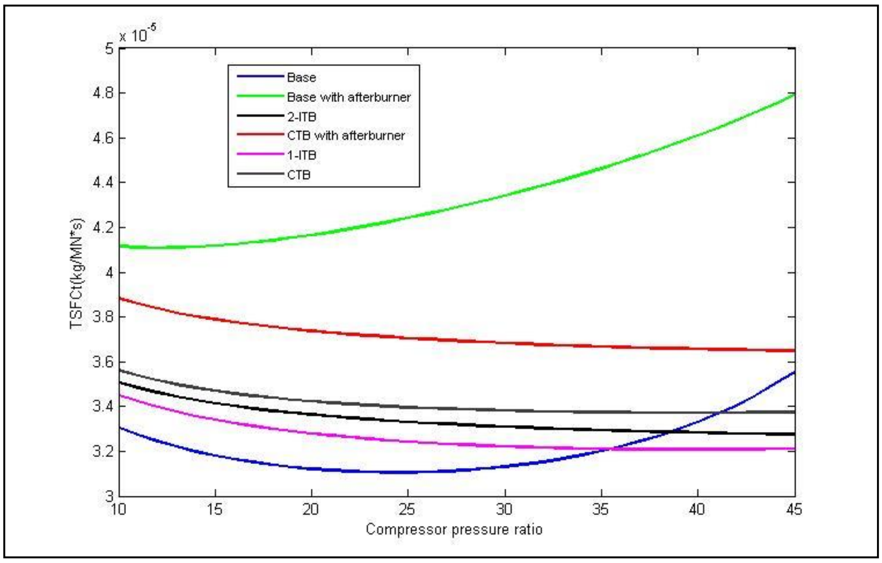
The above calculations were performed assuming the fuel heating value to be 45 MJ/kg. At the sea level, (flight Mach number = 0), ambient pressure considered is 101.3 kPa and ambient temperature as 288.2 K. For flight Mach number = 2, the ambient pressure chosen is 7.170 kPa and ambient temperature as 216.7 K. For flight Mach number = 0.85, the ambient pressure chosen is 18.85 kPa and ambient temperature as 216.7 K (1).

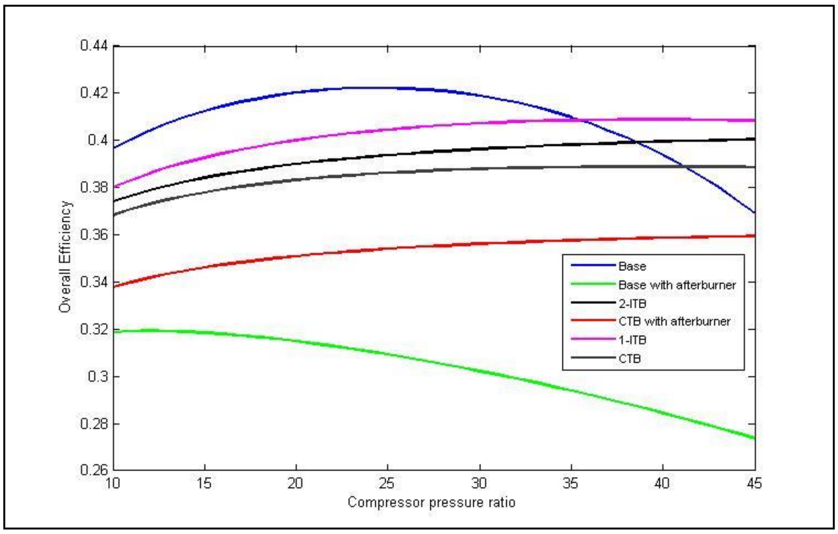
Results and Discussions

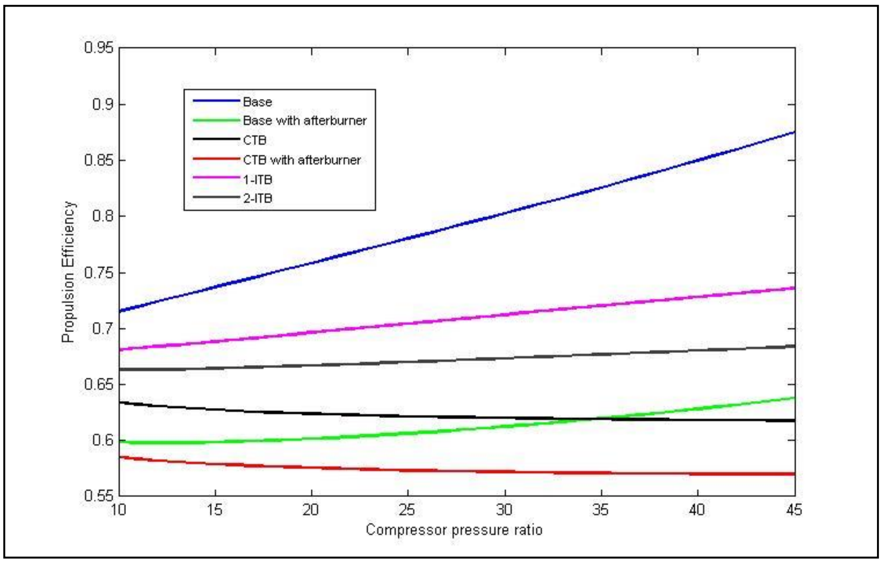
Performance Comparisons of Inter-stage Turbine Burners (ITB) and Continuous Turbine Burners (CTB)

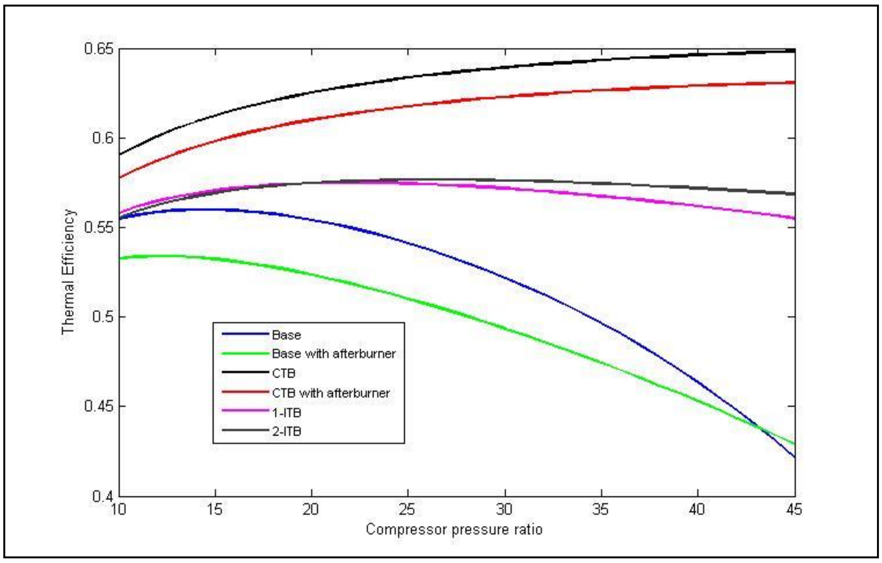
Effects of Compressor Pressure Ratio









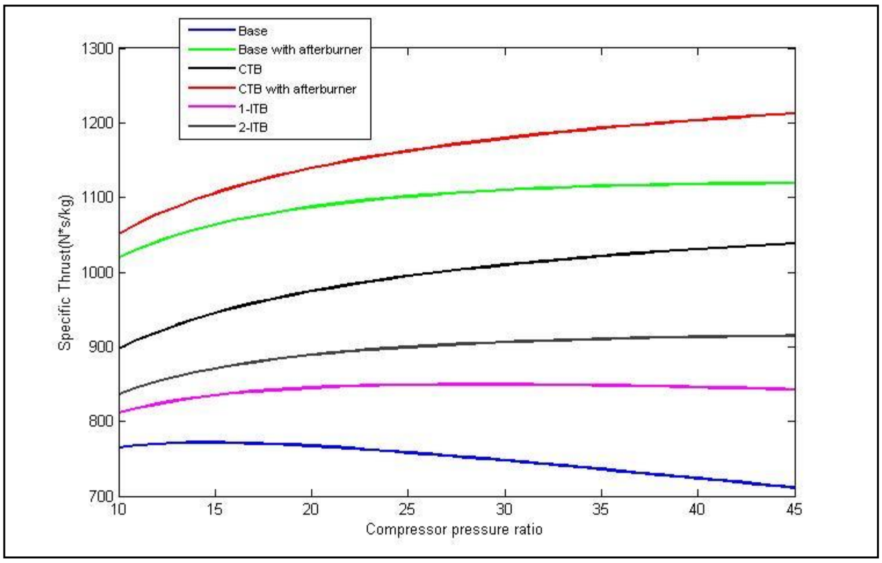


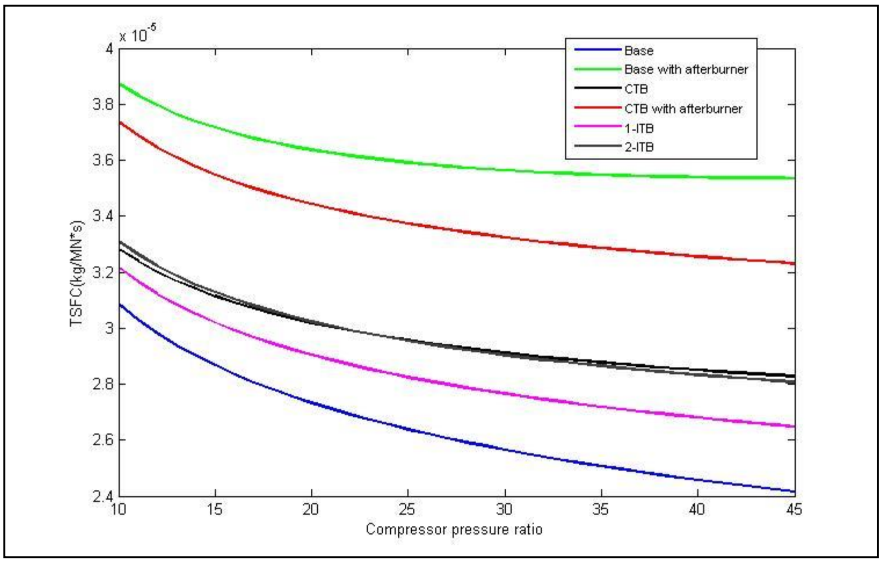
Performance comparisons for turbojet engines with Compressor power ratio assuming Flight Mach number = 2(Supersonic conditions), T_{04} = 1500 K, T_{06} = 1900 K

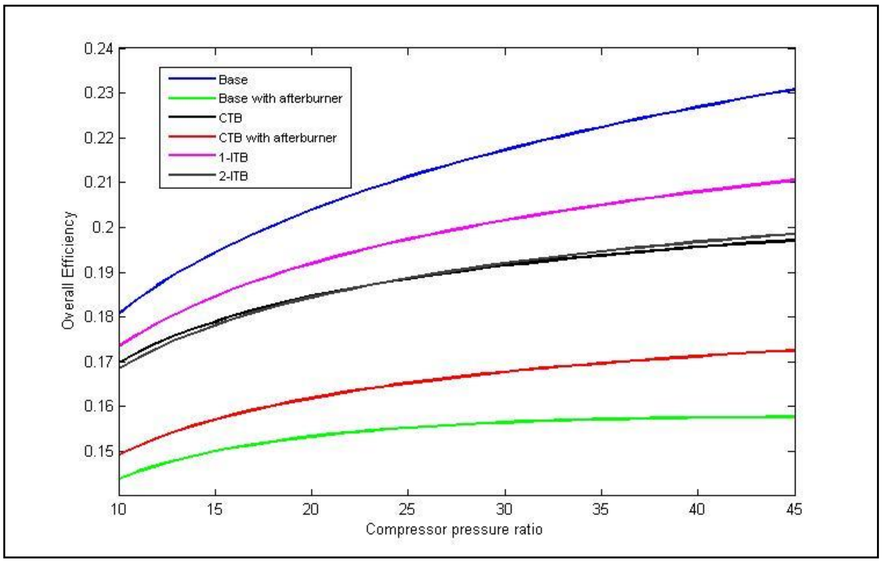
The specific thrust of the base conventional engine drops with the increase in the compression ratio as the increased compression ratio increases the temperature of the gas entering the main combustor which decreases the amount of heat to be added. But the specific fuel consumption decreases initially and later increases rapidly for the conventional engines. The thermal efficiency increases and then decreases at very high compression ratios. The overall efficiency decreases at high compression ratios because of low propulsion efficiencies. Though the afterburner increases the specific thrust, it also increases the specific fuel consumption which is quite undesirable. The thermal and propulsion efficiencies are also less in comparison to turbine burner and base engines. This proves that the afterburner configuration is undesirable to increase the specific thrust for a decrease in other parameters.

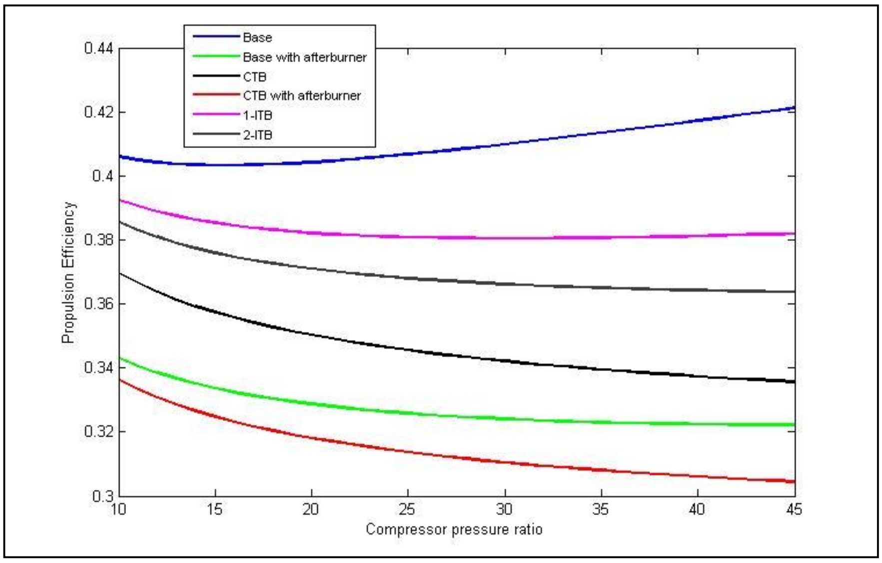
The CTB engine provide the highest specific thrust due to the continuous combustion in the turbine. It also has the highest thermal efficiency. Though the CTB has a high specific thrust, it has a low propulsion efficiency. Thus the overall efficiency and specific fuel consumption is high than the base engine.

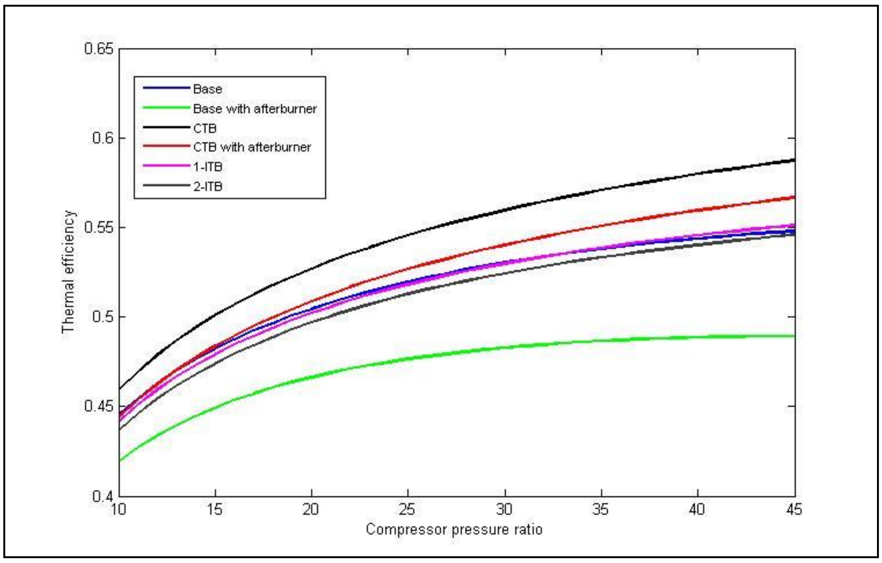
As the number of ITBs increase the performance approaches that of the CTB. Hence to achieve a given ST and TSFC, an optimum number of ITBs have to be selected.




x 10^-5
TSFC(kg/MN*s)
Compressor pressure ratio
Base
Base with afterburner
CTB
CTB with afterburner
1-ITB
2-ITB


Overall Efficiency
Compressor pressure ratio
Base
Base with afterburner
CTB
CTB with afterburner
1-ITB
2-ITB

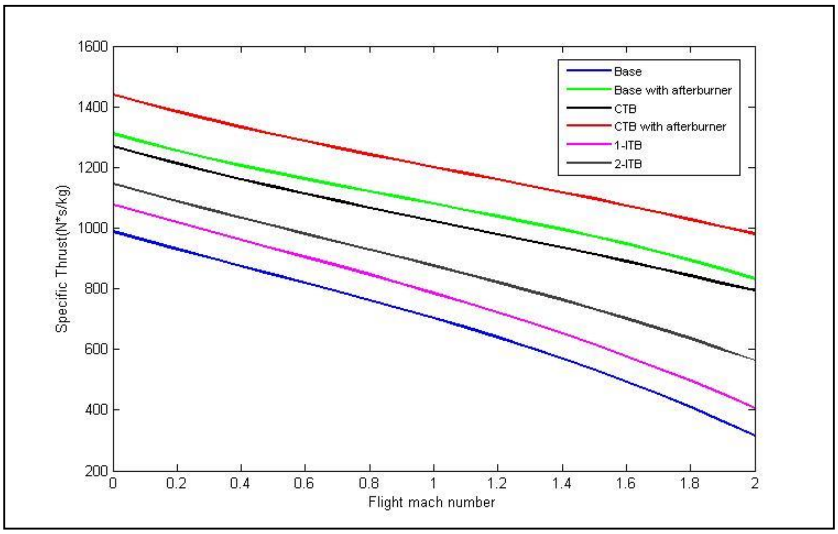

Propulsion Efficiency
Compressor pressure ratio
Base
Base with afterburner
CTB
CTB with afterburner
1-ITB
2-ITB

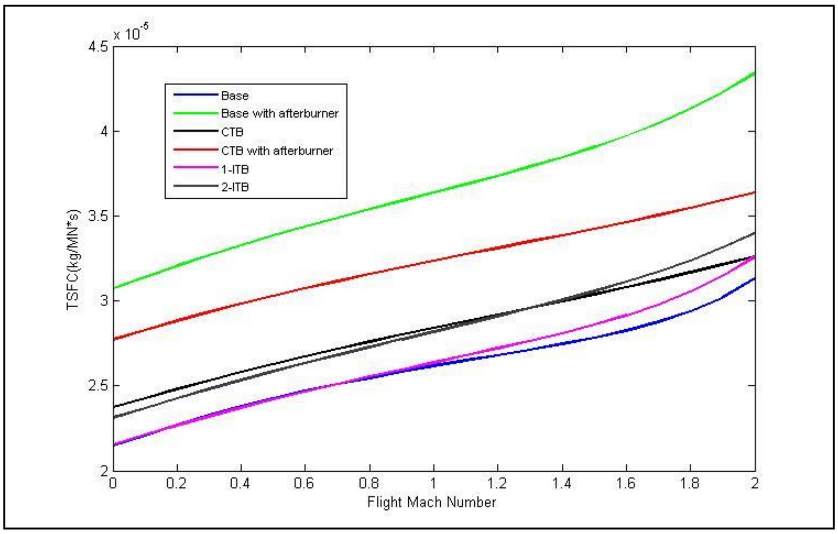


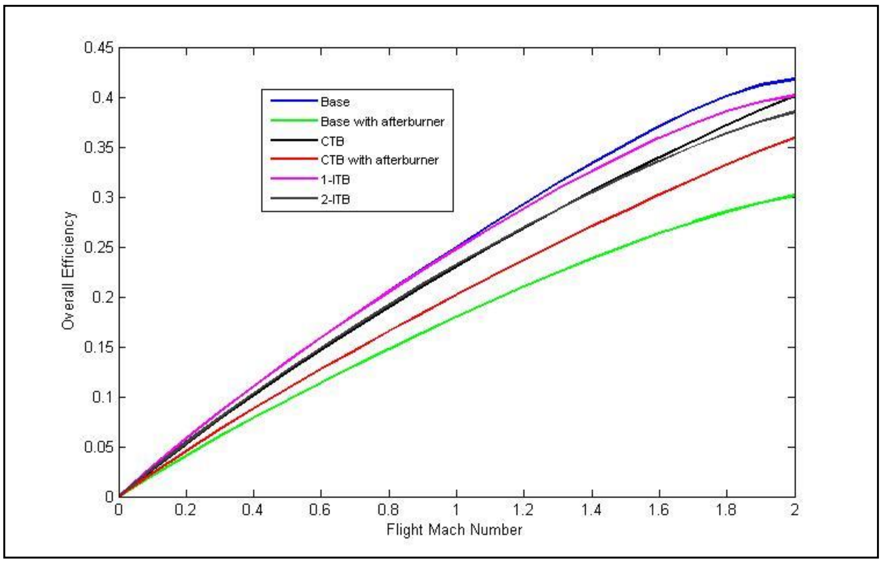
Performance comparisons for turbojet engines with Compressor power ratio assuming Flight Mach number = 0.85(Subsonic conditions), T_{04} = 1500 K, T_{06} = 1900 K

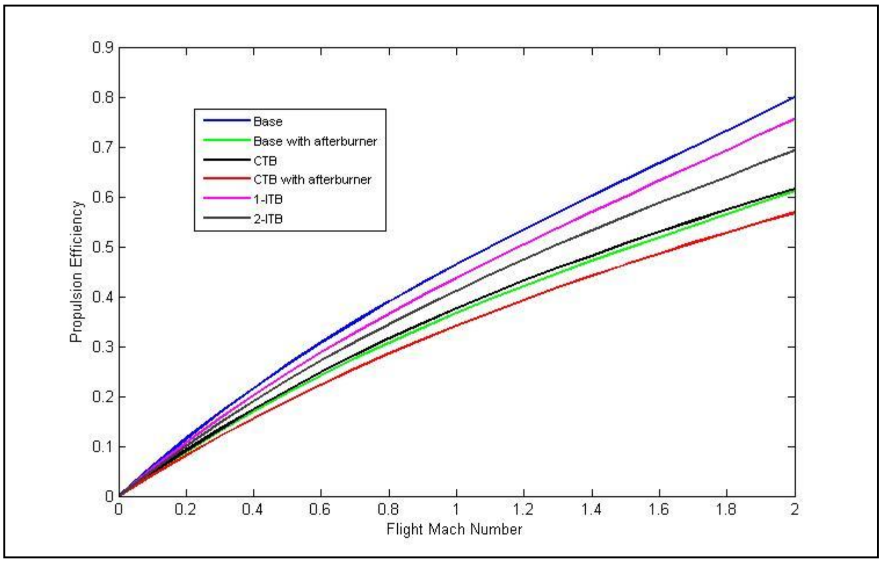
A similar trend of performance behaviour is observed for the subsonic case in comparison to the supersonic case. The only difference between the subsonic and supersonic flight operation of turbojet engines is that the propulsion efficiency is very less at low flight mach numbers.

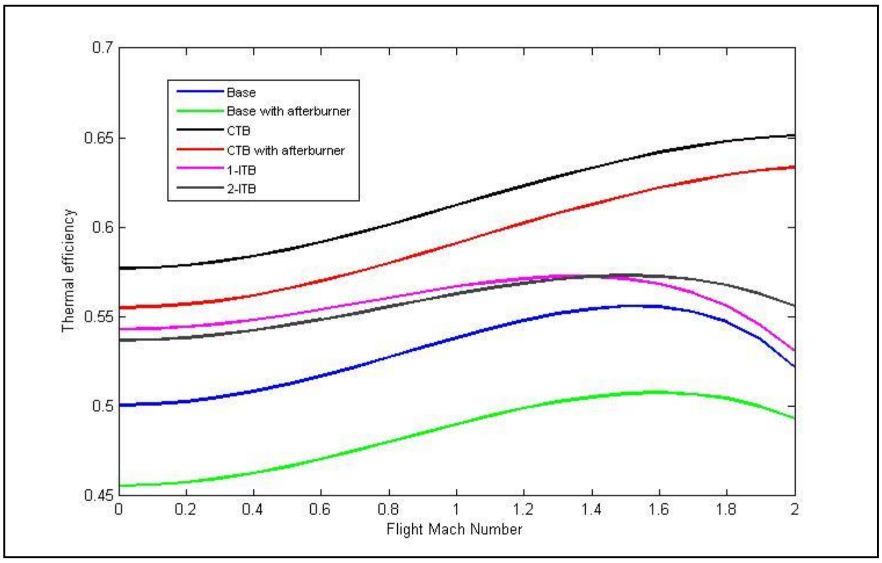
Effects of Flight Mach number




x 10^-5
TSFC(kg/MN*s)
Flight Mach Number
Base
Base with afterburner
CTB
CTB with afterburner
1-ITB
2-ITB


Overall Efficiency
Flight Mach Number
Base
Base with afterburner
CTB
CTB with afterburner
1-ITB
2-ITB

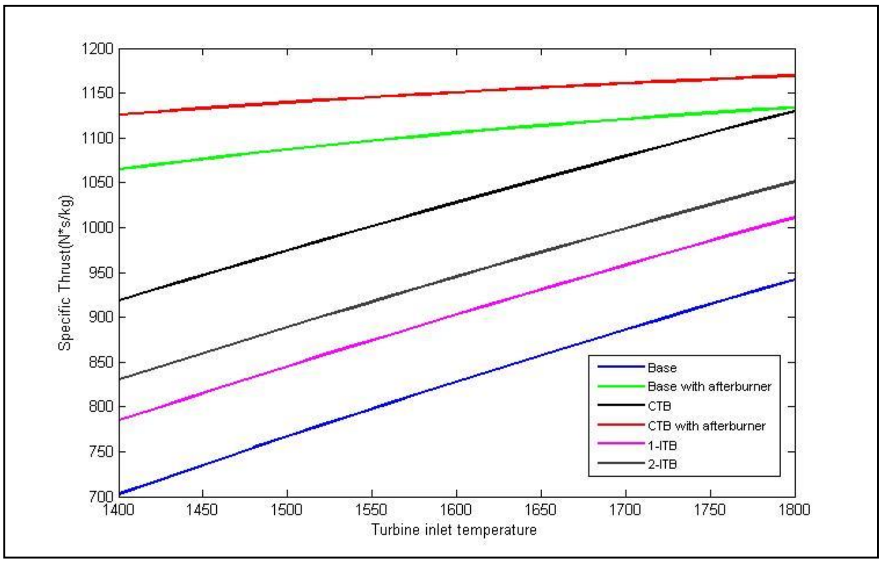

Propulsion Efficiency
Flight Mach Number
Base
Base with afterburner
CTB
CTB with afterburner
1-ITB
2-ITB

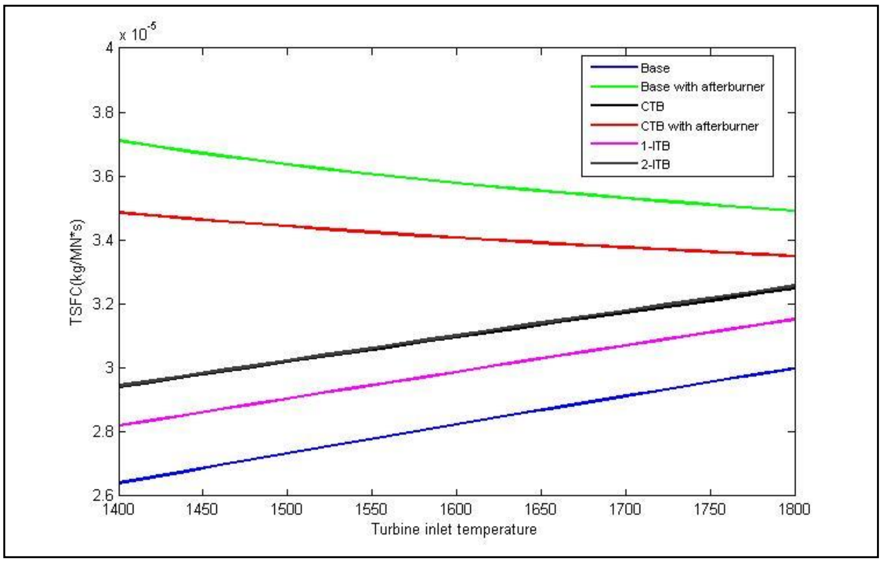


Performance comparisons with Flight Mach Number assuming Compression ratio = 30 for conventional engines and 60 for turbine burners, T_{04} = 1500 K, T_{06} = 1900 K

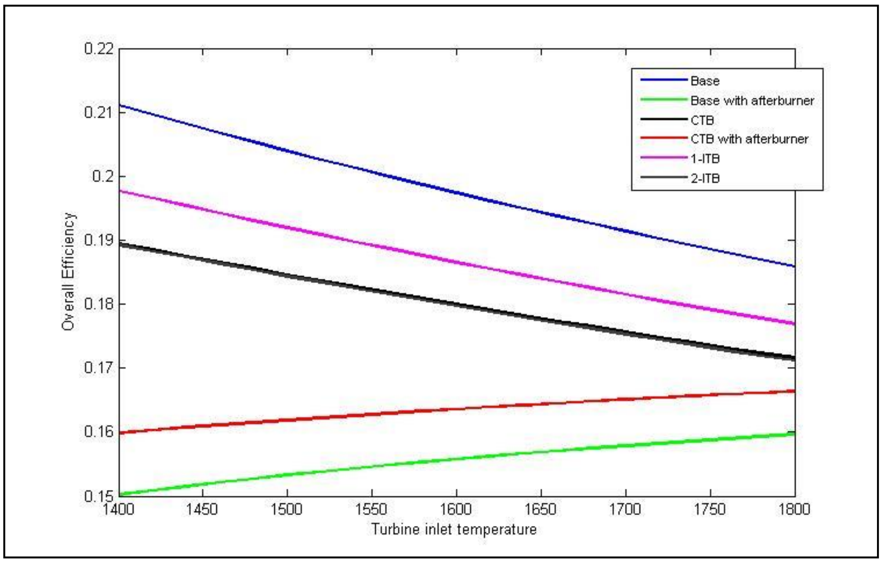
The general trend is that all type of engines depict an increase in specific fuel consumption and decrease in specific thrust as the flight mach number is increased. At higher flight mach numbers, the specific fuel consumption of CTB, 1-ITB and 2-ITB are very close to each other and their specific thrust is higher than the base engines, thus making them desirable for operation at high flight mach numbers.

Effect of turbine inlet temperature

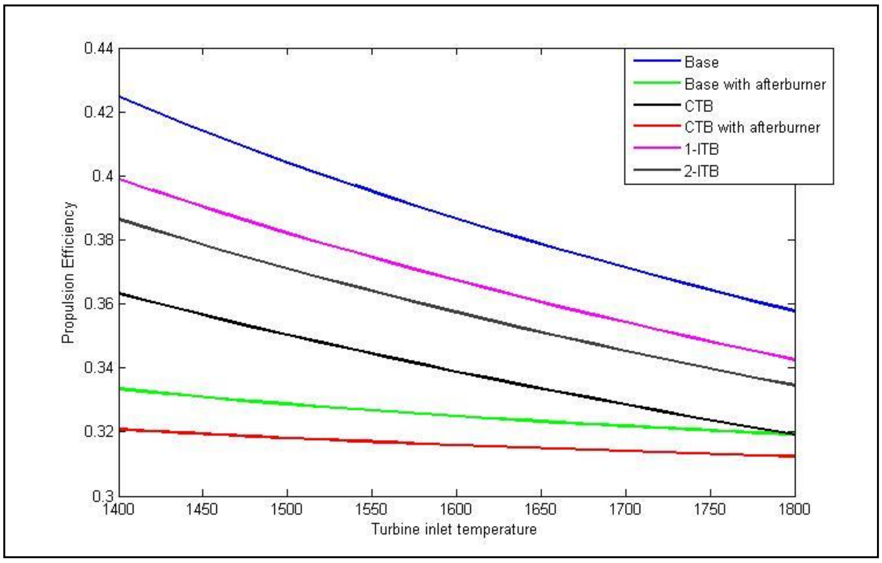




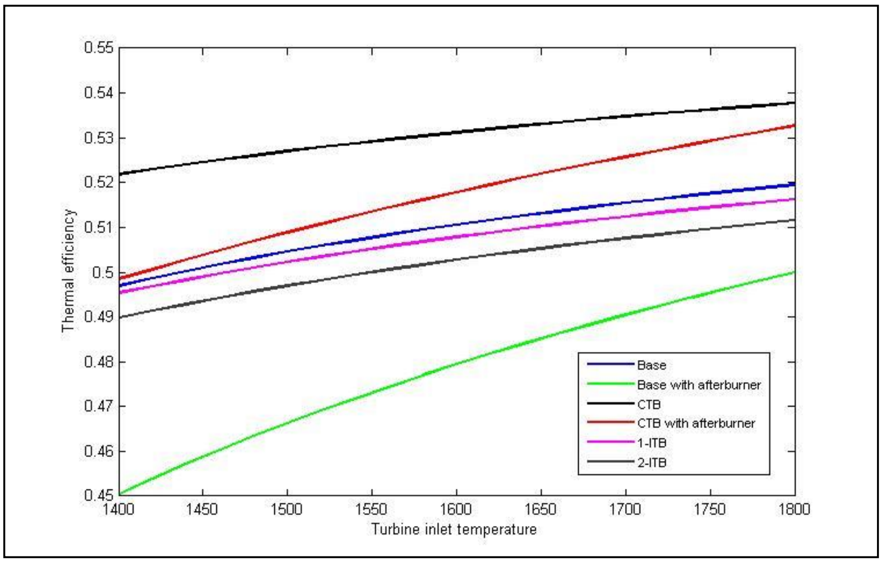
x 10^-5
TSFC(kg/MN*s)
Turbine inlet temperature
Base
Base with afterburner
CTB
CTB with afterburner
1-ITB
2-ITB



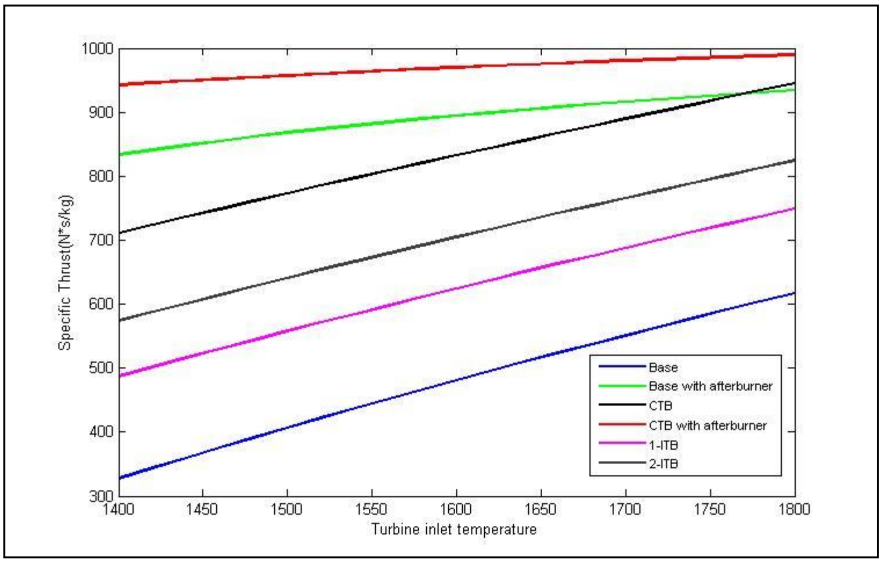
Overall Efficiency
Turbine inlet temperature
Base
Base with afterburner
CTB
CTB with afterburner
1-ITB
2-ITB

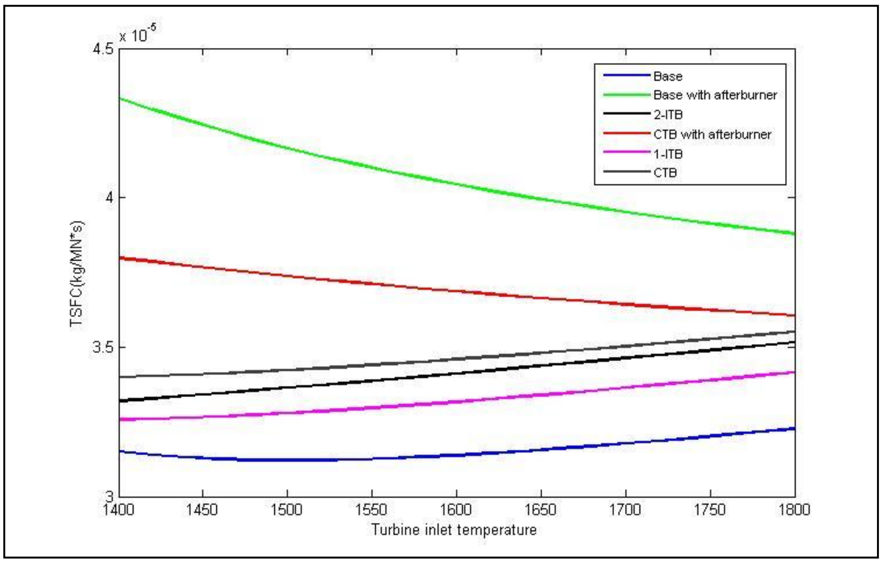


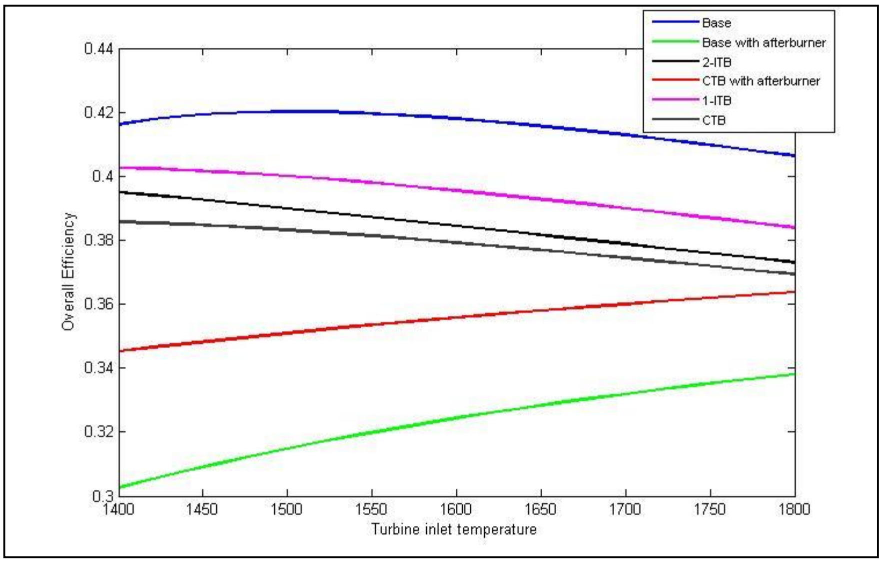
Propulsion Efficiency
Turbine inlet temperature
Base
Base with afterburner
CTB
CTB with afterburner
1-ITB
2-ITB

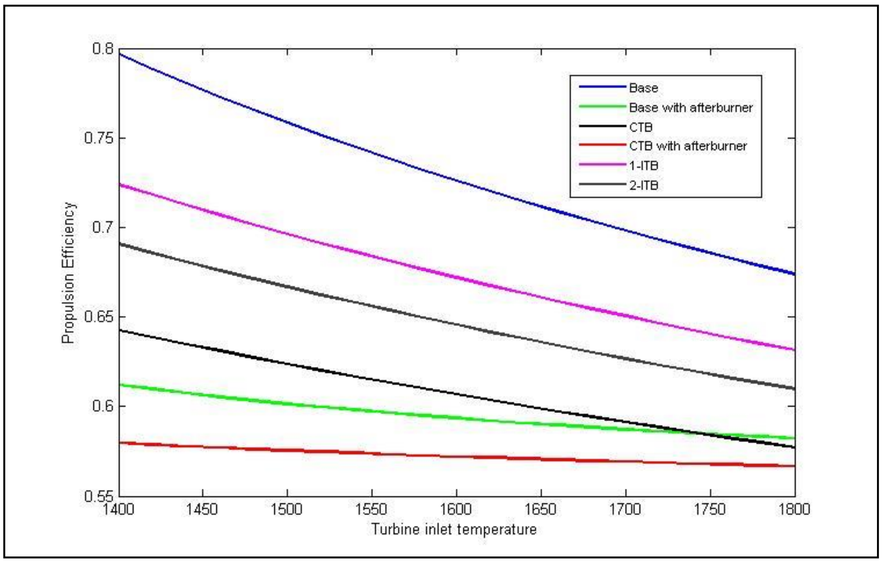


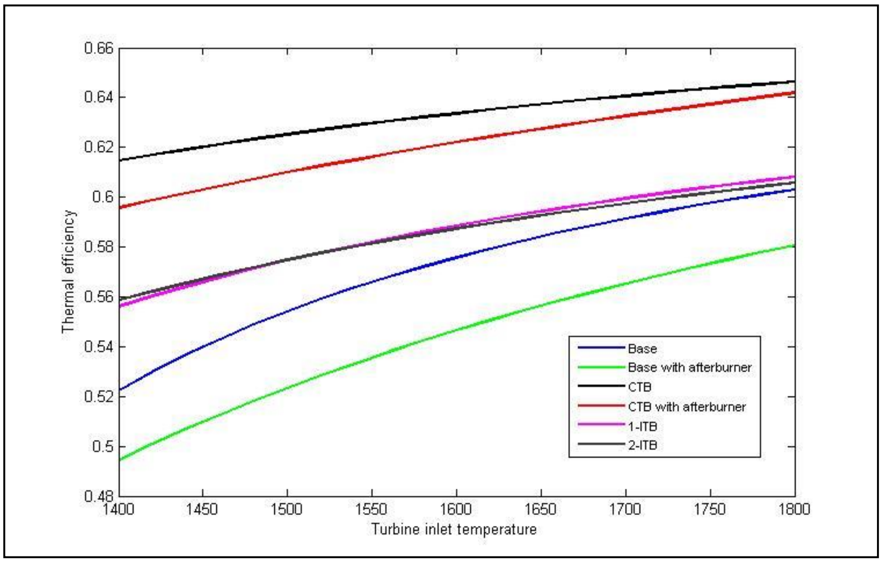
Performance comparisons with Turbine Inlet Temperature assuming Compression ratio = 20, T_{04} = 1500 K, T_{06} = 1900 K and M = 0.85 (Subsonic Conditions)







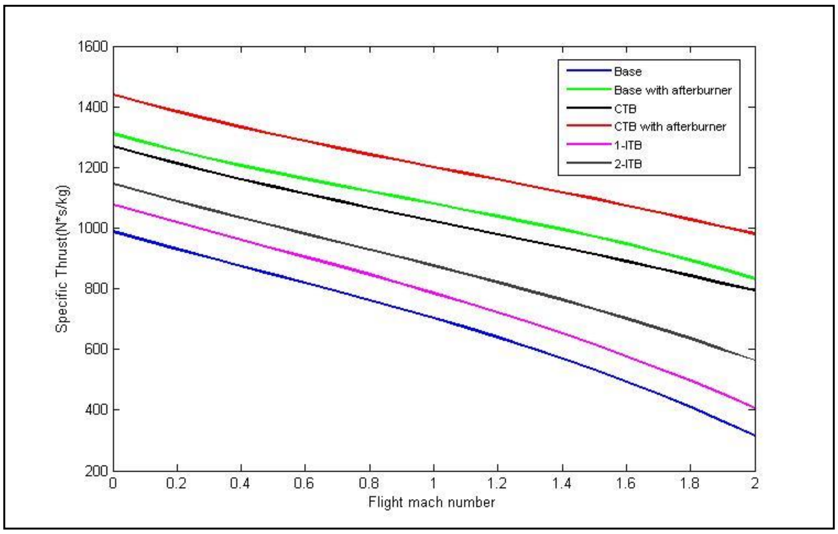


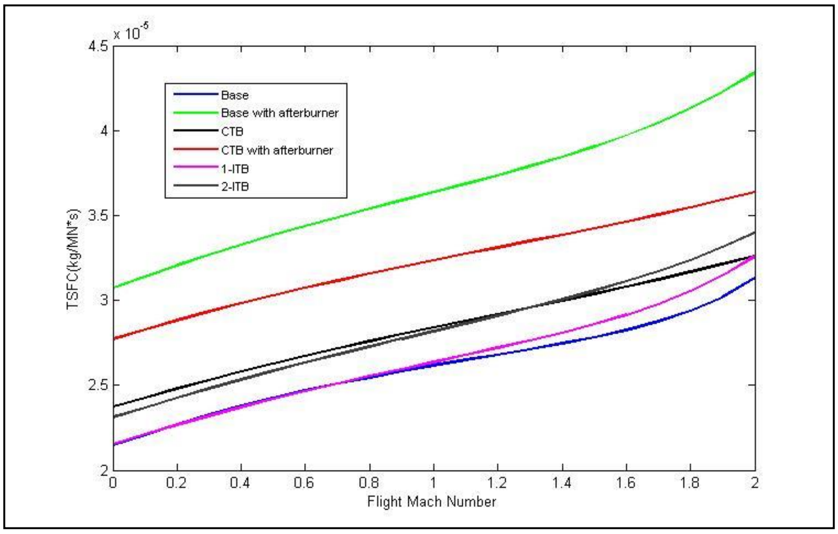


Performance comparisons with Turbine Inlet Temperature assuming Compression ratio = 20, T_{04} = 1500 K, T_{06} = 1900 K and M = 2 (Supersonic Conditions)

The performance of the base engines and turbine burner engines is found to increase with the turbine inlet temperature. Thus the turbine burners are also found to be benefitted with the increase in turbine inlet temperature. For both supersonic and subsonic flights, increasing the turbine inlet temperature is found to increase the specific thrust and causes a small increase in specific fuel consumption. Whereas specific fuel consumption is found to decrease for the afterburner configurations. The general trend is that the overall efficiency and propulsion efficiency decreases with increase in turbine inlet temperature. But the thermal efficiency increases with turbine inlet temperature because the pressure ratio decreases which in turn increases the exhaust velocity(or kinetic energy).

Optimal Design of Conventional Turbojet engines and Turbine burner engines





Performance comparisons with Flight Mach Number assuming Compression ratio = 30 for conventional engines and 60 for turbine burners, T_{04} = 1500 K, T_{06} = 1900 K

Since the base engines and turbine burner engines do not operate at the same parameters, it is quite difficult to compare the relative performances in such a case. The variation of the base engines and turbine burner engines with flight mach number is observed when the base engines and turbine burner engines are operating during their subsequent favourable conditions. It is observed that the difference of specific thrust between the turbine burner engines and base engines is found to increase more at higher flight mach numbers, proving that the turbine burner engines are superior. The specific fuel consumption of the turbine burner engines and base engines is also very close for the entire range of flight mach numbers.

Conclusions

After performing the thermodynamic analysis , we can conclude that

- The turbine burner engines are favourable at high compression ratios/compressor pressure ratios. Although conventional turbojet engines have an optimal compressor pressure ratio between 30-40 for supersonic flight, the turbine burners are capable of operating at high pressure ratios, over 60. High compression ratios increase ST and decrease TSFC of the turbine burner engines.
- Since the turbine burner engines have a high range of compression ratios, and the propulsion efficiency increases at high compression ratios, the performance of the turbine burner engines improves.
- With the advancement and further research in the development of turbine materials and turbine cooling methods, the performance of the turbine burner engines can be further improved.

Future Work

- Design optimization : Optimization of the turbine burner engines (ITB) can be performed during its favourable operating conditions to design the turbine burner engine over a wide range of operating parameters.
- Multistage turbine : Though this work has mainly been concentrated on a single stage turbine, the mathematical equations have to be modified while dealing with a multi-stage turbine as the above thermodynamic analysis does not hold good for the multi-stage turbines.
- Adopting a cooling model : To predict the detailed and realistic behaviour of the turbine, a cooling model of the turbine must be included as a function in the code.
- Multi-spool configuration : The above work has been done for a single spool turbojet engine and must be extended to more than one spool configuration of the turbojet engine.
- Comparison with other cycles : Due to inaccessibility of the journal papers, thermodynamic analysis of the turbojet engine operating on SCC (Sequential Combustion cycles) and CTT(Constant temperature cycles) could not be performed. This work could be done in the future.

References

1. Liu and Sirignano, "Turbojet and Turbofan Engine Perfromance Increases through Turbine Burners", Journal of Propulsion and Power, Vol 17, No. 3, pp 695-705, May-June 2001

2. Liu and Sirignano, "Performance Increases for Gas Turbine Engines through Combustion Inside the Turbine ", Journal of Propulsion and Power, Vol 15, No. 1, pp 695-705, January - February 1999

3. C.R., Hill P.G. and Peterson. Mechanics and Thermodynamics of Propulsion. s.l. : Addison Wesley , Reading, MA, 1992.

4. C.R., Hill P.G. and Peterson**.** Mechanics and Thermodynamics of Propulsion. s.l. : Addison Wesley , Reading, MA, 1992.

5. Liew, Urip, Yang, "Paremetric Cycle analysis of a Turbofan with Interstage Turbine Burner", Journal of Propulsion and Power, Vol 21, No. 3, May-June 2005

Appendix

```
clc
clear
sum_fraction=0.0;
fraction=zeros(100,1);
T_dash_05=zeros(100,1);
p_dash_05=zeros(100,1);
p_dash_06=zeros(100,1);
T_dash_06=zeros(100,1);
m_dot_itb=zeros(100,1);
sum_mitb=0.0;
disp('Thermodynamic design of a turbojet engine');
disp('1.Turbojet engine without an afterburner and turbine burner(conventional configuration');
disp('2.Turbojet engine with an afterburner and no turbine burner');
disp('3.Turbojet engine with a turbine burner and no afterburner');
disp('4.Turbojet engine with an afterburner and a turbine burner');
disp('5.Turbojet engine with a single stage ITB : 1-ITB');
```

```
disp('6.Turbojet engine with a two stage ITB : 2-ITB');
disp('7.Turbojet engine with multi-stage ITB : n-ITB');
x=input('Which configuration of the turbojet engine do you wish to design(enter 1-4)\n');
T_a=input('Enter the ambient temperature at the specific flight mach number\n');
m=input('enter the flightt mach number\n');
p_a=input('enter the ambient pressure at the specific flight mach number\n');
T_04=input('enter the turbine inlet temperature\n');
T_06=input('enter the maximum temperature\n');
eta_d=input('enter the inlet/diffuser adiabatic efficiency\n');
eta_c=input('enter the compressor adiabatic efficiency\n');
eta_b=input('enter the main burner combsution efficiency\n');
pi_b=input('enter the main burner total pressure recovery coefficient\n');
eta_t=input('enter the main turbine adiabatic efficiency\n');
eta_tb=input('enter the turbine burner combustion efficiency\n');
pi_tb=input('enter the additional turbine burner total pressure recovery coefficient\n');
eta_ab=input('enter the afterburner combustion efficiency\n');
pi_ab=input('enter the afterburner total pressure recovery coefficient\n');
eta_n=input('enter the nozzle adiabatic efficiency\n');
pi_c=input('enter the compressor pressure ratio\n');

if isempty(T_a)
    T_a=216.7;
end
if isempty(m)
    m=0.85;
end
if isempty(p_a)
    p_a = 18.75*1000;
end
if isempty(T_04)
     T_04=1500;
end
if isempty(T_06)
    T_06=1900;
```

```
end
if isempty(eta_d)
    eta_d=0.95;
end
if isempty(eta_c)
    eta_c=0.85;
end
if isempty(eta_b)
    eta_b=0.98;
end
if isempty(pi_b)
    pi_b=0.99;
end
if isempty(eta_t)
    eta_t=0.95;
end
if isempty(eta_tb)
    eta_tb=0.96;
end
if isempty(pi_tb)
    pi_tb=0.98;
end
if isempty(eta_ab)
    eta_ab=0.98;
end
if isempty(pi_ab)
    pi_ab=0.99;
end
if isempty(eta_n)
    eta_n=0.97;
end
if isempty(pi_c)
    pi_c=20;
end
```

```
Q_r=4.5*10^7; %heating value of fuel

%values of specific heat ratios

gamma=1.4;

gamma_c=1.37;

gamma_d=1.4;

gamma_t=1.33;

gamma_n=1.36;

c_p=1005;

m_dot_a=1.0;

i=0;


if x==1

    %Turbojet engine without an afterburner and turbine burner(conventional configuration

    T_02=T_a*(1+((gamma-1)/2)*m^2);     %stagnation temperature at the exit of diffuser

    p_02=p_a*(1+eta_d*(T_02/T_a-1))^(gamma_d/(gamma_d-1));  %stagnation pressure at the exit of diffuser

    T_03=T_02*(1+(1/eta_c)*((pi_c)^((gamma_c-1)/gamma_c)-1));   %stagnation temperature at the exit of compressor

    p_03=pi_c*p_02; %stagnation pressure at the exit of compressor

    m_dot_fb=m_dot_a*c_p*(T_04-T_03)/(Q_r*eta_b-c_p*T_04);  %fuel mass flow rate

    p_04=pi_b*p_03; %stagnation pressure at the exit of burner

    T_dash_05=T_04-T_03+T_02; %stagnation temperature at the exit of turbine

    p_dash_05=p_04*(1-(1/eta_t)*(1-(T_dash_05/T_04)))^(gamma_t/(gamma_t-1)); %stagnation pressure at the exit of
turbine

    u_e=sqrt(2*eta_n*c_p*T_dash_05*(1-(p_a/p_dash_05)^((gamma_n-1)/gamma_n))); %exit nozzle velocity

    u=m*sqrt(gamma*287*T_a); %flight velocity dependent on mach number

    st=((1+(m_dot_fb/m_dot_a))*u_e-u);  %specific thrust

    tsfc=m_dot_fb/(st*m_dot_a); %total specific fuel consumption

    t=(m_dot_a+m_dot_fb)*u_e-u; %thrust

    ke_gain=0.5*((m_dot_fb+m_dot_a)*u_e*u_e-m_dot_a*u*u);   %gain in kinetic energy or work output

    eta_p=(t*u)/ke_gain;    %propulsion efficiency

    eta_th=ke_gain/(Q_r*m_dot_fb);   %thermal efficiency

    eta_o=eta_p*eta_th;  %overall efficiency


elseif x==2

    %Turbojet engine with an afterburner and no turbine burner

    T_02=T_a*(1+((gamma-1)/2)*m^2);   %stagnation temperature at the exit of diffuser
```

```
p_02=p_a*(1+eta_d*(T_02/T_a-1))^(gamma_d/(gamma_d-1));  %stagnation pressure at the exit of diffuser

T_03=T_02*(1+(1/eta_c)*((pi_c)^((gamma_c-1)/gamma_c)-1));   %stagnation temperature at the exit of compressor

p_03=pi_c*p_02; %stagnation pressure at the exit of compressor

m_dot_fb=m_dot_a*c_p*(T_04-T_03)/(Q_r*eta_b-c_p*T_04);  %fuel mass flow rate

p_04=pi_b*p_03; %stagnation pressure at the exit of burner

T_dash_05=T_04-T_03+T_02; %stagnation temperature at the exit of turbine

p_dash_05=p_04*(1-(1/eta_t)*(1-(T_dash_05/T_04)))^(gamma_t/(gamma_t-1));  %stagnation pressure at the exit of
turbine

p_dash_06=p_dash_05*pi_ab;  %stagnation pressure at the exit of afterburner

T_dash_06=T_06;

m_dot_fab=(m_dot_a+m_dot_fb)*c_p*(T_dash_06-T_dash_05)/(Q_r*eta_ab-c_p*T_dash_06); %fuel mass flow rate of
afterburner

u_e=sqrt(2*eta_n*c_p*T_dash_06*(1-(p_a/p_dash_06)^((gamma_n-1)/gamma_n))); %exit nozzle velocity

u=m*sqrt(gamma*287*T_a);    %flight velocity dependent on mach number

st=(1+m_dot_fb+m_dot_fab)*u_e/(m_dot_a)-u;  %specific thrust

tsfc=(m_dot_fab+m_dot_fb)/(st*m_dot_a); %total specific fuel consumption

t=(m_dot_a+m_dot_fb+m_dot_fab)*u_e-u; %thrust

ke_gain=0.5*((m_dot_fb+m_dot_fab+m_dot_a)*u_e*u_e-m_dot_a*u*u);   %gain in kinetic energy or work output

eta_p=(t*u)/ke_gain;    %propulsion efficiency

eta_th=ke_gain/(Q_r*(m_dot_fb+m_dot_fab));   %thermal efficiency

eta_o=eta_p*eta_th;  %overall efficiency


elseif x==3

%Turbojet engine with a turbine burner and no afterburner

T_02=T_a*(1+((gamma-1)/2)*m^2);   %stagnation temperature at the exit of diffuser

p_02=p_a*(1+eta_d*(T_02/T_a-1))^(gamma_d/(gamma_d-1));  %stagnation pressure at the exit of diffuser

T_03=T_02*(1+(1/eta_c)*((pi_c)^((gamma_c-1)/gamma_c)-1));   %stagnation temperature at the exit of compressor

p_03=pi_c*p_02; %stagnation pressure at the exit of compressor

m_dot_fb=m_dot_a*c_p*(T_04-T_03)/(Q_r*eta_b-c_p*T_04);  %fuel mass flow rate

p_04=pi_b*p_03; %stagnation pressure at the exit of burner

T_05=T_04;  %stagnation temperature at the exit of turbine burner

P_t=m_dot_a*c_p*(T_03-T_02); %turbine power balanced by a compressor

m_dot_ftb=P_t/(Q_r*eta_tb); %turbine burner fuel mass flow rate

T_av=0.91*T_04; %mean temperature of flow

delta_s=P_t/((m_dot_a+m_dot_fb)*T_av);  %entropy change across the turbine burner
```

```
    p_05=pi_tb*p_04*exp(-delta_s/287);  %stagnation pressure at the end of turbine burner

    u_e=sqrt(2*eta_n*c_p*T_05*(1-(p_a/p_05)^((gamma_n-1)/gamma_n))); %exit nozzle velocity

    u=m*sqrt(gamma*287*T_a);    %flight velocity dependent on mach number

    st=(1+m_dot_fb+m_dot_ftb)*u_e/(m_dot_a)-u;  %specific thrust

    tsfc=(m_dot_fb+m_dot_ftb)/(st*m_dot_a); %total specific fuel consumption

    t=(m_dot_a+m_dot_fb+m_dot_ftb)*u_e-u; %thrust

    ke_gain=0.5*((m_dot_fb+m_dot_ftb+m_dot_a)*u_e*u_e-m_dot_a*u*u);   %gain in kinetic energy or work output

    eta_p=(t*u)/ke_gain;    %propulsion efficiency

    eta_th=ke_gain/(Q_r*(m_dot_fb+m_dot_ftb));   %thermal efficiency

    eta_o=eta_p*eta_th;  %overall efficiency


elseif x==4

    %Turbojet engine with an afterburner and a turbine burner

    T_02=T_a*(1+((gamma-1)/2)*m^2);   %stagnation temperature at the exit of diffuser

    p_02=p_a*(1+eta_d*(T_02/T_a-1))^(gamma_d/(gamma_d-1));  %stagnation pressure at the exit of diffuser

    T_03=T_02*(1+(1/eta_c)*((pi_c)^((gamma_c-1)/gamma_c)-1));   %stagnation temperature at the exit of compressor

    p_03=pi_c*p_02; %stagnation pressure at the exit of compressor

    m_dot_fb=m_dot_a*c_p*(T_04-T_03)/(Q_r*eta_b-c_p*T_04);  %fuel mass flow rate

    p_04=pi_b*p_03; %stagnation pressure at the exit of burner

    T_05=T_04;  %stagnation temperature at the exit of turbine burner

    P_t=m_dot_a*c_p*(T_03-T_02); %turbine power balanced by a compressor

    m_dot_ftb=P_t/(Q_r*eta_tb); %turbine burner fuel mass flow rate

    T_av=0.91*T_04; %mean temperature of flow

    delta_s=P_t/((m_dot_a+m_dot_fb)*T_av);  %entropy change across the turbine burner

    p_05=pi_tb*p_04*exp(-delta_s/287);  %stagnation pressure at the end of turbine burner

    p_06=p_05*pi_ab; %stagnation pressure at the end of afterburner

    m_dot_fab=c_p*(m_dot_a+m_dot_fb+m_dot_ftb)*(T_06-T_05)/(Q_r*eta_ab-c_p*T_06); %afterburner fuel mass flow
rate

    u_e=sqrt(2*eta_n*c_p*T_06*(1-(p_a/p_06)^((gamma_n-1)/gamma_n)));   %exit nozzle velocity

    u=m*sqrt(gamma*287*T_a); %flight velocity dependent on mach number

    st=(1+m_dot_fb+m_dot_ftb+m_dot_fab)*u_e/(m_dot_a)-u; %specific thrust

    tsfc=(m_dot_fb+m_dot_ftb+m_dot_fab)/(st*m_dot_a); %total specific fuel consumption

    t=(m_dot_a+m_dot_fb+m_dot_ftb+m_dot_fab)*u_e-u; %thrust

    ke_gain=0.5*((m_dot_fb+m_dot_ftb+m_dot_fab+m_dot_a)*u_e*u_e-m_dot_a*u*u);   %gain in kinetic energy or work
output
```

```
    eta_p=(t*u)/ke_gain;    %propulsion efficiency

    eta_th=ke_gain/(Q_r*(m_dot_fb+m_dot_ftb+m_dot_fab));   %thermal efficiency

    eta_o=eta_p*eta_th;  %overall efficiency


elseif x==5

    fraction1=input('enter the fraction of power consumed by turbine 1');

    if isempty(fraction1)

        fraction1=0.4;

    end

    %Turbojet engine with a single stage ITB : 1-ITB

    T_02=T_a*(1+((gamma-1)/2)*m^2);     %stagnation temperature at the exit of diffuser

    p_02=p_a*(1+eta_d*(T_02/T_a-1))^(gamma_d/(gamma_d-1));  %stagnation pressure at the exit of diffuser

    T_03=T_02*(1+(1/eta_c)*((pi_c)^((gamma_c-1)/gamma_c)-1));   %stagnation temperature at the exit of compressor

    p_03=pi_c*p_02; %stagnation pressure at the exit of compressor

    m_dot_fb=m_dot_a*c_p*(T_04-T_03)/(Q_r*eta_b-c_p*T_04);  %fuel mass flow rate

    p_04=pi_b*p_03; %stagnation pressure at the exit of burner

    T_dash_05=T_04-fraction1*(T_03-T_02); %stagnaton temperature at the exit of turbine 1

    p_dash_05=p_04*(1-(1/eta_t)*(1-(T_dash_05/T_04)))^(gamma_t/(gamma_t-1));    %stagnation pressure at the exit of
turbine 1

    p_dash_06=p_dash_05*pi_tb;  %stagnation pressure ratio at the inlet to turbine 2

    T_dash_06=T_04;     %constant temperature in the turbine burner

    m_dot_itb=(m_dot_fb*c_p*(T_dash_06-T_dash_05)+m_dot_a*c_p*(T_dash_06-T_dash_05))/(Q_r*eta_tb-c_p*T_04);
%turbine burner fuel mass flow rate

    T_dash_07=T_dash_06-(1-fraction1)*(T_03-T_02);  %stagnaton temperature at the exit of turbine 2

    p_dash_07=p_dash_06*(1-(1/eta_t)*(1-(T_dash_07/T_dash_06)))^(gamma_t/(gamma_t-1));    %stagnation pressure at the
exit of turbine 2

    p_8=p_a;

    T_dash_08=T_dash_07;

    u_e=sqrt(2*eta_n*c_p*T_dash_08*(1-(p_a/p_dash_07)^((gamma_n-1)/gamma_n)));   %exit nozzle velocity

    u=m*sqrt(gamma*287*T_a); %flight velocity dependent on mach number

    st=(1+m_dot_fb+m_dot_itb)*u_e/(m_dot_a)-u; %specific thrust

    tsfc=(m_dot_fb+m_dot_itb)/(st*m_dot_a); %total specific fuel consumption

    t=(m_dot_a+m_dot_fb+m_dot_itb)*u_e-u; %thrust

    ke_gain=0.5*((m_dot_fb+m_dot_itb+m_dot_a)*u_e*u_e-m_dot_a*u*u);   %gain in kinetic energy or work output

    eta_p=(t*u)/ke_gain;    %propulsion efficiency

    eta_th=ke_gain/(Q_r*(m_dot_fb+m_dot_itb));   %thermal efficiency
```

```
    eta_o=eta_p*eta_th;  %overall efficiency


elseif x==6
    fraction1=input('enter the fraction of power consumed by turbine 1');
    if isempty(fraction1)
        fraction1=0.33;
    end
    fraction2=input('enter the fraction of power consumed by turbine 2');
    if isempty(fraction2)
        fraction2=0.33;
    end
    %Turbojet engine with a double stage ITB : 2-ITB
    T_02=T_a*(1+((gamma-1)/2)*m^2);     %stagnation temperature at the exit of diffuser
    p_02=p_a*(1+eta_d*(T_02/T_a-1))^(gamma_d/(gamma_d-1));  %stagnation pressure at the exit of diffuser
    T_03=T_02*(1+(1/eta_c)*((pi_c)^((gamma_c-1)/gamma_c)-1));   %stagnation temperature at the exit of compressor
    p_03=pi_c*p_02; %stagnation pressure at the exit of compressor
    m_dot_fb=m_dot_a*c_p*(T_04-T_03)/(Q_r*eta_b-c_p*T_04);  %fuel mass flow rate
    p_04=pi_b*p_03; %stagnation pressure at the exit of burner
    T_dash_05=T_04-fraction1*(T_03-T_02); %stagnaton temperature at the exit of turbine 1
    p_dash_05=p_04*(1-(1/eta_t)*(1-(T_dash_05/T_04)))^(gamma_t/(gamma_t-1));    %stagnation pressure at the exit of
turbine 1
    p_dash_06=p_dash_05*pi_tb;  %stagnation pressure ratio at the inlet to turbine 2
    T_dash_06=T_04;     %constant temperature in the turbine burner
    m_dot_itb1=(m_dot_fb*c_p*(T_dash_06-T_dash_05)+m_dot_a*c_p*(T_dash_06-T_dash_05))/(Q_r*eta_tb-c_p*T_04);
%turbine burner 1 fuel mass flow rate
    T_dash_07=T_dash_06-fraction2*(T_03-T_02); %stagnaton temperature at the exit of turbine 2
    p_dash_07=p_dash_06*(1-(1/eta_t)*(1-(T_dash_07/T_dash_06)))^(gamma_t/(gamma_t-1));    %stagnation pressure at the
exit of turbine 2
    p_dash_08=p_dash_07*pi_tb;  %stagnation pressure ratio at the inlet to turbine 3
    T_dash_08=T_04;
    m_dot_itb2=c_p*(T_dash_08-T_dash_07)*(m_dot_a+m_dot_itb1+m_dot_fb)/(Q_r*eta_tb-c_p*T_04); %turbine burner 1
fuel mass flow rate
    T_dash_09=T_dash_08-(1-fraction1-fraction2)*(T_03-T_02); %stagnaton temperature at the exit of turbine 3
    p_dash_09=p_dash_08*(1-(1/eta_t)*(1-(T_dash_09/T_dash_08)))^(gamma_t/(gamma_t-1));    %stagnation pressure at the
exit of turbine 3
    p_10=p_a;
    T_dash_10=T_dash_09;
```

```
    u_e=sqrt(2*eta_n*c_p*T_dash_09*(1-(p_a/p_dash_09)^((gamma_n-1)/gamma_n)));   %exit nozzle velocity

    u=m*sqrt(gamma*287*T_a); %flight velocity dependent on mach number

    st=(1+m_dot_fb+m_dot_itb1+m_dot_itb2)*u_e/(m_dot_a)-u; %specific thrust

    tsfc=(m_dot_fb+m_dot_itb1+m_dot_itb2)/(st*m_dot_a); %total specific fuel consumption

    t=(m_dot_a+m_dot_fb+m_dot_itb1+m_dot_itb2)*u_e-u; %thrust

    ke_gain=0.5*((m_dot_fb+m_dot_itb1+m_dot_itb2+m_dot_a)*u_e*u_e-m_dot_a*u*u);   %gain in kinetic energy or work
output

    eta_p=(t*u)/ke_gain;    %propulsion efficiency

    eta_th=ke_gain/(Q_r*(m_dot_fb+m_dot_itb1+m_dot_itb2));   %thermal efficiency

    eta_o=eta_p*eta_th;  %overall efficiency


elseif x==7

    n=input('enter the number of stages of ITB');

    if isempty(n)

        n=3;

    end

    fraction=(ones(n+1,1))/(n+1);

    for i=1:n

    accept=input('do u want to enter a numeric value for fraction of power consumed by turbine i(enter y/Y)','s');

    if strcmpi(accept,'y')~=0

        fraction(i,1)=input('enter the fraction of power consumed by turbine i');

        sum_fraction=sum_fraction+fraction(i);

    else

        fraction(i)=1/(n+1);

        sum_fraction=sum_fraction+fraction(i);

    end

    end

    %Turbojet engine with a n stage ITB : n-ITB

    T_02=T_a*(1+((gamma-1)/2)*m^2);     %stagnation temperature at the exit of diffuser

    p_02=p_a*(1+eta_d*(T_02/T_a-1))^(gamma_d/(gamma_d-1));  %stagnation pressure at the exit of diffuser

    T_03=T_02*(1+(1/eta_c)*((pi_c)^((gamma_c-1)/gamma_c)-1));   %stagnation temperature at the exit of compressor

    p_03=pi_c*p_02; %stagnation pressure at the exit of compressor

    m_dot_fb=m_dot_a*c_p*(T_04-T_03)/(Q_r*eta_b-c_p*T_04);  %fuel mass flow rate

    p_04=pi_b*p_03; %stagnation pressure at the exit of burner

for i=1:n
```

```
    T_dash_05(i)=T_04-fraction(i)*(T_03-T_02); %stagnaton temperature at the exit of turbine

    if i==1

    p_dash_05(i)=p_04*(1-(1/eta_t)*(1-(T_dash_05(i)/T_04)))^(gamma_t/(gamma_t-1));    %stagnation pressure at the exit of turbine

    else

       p_dash_05(i)=p_dash_06(i-1)*(1-(1/eta_t)*(1-(T_dash_05(i)/T_04)))^(gamma_t/(gamma_t-1));    %stagnation pressure at the exit of turbine

    end

    p_dash_06(i)=p_dash_05(i)*pi_tb;  %stagnation pressure ratio at the inlet to turbine

    T_dash_06(i)=T_04;     %constant temperature in the turbine burner

    m_dot_itb(i)=(m_dot_fb*c_p*(T_dash_06(i)-T_dash_05(i))+m_dot_a*c_p*(T_dash_06(i)-T_dash_05(i)))/(Q_r*eta_tb-c_p*T_04); %turbine burner  fuel mass flow rate

    sum_mitb=sum_mitb+m_dot_itb(i);

end

    T_dash_07=T_dash_06(n)-(1-sum_fraction)*(T_03-T_02); %stagnation temperature at the exit of turbine m+1

    p_dash_07=p_dash_06(n)*(1-(1/eta_t)*(1-(T_dash_07/T_dash_06(n))))^(gamma_t/(gamma_t-1));    %stagnation pressure at the exit of turbine

    p_8=p_a;

    T_dash_8=T_dash_07;

    u_e=sqrt(2*eta_n*c_p*T_dash_07*(1-(p_a/p_dash_07)^((gamma_n-1)/gamma_n)));   %exit nozzle velocity

    u=m*sqrt(gamma*287*T_a); %flight velocity dependent on mach number

    st=(1+m_dot_fb+sum_mitb)*u_e/(m_dot_a)-u; %specific thrust

    tsfc=(m_dot_fb+sum_mitb)/(st*m_dot_a); %total specific fuel consumption

    t=(m_dot_a+m_dot_fb+sum_mitb)*u_e-u; %thrust

    ke_gain=0.5*((m_dot_fb+sum_mitb+m_dot_a)*u_e*u_e-m_dot_a*u*u);   %gain in kinetic energy or work output

    eta_p=(t*u)/ke_gain;    %propulsion efficiency

    eta_th=ke_gain/(Q_r*(m_dot_fb+sum_mitb));   %thermal efficiency

    eta_o=eta_p*eta_th;  %overall efficiency


else

    disp('you have entered a wrong option');

end

disp('Output paremeters of the Turbojet engine configuration which u have selected');

disp(['Specific Thrust(Ns/kg):',num2str(st)]);

disp(['Total Specific Fuel Consumption TSFC(kg/MNs):',num2str(tsfc)]);

disp(['Thermal efficiency:',num2str(eta_th)]);
```

```
disp(['Propulsion efficiency:',num2str(eta_p)]);
disp(['Overall efficiency:',num2str(eta_o)]);
disp('Thank you');
```