ECE 340
Lecture 22 : Space Charge at a Junction

Class Outline:

- Space Charge Region

Key Questions

- What is the space charge region?
- What are the important quantities?
- How are the important quantities related to one another?
- How would bias change my analysis?

Space Charge Region

To gain a qualitative understanding of the solution for the electrostatic variables we need **Poisson's equation**:

Most times a simple closed form solution will not be possible, so we need an approximation from which we can derive other relations. Consider the following…

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{K_S \varepsilon_0} = \frac{q}{K_S \varepsilon_0}(p - n + \boxed{N_D - N_A})$$

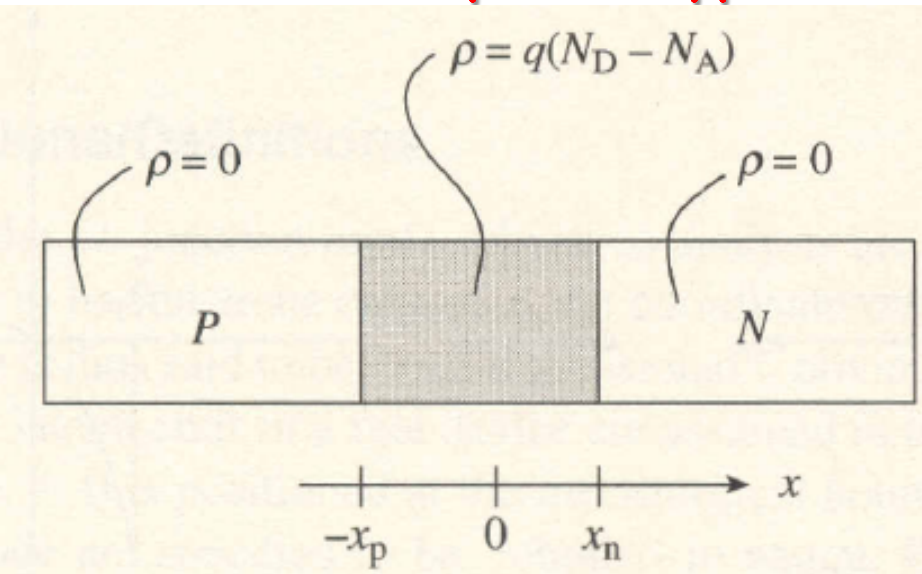
Doping profile is known

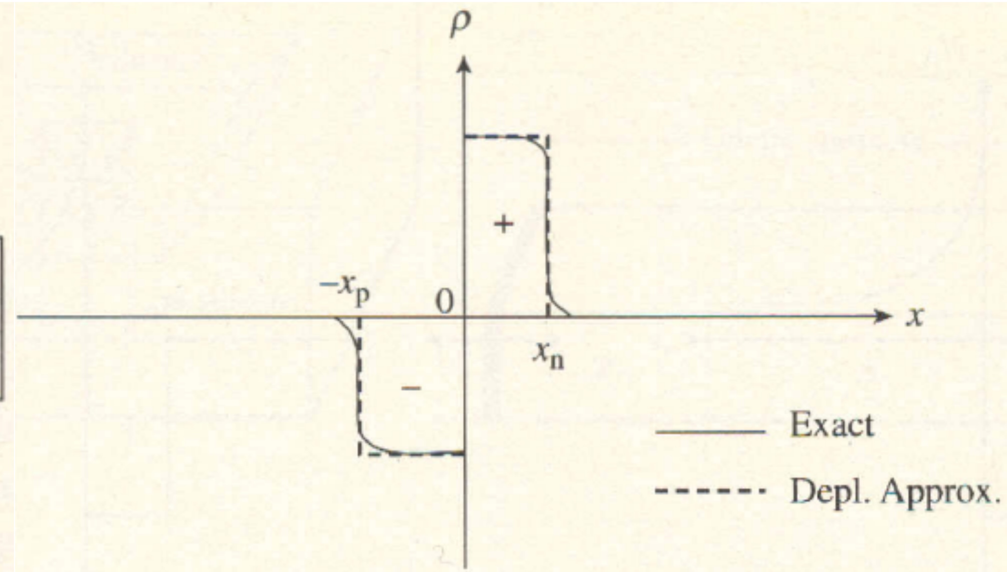
- To obtain the electric field and potential we need to integrate.
- However, we don't know the electron and hole concentrations as a function of x.
- Electron and hole concentrations are a function of the potential which we do not know until we solve Poisson's equation.

Use the **depletion approximation**…

Space Charge Region

What does the **depletion approximation** tell us…





1. The carrier concentrations are assumed to be negligible compared to the net doping concentrations in the junction region.

2. The charge density outside the depletion region is taken to be identically zero.

Poisson equation becomes…

$$\frac{d\mathscr{E}}{dx} \cong \begin{cases} \frac{q}{K_S \varepsilon_0}(N_D - N_A) & \ldots -x_p \le x \le x_n \\ 0 & \ldots x \le -x_p \text{ and } x \ge x_n \end{cases}$$

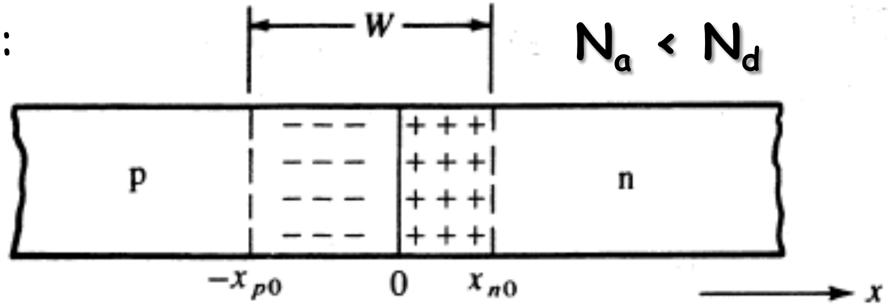
Must $x_p = x_n$?

Space Charge Region

We are already well aware of the formation of the **space charge region**…

The space charge region is characterized by:

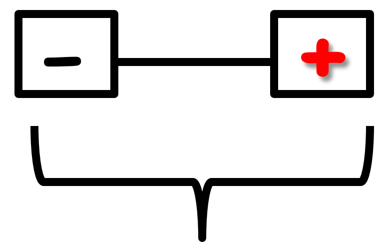
$N_a < N_d$



•Electrons and holes moving across the junction.

•Only a few carriers at a time being in the space charge region (depletion approximation).

•Space charge is primarily composed of uncompensated donors and acceptors.



We are forming a series of **dipoles at the junction**.

The charge on the left of the junction must be balanced by the charge on the right side of the junction.

$$Q_+ = |Q_-|$$ → **The junction can extend unequally into the n and p regions depending on the relative doping.**

$$qAx_{p0}N_a = qAx_{n0}N_d$$

For a given area, A. →

$$W = x_{p0} + x_{n0}$$

Space Charge Region

What about the **electric field** in the **space charge region**?

We again begin with Poisson's equation…

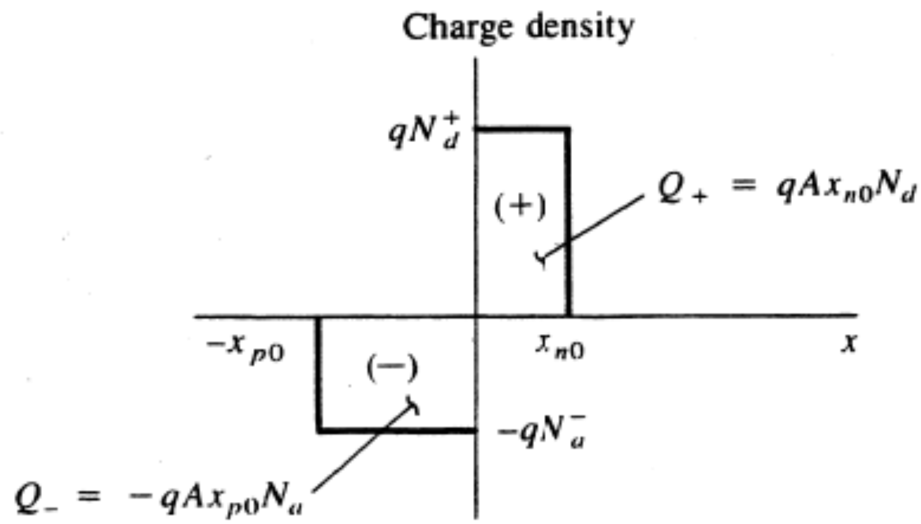
$$\frac{d\xi(x)}{dx} = \frac{q}{\varepsilon}(p - n + N_d^+ - N_a^-)$$

Apply the **depletion approximation**…

$$\frac{d\xi}{dx} = \frac{q}{\varepsilon}N_d \qquad 0 < x < x_{n0}$$

$$\frac{d\xi}{dx} = -\frac{q}{\varepsilon}N_a \qquad -x_{p0} < x < 0$$

Which assumes **complete ionization** of impurities.



So how do we get the electric field out of the charge distribution?

We **integrate**…

Space Charge Region

Let's find the electric field…

What characteristics does the electric field possess…

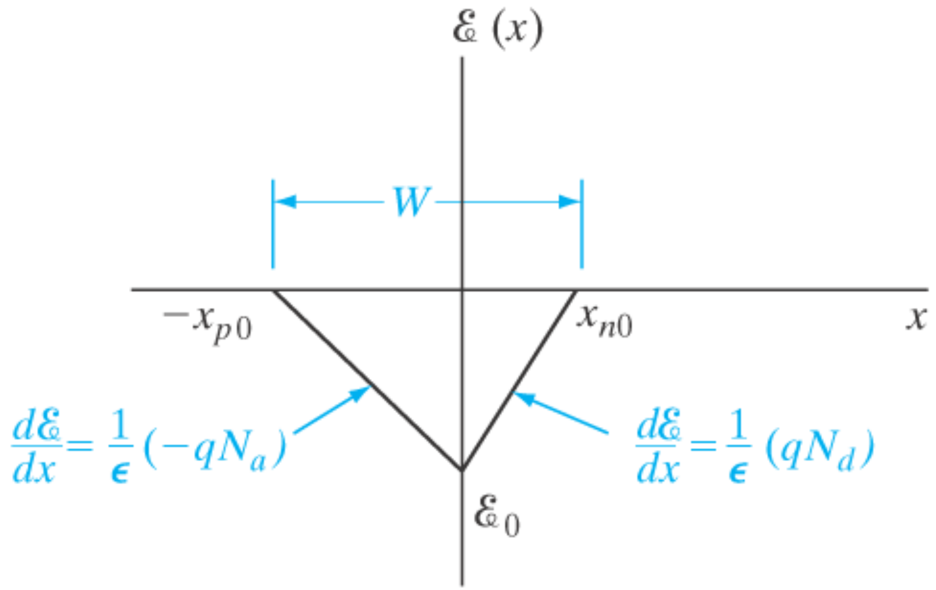
•It should have two slopes, positive on the n-side and negative on the p-side.

•There should be a maximum field at x = 0.

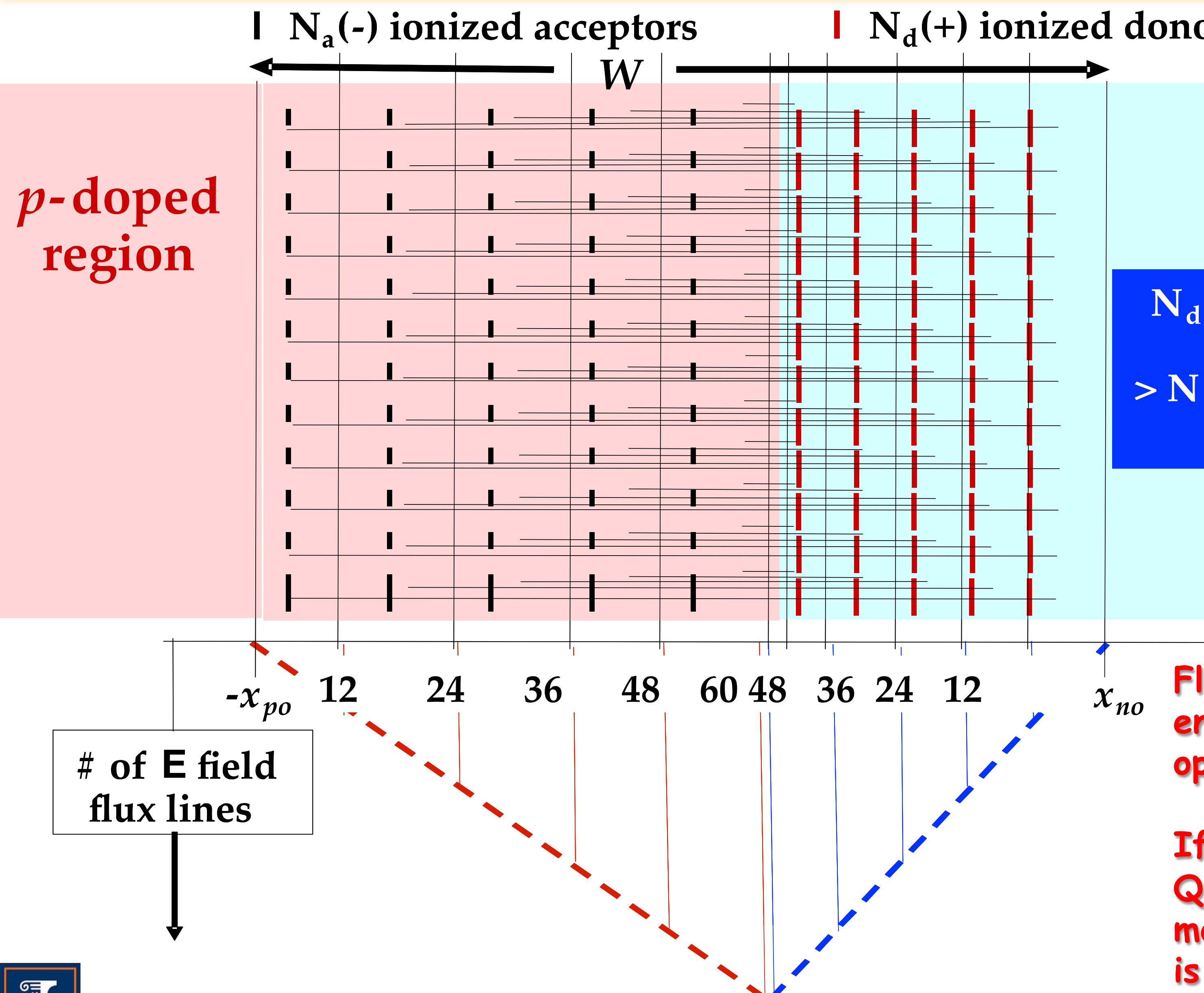
•These characteristics come from Gauss' Law but we could have obtained these qualitatively.

•E(x) should be negative as it points in the –x direction.

•E(x) goes to zero at the edges of the space charge region.



Space Charge Region

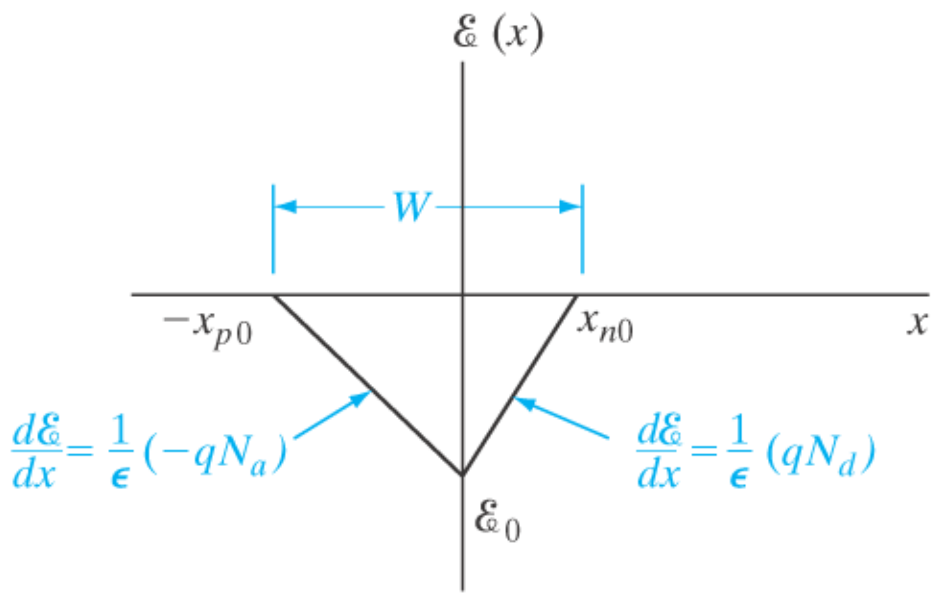


Flux lines begin and end on charges of opposite sign.

If Q_+ does not equal Q_-, we must enclose more area until flux is equal.

Space Charge Region

How do we find the **maximum field**…



Integrate the Poisson solution.

$$\int_{\xi_0}^{0} d\xi = \frac{q}{\varepsilon} N_d \int_{0}^{x_{n0}} dx \qquad 0 < x < x_{n0}$$

$$\int_{0}^{\xi_0} d\xi = -\frac{q}{\varepsilon} N_a \int_{-x_{p0}}^{0} dx \qquad x_{p0} < x < 0$$

Thus we now have the maximum value of the electric field…

$$\xi_0 = -\frac{q}{\varepsilon} N_d x_{n0} = -\frac{q}{\varepsilon} N_a x_{p0}$$

What about the potential?

Space Charge Region

Let's find the **potential**…

It is easy to find the **contact potential** once we have the field in the **space charge region**…

$$\mathscr{E}(x) = -\frac{d\mathscr{V}(x)}{dx} \quad \text{or} \quad -V_0 = \int_{-x_{p0}}^{x_{n0}} \mathscr{E}(x)\,dx$$

The negative of the contact potential is the area under the electric field curve.
We can also relate this to the width of the space charge region…

$$V_0 = -\frac{1}{2}\mathscr{E}_0 W = \frac{1}{2}\frac{q}{\epsilon}N_d x_{n0} W$$

But we can go farther…

Balance of charge requirement

$$x_{n0}N_d = x_{p0}N_a$$

+

Space charge width

$$W = x_{p0} + x_{n0}$$

$$\rightarrow \quad x_{n0} = WN_a/(N_a + N_d)$$

$$V_0 = \frac{1}{2}\frac{q}{\epsilon}\frac{N_aN_d}{N_a + N_d}W^2$$

$$\rightarrow \quad W = \left[\frac{2\epsilon V_0}{q}\left(\frac{N_a + N_d}{N_aN_d}\right)\right]^{1/2} = \left[\frac{2\epsilon V_0}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2}$$

Space Charge Region

Now we have the width of the **space charge region** as a function of contact potential, doping concentrations, and other constants…

However, there are other variations…

$$V_0 = \frac{kT}{q}\ln\frac{N_a}{n_i^2/N_d} = \frac{kT}{q}\ln\frac{N_aN_d}{n_i^2}$$

Simplify using the contact potential relation to obtain an equation which depends on doping only…

$$W = \left[\frac{2\epsilon kT}{q^2}\left(\ln\frac{N_aN_d}{n_i^2}\right)\left(\frac{1}{N_a}+\frac{1}{N_d}\right)\right]^{1/2}$$

And we can also calculate the positions of the space charge region in the p-type and n-type regions…

$$x_{p0} = \frac{WN_d}{N_a+N_d} = \frac{W}{1+N_a/N_d} = \left\{\frac{2\epsilon V_0}{q}\left[\frac{N_d}{N_a(N_a+N_d)}\right]\right\}^{1/2}$$

N-type

$$x_{n0} = \frac{WN_a}{N_a+N_d} = \frac{W}{1+N_d/N_a} = \left\{\frac{2\epsilon V_0}{q}\left[\frac{N_a}{N_d(N_a+N_d)}\right]\right\}^{1/2}$$

P-type

These equations confirm our suspicion that the junction penetrates deeper into the more lightly doped side.

Space Charge Region

Closing thoughts about the space charge region…

$$W = \left[\frac{2\epsilon V_0}{q}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2} = \left[\frac{2\epsilon V_0}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2}$$

What if I vary the externally applied voltage?

•Remember, a positive outside voltage "grabs" the Fermi level on the side it's applied on and drags it down. (negative pulls it up).

•How do we remember this? Think of the simple resistor band diagram, which way the electric field points (external + to -) and which way the electrons "slide down" or holes "bubble up."

•Space charge region width, W, varies as the square root of the potential across the region.

•Thus far, we have only considered the contact or built-in potential but this is also true for an applied bias.

A *forward bias* is + applied to the p-side, which *lowers* the built-in voltage barrier ($V_0 - V_{fwd}$) where $V_{fwd} > 0$.

A *reverse bias* is – applied to the p-side, which *increases* the built-in voltage barrier ($V_0 - V_{rev}$) where $V_{rev} < 0$.

Space Charge Region

Let's solve a problem…

An abrupt p-n junction has $N_a = 10^{18}$ cm^{-3} and $N_d = 5 \times 10^{15}$ cm^{-3} on the other side. Assume that it has a circular cross section with a diameter of 10 μm at 300 K.

(a) Calculate the Fermi level positions at 300 K in the p and n regions.

(b) Draw the equilibrium band diagram and determine the contact potential.

(c) Calculate x_n, x_p, E_0 and Q_+

(d) Sketch the electric field and the charge density.