

A Priori \& Post-Hoc Tests
Statistics

## Hindsight is 20-20

- Although your data may suggest a new relationship, and thus new analyses...
- Theory should guide research and thus comparisons should be decided on before you conduct your experiment.


With hindsight, maybe Colin didn't pick the best time to cover up his acne with a balaclava!

## ŖPIanned \& A Priori Comparisons

- Based on literature review
- Theoretical
- Planned comparisons
- A test that is conducted when there are multiple groups of scores, but specific comparisons have been specified prior to data collection.
- A Priori Comparisons
- If you have planned comparisons...
- Just run $t$ tests
- Subjective Decision about $p$ value
- $p=.05$ ?
- $p=.01$ ?
- Bonferroni Correction?


## Post-Hoc: Tukey HSD

- Tukey Honestly Significant Difference
- Determines differences between means in terms of standard error
- 'Honest' because we adjust for making multiple comparisons
- The HSD is compared to a critical value
- Overview

1. Calculate differences between a pair of means
2. Divide this difference by the standard error

* Basically this is a variant of a $t$ test *


## Tukey HSD

## $S_{M}$

$$
t=\frac{\left(M_{1}-M_{2}\right)}{S_{\text {Difference }}}
$$

- For Tukey HSD, standard error is calculated differently depending on whether your sample sizes are equal.


## Tukey HSD

- Equal Sample Sizes

$N=$ Sample size within each group
- Unequal Sample Sizes

$$
s_{M}=\sqrt{\frac{M S_{\text {within }}}{N^{\prime}}}
$$

$$
N^{\prime}=\frac{N_{\text {Groups }}}{\sum\left(\frac{1}{N}\right)}
$$

## Tukey HSD

- Determine Critical Value from Table
- Make a Decision
- Let's do an example...


## Tukey HSD: Example

- We will use the data from our One-Way ANOVA example:
- Decision: Foreign graduate students in different programs place different importance on financial factors, on average.
- Where are our differences?


## Tukey HSD: Example

## Importance Scores

| Arts \& Sciences | 4 | 5 | 4 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Education | 4 | 3 | 4 | 4 |  |
| Law | 3 | 3 | 2 | 3 |  |
| Business | 4 | 4 | 4 | 3 |  |


| SOURCE | SS | df | MS | F |
| :--- | ---: | ---: | ---: | ---: |
| Between | 3.866 | 3 | 1.289 | 3.94 |
| Within | 4.256 | 13 | 0.327 |  |
| Total | 8.122 | 16 |  |  |

## Tukey HSD: Example

- Standard Error: Unequal Sample Sizes

$$
N^{\prime}=\frac{N_{\text {Groups }}}{\sum\left(\frac{1}{N}\right)}
$$

$$
s_{M}=\sqrt{\frac{M S_{\text {Within }}}{N^{\prime}}}
$$

$$
N^{\prime}=\frac{4}{\frac{1}{5}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}=\frac{4}{.95}=4.211
$$

$$
S_{M}=\sqrt{\frac{.327}{4.211}}=.279
$$

## Tukey HSD: Example

- Arts \& Sciences ( $M=4.00$ ) vs. Education $(M=3.75)$

$$
H S D=\frac{\left(M_{1}-M_{2}\right)}{s_{M}}=\frac{(4-3.75)}{.279}=.896
$$

- Arts \& Sciences ( $M=4.00$ ) vs. Business ( $M=3.75$ )

$$
H S D=\frac{\left(M_{1}-M_{2}\right)}{s_{M}}=\frac{(4-3.75)}{.279}=.896
$$

- Arts \& Sciences ( $M=4.00$ ) vs. Law ( $M=3.75$ )

$$
H S D=\frac{\left(M_{1}-M_{2}\right)}{S_{M}}=\frac{(4-2.75)}{.279}=4.480
$$

## Tukey HSD: Example

- Critical Value

$$
\begin{aligned}
& p=.05,4 \text { groups } \\
& d f_{\text {within }}=13
\end{aligned}
$$

- A\&S-Ed. $q=.896$
- A\&S-Bus.

$$
q=.896
$$

- A\&S Law $q=4.480$


## TABLE 10-13. EXCERPT FROM THE q TABLE

Like the $F$ table, we use the $q$ table to determine critical values for a given $p$ level, based on the number of means being compared and the within groups degrees of freedom. Note that critical values are in regular type for 0.05 and boldface for 0.01 .

WITHIN-GROUPS DEGREES OF FREEDOM
$k=$ NUMBER OF TREATMENTS (LEVELS)
3
4
5

| $\cdot$ |  |  |  |
| :---: | :---: | :---: | :---: |
| $\cdot \cdot$ |  |  |  |
| 12 | 3.77 | 4.20 | 4.51 |
|  | 5.05 | 5.50 | 5.84 |
| 13 | 3.73 | 4.15 | 4.45 |
|  | 4.96 | 5.40 | 5.73 |
| 14 | 3.70 | 4.11 | 4.41 |
|  | 4.89 | $\mathbf{5 . 3 2}$ | $\mathbf{5 . 6 3}$ |
| $\cdot$ |  |  |  |

## Tukey HSD: Example

- Make a Decision
- Foreign graduate students in Arts \& Sciences place more importance on financial factors than students in Law but they do not differ from students in either Education or Business.

