Assignment 5

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March 19, 2007

NERS312/Winter 2007 Assignment 5

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This assignment is due March 19, 2006 at 12:30 pm.

P5.1, 20% Krane, Problem 9.3, p. 332.

P5.2, 20% Krane, Problem 9.9, p. 333.

P5.3, 20% Krane, Problem 9.10, p. 333.

P5.4, 20% Krane, Problem 9.11, p. 333.

P5.5, 20% Krane, Problem 9.14, p. 333.

P5.6, 20% Krane, Problem 9.20, p. 334.

All mass values taken from Krane unless otherwise noted.

P5.1, 20% Krane, Problem 9.3, p. 332.

Find Q for β^-, β^+, and ε for ^{196}Au.

$^{196}\text{Au} \rightarrow {}^{196}\text{Hg} + \beta^- + \overline{\nu}$

For β^--decay:

$$Q_{\beta^-} = [m(^AX) - Zm_e] - [m(^AX') - (Z+1)m_e] - m_e + \sum_{i=1}^{Z} B_i - \sum_{i=1}^{Z+1} B_i$$

We see the masses of the electrons cancels out. We neglect the difference in electron binding energies because this value is very small, thus

$$Q_{\beta^-} = [m(^AX) - m(^AX')]c^2$$

For β^+-decay:

$$Q_{\beta^+} = [m(^AX) - m(^AX') - 2m_e]c^2$$

And for electron capture:

$$Q_\varepsilon = [m(^AX) - m(^AX')]c^2 - B_n$$

Where B_n is the binding energy of the n-shell captured electron. We use the following approximation for all B_n calculations:

$$B_n = 13.6eV \cdot Z^2$$

Some important values:

$$\begin{aligned} m(^{196}Au) &= 195.966544 \text{ u} \\ &= 182543.228 \text{ MeV / c}^2 \\ m(^{196}Hg) &= 195.965807 \text{ u} \\ &= 182542.541 \text{ MeV / c}^2 \end{aligned}$$

From this we calculate Q_{β^-}, Q_{β^+}, Q_ε for ^{196}Au.

$$Q_{\beta^-} = [182543.228 - 182542.541] \text{ MeV}$$

$$\boxed{Q_{\beta^-} = 686.517 \text{ keV}}$$

$$Q_{\beta^+} = [195.966544 - 195.965807 - 2(5.485803 \times 10^{-4})] \text{ MeV}$$

$$\boxed{Q_{\beta^+} = 485.163 \text{ keV}}$$

$$Q_\varepsilon = [195.966544 - 195.964926] \text{ u} * 931.502 \text{ MeV/u} - 13.6 \times 10^{-6} \cdot 79^2 \text{ MeV}$$

$$\boxed{Q_\varepsilon = 1422.292 \text{ keV}}$$

P5.2, 20% Krane, Problem 9.9, p. 333.

Recall the inverse β-decay equations:

$$\begin{aligned} \overline{\nu} + \mathrm{p} &= \mathrm{n} + \mathrm{e}^+ \\ \nu + \mathrm{n} &= \mathrm{p} + \mathrm{e}^- \end{aligned}$$

Using those relations, and the β-decay equations, we can supply the missing components to the equations in the text.

(a) $\overline{\nu} + {}^{3}\mathrm{He} \rightarrow {}^{3}\mathrm{H} + \mathrm{e}^+$
(b) ${}^{6}\mathrm{He} \rightarrow {}^{6}\mathrm{Li} + \mathrm{e}^- + \overline{\nu}$
(c) $\mathrm{e}^- + {}^{8}\mathrm{B} \rightarrow {}^{8}\mathrm{Be} + \nu$
(d) $\nu + {}^{12}\mathrm{C} \rightarrow {}^{12}\mathrm{N} + \mathrm{e}^-$
(e) ${}^{40}\mathrm{K} \rightarrow \nu + \mathrm{e}^+ + {}^{40}\mathrm{Ar}$
(f) ${}^{40}\mathrm{K} \rightarrow \overline{\nu} + \mathrm{e}^- + {}^{40}\mathrm{Ca}$

P5.3, 20% Krane, Problem 9.10, p. 333.

Some important values:

$$\begin{aligned} m_p &= 938.280 \text{ MeV / c}^2 \\ m_e &= 0.511003 \text{ MeV / c}^2 \end{aligned}$$

(a) What is the kinetic energy give to the proton in the decay of the neutron when the electron has negligibly small kinetic energy?

We know that the neutron decay is

$$n \rightarrow p + e^- + \overline{\nu}$$

We also realize that

$$Q = T_p + T_{\overline{\nu}}$$

Where

$$Q = (m_n - m_p - m_e - m_{\overline{\nu}})c^2$$

In the text, Krane calculates this value for us as Q = 0.782.

We realize that the proton will not be moving at any significant fraction of the speed of light, and thus we can treat it nonrelativistically. Thus,

$$\begin{aligned} T_p &= \frac{1}{2}m_p v_p^2 \\ T_{\overline{\nu}} &= m_{\overline{\nu}}c^2(\gamma - 1) \end{aligned}$$

Since the momentum of the electron is negligible, the momentum of the proton must be opposite and equal to that of the neutrino. Thus,

$$\begin{aligned} p_p &= p_{\overline{\nu}} \\ m_p v_p &= \frac{E_{\overline{\nu}}}{c} \end{aligned}$$

Using these equations, we can solve for the kinetic energy of the proton.

We say

$$\begin{aligned} Q &= T_p + T_{\overline{\nu}} \\ &= \frac{1}{2}m_p v_p^2 + E_{\overline{\nu}} \end{aligned}$$

Where $E_{\overline{\nu}}$ is the kinetic energy of the $\overline{\nu}$. Substituting in momentum

$$Q = \frac{1}{2}m_p v_p^2 + p_{\overline{\nu}}c$$

Substitute in the momentum of p_p for $p_{\overline{\nu}}$ because we know their momenta to be equal. Also,

$$Q = \frac{1}{2}m_p v_p^2 + p_p c$$

This can be rewritten as a quadratic

$$0 = \frac{1}{2}m_p v_p^2 + m_p v_p c - Q$$

Using a computer, we can substitute the following converted values into this equation and solve for v_p.

$$\begin{aligned} m_p &= 1.672649059 \times 10^{-27} \text{ kg} \\ c &= 2.99792458 \times 10^{8} \text{ m/s} \\ Q &= 1.252911798 \times 10^{-13} \text{ J} \end{aligned}$$

I used maple and solved for a velocity of $v_p = 2.497549469 \times 10^5$ m/s. Plugging this into $T_p = \frac{1}{2}m_p v_p^2$, we get $5.216786135 \times 10^{-17}$ J. Converting this back to eV gives

$$\boxed{T_p = 325.604 \text{ eV}}$$

P5.3, 20% Krane, Problem 9.10, p. 333.

Some important values:

$$\begin{aligned} m_p &= 938.280 \text{ MeV / c}^2 \\ m_e &= 0.511003 \text{ MeV / c}^2 \end{aligned}$$

(b) What is the kinetic energy give to the proton in the decay of the neutron when the neutrino has negligibly small kinetic energy?

We know that the neutron decay is

$$n \rightarrow p + e^- + \overline{\nu}$$

We also realize that

$$Q = T_p + T_{e^-}$$

Where

$$Q = (m_n - m_p - m_e - m_{\overline{\nu}})c^2$$

In the text, Krane calculates this value for us as Q = 0.782.

We realize that the proton will not be moving at any significant fraction of the speed of light, and thus we can treat it nonrelativistically. Thus,

$$\begin{aligned} T_p &= \frac{1}{2} m_p v_p^2 \\ T_{e^-} &= m_e c^2 (\gamma - 1) \end{aligned}$$

Since the momentum of the neutrino is negligible, the momentum of the proton must be opposite and equal to that of the electron. Thus,

$$\begin{aligned} p_p &= p_{e^-} \\ m_p v_p &= m_e c \beta \gamma \end{aligned}$$

Using these equations, we can solve for the kinetic energy of the proton.
We say

$$Q = T_p + T_{e^-}$$
$$= \frac{1}{2}m_p v_p^2 + m_e c^2(\gamma - 1)$$

Substituting in momentum

$$Q = \frac{1}{2}\frac{p_p^2}{m_p} + m_e c^2(\gamma - 1)$$

Substitute in the momentum of p_{e^-} for p_p because we know their momenta to be equal. Thus,

$$Q = \frac{1}{2}\frac{p_{e^-}^2}{m_p} + m_e c^2(\gamma - 1)$$

This can be rewritten as a quadratic

$$0 = \frac{1}{2}\frac{p_{e^-}^2}{m_p} + m_e c^2(\gamma - 1) - Q$$

We know that

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$
$$\beta = \frac{v^2}{c^2}$$

Using a computer, we can substitute values for γ and solve the following equation for β.

$$0 = \frac{1}{2}\frac{\frac{(m_e c\beta)^2}{1-\beta^2}}{m_p} + m_e c^2(\frac{1}{\sqrt{1-\beta^2}} - 1) - Q$$

Where

$$m_p = 1.672649059 \times 10^{-27} \text{ kg}$$
$$m_e = 9.109537944 \times 10^{-31} \text{ kg}$$
$$c = 2.99792458 \times 10^{8} \text{ m/s}$$
$$Q = 1.252911798 \times 10^{-13} \text{ J}$$

Solving for β gives $\beta = 0.9185$c, meaning that our electron is travelling at 0.9185 times the speed of light. Plugging this β into the momentum equation allows us to calculate a value of $v_p = 3.792379474 \times 10^5$ m/s. Plugging that into T_p yields $1.202813820 \times 10^{-16}$ J. Converting this into eV gives

$$\boxed{\mathrm{T}_p = 750.732 \text{ eV}}$$

P5.4, 20% Krane, Problem 9.11, p. 333.

The decay that we are looking at is

$$^{7}Be + e^{-} \quad \rightarrow \quad ^{7}Li + \nu$$

We know that

$$Q \quad = \quad T_{Li} + T_{\nu}$$

The Q-value of this reaction is equal to the kinetic energy of the ^{7}Li and the KE (or just energy because we assume that the neutrino mass is negligible).

Also,

$$Q_{\varepsilon} \quad = \quad [m(^{A}X) - m(^{A}X')]c^{2} - B_{n}$$

We assume that

$$B_{n} \quad = \quad [13.6 \cdot Z^{2}] \text{ eV}$$

For Be:

$$\begin{aligned} B_{n} &= 13.6 \cdot 4^{2} \\ &= 217 \text{ eV} \end{aligned}$$

Other important values:

$$\begin{aligned} m(^{7}Be) &= 7.016928 \text{ u} \\ &= 6536.282466 \text{ MeV / c}^{2} \\ m(^{7}Li) &= 7.016003 \text{ u} \\ &= 6535.420827 \text{ MeV / c}^{2} \end{aligned}$$

Thus,

$$Q_{\varepsilon} \quad = \quad 0.861422 \text{ MeV}$$

We treat the proton as a classical particle because it will not be moving at any significant fraction of the speed of light. We ignore initial momentum of the electron in our conservation equations. We do this because we view the capture in the zero-momentum frame of the Be nucleus. The moment that the electron is captured, the nucleus is travelling the same speed as the electron in that frame. Thus, the net momentum there is zero. So we say

$$\begin{aligned} p_{Li} &= p_\nu \\ m_{Li}v_{Li} &= \frac{E_\nu}{c} \end{aligned}$$

and

$$\begin{aligned} Q &= T_p + T_\nu \\ &= \frac{1}{2}m_{Li}v_{Li}^2 + E_\nu \end{aligned}$$

Substituting in momentum

$$Q = \frac{1}{2}\frac{p_{Li}^2}{m_p} + p_\nu c$$

Substituting p_{Li} for p_ν gives

$$Q = \frac{1}{2}\frac{p_\nu^2}{m_p} + p_\nu c$$

We can substitute in the value for momentum and then write this in the quadratic form

$$0 = \frac{1}{2}m_{Li}v^2 + m_{Li}vc + Q$$

and then solve. I used maple using the following values.

$$\begin{aligned} m_{Li} &= 1.165207206 \times 10^{-26} \text{ kg} \\ c &= 2.99792458 \times 10^{8} \text{ m/s} \\ Q &= 1.380160853 \times 10^{-13} \text{ J} \end{aligned}$$

Thus,

$$v = 39507.28860 \text{ m/s}$$

Therefore the kinetic energy of the Li nucleus is

$$
\begin{aligned}
T_{Li} &= \frac{1}{2} m_{Li} v^2 \\
&= 9.093427655 \times 10^{-18} \text{ J} \\
&= 56.7563 \text{ eV}
\end{aligned}
$$

And then

$$
\begin{aligned}
E_\nu &= m_{Li} v c \\
&= 1.380069919 \times 10^{-13} \\
&= 0.861365 \text{ MeV}
\end{aligned}
$$

Thus,

$$\boxed{\mathrm{T}_{Li} = 56.7563 \text{ eV}}$$

$$\boxed{\mathrm{E}_\nu = 0.861365 \text{ MeV}}$$

1 P5.5, 20% Krane, Problem 9.14, p. 333.

Classify the following decays according to degree of forbiddenness:

(a)$^{89}\mathrm{Sr}(\frac{5}{2}^{+}) \rightarrow {}^{89}\mathrm{Y}(\frac{1}{2}^{-})$

$$\begin{aligned}\Delta I &= \frac{5}{2} - \frac{1}{2} \\ &= 2\end{aligned}$$

$$\Delta\pi = yes$$

This transition is a first-forbidden transition.

(b)$^{36}\mathrm{Cl}(2^{+}) \rightarrow {}^{36}\mathrm{Ar}(0^{+})$

$$\begin{aligned}\Delta I &= 2 - 0 \\ &= 2\end{aligned}$$

$$\Delta\pi = no$$

This transition is a second-forbidden transition.

(c)$^{26}\mathrm{Al}(5^{+}) \rightarrow {}^{26}\mathrm{Mg}^{*}(2^{+})$

$$\begin{aligned}\Delta I &= 5 - 2 \\ &= 3\end{aligned}$$

$$\Delta\pi = no$$

This transition is a second-forbidden transition.

(d)$^{26}\mathrm{Si}(0^{+}) \rightarrow {}^{26}\mathrm{Al}^{*}(0^{+}) \rightarrow {}^{26}\mathrm{Mg}(0^{+})$

$$\Delta I = 0 - 0$$

$$= \quad 2$$

$$\Delta\pi \quad = \quad no$$

These transitions are both $\boxed{\text{allowed transitions}}$.

(c)^{97}Zr($\frac{1}{2}^{+}$) $\rightarrow$ ^{97}Nb*($\frac{1}{2}^{-}$)

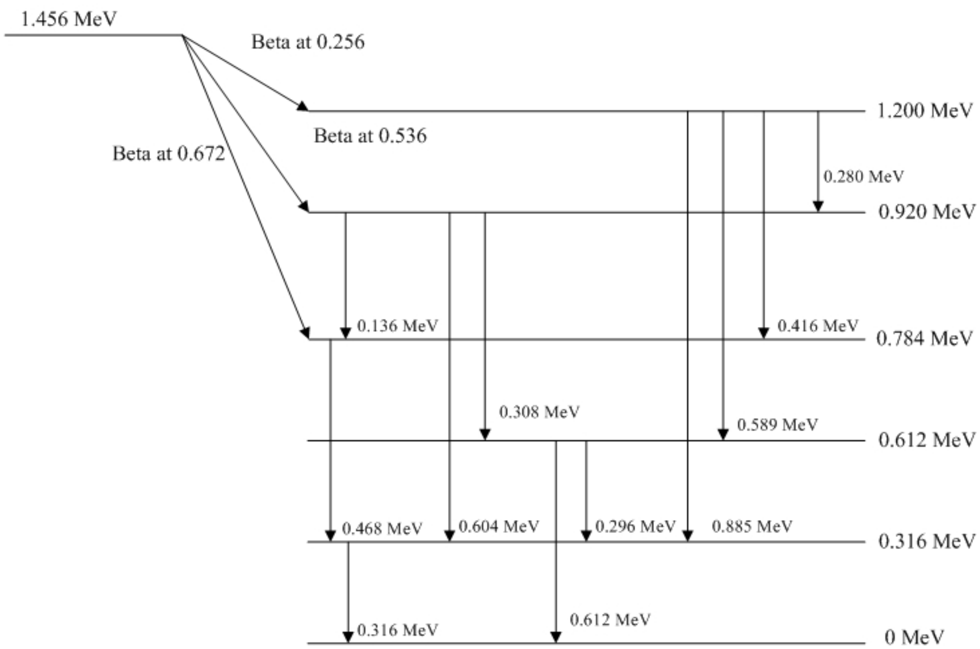
$$\begin{aligned} \Delta I &= \frac{1}{2} - \frac{1}{2} \\ &= 0 \end{aligned}$$

$$\Delta\pi \quad = \quad yes$$

This transition is a $\boxed{\text{first-forbidden transition}}$.

2 P5.6, 20% Krane, Problem 9.20, p. 334.

From the data given in the problem, here is the decay scheme that I constructed.



As you can see from the figure, the mass difference between the two ground states is

$$\boxed{1.456 \text{ MeV}}$$

Or, if you prefer the mass in amu,

$$\boxed{1.563066961 \times 10^{-3} \text{ u}}$$