Lecture Notes on Groundwater Hydrology

Part 1

1. Basic concepts and definitions

1.1   Aquifer, Aquitard, Aquiclude and Aquifuge

Aquifer is a word produced from two Latin words: Aqua, which means water and ferre, which means to bear. Therefore, the term **Aquifer can literally be understood as Water-bearing formation.**

Aquifer can formally be defined as a saturated permeable geological unit that is permeable enough to yield economic quantities of water to wells. In other words, it is defined as a saturated geological unit that can transmit significant quantities of water under hydraulic head. The most important underground water-bearing materials are unconsolidated sand and gravels. But, permeable sedimentary rocks such as sandstone and limestone, and heavily fractured or weathered volcanic and crystalline rocks can also be taken as aquifer (water-bearing) materials.

Aquitard is a geological unit that is permeable enough to transmit water in significant quantities for large area and long period. But, its permeability is not sufficient to justify the construction of production wells to be placed in it. In other words, Aquitard is a geologic formation that can transmit water at a relatively lower rate compared to aquifer. Example includes formations that are predominantly clays, loams and shales.

Aquiclude is an impermeable geological unit which does not transmit water at all. Although this formation is capable of absorbing water slowly. It means that this geological formation can store water, but cannot transmit it easily. In other words, Aquiclude is a saturated geological unit that is incapable of transmitting significant quantities of water under ordinary hydraulic head. Example: metamorphic rocks.

Aquifuge is a geological formation that can neither absorbs nor transmits water.

1.2   Types of Aquifer

There are three types of aquifer: confined, unconfined and leaky and their definition is given as follows.

1.2.1   Confined aquifer

A confined aquifer is an aquifer which is bounded by an aquiclude both at the lower and upper part. In other words, this aquifer is confined between two impervious layers. The confined aquifer is known as pressure aquifer. In a confined aquifer, the pressure of water is higher than atmospheric pressure. The water in a well which is constructed in such an aquifer rises usually above the aquifer and even above the ground surface due to high pressure. By the way, the groundwater pressure can be either equal or greater than atmospheric pressure. Confined aquifer cannot be recharged directly by infiltration.
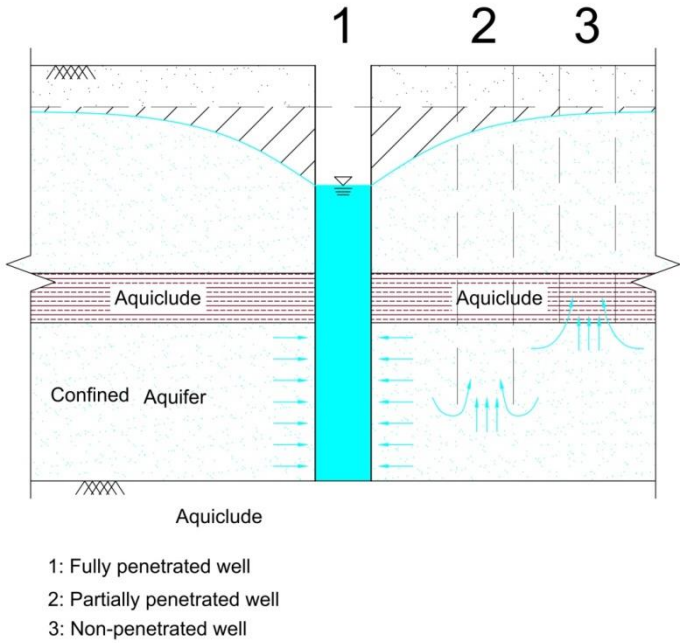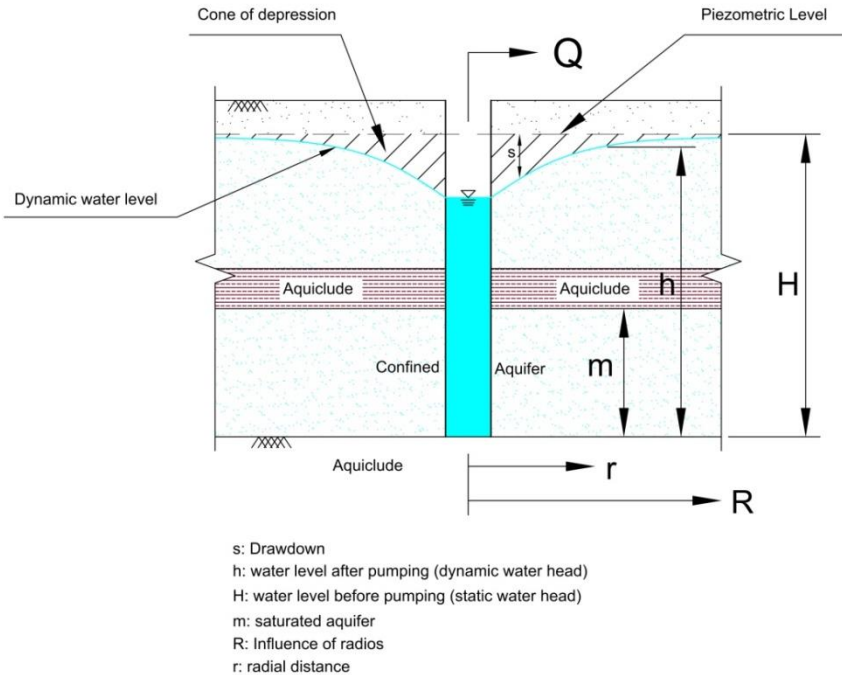

s: Drawdown
h: water level after pumping (dynamic water head)
H: water level before pumping (static water head)
m: saturated aquifer
R: Influence of radios
r: radial distance

1: Fully penetrated well
2: Partially penetrated well
3: Non-penetrated well

Figure 1 Confined Aquifer.

1.2.2 Unconfined aquifer

An unconfined aquifer is an aquifer which is bounded by aquiclude at its lower side and by water table at its upper side. In other words, the flow of water in the upper part of the aquifer is not restricted by any confining layer and that makes the upperpart a bounded free surface. Consequently, the free surface of unconfined aquifer is under atmospheric pressure. Its upper boundary is watertable which is free to rise and fall. Unconfined aquifer is directly recharged by infiltration.
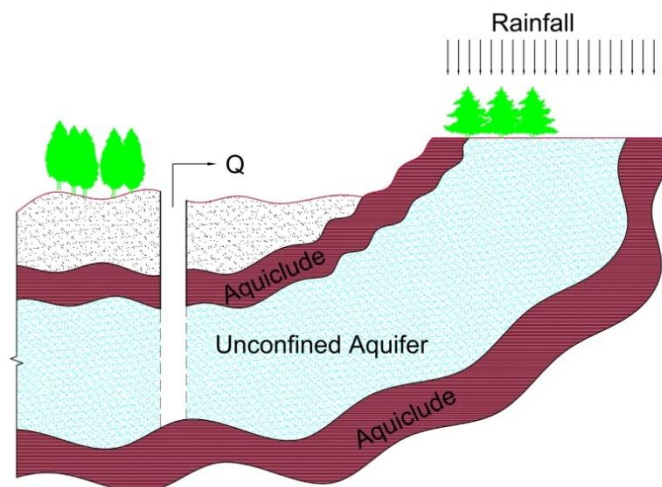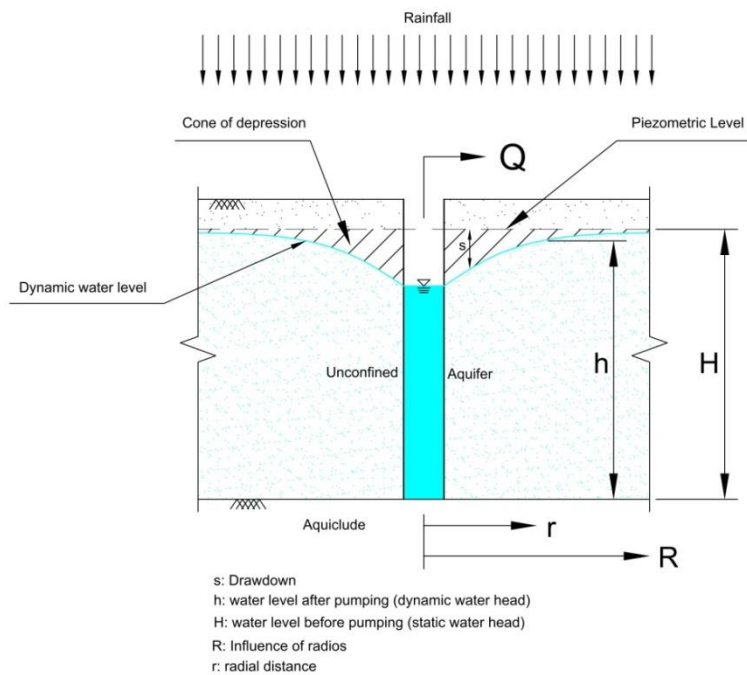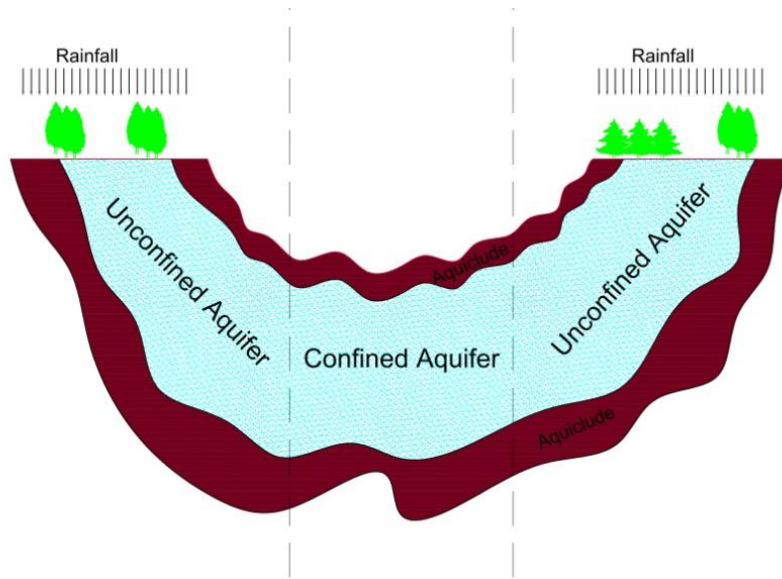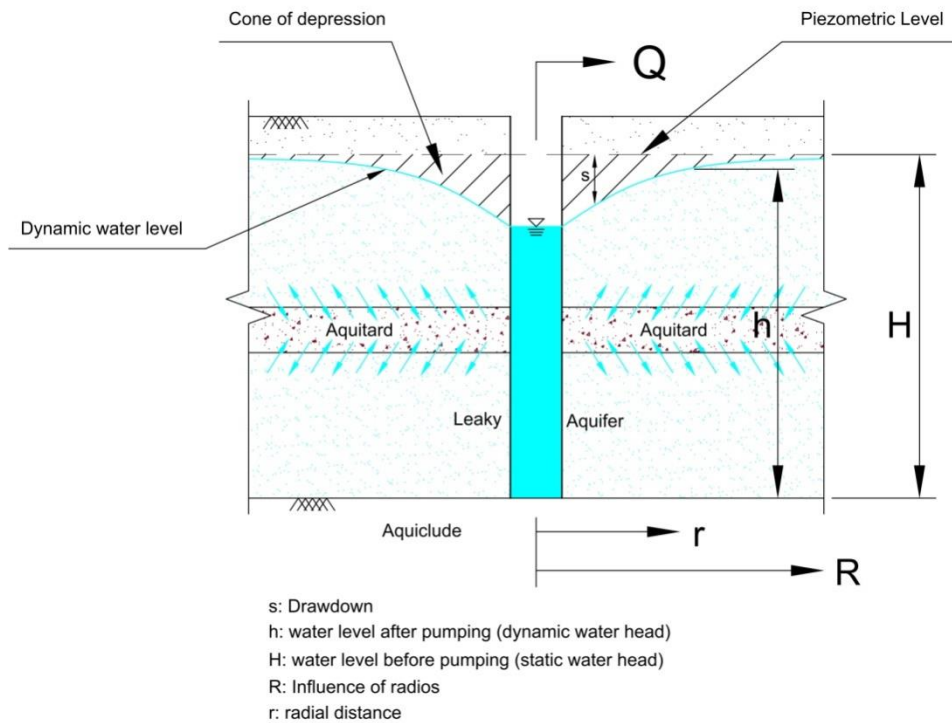

Figure 2 Unconfined Aquifer.

Figure 3 Confined and unconfined Aquifer.

1.2.3 Leaky aquifer

A leaky aquifer is also known as semi-confined aquifer as either both the upper and the lower boundaries are aquitards or one of them is aquiclude and the remaining is aquitard. The water is free to move through aquitards either upward or downward.
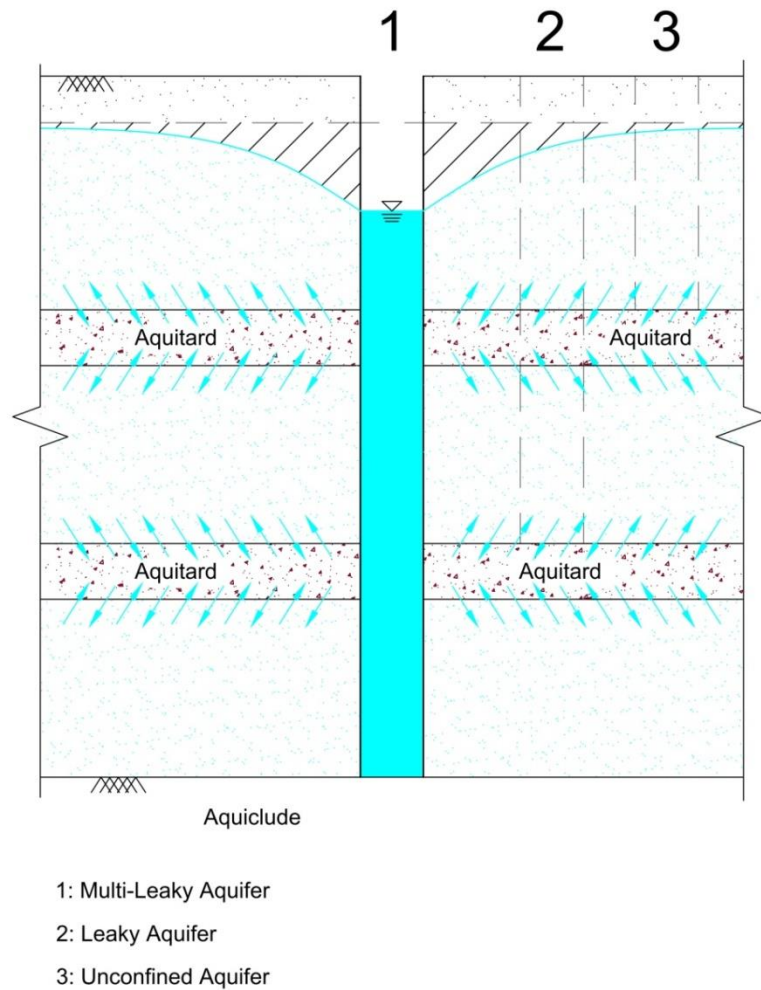

s: Drawdown
h: water level after pumping (dynamic water head)
H: water level before pumping (static water head)
R: Influence of radios
r: radial distance

Figure 4. Leaky Aquifer.

1.3   Anisotropy and heterogeneity

Groundwater hydraulic equations are based on some assumptions. If the hydraulic conductivity, K, is independent of direction at any a point in a geologic formation, the formation is called isotropic at that point. If the hydraulic conductivity, K, is dependent on direction at a point in a geologic formation, the formation is anisotropic. Let, in any xyz coordinate system, K_x, K_y and K_z represent hydraulic conductivity values in the x, y and z directions, respectively. If $K_x = K_y = K_z$ at any point, the formation is isotropic, whereas, for anisotropic condition to occur, $K_x \neq K_y \neq K_z$. In addition, if the hydraulic conductivity is independent of spatial variation (position) within a geologic formation, the formation is termed as homogeneous. And if the hydraulic conductivity is dependent on spatial variation within a geologic formation, the formation is heterogeneous formation. In homogeneous geologic formation, the hydraulic conductivity, K(x,y,z)=C=constant, whereas in heterogeneous geologic formation,

the hydraulic conductivity, K(x,y,z)≠C. We can say that aquifers and aquitards are homogenies and isotropic if we assume that the hydraulic conductivity is same throughout a geologic formation and in all directions. If hydraulic conductivity in the horizontal direction K_h is greater than the hydraulic conductivity in the vertical direction K_v, this phenomenon is called anisotropy. In fact, lithology of geological formation varies significantly horizontally and vertically. For homogeneous, anisotropic formation, $K_x(x, y) = K_1$ at every point and $K_y(x, y) = K_2$ at every point, but $K_1 \neq K_2$.
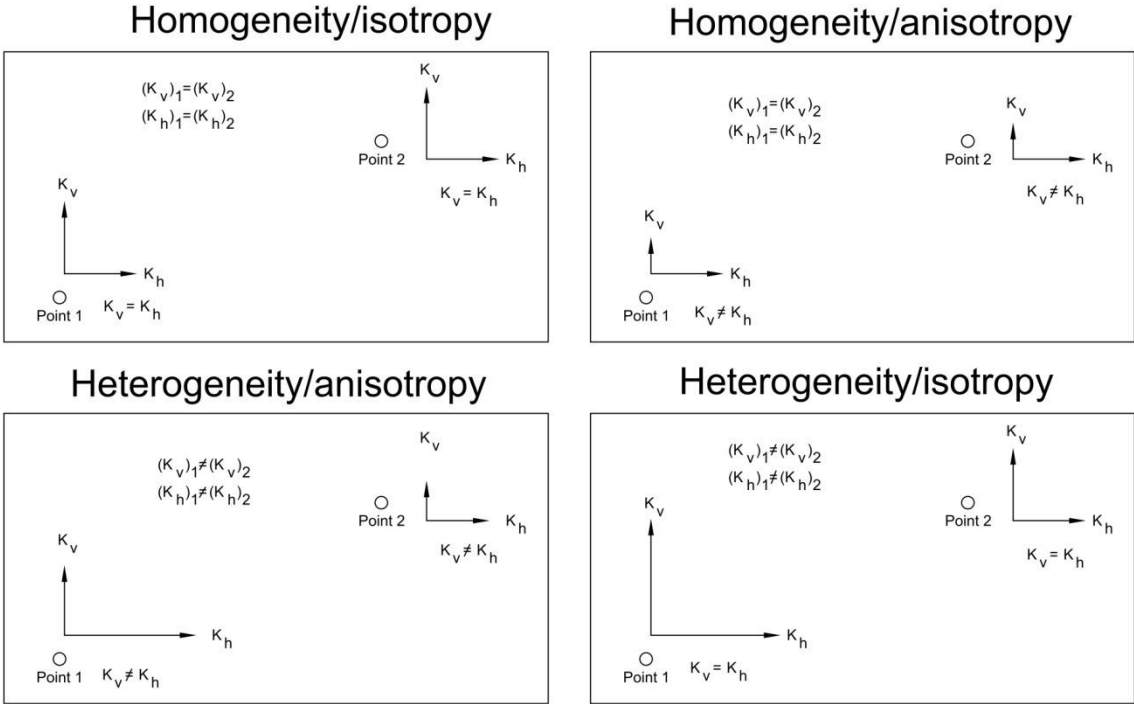
Homogeneity/isotropy

$(K_v)_1 = (K_v)_2$
$(K_h)_1 = (K_h)_2$

Point 2 $K_v = K_h$

Point 1 $K_v = K_h$

Homogeneity/anisotropy

$(K_v)_1 = (K_v)_2$
$(K_h)_1 = (K_h)_2$

Point 2 $K_v \neq K_h$

Point 1 $K_v \neq K_h$

Heterogeneity/anisotropy

$(K_v)_1 \neq (K_v)_2$
$(K_h)_1 \neq (K_h)_2$

Point 2 $K_v \neq K_h$

Point 1 $K_v \neq K_h$

Heterogeneity/isotropy

$(K_v)_1 \neq (K_v)_2$
$(K_h)_1 \neq (K_h)_2$

Point 2 $K_v = K_h$

Point 1 $K_v = K_h$

Figure 5. Anisotrpicity/isotropy and heterogeneity/homogeneity.
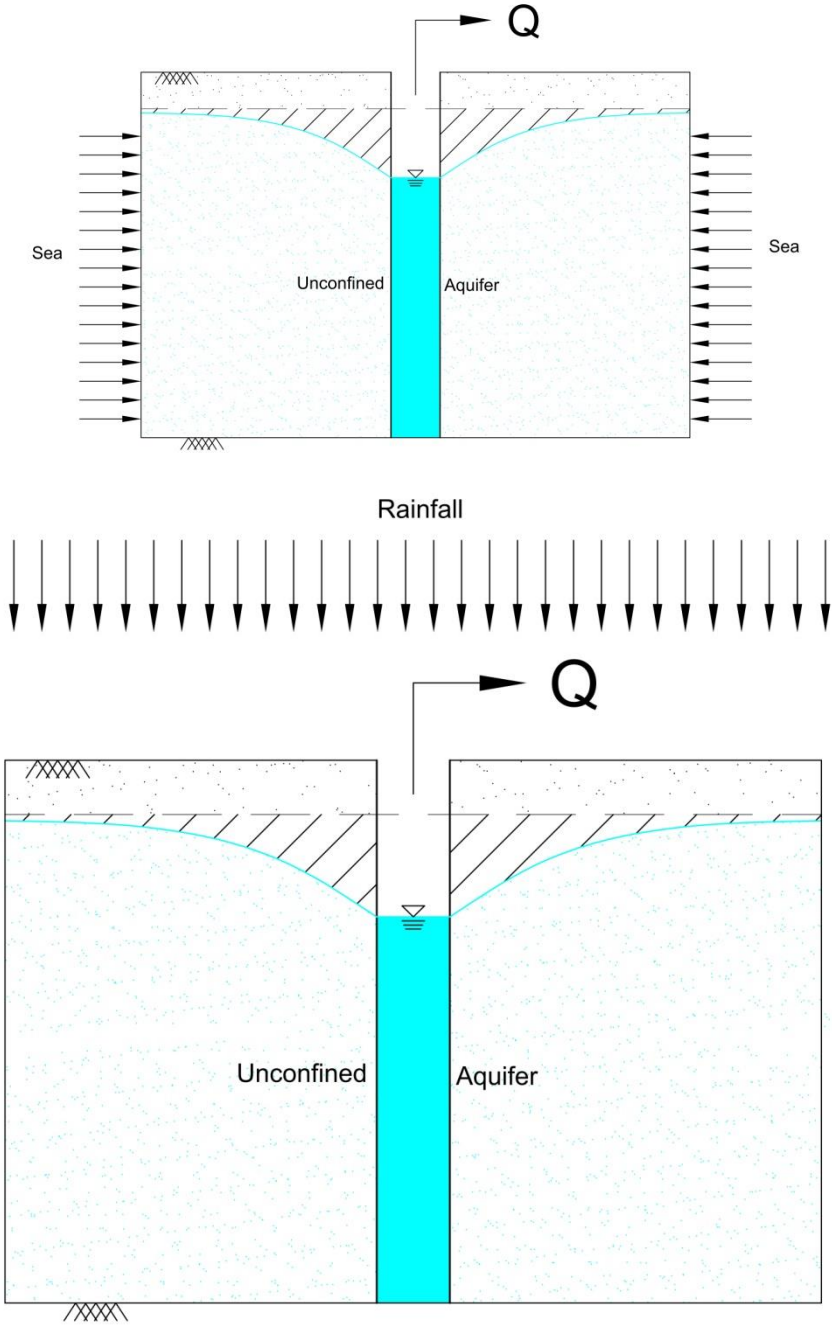
1.4   Steady state and unsteady state flow

There are two types of well-hydraulic equations and these are steady state flow and unsteady state flow equations. The definitions of these flow conditions are given as follows.

Steady state is independent of time. In this flow state, the velocity may differ from point to point, but it will not change with time at any given point in the flow field. As a result, the steady state condition, water level does not change with time. For example, the water level in the pumping well and surrounding piezometers does not vary with time. The steady state flow takes place if pumping aquifer is recharged by outside water resource, rainfall (unconfined

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aquifer), leakage through the aquitard (Leaky aquifer) from upward or downward and directly from open water sources. As a result, we can say that steady state flow is attained if the changes in the water level in wells and piezometers are very small with time that they can be ignored.

Unsteady state occurs from the time of the star of pumping until steady state flow is reached. The flow can be assumed as unsteady state as long as water level changes in the well and piezometers are measurable and cannot ignored.
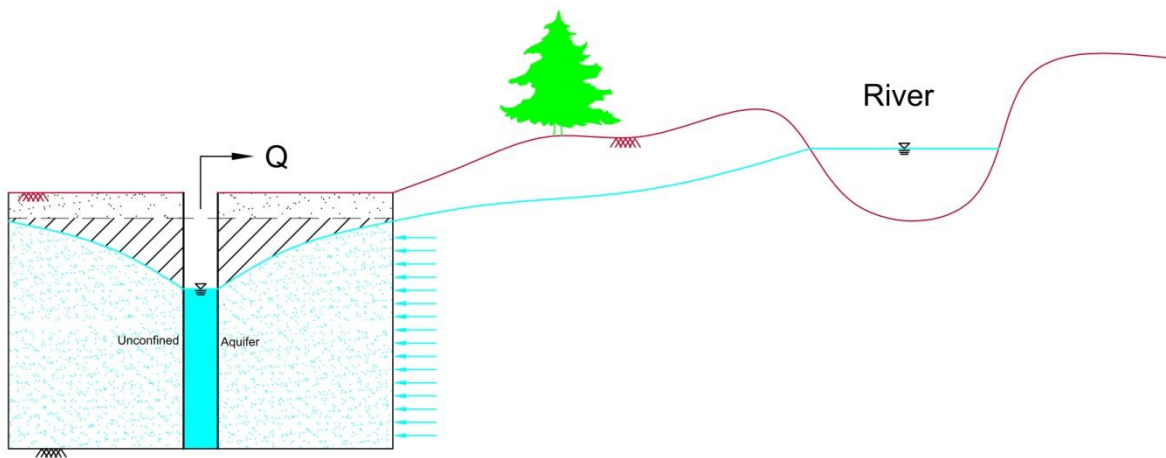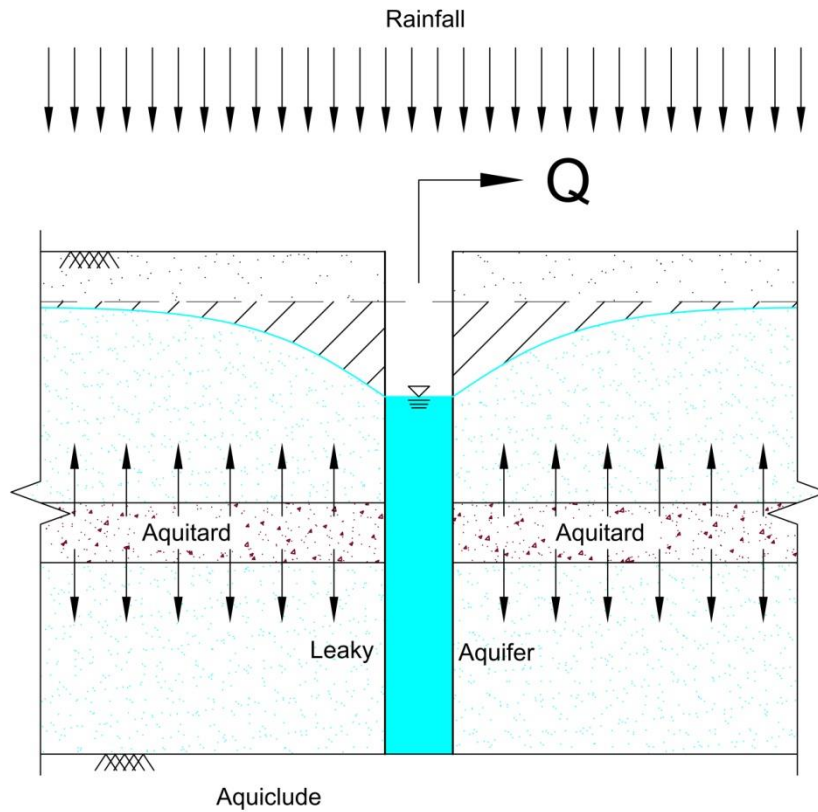
Figure 6. Steady state and unsteady state flow examples.

1.5   Darcy's Law

Let's have two wells as depicted in the figure below. Let h_1 and h_2 refer hydraulic heads in well 1 and well 2, respectively. Let the difference between the two hydraulic heads is Δh. Δh is the cause of groundwater flow. Let the distance between the two wells be Δl.
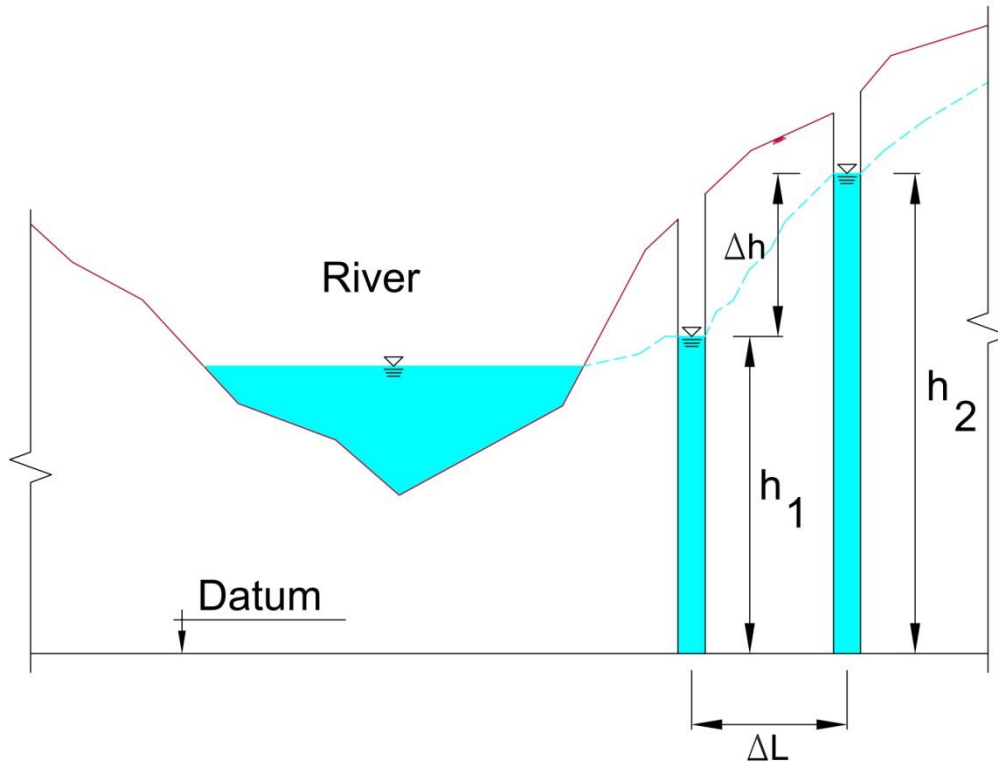
Figure 7. Hydraulic gradient.

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Darcy (1856) expressed that the specific discharge through porous medium is directly proportional to hydraulic head or head loss and inversely proportional to length of flow path. This means that $v \propto \Delta h$ and

$v \propto \frac{1}{\Delta l}$.

 In other words,

$$v \propto \frac{\Delta h}{\Delta l}$$

$\frac{\Delta h}{\Delta l}$ is defined as hydraulic gradient, i.

This means, hydraulic gradient, $i = \frac{\Delta h}{\Delta l}$

Therefore,

$$v \propto i$$

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This proportionality is converted to equality by introducing a constant coefficient, K that has logical physical meaning. This coefficient depends on the characteristics of the porous medium and the groundwater. It refers to a resistance coefficient and is called hydraulic conductivity.

This can be written mathematically as:

$$v = k \frac{\Delta h}{\Delta r} \qquad (1)$$

or in differential form

$$v = k \frac{dh}{dl} \qquad (2)$$

This is what is known as Darcy's Law.

Here $v = \frac{Q}{A}$ is specific discharge (also known as Darcy velocity or Darcy flux (Length/time)), Q= volume rate of flow (Length^3/time), A=cross-sectional area normal to flow direction, Δh= head loss which is the difference between hydraulic heads measured at points 1 and 2 (Length), Δl= the distance between the two wells as indicated earlier. As defined above, $\frac{dh}{dl} = i$, is hydraulic gradient (dimensionless) and k is proportionality constant which is termed as hydraulic conductivity (Length/Time).

In geotechnical engineering, permeability is used in place of hydraulic conductivity. Hydraulic conductivity is a property of both the fluid and the porous medium, whereas permeability depends only on the property of the porous medium.

We have to know that Darcy Law has its range of validity. Darcy's Law is valid for laminar flow, but not for turbulent flow. Turbulent flow can happen in cavernous limestone and fractured basalt. In case of doubt, we can use Reynolds number in order to determine whether a flow is laminar or turbulent flow. The Reynolds number is described as the ratio of inertial forces to viscous forces, and is given as:

$$Re = \frac{\rho.v.d}{\mu}$$

Here, ρ is the specific mass of fluid, v is specific discharge, μ is dynamic viscosity of fluid and d is a representative length of porous medium which is a mean grain diameter (Length) or a mean pore diameter. If d increases, the value of Reynolds number increases, affecting the flow regime.

Darcy concluded that the law's range of validity is between 1<Re<10. Re=10 is upper limit of the validity of Darcy's flow. Most ground water flow occur when Re number is less than 1. Therefore, Darcy's Law applies in ground water flow conditions. Exceptional situations are rock wide opening, vicinity of pumped well and where steep hydraulic head exists. Darcy's Law is invalid at low hydraulic gradient. Consequently, Darcy's low is valid if the Reynolds number stays in range from 1 to 10. All flow through granular media is laminar.
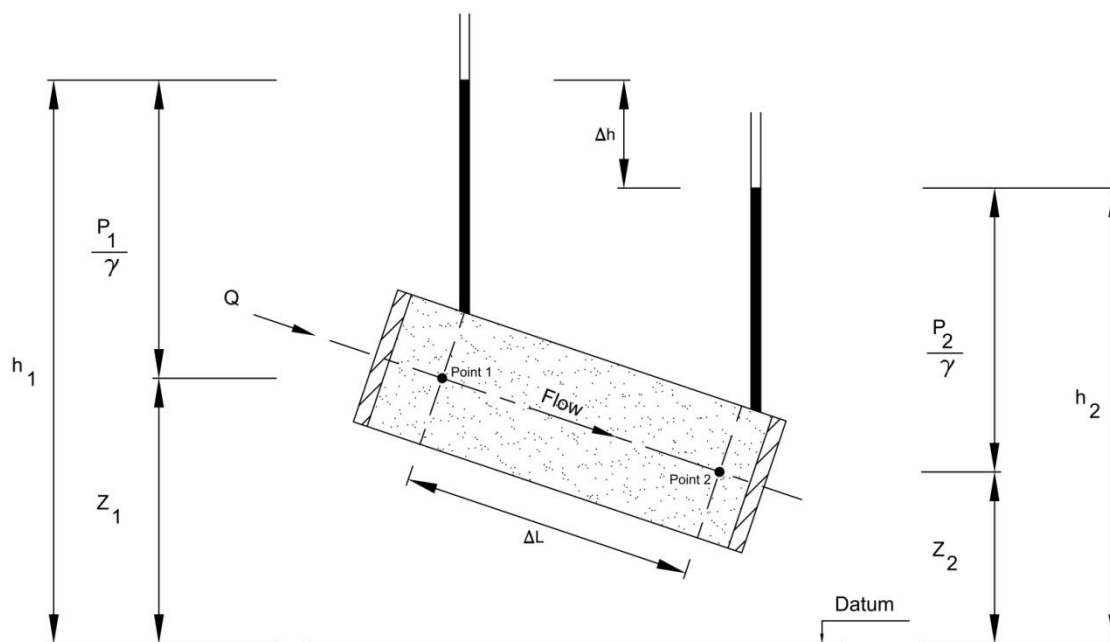
Darcy's Experiment

Figure 8. Darcy Experimental set up.

Darcy filled a cylinder with sand and setup the experiment as shown in the figure given above. He also installed manometers and piezometers at two points 1 and 2. z is elevation of the points above the datum and $\frac{P}{\gamma}$ is pressure head.

Darcy defined velocity of groundwater flow or specific discharge, q, to be directly proportional to the piezometric head difference between the two points ($\Delta h - head\ loss$) and inversely proportional to the length of flow path, Δl. This means:

$q \alpha \frac{\Delta h}{\Delta l}$. This proportionality is converted to equality by introducing a resistance coefficient, k, which is a property of both the fluid and the porous medium as discussed earlier. Therefore,

$$q = k\frac{\Delta h}{\Delta l}$$

Since $\frac{\Delta h}{\Delta l} = i$ (hydraulic gradient) then, $q = ki$

The differential form of the equation is given as $q = -k\frac{dh}{dl}$

Negative sign is put in front because the difference between piezometric head 2 and piezometric head 1 is negative. We should multiply the value by negative so that the specific discharge will be positive with a physical meaning.

In addition, we know that

$$s + h = H$$

,where s is drawdown, h is hydraulic head and H is static water level.

In differential form, the equation can be rewritten as:

$$ds + dh = dH$$

Dividing both sides of the equation by the differential distance, dr, yields:

$$\frac{ds}{dr} + \frac{dh}{dr} = \frac{dH}{dr}$$

But, $\frac{dH}{dr}$ is zero as H is constant.

Therefore,

$$\frac{ds}{dr} + \frac{dh}{dr} = 0$$

This implies that

$$\frac{ds}{dr} = -\frac{dh}{dr}$$

Bernoulli's Equation can be used to drive Darcy's Equation. Bernoulli's Eq. states that, at any point in a flow field, the sum of elevation head (Z), pressure head ($\frac{P}{\gamma}$) and velocity head ($\frac{v^2}{2g}$) remain constant. This means:

$$Z_1 + \frac{P_1}{\gamma} + \frac{v_1^2}{2g} = Z_2 + \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + \Delta h$$

,where Δh is head loss as a result of friction.

Since the flow in porous medium, groundwater flow being one of a kind, is very slow, the velocity head in the above equation can be ignored. Therefore,

$$Z_1 + \frac{P_1}{\gamma} = Z_2 + \frac{P_2}{\gamma} + \Delta h$$

Rearranging this equation, we can find the following equation.

$$\left(Z_1 + \frac{P_1}{\gamma}\right) - \left(Z_2 + \frac{P_2}{\gamma}\right) = \Delta h$$

We know that $\left(Z_1 + \frac{P_1}{\gamma}\right)$ and $\left(Z_2 + \frac{P_2}{\gamma}\right)$ are piezometric heads, h_2 and h_2, respectively. Therefore, Eq. x can be written as

$$(h_1 - h_2) = \Delta h$$

Dividing both sides of Eq. x by Δl, where Δl is the distance between the two points of interest, we will find:

$$\frac{\left(Z_1 + \frac{P_1}{\gamma}\right)}{\Delta l} - \frac{\left(Z_2 + \frac{P_2}{\gamma}\right)}{\Delta l} = \frac{\Delta h}{\Delta l}$$

We know, from earlier definition that $\frac{\Delta h}{\Delta l}$ is hydraulic gradient.

Based on the definition given by Darcy, as written above, this hydraulic gradient is proportional to velocity of groundwater flow or specific discharge, q.

This means that $\frac{\Delta h}{\Delta l} \alpha \ q$.

Following the same logic given earlier, this proportionality is converted to equality by introducing a resistance coefficient, k, which is a property of both the fluid and the porous medium. Therefore,

$$q = k \frac{\Delta h}{\Delta l}$$

1.6　Hydraulic head and Fluid potential

It is known that heat flows through solids from higher to lower values of temperature. Electrical current flows through electrical circuits from higher voltage to lower voltage value. Tem-

perature and voltage are potential quantities, and the rates of flows of heat and electricity are proportional to temperature gradient and voltage gradient, respectively. In the same manner, the fluid potential, which is responsible for flow through porous media is mechanical energy per unit mass of fluid.

If we consider a particle in a flow, work is required to lift the particle from the datum to a certain elevation as depicted in the figure below.
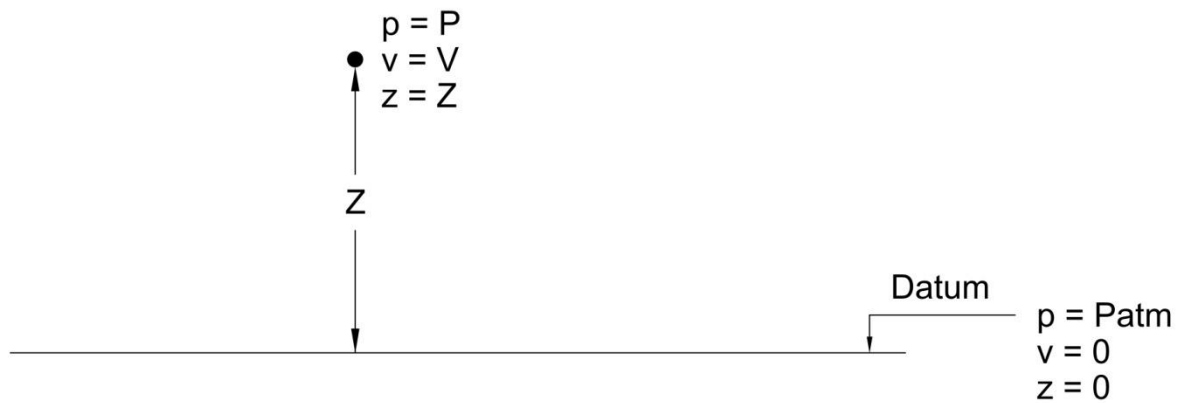

Figure 9. Particle position.

There are three required components of the work to be calculated. First, there is work required to lift the mass from elevation, z=0 from elevation=z.

$$w_1 = mgz$$

Second, there is work required to accelerate the fluid from velocity, v=0 to velocity, v.

$$w_2 = \frac{1}{2}mv^2$$

Third, there is work required to rise the pressure from $p=p_o$=atmospheric to pressure, p.

$$w_3 = m \int_{p_0}^{p} \frac{dp}{\rho}$$

The total mechanical energy per unit mass is the sum of these three components of work. For a unit mass of fluid, m=1. The total mechanical energy can, therefore, be written as:

$$w = gz + \frac{1}{2}v^2 + \int_{p_o}^{p} \frac{dp}{\rho}$$

After mathematical processing,

$$w = gz + \frac{1}{2}v^2 + \frac{p-p_o}{\rho}$$

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We know that the velocity of groundwater flow is extremely low. Therefore, the second term of the right hand side of the above equation can be ignored. By the way, fluids are incompressible. This means that specific mass is constant and does not change with pressure.

The equation can, therefore, be simplified as:

$w = gz + \frac{p - p_o}{\rho}$

As can be seen in the above equation, the first term in the right hand side of the equation involves elevation and second term involves pressure term.

How can we relate these terms to hydraulic head, h. Let's think about Darcy manometer.
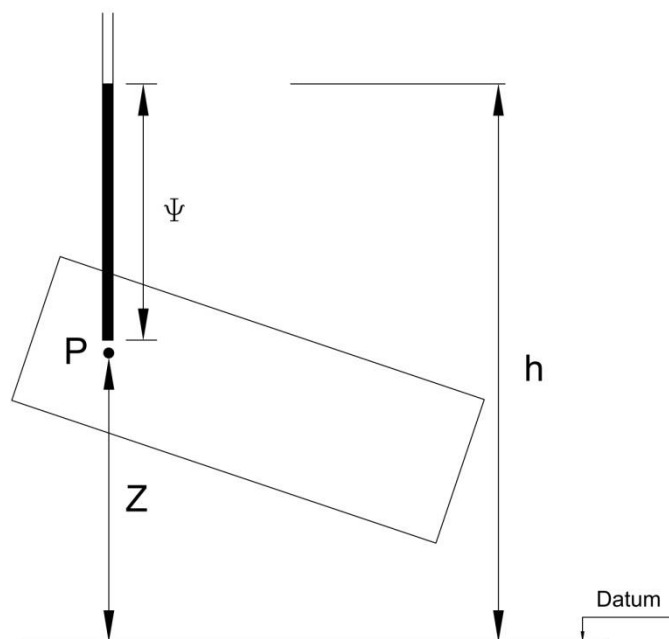

Figure 10. Piezometer.

The pressure at the manometer located at point1 is expressed as:

$p = p_o + \rho g\, \psi$

Here, p_o is atmospheric pressure and ψ is height of fluid column, which is equal to (h-z).

After algebraic manipulation,

$p - p_0 = \rho g(h - z)$

The substitution of this equation into the pervious one yields:

$$w = gz + \frac{\rho g(h-z)}{\rho}$$

After simplification, the following equation can be obtained.

$$w = gh$$

From here, it can beconcluded that the fluid potential, w, at any point 1 in a porous medium is the product of hydraulic head and acceleration of gravity. This means that fluid potential is proportional to hydraulic head. In other words, there is strong relationship between fluid potential and hydraulic head. According to Hubbert's definition, the groundwater flow occurs from higher hydraulic head toward lower hydraulic head.The last equation indicates that the fluid potential is energy per unit mass, and h is energy per unit weight.

We know that $w = gz + \frac{p}{\rho}$

In practical works, however, relative pressure is used. In this case atmospheric pressure is equal to zero. Therefore, fluid potential can be written as:

$$w = gz + \frac{p}{\rho} = gh$$

Dividing both side of equation by g leads to:

$$z + \frac{p}{\gamma} = h$$

Or

$$z + \frac{\rho g \psi}{\rho g} = h$$

After simplification, the equation can bewritten as:

$$h = z + \psi$$

This shows that the hydraulic head involves two components: elevation head, z and pressure head, ψ.

Actually, we are familiar with all terms and can be obtained from elementary fluid mechanics as Bernoulli equation.

We can write concepts of fluid potential and hydraulic head in terms of Bernoulli's equation. The total head is given as:

$$h_t = h_z + h_p + h_v$$

Here h_z is elevation head, h_p is pressure head and h_v is velocity head. Therefore,

$$h_t = z + \frac{p}{\gamma} + \frac{v^2}{2g}$$

In case of groundwater flow, velocities are extremely low. Because of this, velocity head is equal to zero, and the above equation becomes:

$$h = z + \frac{\rho g \psi}{\rho g}$$

After simplification, the equation becomes:

$$h = z + \psi$$

This relationship shows that, if we consider a pipe positioned horizontally, flow occurs as a result of pressure head, and if we consider vertically positioned pipe, elevation head is responsible for flow to occur.

Classification of sub-surface

Let's consider the following figure.

The sub-surface can generally be divided into two zones: Unsaturated zone and saturated zone. In the unsaturated zone, pore spaces are partially filled by water. However, saturated zone is a zone where all the void space is filled by water. The boundary between the saturated and unsaturated zones is called as the water table, which is the surface of the saturated zone, the fluid pressure at this surface is atmospheric pressure. Unsaturated zone can further be divided into Soil water zone, Vadose zone and Capillary zone.

In the capillary zone, there is a negative pressure (suction), and considering the following figure, the pressure distribution will have the following form.
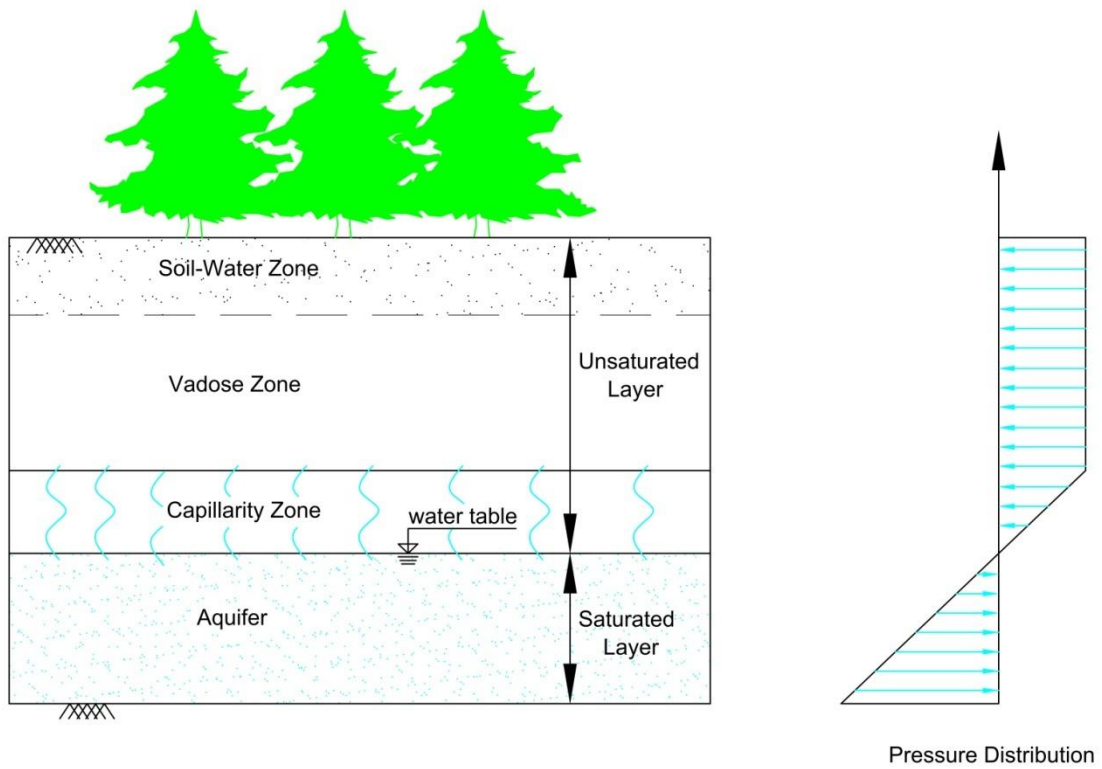
Figure 11. Classes of Sub-surface and pressure distribution.

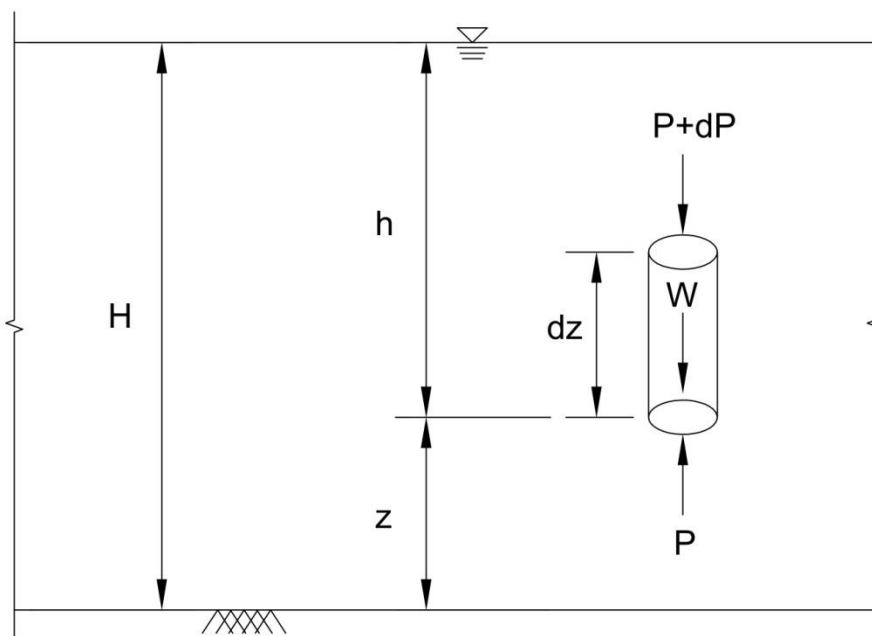Note on concepts of hydrostatic pressure

Figure 12. Hydrostatic pressure.

Assumption

Let's assume here that the volumetric force is as a result of gravitational acceleration, g. There is pressure acting on the surfaces of the cylinder along the z direction. It should not be forgotten that there is no frictional force as the condition is hydrostatic.

Therefore,

$\sum \vec{F}_z = m.a_z = m.g = 0$ as the fluid is not moving and this implies that:

$P.dA - (P+dp)dA - \gamma.dzdA = 0$

After the required mathematical manipulations, we will arrive at:

$dP = -\gamma\, dz$

Integrating both sides of the above equation yields:

$\int dP = -\gamma \int dz$

This results in:

$P = -\gamma.z + C$

We have to determine the value of C by taking boundary conditions in to consideration. Looking at the figure, when z=H, the pressure, P, acts on the surface of water. We call this pressure **atmospheric pressure**, P_a.

Therefore, for z=H,

$C = P_a + \gamma\, H$

Substituting the value of C in the above given basic equation:

$P = -\gamma\, z + P_a + \gamma\, H$

Therefore,

$P = P_a + \gamma(H - z) = P_a + \gamma\, h$

This is what is called the equation of **hydrostatic pressure**.
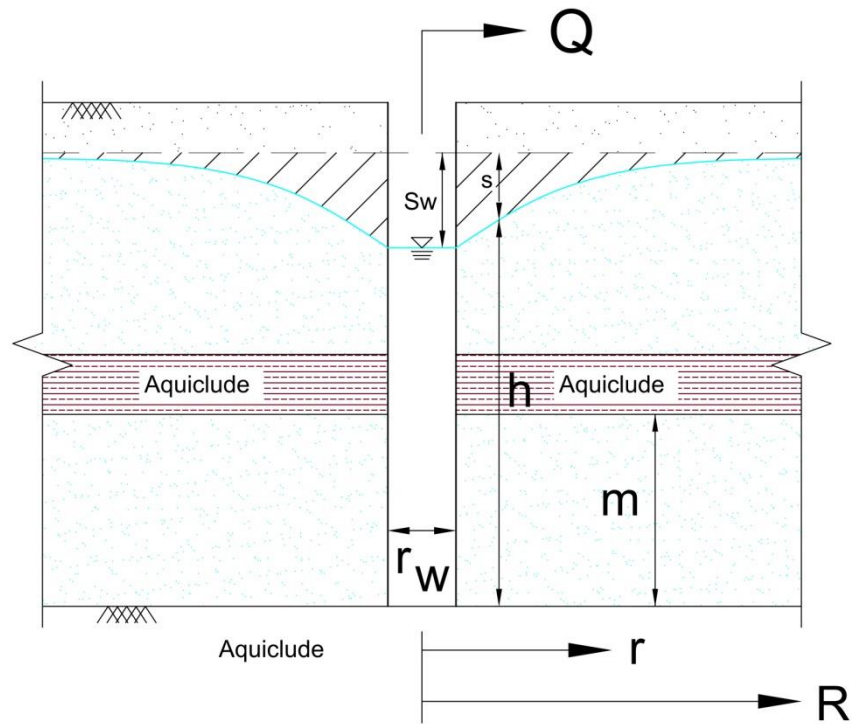
Important Terminologies

Figure 13. Aquifer and related parameters

Q=rate of flow (discharge)

S= Drawdown- the difference between the static and dynamic water levels

r=radial distance

R=radius of influence

r_w= well radius

S_w=Drawdown in the well

h=hydraulic head

m=thickness of saturated zone

When r=R, S=0 and when r=r_w, S=S_w.

1.7 Physical properties

1.7.1 Porosity (n)

If we consider a certain volume of unconsolidated material, the total unit volume, V_T, of the unconsolidated material can be divided into the volume of its solid portion, V_s, and the volume of its voids, V_v. The total volume of the unconsolidated material can be written in terms of the volumes of both solid and void as:

$$V_T = V_S + V_v$$

Let's divide both of sides of the equation by total volume, V_T. This yields:

$$1 = \frac{V_S}{V_T} + \frac{V_v}{V_T}$$

The second term in right hand side of this equation represents porosity. It is defined as the ratio of volume of void space to total volume of rock mass. Porosity is strongly related to void space.

$$n = \frac{V_v}{V_T}$$

Porosity is usually expressed as a percentage. The porosity values take in range from 0 to 1 based on geometrical formation. The porosity indicates the capability of the medium to store water. In other words, it measures water bearing capacity. Therefore, the total potential of groundwater can be calculated by using the following expression.

$$V_p = n \, V_a$$

Here V_p is potential of ground water, V_a is volume of aquifer.

As a result, porosity is one of the most important parameters in order to determine hydraulic conductivity. We can evaluate the water potential in the groundwater by porosity if there is no pressure. Not all the water found in the pore space can be extracted as some of the water will remain in the void space because of surface tension forces. This makes porosity not to be a good measure of water that can be extracted. In general, the rocks have lower porosities than soils, gravels, sands and silts.

1.7.2 Specific yield and retention

As indicated earlier, all of the water found in an aquifer cannot be abstracted from the aquifer making porosity not to be a good indicator of water potential. Some of the water will be kept in the void by molecular and surface tension forces. A good indicator of water potential is

specific yield. The specific yield can be defined as the ratio of the volume of yielded water to the total volume of rock mass. The specific yield is also known as effective porosity.

We know that $V_v = V_y + V_r$

Where V_v is volume of voids, V_y is volume of water that can be yielded and V_r is volume of water retained in the porous medium.

If we divide both sides of the equation by the total volume of the rock mass, V_T, we will find:

$$\frac{V_v}{V_T} = \frac{V_y}{V_T} + \frac{V_r}{V_T}$$

Obviously, $\frac{V_v}{V_T}$ is porosity, $\frac{V_y}{V_T}$ is called specific yield and $\frac{V_r}{V_T}$ s specific retention.

Therefore, total porosity can be written as

$$n = S_y + S_r$$

Here S_y is specific yield and S_r is specific retention. Specific yield indicates the amount of water available to use and it is more important than porosity value.

In general, rocks have lower porosities than soils, gravels, sands and silts. Clays have higher porosities than sand and gravels, but lower hydraulic conductivities. For example, the porosity of volcanic rocks is approximately 2 %. But, they yield all water that exists in their voids. Oppositely, clays have very high porosity values, but they can yield less than 5 % of water that exists in their voids.

It can be concluded that finer grain sizes have greater specific retention values compared to coarse-grained materials. Specific yield of aquifer is always less than the porosity. In fine grained porous formation, specific yield values are significantly different from their porosity values, whereas, in course grained materials, specific yield and porosity values are very close to each other. Specific yield can also be defined as the volume of water that unconfined aquifer release from storage per unit surface area of aquifer per unit decline of the hydraulic head. Specific yield of unconfined aquifer is much higher than storage coefficient of confined aquifer. The specific yield values vary in range from 0.01 through 0.3, which are much higher than the storativities of confined aquifers. In other words, the specific yield is known as storage coefficient in unconfined aquifer.

Specific yield indicates the quantity of abstracted water and specific retention shows how much water is retained in the aquifer after the water is yielded by gravity. The specific yield and specific retention can be written, respectively, as

$$S_y = \frac{V_y}{V_T}$$

and

$$S_r = \frac{V_r}{V_T}$$

Here, V_y is volume of yielded water and V_r is volume of retained water. Specific retention measures the amount of water in the void space against gravity by capillarity and hygroscopic forces when the hydraulic head in the confined aquifer is declined. This phenomenon is called specific retention.

1.7.3 Hydraulic conductivity

The hydraulic conductivity is constant of proportionality in Darcy's law. It is defined as the volume of water that moves through the porous medium in unit time under a unit hydraulic gradient. The hydraulic conductivity has dimension of Length/Time (L/T). Hydraulic conductivity is a function of properties of both the porous medium and fluid. The hydraulic conductivity is based on the pore and fracture sizes within aquifer. It is well known that horizontal hydraulic conductivity is approximately three times greater than vertical hydraulic conductivity based on formation of the porous medium. Hydraulic conductivity and hydraulic head are used in saturated porous media. Saturated porous media are media where all voids are filled with water. But, in practice, some of the voids are partially filled with water. This zone is referred as unsaturated zone.

1.7.4 Permeability

In geotechnical engineering, engineers use permeability coefficient concept in place of hydraulic conductivity. Actually, hydraulic conductivity is different from permeability coefficient. Because, hydraulic conductivity is based on properties of both porous medium and fluid flowing through the formation, whereas, permeability depends only on properties of the porous medium.

The properties of fluid are specific weight (γ) and dynamic viscosity (μ). The dynamic viscosity of fluid can be taken as resistive force within pores of formation. Also, specific weight of fluid acts as driving force.

The hydraulic conductivity is a function of permeability, k, specific weight (γ) and dynamic viscosity (μ). It can be written as:

$$K = f(k, \mu, \rho, g)$$

A relationship can be obtained between hydraulic conductivity and these parameters by using dimensional analysis.

$$K = \frac{k\,\gamma}{\mu}$$

k permeability and is based on only properties of porous medium. Permeability is proportional with square of a mean grain diameter. This means:

$$k \; \alpha \; d^2$$

where d is a mean grain diameter.

This proportionality is converted to equality by introducing a constant coefficient, c.

Therefore, the permeability coefficient can be expressed as:

$$k = c\,d^2$$

Here c is constant of proportionality and is based on porosity of the medium which are distribution of grain sizes, the sphericity and roundness of grain.

1.7.5   Transmissivity

There are six basic properties of fluid and porous media that must be known in order to describe hydraulic aspects of groundwater. These are: specific mass (ρ), dynamic viscosity (μ) and compressibility (β) for water, whereas porosity (n), permeability (k) and compressibility (α) for porous media. In unconfined aquifer, the transmissivity is not as well defined as in confined aquifer.

Transmissivity is the product of the average hydraulic conductivity, K, and the saturated thickness of the aquifer, m, (Freeze and Cherry, 1979). Consequently, transmissivity is the rate of flow under a unit hydraulic gradient through a cross-section of unit width over the whole saturated thickness of the aquifer (Bear, 1979). This definition is invalid if groundwater flow is non-linear. In other words, this definition is valid only for Darcian flow. This means that this definition is valid for neither fractured medium nor karstic medium, but just for only porous medium. Based on the definition given above, Transmissivity can be written as:

$$T = m.K$$

The equation to determine Transmissivity can also be given in another form. We know that the discharge, Q=A.v, where A is the area given as a product of the saturated thickness of the aquifer (m) and width of the aquifer (w), and v is the groundwater flow velocity.

Therefore $Q = m.w.v$

But, since $v = Ki$, $Q = m.w.K.i$

We also know that $m.k = T$

Substituting this in the above equation gives:

$$Q = T.w.i$$

Therefore, the transmissivity, T can be given as:

$T = \frac{Q}{w\,i}$

Here, T is transmissivity coefficient, Q is rate of flow, w is width of saturated aquifer, and i is hydraulic gradient. Transmissivity coefficient can be defined as the amount of water transmitted through the whole saturated thickness under unit width and unit change of hydraulic gradient. The transmissivity is defined well in confined aquifer, whereas in unconfined aquifer, it is not well defined. This is because the saturated thickness of aquifer is well defined in confined aquifer. Transmissivity and storage coefficients are defined for using well hydraulics in confined aquifer.

1.7.6 Hydraulic resistance (H_R)

Hydraulic resistance measures the resistance of vertical flow (upward or downward) through aquitard. In other words, it characterizes the amount of leakage through aquitard.The hydraulic resistance can be defined as:

$H_R = \frac{D_v}{K_v}$

Here, K_v is hydraulic conductivity of aquitard in vertical direction, and D_v is thickness of aquitard. It is obvious that, for impervious medium, K=0. Therefore, H_R goes to infinity. As a result, this parameter can measure the resistance of aquitard (semi-previous) formation to upward or downward leakage in leaky aquifers. Hydraulic resistance has dimension of time (Time).

1.7.7 Leakage factor

Leakage factor measures spatial variation of leakage through an aquitard in a leaky aquifer. It is defined as:

$L = \sqrt{T\,H_R}$

Lower values of L show high leakage rate through the aquitard, whereas, high values of L show low leakage rate. Leakage factor has dimension of Length.

1.7.8 Compressibility (α and β)

It is required to define compressibility of water and porous media separately. Compressibility of porous media describes the change in volume caused in an aquifer under a given stress and is given as:

$$\alpha = -\frac{\frac{dV_T}{V_T}}{d\sigma_e}$$

Here, V_T is the total volume of a given mass of material and $d\sigma_e$, is the change in effective stress.

The compressibility of water is defined as:

$$\beta = -\frac{\frac{dV_w}{V_w}}{dp}$$

The negative sign is required because of pressure, p, and to make β a positive number. An increase in pressure, dp, leads to a decrease in the volume V_w of a given mass of water. For incompressible water, since specific mass $=\rho=\rho_0=$ constant, then $\beta=0$.

1.7.9 Specific storage

Specific storage is defined as the volume of water that unit volume of aquifer release from storage under a unit decline in hydraulic head. It is well-known that decrease in hydraulic head, h, lead to decrease in fluid pressure and increase in effective stress σ_e. The decrease in hydraulic head causes two results: 1) increase in effective stress 2) decrease in pressure.

The first one is controlled by aquifer compressibility, α, and the second one is controlled by fluid compressibility, β. As a result, Specific storage is given as:

$$S_s = \rho.\,g(\alpha + n\beta)$$

And storage coefficient can be written as:

$$S = S_s\,D$$

Here, D is saturated thickness, and S_s is specific storage coefficient. Transmissivity, T, and storage coefficient, S, (storativity) were developed for the analysis of well hydraulics in confined aquifer.

1.8.1   Moisture Content

In unsaturated condition, the total volume of rock mass, V_T is divided into three parts: volume of solid part, V_s, volume of water, V_w and volume of air V_a . The total volume of rock mass can be expressed mathematically as:

$$V_T = V_s + V_w + V_a$$

Dividing both side of the equation by total volume of mass rock yields:

$$1 = \frac{V_s}{V_T} + \frac{V_w}{V_T} + \frac{V_a}{V_T}$$

The second term in the right hand side is referred as moisture content. It can be written as

$$n_w = \theta = \frac{V_w}{V_T}$$

This term is defined as the ratio of volume of water to total volume of rock mass. In other words, it is expressed as percentage like porosity, n. For saturated flow θ=n and for unsaturated flow θ<n.

1.8.2 Water table

Water table refers to the boundary between saturated zone and unsaturated zone. In other words, it is the upper surface of saturated zone. On this surface, the fluid pressure in pores of porous media is atmospheric (p=0). This implies that ψ=0, hence $h = \psi + z$, hydraulic head at any point on the water table must be equal to elevation of the water table. Therefore,

$$h = z$$

1.8.3 Negative pressure

Ψ=0   at any point on the water table (boundary)

Ψ>0 at any point under water table (saturated zone)

Ψ<0 at any point above water table (unsaturated zone)

Since water in the unsaturated zone is kept in the soil under surface-tension forces, the pressure head, ψ is taken as tension head or suction head when ψ<0. As mentioned before, hydraulic head is algebraic sum of elevation, z and pressure head ψ. Above the water table, where ψ is taken as tension head or suction head, it is not appropriate to measure hydraulic head with piezometers. But it can be measured with tensiometer.

1.8.4 Saturated, Unsaturated, and Tension-Saturated Zone

Saturated zone:

1) The saturated zone occurs under water tables (ψ>0)
2) The soil pores are filled fully with water. The moisture content, θ is equal to porosity, n (θ=n)
3) The fluid pressure is greater than atmospheric pressure and the pressure head, ψ is greater than zero (ψ>0).

4) The hydraulic head must be measured with a piezometer.

5) The hydraulic conductivity is a constant. It is not a function of pressure head ψ.

Unsaturated zone

1) It occurs above the water table and above the capillary fringe.

2) The soil pores are only partially filled with water. The moisture content is less than the porosity, n.

3) The fluid pressure is less than atmospheric pressure. This implies that the pressure head is less than zero.

4) The hydraulic head, h must be measured with a tensiometer.

5) The hydraulic conductivity, K and moisture content, θ are both functions of the pressure head, ψ.

In Summary

For saturated flow: $\psi > 0$, $\theta = n$, $K = K_0$

For unsaturated flow: $\psi < 0$, $\theta = \theta(\psi)$, and $K = K(\psi)$.