CHAPTER 27

Continuum Emission Mechanisms

Continuum radiation is any radiation that forms a continuous spectrum and is not restricted to a narrow frequency range. In what follows we briefly describe five continuum emission mechanisms:

- **Thermal (Black Body) Radiation**

- **Bremsstrahlung (free-free emission)**

- **Recombination (free-bound emission)**

- **Two-Photon emission**

- **Synchrotron emission**

In general, the way to proceed is to 'derive' the emission coeffient, j_ν, the absorption coefficient, α_ν, and then use the equation of **radiative transfer** to compute the specific intensity, I_ν, (i.e., the 'spectrum'), for a cloud of gas emitting continuum radiation using any one of those mechanisms.

First some general remarks: when talking about continuum processes it is important to distinguish **thermal emission**, in which the radiation is generated by the thermal motion of charged particles and in which the intensity therefore depends (at least) on temperature, i.e., $I_\nu = I_\nu(T, ..)$, from **non-thermal emission**, which is everything else.

Examples of **thermal continuum emission** are *black body radiation* and *(thermal) bremsstrahlung*, while *synchrotron radiation* is an example of *non-thermal emission*. Another non-thermal continuum mechanism is *inverse compton radiation*. However, since this is basically an incoherent photon-scattering mechanism, rather than a photon-production mechanism, we will not discuss IC scattering any further here (see Chapter 22 instead).

Characteristics of Thermal Continuum Emission:

• **Low Brightness Temperatures:** Since one rarely encouters gases with kinetic temperatures $T > 10^7 - 10^8$K, and since $T_B \leq T$ (see Chapter 25), if the brightness temperature of the radiation exceeds $\sim 10^8$K it is most likely non-thermal in origin (or has experienced IC scattering).

• **No Polarization:** Since these is no particular directionality to the thermal motion of particles, thermal emission is essentially unpolarized. In other words, if emission is found to be polarized, it is either non-thermal, or the signal became polarized after it was emitted (i.e., via Thomson scattering).

Thermal Radiation & Black Body Radiation: Thermal radiation is the continuum emission arising from particles colliding, which causes acceleration of charges (atoms typically have electric or magnetic dipole moments, and colliding those results in the emission of photons). This thermal radiation tries to establish **thermal equilibrium** with the matter that produces it via photon-matter interactions. If thermal equilibrium is established (locally), then the **source function** $S_\nu \equiv j_\nu/\alpha_\nu = B_\nu(T)$ (**Kirchoff's law**).
As we have seen in Chapter 25;

$$I_\nu = \begin{cases} B_\nu(T) & \text{if } \tau_\nu \gg 1 \\ \tau_\nu\, B_\nu(T) & \text{if } \tau_\nu \ll 1 \end{cases}$$

where $\tau_\nu = \alpha_\nu\, l$ is the **optical depth** through the cloud, which has a dimension l along the line-of-sight.

Free-free emission (Bremsstrahlung): Bremsstrahlung (German for 'braking radiation') arises when a charged particle (i.e., an electron) is accelerated though the Coulomb interaction with another charged particle (i.e., an ion of charge Ze). Effectively what happens is that the two charges make up an electric dipole which, due to the motion of the charges, is time variable. A variable dipole is basically an antenna, and emits electromagnetic waves. The energy in these EM waves (photons) emitted is lost to the electron, which therefore loses (kinetic) energy (the electron is 'braking').

It is fairly straightforward to compute the amount of energy radiated by a single electron moving with velocity v when experiencing a Coulomb interaction with a charge Ze over an impact parameter b (see `Rybicki & Lightmann 1979` for a detailed derivation).

The next step is to integrate over all possible impact parameters. This are all impact parameters $b > b_{\min}$, where from a classical perspective $b_{\min}$ is set by the requirement that the kinetic energy of the electron, $E_k = \frac{1}{2} m_e v^2$, is larger than the binding energy, $E_b = Ze^2/b$ (otherwise we are in the regime of **recombination**; see below). However, there are some quantum mechanical corrections one needs to make to this $b_{\min}$ which arise from Heisenberg's Uncertainty Principle ($\Delta x \, \Delta p \geq \hbar/2$). This correction factor is called the **free-free Gaunt factor**, $g_{ff}(\nu, T_e)$, which is close to unity, and has only a weak frequency dependence. The final step in obtaining the emission coeffient is the integration over the **Maxwellian velocity distribution** of the electrons, characterized by T_e. The result (in $\mathrm{erg\,s^{-1}\,cm^{-3}\,Hz^{-1}\,sr^{-1}}$) is:

$$j_\nu = 5.44 \times 10^{-39} \left(\frac{Z^2}{T_e^{1/2}} \right) n_e \, n_i \, g_{ff}(\nu, T_e) \, e^{-h\nu/k_B T_e}$$

In the case of a pure (ionized) hydrogen gas, $Z = 1$ and $n_i = n_e$. Upon inspection, it is clear that free-free emission has a flat spectrum $j_\nu \propto \nu^\alpha$ with $\alpha \sim 0$ (controlled by the weak frequency dependence of the Gaunt factor) with an exponential cut-off for $h\nu > k_B T_e$ (the maximum photon energy is set by the temperature of the electrons). This reveals that a measurement of the exponential cut-off is a direct measure of the electron temperature.

The above emission coefficient tells us the emissive behavior of a pocket of gas without allowance for the internal absorption. Accounting for the latter requires radiative transfer. Since Bremsstrahlung arises from collisions, we may use the LTE approximation. Hence, **Kirchoff's law** tells us that $\alpha_\nu = j_\nu/B_\nu(T)$, which allows us to compute the absorption coeffient, and thus the optical depth $\tau_\nu = \alpha_\nu l$. Substitution of $B_\nu(T)$, with $T = T_e$, yields

$$\tau_\nu \simeq 3.7 \times 10^8 \, Z^2 \, T_e^{-1/2} \, \nu^{-3} \left[1 - e^{-h\nu/k_B T_e} \right] g_{ff}(\nu, T_e) \, \mathcal{E}$$

where

$$\mathcal{E} \equiv \int n_e^2 \, \mathrm{d}l \simeq n_e^2 \, l$$

is called the **emission measure**, and we have assumed that $n_{\mathrm{e}} = n_{\mathrm{i}}$. Upon inspection, one notices that $\tau_\nu \propto \nu^{-2}$ (for $h\nu \ll k_{\mathrm{B}}T_{\mathrm{e}}$), indicating that the opacity of the cloud increases with decreasing frequency. The opacity arises from **free-free absorption**, which is simply the inverse process of free-free emission; a photon is absorbed by an electron that is experiencing a Coulomb interaction.

If we now substitute our results in the **equation of radiative transfer** (without background source),

$$I_\nu = B_\nu(T) \left[1 - e^{-\tau_\nu}\right]$$

then we obtain that

$$I_\nu = \begin{cases} B_\nu(T_{\mathrm{e}}) & \text{if } \tau_\nu \gg 1 \\ \tau_\nu \, B_\nu(T) = j_\nu \, l & \text{if } \tau_\nu \ll 1 \end{cases}$$

Fig. 25 shows an illustration of a typical free-free emission spectrum: at low frequency the gas is optically thick, and one probes the Rayleigh-Jeans part of the Planck curve corresponding to the electron temperature ($I_\nu \propto \nu^2 T_{\mathrm{e}}$). At intermediate frequencies, where the cloud is optically thin, the spectrum is flat ($I_\nu \propto \mathcal{E} T_{\mathrm{e}}^{-1/2}$), and at the high-frequency end there is an exponential cut-off ($I_\nu \propto \exp[-h\nu/k_{\mathrm{B}}T_{\mathrm{e}}]$).

Free-Bound emission (Recombination): this involves the capture of a free electron by a nucleus into a quantized bound state. Hence, this requires the medium to be ionized, similar to free-free emission, and in general both will occur (complicating the picture). Free-bound emission is basically the same as free-free emission (they have the same emission coefficient, j_ν), except that they involve different integration ranges for the impact parameter b, and therefore different Gaunt factors; the **free-bound Gaunt factor** $g_{\mathrm{fb}}(\nu, T_{\mathrm{e}})$ has a different temperature dependence than $g_{\mathrm{ff}}(\nu, T_{\mathrm{e}})$, and also has more 'structure' in its frequency dependence; in the limit where the bound state has a large quantum number (i.e., the electron is weakly bound), we have that $g_{\mathrm{fb}} \sim g_{\mathrm{ff}}$. However, for more bound states the frequency dependence of g_{fb} reveals sharp 'edges' associated with the discrete bound states.
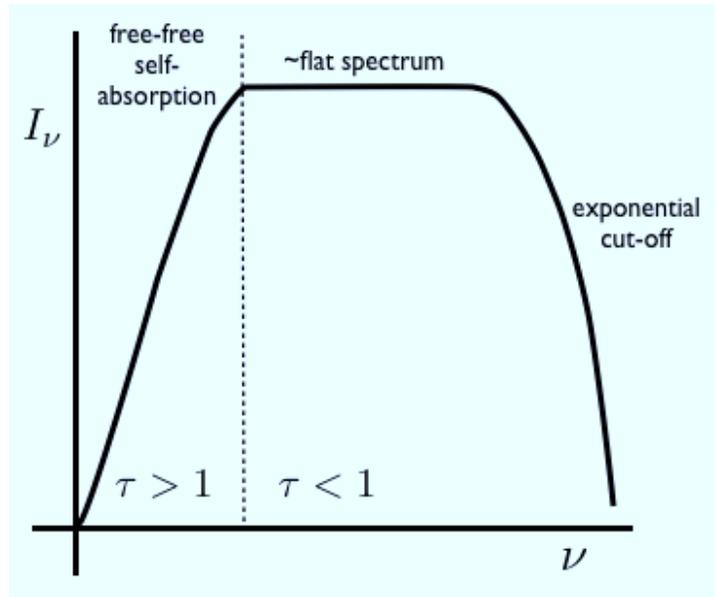
Figure 25: *Specific intensity of free-free emission (Bremsssstrahlung), including the effect of free-free self absorption at low frequencies, where the optical depth exceeds unity. At low frequencies, one probes the Rayleigh-Jeans part of the Planck curve corresponding to the electron temperature. At intermediate frequencies, where the cloud is optically thin, the spectrum is flat, followed by an exponential cut-off at the high-frequency end.*

When $h\nu \ll k_B T_e$ recombination is negligible (electrons are moving too fast to become bound), and the emission is dominated by the free-free process. At higher frequencies (or, equivalently, lower electron temperatures), recombination becomes more and more important, and often will dominate over bremsstrahlung.

Two-Photon Emission: two photon emission occurs between *bound* states in an atom, but it produces *continuum* emission rather than line emission.

Two photon emission occurs when an electron finds itself in a quantum level for which *any* downward transition would violate quantum mechanical selection rules. Each transition is therefore highly forbidden. However, there is a chance that the electron decays exponentially under the emission of two, rather than one, photons. Energy conservation guarantees that

$\nu_1 + \nu_2 = \nu_{\rm tr} = \Delta E_{\rm tr}/h$, where $\Delta E_{\rm tr}$ is the energy difference associated with the transition. The *most probable* configuration is the one in which $\nu_1 = \nu_2 = \nu_{\rm tr}/2$, but all configurations that satisfy the above energy conservation are possible; they become less likely the larger $|\nu_1 - \nu_{\rm tr}/2|$, resulting in a 'continuum' emission that appears as an extremely broad 'emission line'. In fact, whereas the *number* of photons with $0 < \nu < \nu_{\rm tr/2}$ is equal to that with $\nu_{\rm tr/2} < \nu < \nu_{\rm tr}$, the latter have more *energy* (i.e., $E_\gamma = h\nu$). Consequently, the spectral energy distribution, L_ν ($\rm erg\,s^{-1}\,Hz^{-1}$) is skewed towards higher frequency.

For two photon emission to occur, we require that spontaneous emission happens before collisional de-excitation has a chance. Consequently, two-photon emission occurs in low density ionized gas. The strength of the two photon emission depends on the number of particles in the excited states. This in turn depends on the recombination rate; although two-photon emission is quantum-mechanical in nature, it can still be throught of as 'thermal emission', and the density dependence is the same as for free-free and free-bound emission (i.e., $j_\nu \propto n_{\rm e}^2$).

An important example of two-photon emission is associated with the Lyα recombination line, which results from a de-excitation of an electron from the $n = 2$ to $n = 1$ energy level in a Hydrogen atom. As it turns out, the $n = 2$ quantum level consists of both $2s$ and $2p$ states. The transition $2p \rightarrow 1s$ is a permitted transition with $A_{2p \rightarrow 1s} = 6.27 \times 10^8 {\rm s}^{-1}$. However, the $2s \rightarrow 1s$ transition is highly forbidden, and has a two-photon-emission rate coefficient of $A_{2s \rightarrow 1s} = 8.2 {\rm s}^{-1}$. Although this is orders of magnitude lower than for the $2p \rightarrow 1s$ transition, the two photon emission is still important in low-density nebulae ($n \lesssim 10^4 {\rm cm}^{-3}$).

Synchrotron & Cyclotron Emission: A free electron moving in a magnetic field experiences a **Lorentz force:**

$$\vec{F_{\rm e}} = e \left(\frac{\vec{v}}{c} \times \vec{B} \right) = \frac{e\,v}{c} B \sin\phi = \frac{e\,v}{c} B_\perp = \frac{e\,v_\perp}{c} B$$

where ϕ is the *pitch angle* between $\vec{v}$ and $\vec{B}$. If $\phi = 0$ the particle moves along the magnetic field, and the Lorentz force is zero. If $\phi = 90^{\rm o}$ the particle will

move in a circle around the magnetic field line, while for $0° < \phi < 90°$ the electron will spiral ('cork-screw') around the magnetic field line. In the latter two cases, the electron is being accelerated, which causes the emission of photons. Note that this applies to both electrons and ions. However, since the cyclotron (synchrotron) emission from ions is negligble compared to that from electrons, we will focus on the latter.

If the particle is *non-relativistic*, then the emission is called **cyclotron emission**. If, on the other hand, the particles are *relativistic*, the emission is called **synchrotron emission**. We will first focus on the former.

Cyclotron emission: the gyrating electron emits dipolar emission that (i) has the frequency of **gyration**, and (ii) is highly polarized. Depending on the viewing angle the observer can see circular polarization (if line-of-sight is alined with $\vec{B}$), linear polarization, if line of sight is perpendicular to $\vec{B}$, or elliptical polarization (for any other orientation).

The gyration frequency can be obtained by equating the Lorentz force with the centripetal force:
$$F_e = \frac{e\, v_\perp}{c}\, B = \frac{m_e\, v_\perp^2}{r_0}$$
where $v_\perp = v \sin \phi$, which results in

$$r_0 = \frac{m_e\, v_\perp\, c}{e\, B}$$

where $B = |\vec{B}|$. This is called the **gyration radius** (or gyro-radius). The period of gyration is $T = 2\pi r_0/v_\perp$, which implies a **gyration frequency** (i.e., the frequency of the emitted photons) of

$$\nu_0 = \frac{1}{T} = \frac{e\, B}{2\pi\, m_e\, c}$$

Note that this frequency is *independent of the velocity of the electron*! It only depends on the magnetic field strength B;

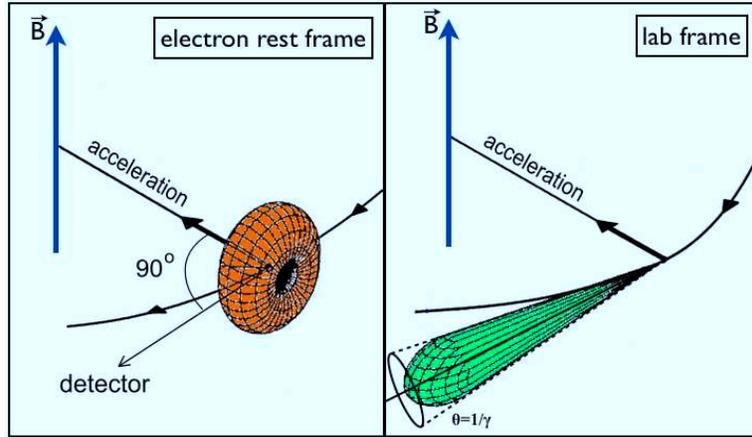$$\frac{\nu_0}{\mathrm{MHz}} = 2.8\, \frac{|\vec{B}|}{\mathrm{Gauss}}$$

Figure 26: *Illustration of how the Lorentz transformation from the electron rest frame to the lab frame introduce relativistic beaming with an opening angle $\theta = 1/\gamma$. Note that in the electron rest frame, the synchrotron emission is dipole emission.*

We thus see that **cyclotron emission** really is line emission, rather than continuum emission. The nature of this line emission is very different though, from 'normal' spectral lines which result from quantum transitions within atoms or molecules. Note, though, that if the 'source' has a smoothly varying magnetic field, then the variance in B will result in a 'broadening' of the line, which, if sufficiently large, may appear as 'continuum emission'.

In principle, observing cyclotron emission immediately yields the magnetic field strength. However, unless B is extremely large, the frequency of the cyclotron emission is extremely low; typical magnetic field strengths in the IGM are of the order of several μG, which implies cyclotron frequencies in the few Hz regime. The problem is that such low frequency radiation will not be able to travel through an astrophysical plasma, because the frequency is lower than the **plasma frequency**, which is the natural frequency of a plasma (see Appendix E of Irwin). In addition, the Earth's ionosphere blocks radiation with a frequency $\nu \lesssim 10$MHz, so that we can only observe cyclotron emission from the Earth's surface if it originates from objects with $B \gtrsim 3.5$G. For this reason, cyclotron emission is rarely observed, with the exception of the Sun, some of the planets in our Solar System, and an occasional pulsar.

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Synchrotron Emission: this is the same as cyclotron emission, but in the limit in which the electrons are relativistic. As we demonstrate below, this has two important effects: it makes the gyration frequency dependent on the energy (velocity) of the electron, and it causes strong beaming of the electron's dipole emission.

In the relativistic regime, the electron energy becomes $E_e = \gamma\, m_e\, c^2$, where $\gamma = (1 - v^2/c^2)^{-1/2}$ is the **Lorentz factor**. This boosts the gyration radius by a factor γ, and reduces the gyration frequency by $1/\gamma$:

$$
\begin{aligned}
r_0 &= \frac{\gamma\, m_e\, v_\perp\, c}{e\, B} \simeq \frac{\gamma\, m_e\, c^2}{e\, B} \\
\nu_0 &= \frac{e\, B}{2\pi\, \gamma\, m_e\, c}
\end{aligned}
$$

Note that now the gyration frequency *does* depend on the velocity (energy) of the (relativistic) electrons, which in principle implies that because the electrons will have a distribution in energies, the synchrotron emission is going to be continuum emission. However, you can also see that the gyration frequency is even **lower** than in the case of cyclotron emission, by a factor $1/\gamma$. For the record, Lorentz factors of up to $\sim 10^{11}$ have been measured, indicating that γ can be extremely large! Hence, if the photon emission were to be at the gyration frequency, we would never be able to see it, because of the plasme-frequency-shielding.

However, the gyration frequency is not the only frequency in this problem. Because of the relativistic motion, the dipole emission from the electron, as seen from the observer's frame, is highly beamed (see Fig. 26), with an opening angle $\sim 1/\gamma$ (which can thus be tiny). Consequently, the observer does not have a continuous view of the electron, but only sees EM radiation when the beam sweeps over the line-of-sight. The width of these 'pulses' are a factor $1/\gamma^3$ shorter than the gyration period. The corresponding frequency, called the **critical frequency**, is given by

$$
\nu_{\mathrm{crit}} = \frac{3\, e}{4\, \pi\, m_e\, c}\, \gamma^2\, B_\perp
$$

which translates into

$$\frac{\nu_{\text{crit}}}{\text{MHz}} = 4.2 \, \gamma^2 \, \frac{B_\perp}{\text{Gauss}}$$

So although the gyration frequency will be small, the critical frequency can be extremely large. This critical frequency corresponds to the shortest time period (the pulse duration), and therefore represents the largest frequency, above which the emission is negligble. The longest time period, which is related to the gyration period, determines the **fundamental frequency**

$$\frac{\nu_{\text{f}}}{\text{MHz}} = \frac{2.8}{\gamma \sin^2 \phi} \frac{|\vec{B}|}{\text{Gauss}}$$

The emission spectrum due to synchrotron radiation will contain this fundamental frequency plus all its harmonics up to ν_{crit}. Since these harmonics are extremely closely spaced (after all, the gyration frequency is extremely small), the synchrotron spectrum *for one value of* γ looks essentially continuum. When taking the γ-distribution into account (which is related to the energy distribution of the relativistic electrons), the distribution becomes trully continuum, and the critical and fundamental frequencies no longer can be discerned (because they depend on γ).

After integrating over the energy distribution of the relativistic electrons, which typically has a power-law distribution $N(E) \propto E^{-\Gamma}$ one obtains the following **emission and absorption coefficients:**

$$\begin{aligned} j_\nu &\propto B_\perp^{(\Gamma+1)/2} \, \nu^{-(\Gamma-1)/2} \\ \alpha_\nu &\propto B_\perp^{(\Gamma+2)/2} \, \nu^{-(\Gamma+4)/2} \end{aligned}$$

Note that α_ν describes **synchrotron self-absorption**. The resulting **source function** and **optical depth** are

$$\begin{aligned} S_\nu = \frac{j_\nu}{\alpha_\nu} &\propto B_\perp^{-1/2} \, \nu^{5/2} \\ \tau_\nu = \alpha_\nu \, l &\propto B_\perp^{(\Gamma+2)/2} \, \nu^{-(\Gamma+4)/2} l \end{aligned}$$

Using that typically $\Gamma > 0$, we have that $\tau_\nu \propto \nu^a$ with $a < 0$; synchrotron self-absorption becomes more important at lower frequencies.
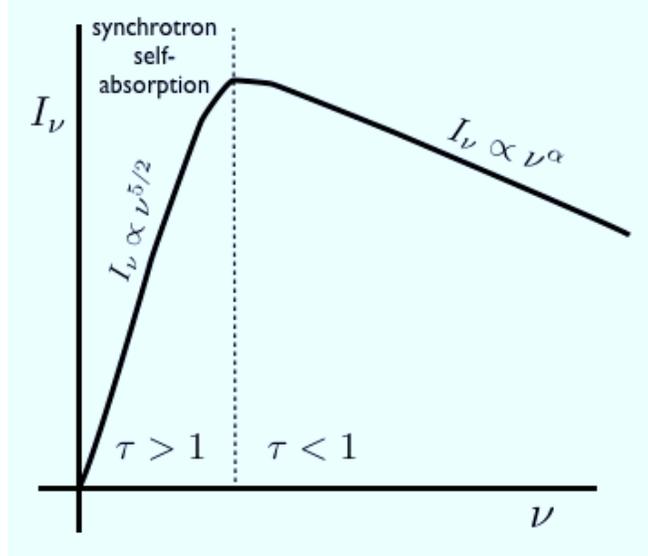
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Figure 27: *Specific intensity of synchrotron emission, including the effect of synchrotron self absorption at low frequencies, where the optical depth exceeds unity.*

Application of the equation of radiative transfer, $I_\nu = S_\nu \left(1 - e^{-\tau_\nu}\right)$, yields

$$I_\nu = \begin{cases} S_\nu \propto \nu^{5/2} & \text{if } \tau_\nu \gg 1 \\ j_\nu l \propto \nu^\alpha & \text{if } \tau_\nu \ll 1 \end{cases}$$

where $\alpha \equiv -\frac{\Gamma-1}{2}$. Fig. 27 shown an illustration of a typical synchrotron spectrum: at low frequencies, where $\tau_\nu \gg 1$, we have that $I_\nu \propto \nu^{5/2}$, which transits to $I_\nu \propto \nu^{-(\Gamma-1)/2}$ once the emitting medium becomes optically thin for synchroton self-absorption. Note that there is no cut-off related to the critical frequency, since $\nu_{\text{crit}} = \nu_{\text{crit}}(E)$.