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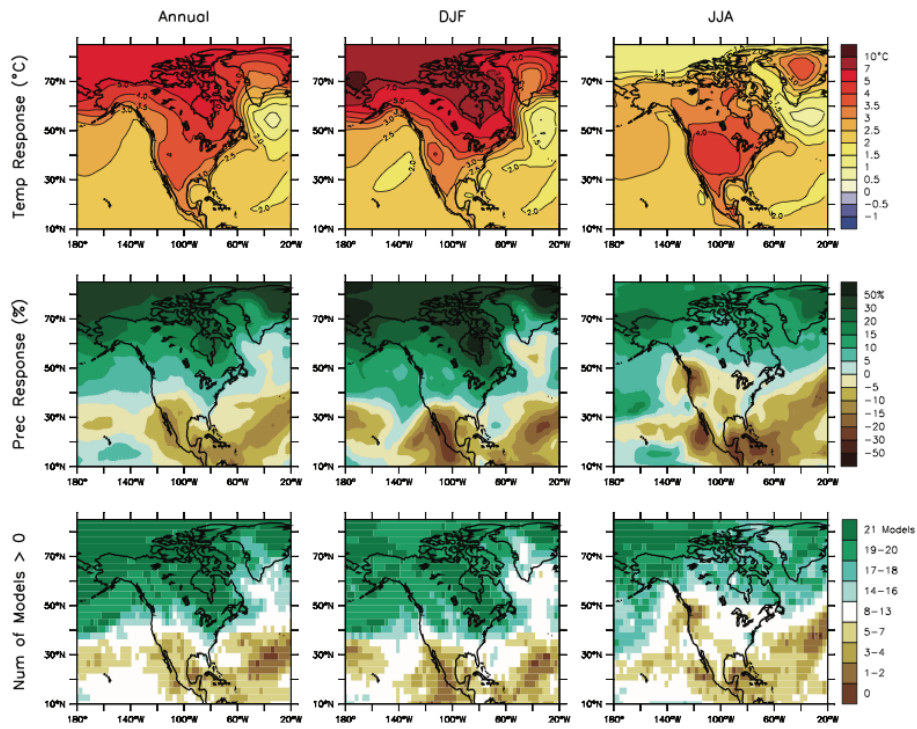


Figure 11.12. Temperature and precipitation changes over North America from the MMD-A1B simulations. Top row: Annual mean, DJF and JJA temperature change between 1980 to 1999 and 2080 to 2099, averaged over 21 models. Middle row: same as top, but for fractional change in precipitation. Bottom row: number of models out of 21 that project increases in precipitation.

Figure 1: Projections for climate change in North America based on the IPCC AR4 report.

1. Water in the Atmosphere

The earth's surface extends from the surface to about 700km. Hydrologic processes occur almost exclusively in the troposphere. This layer extends approximately 12km above the surface and the lowest km is the boundary layer, the layer that is directly affected by the surface. While vertical temp and pressure gradients vary with latitude, season and local weather patterns, averaged spatially and temporally the following temperature structure arises:

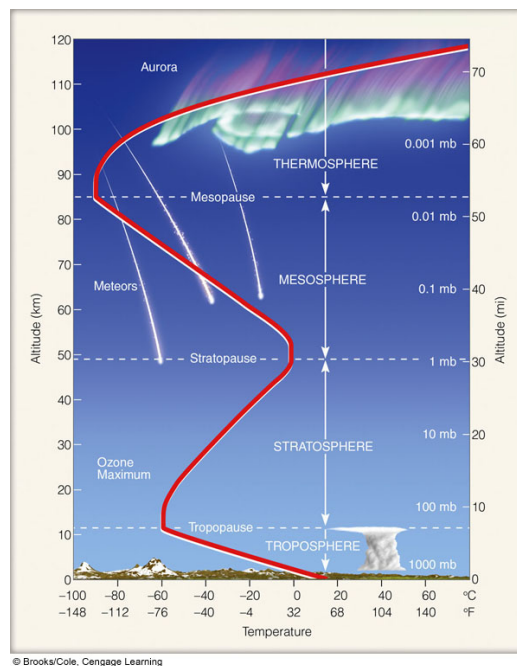


Figure 2: Layers of the atmosphere and their mean temperature. From Ahrens

As can be seen, the vertical temperature gradient in the troposphere, called a **lapse rate**, has an average value of $-6.5\text{ }^{\circ}\text{Ckm}^{-1}$. In some cases, like during cold nights, the temperature gradient near the surface may temporarily reverse direction - this is called an **inversion**.

1a. Temperature-pressure relations

Temperature, pressure and density in the atmosphere are related via the Ideal Gas Law:

$$\frac{P}{T_a \rho_a} = R_a \quad (1)$$

where P is the atmospheric pressure, T_a is air temperature, ρ_a is mass density of air in kg/m^3 and R_a is the gas constant [for dry air $R_a = R_d = 287 Jkg^{-1}K^{-1}$]. This law states that an increase (decrease) in pressure is always accompanied by an increase (decrease) in temperature and density. So when a parcel moves to a region of lower pressure it cools. These processes don't have an associated input or removal of heat so they are **adiabatic** processes.

1b. Water Vapor

All of the molecules that compose air (including H_2O) are in constant motion with velocities on the order of $550 ms^{-1}$. Each of the gases in a gas mixture (such as the atmosphere) exerts a pressure called a **partial pressure**, which is proportional to its concentration. The sum of the partial pressures of the gasses equals the total atmospheric pressure. The partial pressure of water vapor is called the **vapor pressure** and is designated e .

Saturation vapor pressure (e^*) is the partial pressure of the water vapor in equilibrium with a plane surface of pure water. It is a function of temperature alone, so *it doesn't depend on the vapor content of the air*. e^* is highly nonlinear, as it increases rapidly with temperature. This behavior is extremely important and explains why the amount of atmospheric water vapor will likely increase with global warming.

$$e^* = .611 \exp\left(\frac{17.3T}{T + 237.3}\right) \quad (2)$$

where e^* is in kPa and T is in $^{\circ}C$. This is an empirical relation, and there are many of these relations. This is the one we will use in this class.

The **absolute humidity** or **vapor density** is the mass concentration of water vapor in a volume of air. Using the ideal gas law:

$$\frac{e}{T_a \rho_v} = R_v \quad (3)$$

Specific humidity (q) is the concentration of water vapor expressed as the mass of water vapor per unit mass of moist air. Thus we have:

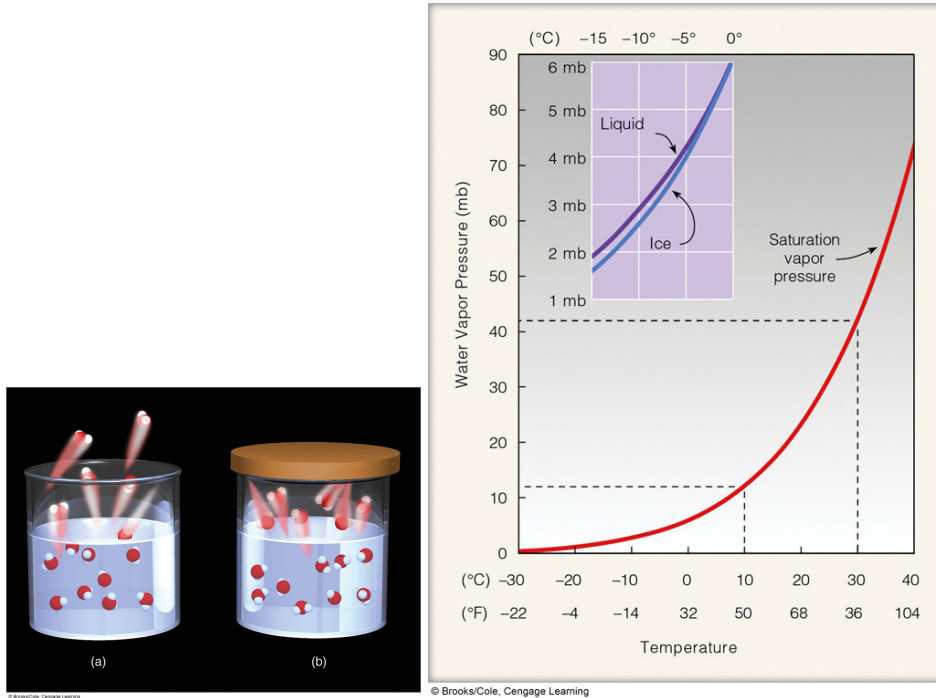


Figure 3: Explanation of Saturation vapor pressure. From Ahrens

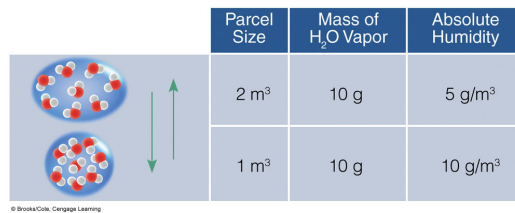
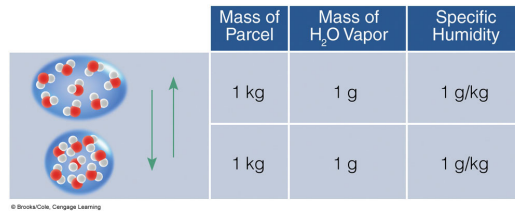


Figure 4: Absolute humidity. From Ahrens

$$q = \frac{m_v}{m_v + m_d} = \frac{\rho_v}{\rho_d + \rho_v} \quad (4)$$

$$= \frac{e/R_v T_a}{(P - e)R_d T_a + e/R_v T_a} = 0.622 \frac{e}{P - (1 - 0.622)e} \approx 0.622 \frac{e}{P}$$



	Mass of Parcel	Mass of H ₂ O Vapor	Specific Humidity
Parcel 1	1 kg	1 g	1 g/kg
Parcel 2	1 kg	1 g	1 g/kg

Figure 5: Specific humidity. From Ahrens

The **saturation specific humidity** (q^*) is defined as:

$$q^* = 0.622 \frac{e^*}{P - (1 - .622)e^*} \approx 0.622 \frac{e^*}{P} \quad (5)$$

Mixing ratio is the mass of water vapor per unit mass of dry air:

$$r = \frac{m_v}{m_d} = \frac{\rho_v}{\rho_d} = \frac{e/R_v T_a}{(P - e)/R_d T_a} = 0.622 \frac{e}{P - e} \approx 0.622 \frac{e}{P} \quad (6)$$

The **saturation mixing ratio** (r^*) is defined as:

$$r^* = 0.622 \frac{e^*}{P - e^*} \approx 0.622 \frac{e^*}{P} \quad (7)$$

Relative humidity can be defined in one of three different ways:

$$Wa = 100 \frac{e}{e^*} \text{ or } Wa = 100 \frac{q}{q^*} \text{ or } Wa = 100 \frac{r}{r^*} \quad (8)$$

it is expressed as a percent. It is the degree to which the air is saturated. Any value less than 100% means that the air is unsaturated. Cooling a parcel brings it closer to saturation. Air can have Wa values in excess of 100%, at these points the air is **supersaturated**.

Dew Point (Td) is the temperature to which a parcel with a given vapor pressure has to be cooled in order to reach saturation. From its definition:

$$r = r^*(T_d, P) = 0.622 \frac{e^*(T_d)}{P - e^*(T_d)} \quad (9)$$

This is an implicit equation, which we can solve by plotting or by Newton-Raphson iterations. However, we can calculate it using an approximation as:

$$T_d = \frac{\ln(e) + 0.4926}{.0708 - .00421 \ln(e)} \quad (10)$$

where T_d is in $^{\circ}C$ and e is in kPa .