Pietro Mengoli's Theory of Perception of Musical Intervals: A Mathematical Approach to the Sense of Hearing in the Scientific Revolution

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Introduction

What is the agent of recognition of musical intervals – reason or the sense of hearing? This was one of the most important questions in Western music theory from antiquity to the early modern period. On the one hand, music was thought to be basically mathematical, because music theory was one of the four mathematical sciences (*Quadrivium*) in medieval times and it used ratios to define musical intervals. In this sense, musical intervals were objects of rational recognition. On the other hand, they were also associated with the sense of hearing because they could be heard as sounds in reality.

For musical theorists of that period, music was primarily a mathematical science because they firmly believed in the Pythagorean view of music. This view maintained that music should be judged not by the fallible sense of hearing but by reason using numbers as the subject of music. That is, musical intervals were to be understood through mathematical equations that assured the certainty of recognition.

We can find an example of this attitude in the works of Giovanni Maria Artusi (ca.1540-1613), an Italian musical theorist known as a critic of Claudio Monteverdi. In the first part of his *L'Artusi overo delle imperfettioni della moderna musica* (1600), Vario, Artusi's spokesman, says:

> I admit you that the sense of hearing is the first that rejoices at that part to which it belongs: that is sound, its proper object and the matter of the songs. But I also tell you that the sense without intelligence cannot judge anything by itself, because it is subject to many deficiencies and many imperfections, and it does not receive things unless [those received things are] somewhat vague, even if [they are] close to their true [states].[1]

According to Artusi, the sense of hearing could receive sounds as the matter of musical intervals and songs, but it could not judge how they relate with each other. He believed that using only hearing to perceive musical intervals would only result in confusion. Therefore, Artusi insisted on the necessity of intelligence for the clear and distinct judgment of music. Thus, the primacy of intelligence/reason over the sense of hearing was maintained in the tradition of music as a mathematical science.

However, in the latter half of the seventeenth century, a theoretical work appeared that focused on the function of the sense of hearing: the *Speculationi di musica* (1670) by Pietro

Mengoli (1625-86), an Italian mathematician and professor at the University of Bologna. In this study I examine Mengoli's theory of interval perception and the functions of the sense of hearing. Specifically I focus on Mengoli's discussion of the recognition of errors in intervals, which was a subject that was discussed very rarely at that time[2].

1. Mengoli and *Speculationi*

Mengoli was active as a mathematician of the Galilean school, which promoted new natural sciences. However, when he was ordained in 1660, he began to waver between the religious doctrines of the Roman Catholic Church and the new scientific trends of the scientific revolution. After all, he represented Catholicism and opposed the Galilean doctrines (for instance, he supported the geocentric theory). Due to the fact that he developed his original metaphysics while attempting to reconcile the Catholic faith with the new scientific view of nature, the terms and logic in his works were often difficult to understand. Mengoli's inclination toward speculation prevented his works from having much influence on future generations. It was not until the beginning of the twentieth century that his mathematical achievements were reevaluated[3].

In addition to his works of pure mathematics (arithmetic and geometry), Mengoli wrote about "mixed mathematics" (astronomy, optics, and music, etc.), in which natural phenomena were examined by means of pure mathematics. *Speculationi* is one of his works on "mixed mathematics" and was written in Italian. It consists of the following sections: a dedication, preface, "Natural History of Music" (*Historia Naturale della Musica*; a summary of the whole work), a "Protestation of the author" (*Protesta dell'autore*; a declaration that the standpoint of the work is not against the doctrines of the Roman Catholic Church), and twenty-five speculations (*Speculationi*) – the main parts of the work.

In 1673, three years after its publication, *Speculationi* was discussed in articles in *Philosophical Transactions of the Royal Society*, and *Giornale de' letterati*, a book review journal issued in Rome[4]. Both articles mentioned that *Speculationi* contained new knowledge about the auditory organ, which reflected the prevalence of empiricism in those days. In the former article, after a short introduction of the work, the "Natural History of Music" was wholly translated into English. Henry Oldenburg (1619-77), the first secretary-general of the Royal Society and the supposed author of this article, pointed out that *Speculationi* was "somewhat obscurely written", indicating his difficulty understanding the work[5]. In the latter article, the reviewer stated that Mengoli's main purpose was to argue the nature of the soul (*anima*) rather than to teach music theory[6]. This remark is particularly astute, because *Speculationi* deals exclusively with the recognition of musical intervals by the human soul, not with the practice of music.

2. The Physiological Structure of Hearing

Mengoli's theory of perception of musical intervals is based on the anatomical knowledge of the auditory organ offered by Giovanni Galeazzo Manzi, an anatomist at the University of

Bologna. The anatomy described in the first *Speculatione* is not far from the current knowledge of the ear for the most part, with exception that there are two eardrums – one inside and one outside of the middle ear. According to Mengoli, the inner eardrum vibrates two times while the outer eardrum vibrates once; in other words, the ratio of the vibrational frequencies of the two eardrums is 2:1[7]. This is because the weight and velocity of the air confined in the inner ear is twice that of free air in the external ear. Consequently, the load added to the inner eardrum is twice that of the outer, and the tension of the inner eardrum is also twice that of the outer[8]. This assumption, which is naturally false according to the current knowledge of anatomy, played an important role in Mengoli's theory of musical perception.

Additionally, sound is considered to be the wavy motion of air, which is believed to be particulate matter, and the space between the particles is filled with "aura", which is subtle matter like ether. This assumption must have been taken from the scholastic conception of *horror vacui*. Aura is matter that reacts passively to the motions of the objects that surround it and it reflects their motions exactly; therefore, it is a medium that can accurately transmit the motions of air particles perceived as sound. Furthermore, it is so subtle that it can pass through the eardrum (the middle and inner ear where air particles cannot reach) and transmit the movement of air particles from the outer eardrum to the soul. According to Mengoli, "Aura is the immediate instrument of the soul, in which it receives and senses, and for which it operates and moves"[9]. That is, aura is a medium through which the soul works both passively and actively. It should be noted that aura was conceived to be matter with functions similar to both ether (transmitting the external phenomena of nature) and animal spirits (mediating between the mind and body inside a human being). Aura bridged the gap between the physical phenomena of the motions of air particles and the subjective perception of sound.

Next, Mengoli developed his ideas about the soul's function in hearing. The rational human soul spiritualizes all corporeal things that vanish away in process of time and keeps them in intelligence eternally. Sound is also abstracted as an intellectual object, which is, according to Mengoli, the logarithm of the ratio of the durations that the outer and inner eardrums vibrate (the two eardrums are linked to each other through the aura that fills the space between them)[10]. Then, the soul perfects the knowledge about the received things by reflecting on such abstracts as logarithms. However, Mengoli declared that he focused only on naturally executed abstractions, not the soul's reflection on the results of those abstractions[11]. This suggests that he intended to focus on the way the sense of hearing perceived sounds, not the rational judgments on what was perceived.

In addition, Mengoli distinguished the real motion of the eardrum and the "inclination" to that motion. On the basis of Galileo's theory that the ratio between the tensions of two chords emitting an octave is not 2:1 but 4:1 (the square of 2:1), he asserted that if the ratio between each vibrational frequency of the two eardrums was 2:1, the ratio between the each inclination to the motion was $\sqrt{2}$:1 (*dimidiata della doppia*) and the soul abstracted the logarithms of both ratios. This assumption makes his argument rather confusing. However, because the "inclination" Mengoli referred to can be interpreted as the impression the real motion leaves on the soul, it seems that the ratio perceived by the soul is in fact only that of the inclination. About the interval of the ratio $\sqrt{2}$:1, half-octave or tritone, Mengoli says:

> [...] this interval is the first ratio, according to which the soul is made to perceive what is done by sound in the eardrum of the ear and in the ethereal aura between the [two] eardrums. Therefore it is right that it should be the first of all the ratios to be found out and also the gauge of all the other [intervals], [...] [12]

Because the ratio between the vibrational frequencies of the inner and the outer eardrums is fixed at 2:1, the sound perceived by the outer eardrum ought to be perceived as being an octave higher by the inner eardrum. However, according to his idea of inclination, the soul perceives it as a half-octave; whenever the ear receives a certain sound, the soul abstracts the logarithm of $\sqrt{2}:1$, the ratio of a half-octave. Therefore Mengoli believed that a half-octave was the first interval perceived when hearing any sound, and he believed this to be the basis of every auditory experience at the physiological level. He called this interval "musical measure (*misura musica*)" and used it to gauge the errors of intervals.

3. The Mechanism of Interval Perception and the Logarithm

When hearing two differently pitched sounds, how is the interval between these pitches recognized? This was the main problem addressed in *Speculationi*; Mengoli provided a mechanical interpretation by defining the interval as the difference between the frequencies of contact the air particles have with the eardrum. When sounds are emitted at a certain pitch, they cause air particles to make contact with the outer eardrum, making it vibrate. If there are two tones, one high-pitched and the other low-pitched, the air particles from the high-pitched tone moves faster than those of the low-pitched tone, so they make contact with the eardrum more frequently, causing more frequent vibrations; the opposite is true for the low-pitched tone. Even when those two tones are in harmony, the air particles emitted from each tone strike the eardrum alternately, never at the same time. This theory is particular to Mengoli and is decidedly different from Galileo's theory, which states that the more frequently the pulses of air from two tones coincide, the more consonant the interval between the two tones is[13]. Mengoli may have designed this theory to accommodate the idea that the soul could recognize the frequencies of both high and low tones and therefore understand the ratio of the interval between those tones with mathematical accuracy, even when hearing sounds that were perceived to be harmonious. He explains this idea:

> Now, listening to two sounds is, in my opinion, none other than that the soul is attentive to the two strokes that are alternately hitting the eardrum of the ear, and perceives the alternation of the strokes, the period of the alternation, and the return to the same period, and is counting how many times a sound gives a stroke and how many times in the same period of alternation another gives a stroke at intervals. [14]

Mengoli showed the relationship between alternating strokes of air and intervals in Fig.1[15]. The lines of numbers on the left represent the periodic patterns of the numbers of strokes by

Alternationi di due ſuoni proportionali , come

1.	1 ad 1
2. 1. 1. 1. 1. 1. 1. 1	9 ad 8
2. 1. 1. 1. 1. 1. 1	8 a 7
2. 1. 1. 1. 1. 1	7 a 6
2. 1. 1. 1. 1	6 a 5
2. 1. 1. 1	5 a 4
2. 1. 1	4 a 3
2. 1	3 a 2
2. 2. 1	5 a 3
2. 2. 2. 1	7 a 4
2. 2. 2. 2. 1	9 a 5
2. 2. 2. 2. 2. 1	11 a 6
2. 2. 2. 2. 2. 2. 1	13 a 7
2. 2. 2. 2. 2. 2. 2. 1	15 ad 8
2.	2 ad 1

Fig. 1 Periodic patterns of the numbers of strokes by high tones and the ratios of each stroke by high and low tones

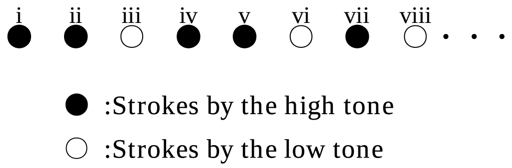

Fig. 2 The pattern of the alternation of the strokes for a major sixth chord (5:3)

high tones; the ratios on the right represent those of each stroke by high and low tones.

For example, the number of strokes produced by high tone 2. 2. 1 corresponds to the ratio of a major sixth 5:3. This means (see Fig.2) that since air particles from the high tone reach the eardrum faster than those from the low tone, the high tone strikes the eardrum twice (i, ii), then the low tone strikes once (iii), followed by the high tone twice (iv, v), the low tone once (vi), the high tone once (vii), the low tone once (viii), and so on. This alternating pattern of strokes will be repeated. In this example, we can see that five strokes by the high tone correspond to three strokes by the low tone. Thus, in the perception of two contemporaneous sounds, the soul counts the strokes that each sound transmits to the eardrum alternately and then grasps the periodic pattern of their regular alternation[16]. Finally, the soul obtains the logarithm from the ratio of the strokes and recognizes the size of the interval.

Now, we must consider the significance of Mengoli's idea that the soul gets the logarithm from the ratio as a cognitive function. The logarithm, invented in England around 1600, was a rather new mathematical tool in Mengoli's time and was useful calculating large figures, such as those used in astronomy, because it could replace multiplication or division with addition or subtraction. Therefore, its use spread rapidly in various mathematical fields, including music theory. In addition to its utility, introducing logarithms into the music theory was significant for another reason: it radically changed the view of musical intervals. While an interval was formerly understood to be the "relationship" between two pitches shown by the ratio of integers in the Pythagorean tradition of music, the logarithm, which replaced the ratio of two numbers with one real number, played a large role in establishing the new view of intervals as the "distance" between two pitches. This concept of an interval as a continuum became the premise for the division of the "musical measure" in Mengoli's arguments concerning the errors of intervals.

4. Counting through the Sense and Errors of Intervals

Although we can find novelty in his understanding of musical intervals, Mengoli's idea of the mechanism of their perception is not far from the traditional doctrine that reason should judge the size of an interval. That is, he treated intervals as a mathematical object to be understood by reason. Nevertheless, he attempted to explain the perception of intervals as a function of the sense that receives sound directly, not of reason. In the "Protestation of the author", Mengoli expressed his basic attitude:

> I talk about some operations of the soul: count, division, and about the errors which happen to the soul in these operations, and about its active attention, [...] [those are] always [done] according to the sense and without abstraction by mind, as far as all these things belong to the inferior part of human being, which is sensitive, never to the superior part, which is rational. For this I don't divide human being in two forms or natures, but ignoring only that human being is rational, I consider it as sensitive being, and no more. [17]

The soul perceived an interval by counting the strokes on the eardrum and Mengoli attributed this function not to reason, which is the superior cognitional faculty of human beings, but to the sense of hearing, which is its inferior cognitional faculty. He believed that since sound was the proper object of the sense of hearing, there must be a system of musical perception that relied on that sense, not on reason. Therefore, since interval perception is executed by counting strokes of air particles, it is necessary that the soul count them only through the sense of hearing without reason. Mengoli had the following to say regarding this idea:

> Now I say that the rational soul can count according to the sense and make sure of the number <u>without any effort</u> until it finds how many counts have been done in itself, independently of its every operation. And this [count] is one, two, three, up to three, and no more. Because the rational soul has one in itself for its natural faculty of learning one thing, and no more, at one time; it has two for its natural faculty of putting two things in comparison with each other and judging them; it has three for the natural faculty of discussing three things by putting the first in comparison with the third through the second to which the both are compared. I don't refer to the very acts of judgment and discussion, but to its natural faculties of having two and three <u>without any expertness</u>. [18](Emphasis added.)

It should be noted that, according to Mengoli, the soul can count from one to three through the sense "without any effort" or "without any expertness" and that this function is carried out by the "natural faculty" of the human soul. Mengoli emphasized that the function is based on the innate faculties of the soul and that it is clearly distinguished from rational acts such as judgment and discussion as if it were carried out unconsciously[19]. According to this principle, Mengoli limited the numbers the soul can count using the sense of hearing to 1 and multiples of 2 and 3 up to 36: 1, 2, 3, 4, 6, 9, 12, 18, 27, 36. He added 5 to them for the seemingly naïve reason that there are five fingers on one hand. The limitation of numbers is indicative of the limitation of intervals the sense of hearing can perceive. Therefore the perceptible intervals are limited to those of multiple ratios (n:1, where n is natural number) and superparticular ratios (n+1:n) as shown in Fig.3. All of the antecedents of these ratios are the numbers that are countable using the sense of hearing; the smaller the number is, the easier the perception of the interval (unison (1:1) and octave (2:1) are equally easy to perceive).

Multiple Ratios	1:1(=)2:1, 3:1, 4:1, 6:1, 9:1, 5:1
Superparticular Ratios	3:2, 4:3, 6:5, 9:8, 5:4, 12:11, 18:17, 27:26, 36:35

Fig.3 The Ratios of Intervals Perceptible to the Sense of Hearing

Mengoli adapted this theory of counting using the sense of hearing to his thoughts on the errors of intervals. He distinguished intervals that contained errors from pure intervals (i.e. the intervals given in Fig.3) into three categories according to tolerance the sense of hearing must have to identify the intervals: 1. Interval with an error that is unrecognizable to the sense of hearing (*Apparente*), 2. Interval with an error that is recognizable but acceptable to the sense of hearing (*Aggradevole*), 3. Interval with an error that is recognizable and barely tolerable to the sense of hearing (*Tollerabile*). The larger the error becomes from Category 1 to Category 3, the more difficulty the soul has in identifying the interval, and the less pleasure it gets from understanding it.

Mengoli carried out the categorization of errors in two ways. The first was through the equal division of a half-octave he called "musical measure". The second was through the ratios of the strokes created by two sounds.

In the first method of error categorization, the sizes of errors are defined by the parts of "musical measure" that can be divided equally. Since the biggest countable number using the sense of hearing was 36, he used 1/36 of a half-octave as the smallest perceptible error. If the error is smaller than this interval, it is "unrecognizable" (Category 1). If the error is between 1/36 and 1/18 of a half-octave, it is "recognizable but acceptable" (Category 2). If the error is between 1/18 and 1/12 of a half-tone, it is "tolerable" (Category 3). In a case where the error is bigger than 1/12 of a half-octave, that is a quarter of a tone, it is no longer tolerable. Needless to say, 18 and 12 are numbers that the soul can count only through the sense of hearing.

On the other hand, in the second method of error categorization, errors are related to confusions in the periodic regular pattern of each interval according to which air particles from two tones alternate. The confusions are caused by mixtures of foreign patterns from other intervals that break the regular pattern of the alternation. That is, when a certain interval has an error, the ratio of that interval is mixed with its proximate ratios. For example, an error in a fifth chord, whose ratio is 3:2, is produced by mixtures with the ratios 2:1 and 4:3. The sizes of errors are defined by the frequencies of those mixtures in the strokes by the low tone. If there is a mixture in more than 36 strokes by the low tone, the error is "unrecognizable" (Category 1). If there is a mixture in more than 24 strokes, the error is "recognizable but acceptable" (Category 2). If there is a mixture in more than 12 strokes, the error is "tolerable" (Category 3). In the case of a fifth, intervals with an error of Category 1 are within 56:37 ((54 + 2):(36 + 1); 54:36=3:2) and 58:39 ((54 + 4):(36 + 3)); intervals with an error of Category 2 are within 38:25 ((36 + 2):(24 + 1); 36:24=3:2) and 40:27 ((36 + 4):(24 + 3)); and intervals with an error of Category 3 are within 20:13 ((18 + 2):(12 + 1); 18:12=3:2) and 22:15 ((18 + 4):(12 + 3)). If there is a mixture in less than 12 strokes by the low tones, the error is no longer tolerable. Errors caused by a mixture in more than 36 strokes are not perceptible, because the soul cannot count numbers over 36 through the sense of hearing. 24 and 12 are two thirds and one third of 36, respectively, and both are countable numbers only through the sense of hearing. Thus, the more frequent confusions in the patterns of alternations are, the less tolerant the errors are. Fig.4 sums up the above-mentioned argument.

Category of Errors	1. Unrecognizable	2. Recognizable but acceptable	3. Tolerable
Parts of "musical measure"	Less than 1/36	1/36 ~ 1/18	1/18 ~ 1/12
Frequency of mixture of foreign intervals in the strokes by the low tone	Once in more than 36 strokes	Once in more than 24 strokes	Once in more than 12 strokes

Fig.4 Categorization of the Tolerance of Errors

Why did Mengoli approach interval errors in these two ways? He did not provide an answer to this question, but the very fact that he set up such double standards might show his ambivalent attitude toward the understanding of musical intervals. From the viewpoint of temperament, the division of "musical measure" corresponds to the method of creating an equal temperament. The ratio of strokes corresponds to just intonation in which the intervals are defined by simple ratios of integers. The latter method for categorizing errors in intervals is based on the Pythagorean concept that a musical interval should be considered as the "relationship" between two pitches. This relationship is indicated by a ratio that compares two quantities, whether they are lengths of monochord or frequencies of the strokes of air particles. On the other hand, the former method is in accordance with the idea that a musical interval should be considered as the "distance" (continuum) between two pitches. In order to equally divide a half-octave, indicated by the ratio $\sqrt{2}:1$, it is necessary to treat it as a continuum. This is because $\sqrt{2}:1$ is an irrational ratio, which is impossible to deal with according to the Pythagorean theory of intervals because it can only handle rational numbers. Although the idea of dividing an interval into equal parts in the manner of geometry had already appeared in music theory by the late sixteenth century[20], Mengoli's originality is indicated by the fact that he related his view of musical intervals as continua to the physiological structures of hearing and to the functions of the sense of hearing. In other words, Mengoli considered the nature of musical intervals from the way human beings perceive them.

In this sense, the examination of interval errors through the divisions of "musical measure" seems to be more suitable to Mengoli's purpose. Nevertheless, when considering a musical interval as a physical phenomenon external to humans, he had no choice but to define it using the ratio of strokes of air particles, and he also had to believe that the soul recognized the size of the interval errors through the frequency of confusions in the alternation pattern of strokes. Consequently, we suppose that Mengoli must have considered interval errors from the both points of view, and this gap in his view of intervals indicates his failure to connect their external factors and the physiological structures of hearing.

As we have seen so far, Mengoli's argument about counting through the sense of hearing developed in relation to the problem of determining the errors of interval. We should focus less on the seemingly arbitrary numbers that he used and more on his assumption that the soul can only count the numbers through the sense of hearing, which is very limited. The confusion of sense perception is caused by this limitation. Nevertheless, the rational human soul requires mathematical evidences based on the limited faculty of the sense of hearing when perceiving musical intervals. We will examine this aspect of sense perception in the next two sections.

5. The Active Attention of the Soul

In the nineteenth *Speculatione*, Mengoli emphasized the activeness of the soul's reaction in hearing. He asserted that when perceiving a certain interval, due to its natural desire for the interval to be understandable through the sense of hearing, the soul actively affected the body and changed the tension of the eardrum; that is, it changes the real ratio of the interval.

He used the case of perceiving the interval of 36:35, which is the smallest and most difficult interval to recognize using the sense of hearing as shown in Fig.3 above. First, when receiving the interval of 39:38, the soul relaxes the eardrum and decreases the vibrational frequencies caused by this interval to those caused by the interval of 36:35. Then, it changes the interval of 39:38 to the interval of 36:35. In other words, it perceives the interval of 39:38 as the interval of 36:35 in order to satisfy the soul's desire for clear understanding. On the other hand, when receiving the interval of 34:33, the soul strains the eardrum and increases the vibrational frequencies caused by this interval to those caused by the interval of 36:35. In this case, it also perceives the interval of 34:33 as the interval of 36:35.

With regard to this function of the soul, Mengoli said, "It is necessary to say that the rational soul, by its active attention, gives the true number to the first sound offered to the ear"[21]. The "true number" mentioned here must be one of the numbers that constitute the ratios of pure intervals; consequently, it becomes one of the numbers the soul can understand only through the sense of hearing. Therefore, we can conclude that the soul's active attention is the function of perceiving the provided intervals as intervals of the ratios that can be understood clearly through the sense of hearing. If the provided interval has a ratio that cannot be understood through the sense of hearing, the soul corrects that ratio by affecting the appropriate part of the body – the eardrum. Thus, Mengoli believed that the essential element of interval perception was the mathematical evidence based on the limited counting faculty of the sense of hearing. Therefore, the natural desire for mathematical evidence caused the soul to directly affect the structures of the body. This means that he considered sense perception not as a passive function where the soul received external stimuli one-sidedly, but as an active function where the soul understood the stimuli according to its own inner principles[22].

However, it is difficult to imagine that such phenomena really occur in the eardrum. Can we suppose that Mengoli's intention was to assert that the human sense of hearing must correct the ratios of intervals because of the imperfections of its cognitional faculty? A reexamination of his classification of intervals according to the sizes of their errors should provide an answer to this question.

6. The Tolerance of the Sense of Hearing concerning the Errors of Intervals

In the sixteenth *Speculatione* entitled "Definitions of the intervals", Mengoli divided all intervals into two categories, "natural intervals" and "artificial intervals". Then, he applied them to the three genera of tetrachords in Ancient Greek music theory. "Natural intervals" are equivalent to intervals in the diatonic genus, and "artificial intervals" are all the other intervals, including those in the chromatic and enharmonic genera. Diatonic intervals, which are supposed to be "natural" and the most easily perceived, are characterized by unrecognizable errors (that is, they are intervals in Category 1: *Apparente*). They are unrecognizable with regard to the alternation pattern of the strokes and the divisions of "musical measure". Mengoli referred to "natural intervals" as follows:

> [I call these intervals "natural"] because they are made easily and recognized easily by the sense [...], and because the errors of these intervals from their true forms are within certain confines where the sense cannot discern them. Therefore [the soul] considers the intervals with unrecognizable errors (*gli apparenti*) as true intervals without any discrepancy. And like true intervals it understands the intervals with unrecognizable errors completely both in terms of the number [of the strokes] and the [musical] measure they have, in which consists the complete satisfaction of the soul, according to the sense. [23]

Since "natural intervals" contain errors that fall within the "unrecognizable" category, the soul considers them to be pure intervals and is satisfied with the completion of its recognition; this satisfaction comes from the mathematical evidence based on the limited counting faculty of the sense of hearing.

On the other hand, "artificial intervals" contain clearly perceptible errors. According to Mengoli, errors must fall within the confines of the "unrecognizable" category, both in terms of the ratio of strokes and the divisions of "musical measure" in order for an interval to be understood completely using the sense of hearing only. However, it is necessary to use both the sense of hearing and reason to recognize "artificial intervals", because they contain perceptible errors in the ratio of strokes and/or the divisions of "musical measure"[24].

In his argument concerning which intervals are suitable to melodies, Mengoli includes intervals from the chromatic and enharmonic genera in the category of intervals that the soul can understand through the sense of hearing. According to Mengoli, intervals from the chromatic and enharmonic genera cannot be understood fully through the cognitive faculty of the sense of hearing alone; however, the soul imagines understanding them and it satisfies its desire to recognize them (he compared this satisfaction of the soul with that of the person who imagined understanding fully the contents of a book only by glancing at its table of contents)[25]. This means that intervals from these genera belong to intervals that the soul can understand through the sense of hearing in the same way as it understands intervals from the diatonic genus. He considered "artificial intervals" except those from the chromatic and enharmonic genera to be unsuitable to melodies, because they contained "recognizable but acceptable" errors (Category 2) both in the ratio of strokes and the divisions of "musical measure" and consequently their errors were clearly recognized[26]. Therefore, the errors in the "artificial intervals" suitable to melodies must be "recognizable but acceptable" for only one of two criteria. Thus, Mengoli concluded that "artificial intervals" from the chromatic and enharmonic genera as well as "natural intervals" from the diatonic genus were suitable for melodies.

It should be noted that the intervals suitable for melodies are allowed to have certain errors that are tolerable to the sense of hearing; in other words, a note in a melody does not necessarily have a single allowable pitch, it can represent a certain range of pitches. We perceive each interval that constitutes a melody as a category containing a certain error. If each interval in a melody has an error within this category, we perceive it in its pure form and the melody is not spoiled.

From these observations, we can infer that the correction of the interval ratios by the soul's active attention indicates the categorical perception of intervals. When the soul recognizes (or when it imagines recognizing) an interval through the sense of hearing, the interval is within the category where it is identifiable. At the same time it is perceived in its pure form under the correction of the soul. Mengoli must have intended to explain this way of perception by reducing it into the physical function of changing tension in the eardrum. Moreover, the categorical perception indicates the limitation of the capacity of human sense perception, and the correction by the soul's active attention might be a form of mind-body relationship that was invented in order to make up for this limitation.

Conclusion

Mengoli's *Speculationi* is an unusual work on music. Although it deals with musical intervals and melodies, they are argued exclusively from the physiological aspect of musical hearing without notes. Its main purpose was to explicate how the soul recognizes the size of an interval when it is heard. Therefore, the work is a kind of epistemology that considers musical perception rather than music theory, as was pointed out by the article in *Giornale de' letterati*. Certainly many of Mengoli's theories appear very strange to us because they are deduced from his own speculations and do not involve any actual proof, however, his focus on the cognitive functions of the sense of hearing itself was significant. In this sense, we can regard *Speculationi* as a major milestone in the birth of aesthetics advocated by Alexander Gottlieb Baumgarten (1714-62) as a study of sensuous recognition and its proper objects: beauty and art[27].

Notes

[1] Giovanni Maria Artusi, *L'Artusi overo delle imperfettioni della moderna musica* (prima parte), Venezia: Giacomo Vicenti, 1600 (fac., Arnaldo Forni Editore, 2007), fol.11v.: "Vi confermo, che l'udito è il primo, che gode, & si diletta di quella parte che à lui s'appartiene; cioè del suono, suo proprio oggetto, & è la materia delle Cantilene; ma vi dico ancora, che il senso senza l'intelletto, non può da se solo iudicare cosa alcuna, perche è sottoposto à molti difetti, & à molte imperfettioni; nè riceue le cose, se non in vn certo modo confuse, se bene vicine al vero:"

[2] Gozza pioneered the study of Mengoli's *Speculationi* by putting it in the historical context of the science of his time: Paolo Gozza, "A Mechanical Account of Hearing from the 'Galilean School': Pietro Mengoli's *Speculationi di Musica* of 1670", C. Burnett, M. Fend, P. Gouk (eds.), *The Second Sense: Studies in Hearing and Musical Judgement from Antiquity to the Seventeenth Century*, The Warburg Institute: University of London, 1991, pp.115-136. However, he hardly deals with the mathematics that Mengoli used to explicate musical perception. Wardhaugh's paper is rather satisfying in this respect: Benjamin Wardhaugh, "The Logarithmic Ear: Pietro Mengoli's Mathematics of Music", *Annals of Science*, Vol.64, No.3 (2007), pp.327-348.

[3] Studies of the history of mathematics in the 1910s demonstrated that Mengoli was the first to state the concept of "convergence" and "divergence" in infinite series, and he was the first to demonstrate that the harmonic series is divergent. The achievement concerning harmonic series was attributed to Jakob Bernoulli (1654-1705), a Swiss mathematician, but in reality Mengoli was the one who discovered it. For Mengoli's bibliographical informations, see Marta Cavazza "Introduzione", *La Corrispondenza di Pietro Mengoli*, a cura di G. Baroncini e M. Cavazza, Firenze: Olschki, 1986, pp.1-22.

[4] *Philosophical Transactions*, Royal Society of London, Vol.8, 1673, pp.6194-7000; *Giornale de' letterati*, Roma: Nicol'Angelo Tinassi, 1673, pp.131-133.

[5] *Philosophical Transactions* (note 4), p.6195.

[6] *Giornale de' letterati* (note 4), p.131.

[7] Pietro Mengoli, *Speculationi di musica*, Bologna: l'Herede del Benacci, 1670, p.4.

[8] ibid., p.16.

[9] ibid., p.14: "l'aura è l'instromento immediato dell'anima, nel quale riceuendo, sente; e per lo quale operando, moue."

[10] ibid., pp.22-24.

[11] ibid., p.22.

[12] ibid., p.51: "[...] questo interuallo è la prima ragione, secondo la quale l'anima è costretta ad auuertire quel che ne' timpani dell'orecchio, e nell'aura eteria trà gli timpani per il suono si fà. Onde hà ragione per essere il primo di tutte le ragioni à scoprirsi, di essere anche la misura di tutti gli altri, [...]"

[13] Galileo's "coincidence theory of consonance" appears at the end of the First Day in his *Discourses and Mathematical Demonstrations Concerning Two New Sciences* published in 1638. For more details on this theory, see H. F. Cohen, *Quantifying Music*, Reidel, 1984, pp.85-97.

[14] *Speculationi* (note 7), p.40: "Hor l'vdire due suoni, non è altro, per mio credere, se non che l'anima stà attenta a i due percutienti, che vanno alternatamente battendo il timpano dell'orecchio; ed auuerte l'alternatione de' battimenti, e il periodo dell'alternatione, e il ritorno allo stesso periodo; e và contando, quante volte vn suono percuote, e quante volte nel tempo dello stesso periodo d'alternatione, interuallatamente percuote l'altro."

[15] ibid., p.37.

[16] As Wardhaugh pointed out, this explanation has a number of problems. First, unless air particles reach the eardrum at a fixed distance from one another, they cannot strike the eardrum at a determinate frequency, so they cannot keep a determinate pitch. However, that is only under ideal

circumstances. Also, even if air particles reach the eardrum at a fixed distance from one another, in order for the soul to distinguish between the two kinds of particles, there should be some differences in their shapes or sizes, which is not addressed by Mengoli. See Wardhaugh, *op.cit.* (note 2), p.336.

[17] *Speculationi* (note 7), n.pag. [xix-xx]: "Parlo di alcune operationi dell'anima, numerare, partire, e de gli errori, che in queste operationi gli occorrono, e del suo attendere attiuo, [...] sempre secondo il senso, e senza astrattione di mente, e in quanto tutte queste cose appartengono alla parte inferiore dell'huomo, come che è sensitiuo, e non mai alla superiore, come che è ragioneuole: non perciò io diuido l'huomo in due forme, ò nature, ma prescindendo solo dall'eßer suo ragioneuole, lo considero come sensitiuo, e niente più."

[18] ibid., p.54: "Hor'io dico, che può l'anima rationale numerare secondo il senso, ed accertare il numero senza punto d'applicatione, sino à quanto troua in se la numeratione già fatta, independente da ogni sua operatione. E questa è, vno, due, tre, sino al tre, e non più oltre. Impercioche l'anima ragioneuole hà l'vno in se, per la facoltà naturale, che hà d'apprendere vna cosa alla volta, e non più: hà il due, per la facoltà, che hà naturale di giudicare, e mettere due cose l'vna à paragone dell'altra: hà il tre, per la facoltà naturale di discorrere per tre cose, mettendo à paragone la prima della terza, mediante la seconda, à cui si paragonano tutte due, non dico per gli stessi atti di giudicare, e discorrere, ma per le facoltà, che hà naturali, nelle quali hà il due, e il tre, senz'alcun magistero."

[19] Mengoli's idea of counting through the sense reminds us of the famous definition of music by Leibniz: "Music is a secret exercise of arithmetic by the soul not knowing that it counts (*Musica est exercitium arithmeticae occultum nescientis se numerare animi*)." Leibniz regarded musical perception as the result of the soul's unconscious counting. Unconscious perception, which was called *petite perceptions* by Leibniz, is the basis of sense perception that is clear but not distinct (i.e. its cause is unknown). This "inferior cognition" was addressed by Christian Wolff (1679-1754), which led to the birth of aesthetics in the middle of the eighteenth century.

[20] Gioseffo Zarlino (1517-89) and the other theorists of his time attempted to find a way to divide a line into equal parts geometrically in order to construct equal temperament. See J. Murray Barbour, *Tuning and Temperaments: A Historical Survey*, New York: Dover, 2004 (originally published in 1950), p.49 ff.

[21] *Speculationi* (note 7), p.162: "[...] bisogna dire, che l'anima rationale, con l'attentione attiua, dia il vero numero al primo suono, che gli si rappresenta nell'orecchio."

[22] The influence of St. Augustine is evident here. According to him, perception is the soul's acting on the passive state of the body to adjust it to the soul itself. *De musica*, Book VI, Chap.V.

[23] *Speculationi* (note 7), pp.105-106: "perche facilmente si fanno, e facilmente col senso si riconoscono, [...] e perche gli errori di questi dalle vere loro forme, sono dentro à certi confini, che non può il senso discernerli; onde prende gli apparenti per veri, senz'alcuna discrepanza; e come i veri, così comprende gli apparenti del tutto, e quanto al numero, e quanto alla misura, che hanno: nel che consiste tutto l'intiero gusto dell'anima, secondo il senso."

[24] ibid., p.106.

[25] ibid.

[26] ibid., pp.106-107.

[27] See also note 19.