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COMMINUTION KINETICS

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INTRODUCTION

The object of this paper is to present an analysis of the kinetics of comminution of quartz and limestone in ball mills and rod mills. In this investigation, the kinetics of size reduction has been studied for these materials being ground separately as well as part of a mixture.

Arbiter and Bhrany[1] recently observed that the initial rate of production of fine particles in a ball mill is a zero order rate phenomenon, that is the amount of fine particles produced is directly proportional to the time of grinding. As pointed out by these authors, if fine particles are produced at a constant rate, comminution of these fine particles themselves must be negligible. Arbiter and Bhrany suggest that even in ball milling the coarsest particles are ground selectively, these particles acting as shields for the finer particles. Intermediate sized particles are produced at a constant rate initially but as the coarser particles disappear, these particles themselves are ground. Measurement of the initial rates of formation of particles provides an exceedingly useful tool for studying the comminution action in a tumbling mill and this will be the basis for the investigation reported in this paper.

BASIC PRINCIPLES

Schuhmann[2] has considered that a complex comminution operation, such as ball milling, can be considered to be the summation of many individual comminution events. Schuhmann has assumed that each event produces fragments whose size distribution is characterized by the following size distribution:

$$y_i = 100\left(\frac{x}{k_i}\right)^{\alpha} \qquad (1)$$

where y_i is the percentage of the fragments by weight from the single comminution event that are finer than size x, k_i is the size modulus of the fragments (the theoretical maximum size in the assembly of fragments), and α is the distribution modulus that is characteristic of the material and comminution method. The size modulus k_i is determined by the energy expended per unit mass of the particle being broken during the comminution event.[3] For a given material and comminution method, the distribution modulus is constant.[2] The size distribution of the total product must then be the summation of the products from all of the individual comminution events:

$$y = \sum_i z_i \delta w_i \left(\frac{x}{k_i}\right)^{\alpha} \qquad (2)$$

where y is the cumulative percentage of total material finer than size x, z_i is the number of comminution events of kind i, and δw_i is the weight of particle broken by the given event. The more nearly alike the different comminution events are, the less will be the deviation of the size distribution from equation (2a):

$$y = 100\left(\frac{x}{k}\right)^{\alpha} \qquad (2a)$$

where k is the size modulus of the total product.

For comminution events that are nearly alike in a mill that consumes power at a constant rate, the rate of formation of particles finer than any stated size must be constant, as observed experimentally by Arbiter and Bhrany.[1] Considering the case where uniformly sized feed particles are being comminuted, the rate at which particles finer than size x are being formed is given in equation (3):

$$W_x = K_x t \qquad (3)$$

where W_x is the cumulative weight of particles finer than size x, K_x is the cumulative rate constant for particles finer than size x, and t is the grinding

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time. If W_0 is the weight of material in the mill, y is simply $100W_x/W_0$. Thus, with comminution events that are quite similar, K_x must be related to x^α. Arbiter and Bhrany[1] experimentally found that log–log plot of the initial rate of formation of particles finer than a given size versus the size yields a straight line of slope α. The equation of this straight line is:

$$K_x = K_0\left(\frac{x}{x_0}\right)^\alpha \tag{4}$$

where K_0 and x_0 refer to some reference size.

For a particular grinding time, t_g, for all sizes that follow equation (3), the following can be written by substituting equation (4) into equation (3)

$$W_x = K_0 t_g\left(\frac{x}{x_0}\right)^\alpha \tag{5}$$

This equation gives the weight–size relationship among the various sizes after a grinding time t_g. Equation (5) becomes equation (2a), if the quantity $K_0 t_g$ is 100, making x_0 equal to k.

In a ball mill, it is quite probable that any particle or particles can be impacted by a ball and be broken. Thus, not only feed particles but also the fragments from broken feed will be comminuted by the balls. Thus the product from a ball mill will be a mixture of the products from these different kinds of comminution events, and may even contain untouched feed.[2,4] On the other hand, in a rod mill the rods are wedged apart by the largest particles and there is little chance of finer particles being ground until the coarse particles have been broken. Thus, in the rod mill the kinds of comminution events are more nearly alike, and the size distribution of the rod mill product then follows equation (2a) over a much greater range. Because of the wide variation in kinds of comminution events in the ball mill and because of possible untouched feed, the ball mill product follows equation (2a) only in the finer sizes, with large deviations in the coarser sizes. In terms of the kinetics of particle formation in a ball mill, one might then expect deviations from a straight line on the log–log plot of K_x versus x in the coarser sizes, whereas in the case of rod milling, the log–log plot of K_x versus x may be linear up to coarse sizes.

Recently extensive investigations of the comminution of mixtures of limestone and quartz in laboratory ball mills and rod mills have been reported.[5,6] These investigations showed that the distribution modulus of quartz and limestone is the same whether it is ground separately or as part of a mixture, which is in accord with Schuhmann's single comminution event hypothesis. The size modulus of each component in the ground product depends upon the fraction of energy consumed by the particular mineral. In the case of ball milling, the fraction of energy that each component consumes is approximately determined by the volume fraction of that component in the mixture.[6] On the other hand, in rod milling, the wedge action of coarse particles between the rods causes the material with the least grindability to consume a greater quantity of energy. Thus, in a rod mill, quartz will consume an increasingly greater amount of energy when ground as a mixture with limestone.[5] Hence, in the case of rod milling mixtures, the rate of formation of fine quartz might be expected to increase with grinding time, with the rate of formation of fine limestone decreasing with time. In the case of ball milling, however, these deviations from linearity would be expected to be less.

To test these hypotheses, this investigation will be concerned with the rate of formation of fine quartz and limestone in a laboratory rod mill and ball mill under conditions where each material is ground separately and as part of a binary mixture. Since we are concerned primarily with initial rates of formation, these experiments will have to be for short grinding times.

MATERIALS AND METHOD

For ease of differentiating the comminution characteristics of two minerals in a mixture, Brazil quartz and California limestone were selected for this study for several reasons. Quartz has a distribution modulus of about 0.9 whereas that of limestone is about 0.6. Since the limestone analyzed 99.8 per cent $CaCO_3$, 0.1 per cent SiO_2, and 0.1 per cent R_2O_3, decomposition with acid affords an easy method for differentiating between the two minerals.

In each test, 1 kg of 4 × 8 mesh material was ground at 60 per cent solids by weight[7] for short time periods (1, 2, 3, and 4 min) in an 8 × 9½ in. mill that contained either 17.6 kg of rods (26 rods ⅝ in. diameter, 10 rods ¾ in. diameter and 5 rods ⅞ in. diameter) or 17.6 kg of balls ranging from ⅞ to

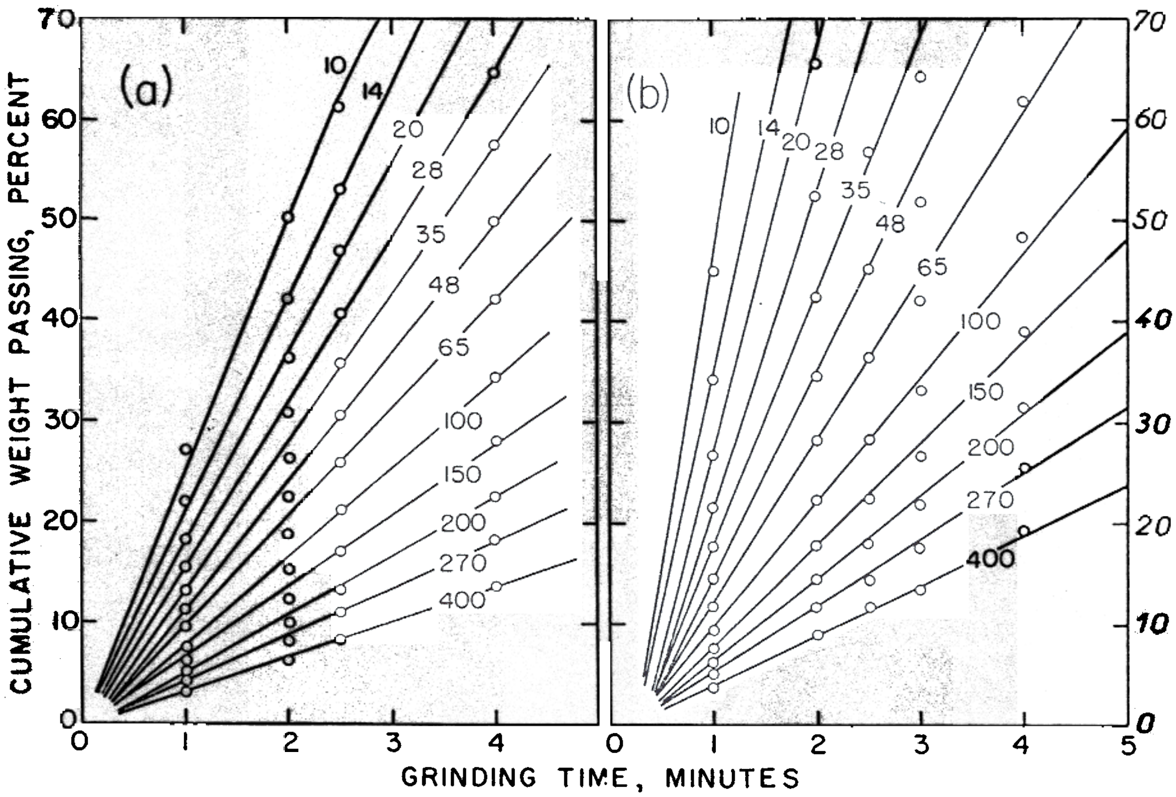

FIG. 2. Plot of cumulative percentage of limestone finer than each sieve size versus grinding time in (a) the ball mill and (b) the rod mill.

In Fig. 4a, the initial rates of formation (given as cumulative percentage of material finer than any stated size per minute) of limestone and quartz are plotted semilogarithmically as a function of particle size in mesh (Tyler scale) for these materials ground separately in the ball mill and rod mill. The slope of the log-log plot of initial rate of formation, K_x, versus particle size (a linear plot of size in mesh being equivalent to a logarithmic plot of size in microns) is 0.61 and 0.63 for limestone ground in the ball mill and rod mill, respectively, and 0.87 for quartz ground in the ball mill and rod mill. These slopes compare quite well experimentally with the values of α from the size distribution of these materials, as would be expected from equations (4) and (5). The log K_x — log x plot for rod milling is a straight line to about 20 mesh for both quartz and limestone, whereas each curve is a straight line only up to about 65 mesh for the ball milling of these materials. This deviation in the case of ball milling can be ascribed to wide variation in the kinds of comminution events occurring in the mill, that is, the grinding of fragments while untouched feed remains. As mentioned earlier, the wedge action of the largest particles between the rods within a rod mill prevents appreciable grinding of any particles finer than the coarsest material in the mill. These experiments show that during grinding a certain size fraction of material is formed at a constant rate which is characteristic of the material, the size being considered, and the comminution method.

Observation of Fig. 4a shows that the rate of formation of fine limestone particles in the rod mill exceeds the rate in the ball mill whereas the rate of formation of fine quartz particles in the ball mill exceeds the rate in the rod mill. The increased grindability of limestone in the rod mill could possibly be due to the greater attrition effect in the rod mill. Where the grindability of the material (quartz) is about the same in either type of mill, the rate of formation of fine particles in the ball mill

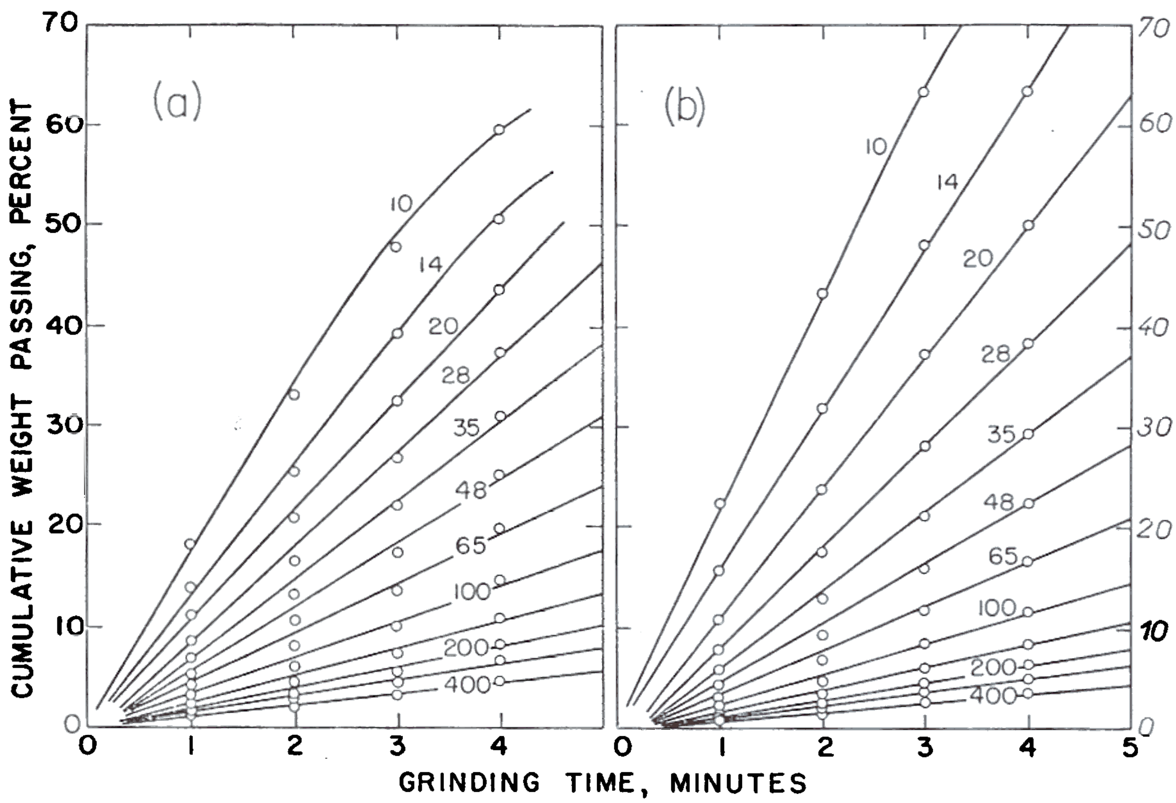

FIG. 3. Plot of cumulative percentage of quartz finer than each sieve size versus grinding time in (a) the ball mill and (b) the rod mill.

might be expected to be greater than that in a rod mill because of intermediate sized particles being ground themselves.

The size distributions of quartz and limestone, when ground as 1 : 1 mixtures, are given in Figs. 5a and 5b for ball milling and rod milling, respectively. The values of α for quartz ground as a component of a mixture in the ball mill and rod mill are 0.87 ± 0.03 and 0.92 ± 0.02, respectively. For limestone, the corresponding values are 0.61 ± 0.01 in both cases. This is in accord with Schuhmann's single comminution event hypothesis.

Figures 6a, 6b, 7a, and 7b present the cumulative percentage of material finer than each sieve size as a function of time of grinding of mixtures in the ball mill and rod mill. Data for ball milling the mixtures for 5 and 10 min are taken from the experiments of Fuerstenau and Sullivan[6] and data for rod milling the mixtures for 5, 7.5, and 10 min are also taken from experiments of Fuerstenau and Sullivan.[5] These figures show that the rate of formation of fine quartz increases with time and that of fine limestone decreases with the time for which the mixtures are ground. Although Fuerstenau and Sullivan's data for the extended grinding of quartz and limestone separately are not presented in Figs. 2 and 3, their data lie on these same straight lines. In the case of grinding mixtures, the initial rate of formation is linear because comminution will be independent of the grindability of the constituents in the beginning. Because of the greater grindability of limestone, quartz will remain larger in size as comminution proceeds and these larger quartz particles, therefore, will consume more of the energy than the protected, finer limestone particles. This effect becomes more pronounced as grinding progresses, with the rate of fine quartz formation increasing and that of fine limestone decreasing with time. The wedge action in the rod mill should augment this phenomenon. It also occurs in a ball mill, because of the existence of a protective action, to some extent.

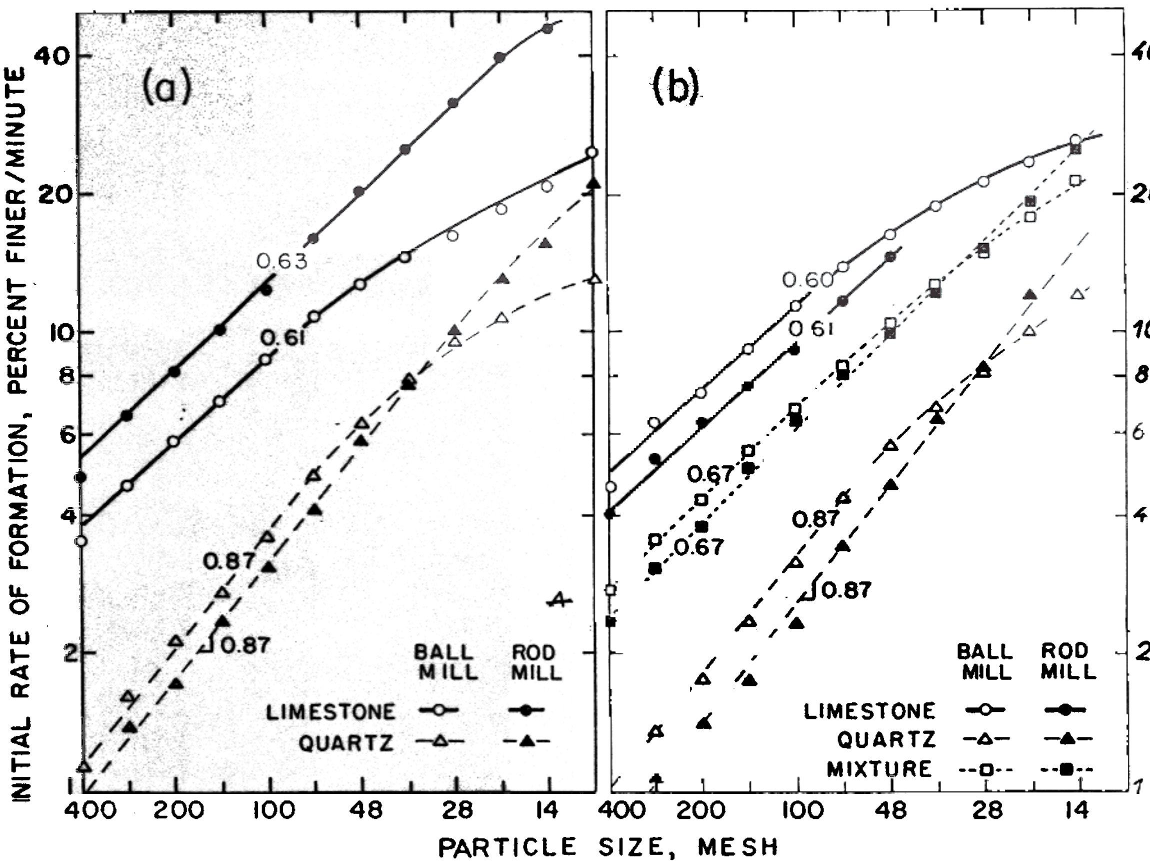

FIG. 4. Variation of initial rates of formation with particle size in the ball mill and rod mill for (a) grinding limestone and quartz separately and (b) grinding limestone and quartz as mixtures.

The *initial* rates of formation of fine quartz and fine limestone when ground as mixtures in the ball mill and rod mill are presented in Fig. 4b as a function of particle size. These results are somewhat similar to those obtained when the minerals are ground alone in that the slopes of the linear portions of the $\log K - \log x$ plots are again about equal to α. This shows that the mode of comminution of one mineral is independent of the presence of other minerals. However, because more energy may be going to the harder material, even in the initial stages, the rate of formation of the softer mineral (limestone) in the rod mill has decreased. Because the initial rate of formation of quartz particles is relatively low, changes in the rate may be difficult to detect.

Figure 4b gives only the initial rates of formation of limestone and quartz particles in mixtures. If the instantaneous rate of formation of limestone and quartz particles is taken at higher grinding times, a log–log plot of rate of formation of particles versus size yields lines of slopes of about α. Such plots are not given in this paper.

The initial rates of formation of both limestone and quartz particles are found to be less when ground as a mixture than when ground alone in a rod mill. This possibly could be due to the protection of limestone particles by coarser quartz particles and protection of quartz particles by the limestone dust coating on the quartz particles. The kinetics of grinding mixtures may be difficult to interpret because of the possible change in power required to move the ball or rod charge, dependent upon the size and hardness of material in the mill.[8] Accurate determination of power consumed in comminution may help in analyzing comminution during short grinding times, since this would tell us how valid is the assumption that energy expended is perfectly proportional to the grinding time.

The rate of particle formation of a mixture as a whole behaves similarly to the case obtained when minerals are ground separately, that is a plot of cumulative percent finer versus grinding time is a straight line. The slope of the log K versus log x plot for the mixture as a whole is 0.67 (see Fig. 4b). This agrees with the distribution modulus α for the mixture if a weighted average is taken based on the relative grindabilities of the two constituents.

SUMMARY

This study of the initial rates of formation of quartz and limestone particles, when ground separately in ball mills and rod mills, shows that each mineral follows the relation:

$$K_x = K_0\left(\frac{x}{x_0}\right)^\alpha$$

where K_x is the initial rate of formation of particles finer than size x, K_0 and x_0 refer to some reference size, and α is the distribution modulus of the material. For this equation to hold, the grinding of these fine particles themselves must be negligible. In the ball mill, where probability has a great role governing a ball striking any particle, deviations occur for sizes greater than 65 mesh. However, in the rod mill where coarse particles wedge the rods apart, a particle cannot be ground until the coarse one has been considerably reduced in size. Consequently, the initial rate of formation of particles in rod mills satisfies the above equation to the region of 20 mesh.

In the case of grinding of mixtures, the material with least grindability remains coarser and hence more of the energy will go for grinding quartz than limestone after the first stages of grinding. Thus the rate of formation of fine quartz increases as grinding of the mixture proceeds and the rate of formation of limestone particles in the mixture decreases. This effect is greater for the rod mill than for the ball mill.

This work has shown that during grinding, a

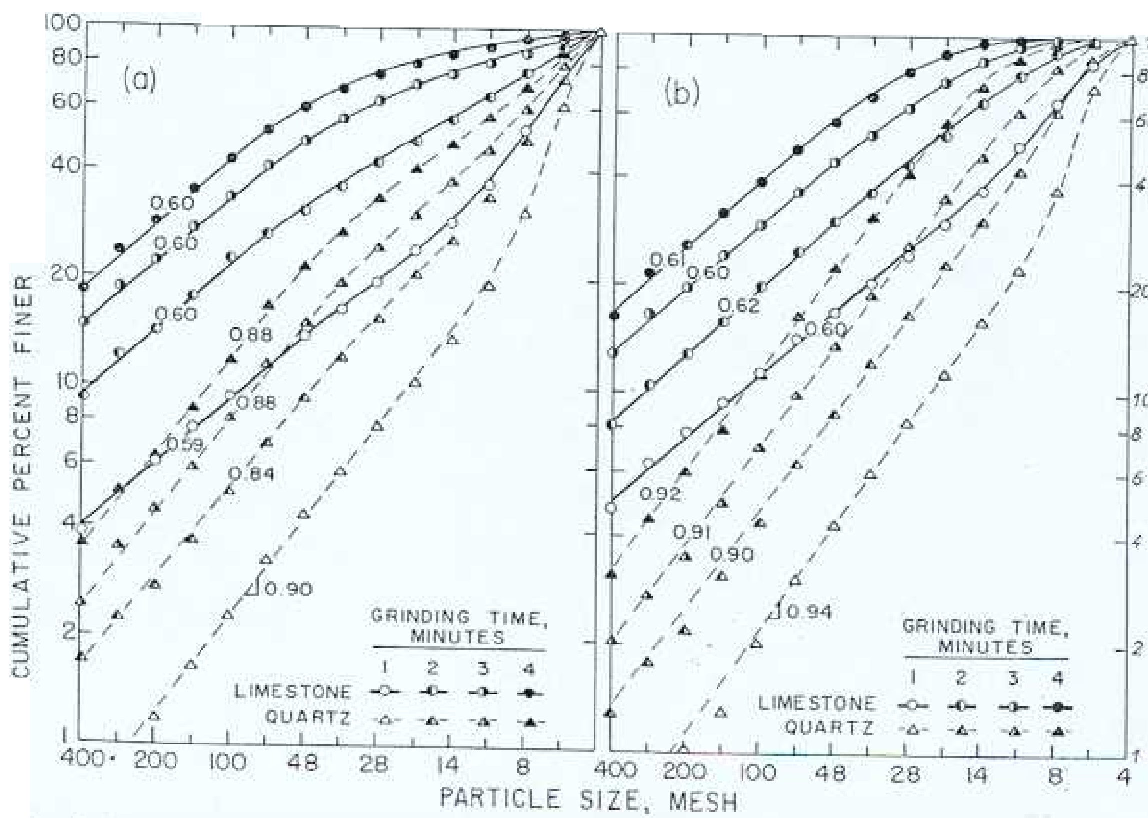

FIG. 5. Size distributions of the limestone and quartz fractions ground as 1:1 mixtures in (a) the ball mill and (b) the rod mill.

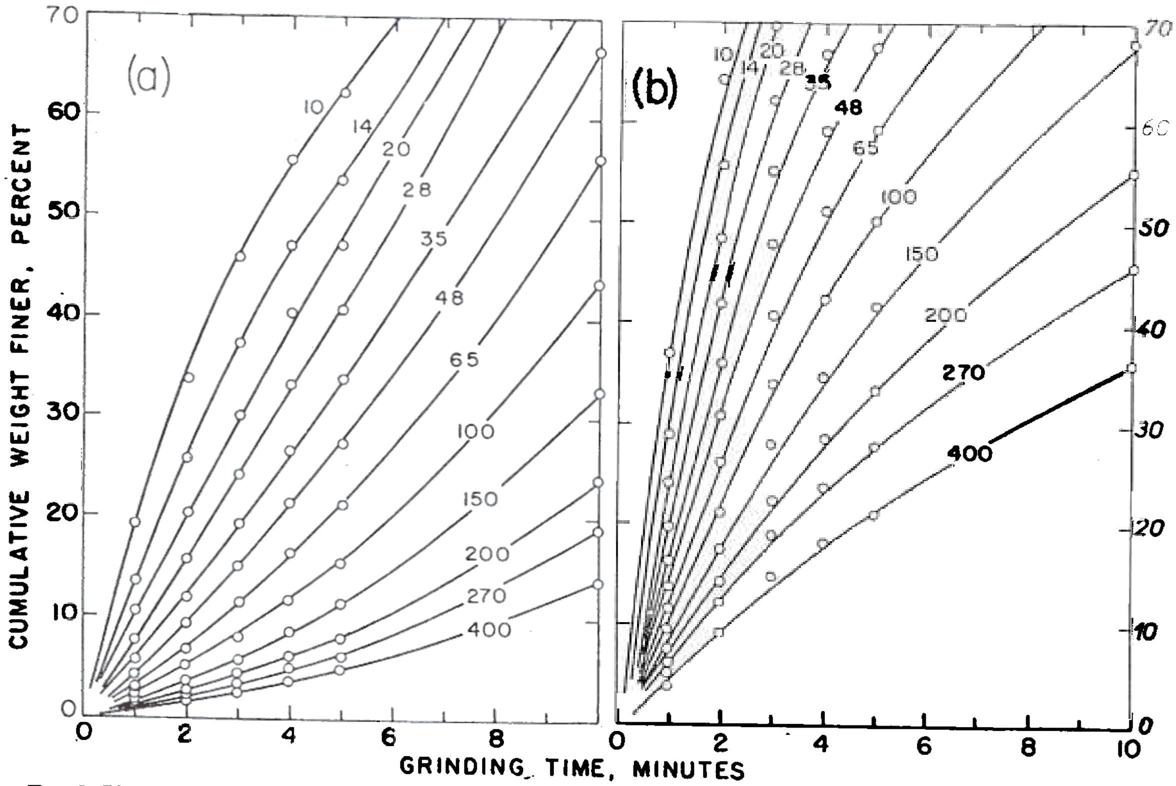

Fig. 6. Plot of cumulative percentage of material finer than each sieve size versus grinding time for (a) quartz fractions and (b) limestone fractions of the 1:1 mixtures ground in a ball mill.

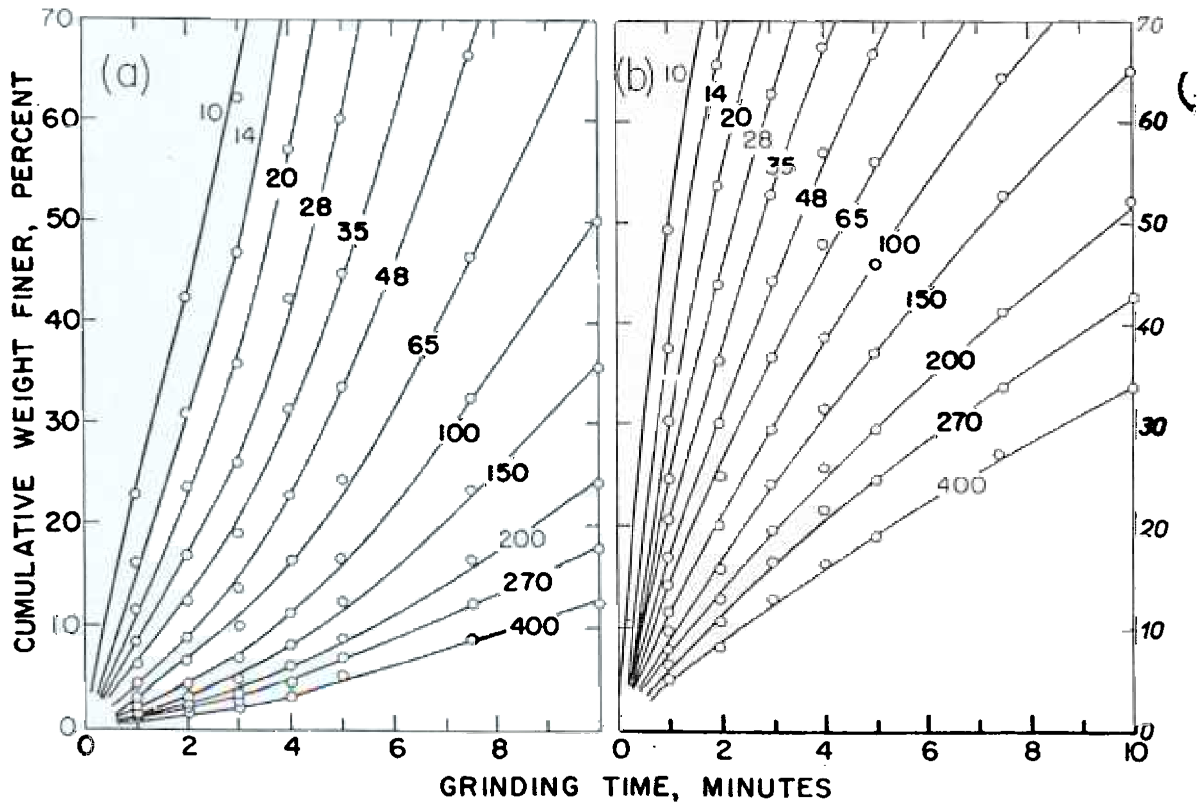

Fig. 7. Plot of cumulative percentage of material finer than each sieve size versus grinding time for (a) quartz fractions and (b) limestone fractions of the 1:1 mixtures ground in a rod mill

certain size fraction of material is formed at a constant rate, which is characteristic of the material, the size being considered and the comminution method.

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DISCUSSION

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This is an important contribution to further understanding of grinding kinetics. In the investigation by Bhrany, data were obtained only on single minerals, although data by others on heterogeneous materials did appear to exhibit constancy of initial grinding rates. Now in the present study it has been shown that a mineral has the same fragment size dispersion characteristic in a mixture as it has alone (Figs. 4, 5). Also important is the authors' finding that the initial rates for the mixtures are constant although obviously different from those for the components. Thus the validity of Bhrany's thesis that the rates of formation of the product sizes are constant over practical grinding times receives powerful support.

The implication of this constancy is large, both theoretically and practically. Thus, the slope in Fig. 4b is the proper exponent for the energy versus product size relationship and this exponent is *not* constant for different materials but varies approximately from 0.3 to 1.0, depending on the material and, to some extent, on the method of grinding.

The constant in equation 3 is a true grindability, with the dimensions: weight per unit time and per unit energy.

The original observation of the simplicity of grind kinetics and its usefulness was made about 15 years ago in a large Arizona copper ore grinding circuit. The capacity of 10 × 10 ft grate mills to produce a desired constant grind varied from 1500 to 2000 tons per day. Samples of mill feed, showing this variation, were tested in a 1 foot batch mill. The rates of formation of sizes from 65 to 200 mesh plotted against corresponding daily mill capacities showed a linear relationship. Thus the large and small scale grindabilities, as defined above, were not the same, because of the differences between the mills, but nevertheless were exactly proportional.

DR. G. AGAR
(International Minerals and Chemicals Corporation, U.S.A.)

This paper is based on the excellent work of Arbiter and Bhrany which was presented under a title that disguised the true worth of their contribution. We have here one of the few public applications of their findings to the practical problem of comminution. One of the significant points, that has not been stressed in the presentation here, is the unique value of the parameter α associated with each mineral. This conflicts radically with some recent proposals on fracture and particle size distributions. The authors' comments would be appreciated.

Also this analysis can be extended, as Arbiter and Bhrany did, to show that the energy consumed E is equal to Ak^{-a}. The analysis seems to indicate therefore, that the size of the feed is without effect on the energy consumed.

Can the writers reconcile this with the proposal of Svensson and Murkes[1] who account for the feed size in their energy-particle size relationship?

AUTHORS' REPLY TO G. AGAR'S COMMENTS

Our views on the parameter α, as obtained from size distribution or from kinetic data, are that the value is not unique for any material but that it depends on the mineral and comminution method. For example, we have found in our laboratories that α for numerous minerals is about 1.0 for impact and compression fracture of individual specimens; but when the same minerals are ground in a rod mill or a ball mill, the values of the parameter α become less than unity. In some cases, the values obtained in a ball mill may differ considerably from those in a rod mill. The explanation for these differences is that they appear to result from differences in comminution mechanisms. Although Schuhmann implied only impact events, we have now postulated that comminution events other than impact events occur in a mill, these events being abrasion events (surface attrition) and chipping events (nonfruitful impact that fractures only corners and edges). When abrasion and chipping are appreciable, the value of α decreases because of the production of greater quantities of fines. Hence, the size distribution produced by any device will depend upon the relative abundance of impact, chipping, and abrasion events. This in turn depends upon the machine and such properties as hardness, cleavage, and grain size of the material being ground.

The problem of feed size in analysis of comminution systems is a difficult one. However, we designed our experiments so that the feed size effect (if there were any) would be constant by using narrowly sized (4 × 8 mesh) feed particles. Dr. Agar himself has written a paper in which he suggested that a feed size term does not enter into comminution of fairly uniformly sized materials and he suggested that the observed effects in his own experiments were due to efficiency of energy transfer from grinding balls to feed particles (see *Trans. AIME*, 220, 390 (1961)). For those who are interested in this problem we would like to suggest that they read the following papers: *Trans. AIME*, 217, 22 (1960) and *Trans. AIME*, 223, 62 (1962).

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[1] Stockholm Congress (1957).

Dr. G. Agar

The authors' reply to one of the questions indicated that they believe that all single particle fracture results in a size distribution with $\alpha = 1$.

This is not supported by the extensive measurements of Hukki and Charles and would negate Schuhmann's arguments.

Prof. M. Rey

(Ecole Nationale Supérieure des Mines de Paris)

Le broyage de minéraux purs, seuls ou en mélange, entrepris récemment par le professeur Fuerstenau, est extrêmement utile pour faire progresser nos connaissances dans un domaine jusqu'ici mal connu, malgré son importance fondamentale.

Nous avons poursuivi, pendant l'année 1962, au laboratoire de Minerais et Métaux, et pour le Bureau de Recherches Géologiques et Minières (BRGM), des études de broyage très fin sur des mélanges de galène avec divers minéraux tels que la calcite, le quartz ou la pyrite, en dosant le plomb dans les fractions granulométriques de façon à pouvoir établir la granulométrie des minéraux séparés. Ces essais ont été faits à diverses densités de pulpe et avec diverses proportions des minéraux dans des broyeurs à boulets et des broyeurs à barres de laboratoire.

Comme les auteurs, et comme Arbiter et Bhrany ou Agar et Charles (réf. dans le mémoire) nous avons trouvé que les granulométries obtenues s'exprimaient, dans le domaine des fines, par un modulo de distribution (α) caractéristique du minéral et sur lequel on a extrêmement peu d'action. Dans le domaine des grenus le broyeur à barres donne une déviation faible et le broyeur à boulets une déviation importante, correspondant à un reliquat de grenus qui se broient lentement.

Là où nous différons des auteurs, c'est qu'ils ne trouvent qu'une variation assez faible des granulométries, lorsque les minéraux sont séparés ou en mélange.

Nous avons au contraire observé des interactions très notables entre les minéraux qui dépendent de leurs duretés respectives, de leurs proportions, de la densité de pulpe et du type de broyeur.

Considérons le broyage des deux minéraux purs séparément ou en mélange.

Si l'on appelle « sélectivité du broyage » le rapport des d_{50} des deux minéraux, c'est-à-dire des dimensions pour lesquelles on a 50 pour cent de refus et 50 pour cent de passé, on constate que celle-ci est notablement plus faible lorsque les minéraux sont broyés en mélange, que lorsqu'ils sont broyés séparément. En effet le minéral dur protège le minéral tendre surtout s'il est en proportion élevée, facilite le broyage du minéral dur.

D'autre part, contrairement à ce qu'on pourrait croire, la sélectivité est plus élevée au broyeur à barres qu'au broyeur à boulets et elle est plus élevée en pulpe épaisse qu'en pulpe diluée.

Si on n'a pas d'action sur le module de distribution des minéraux, on peut donc par contre obtenir un broyage plus ou moins différentiel de deux minéraux selon les conditions.

Les données qui précèdent sont relatives à des broyages discontinus, donc en circuit ouvert. Il faudrait ajouter les effets dus au classificateur ou au cyclone pour établir ce qui se passe dans un circuit fermé.